# Appendix for Chapter 14

## Calculation of $E^{-1}$

$$E = \begin{pmatrix} 51 & 13 \\ 13 & 122 \end{pmatrix}$$

$$\text{determinant of } E, |E| = (51 \times 122) - (13 \times 13) = 6053$$

$$\text{matrix of minors for } E = \begin{pmatrix} 122 & 13 \\ 13 & 51 \end{pmatrix}$$

$$\text{pattern of signs for } 2 \times 2 \text{ matrix} = \begin{pmatrix} + & - \\ - & + \end{pmatrix}$$

$$\text{matrix of cofactors} = \begin{pmatrix} 122 & -13 \\ -13 & 51 \end{pmatrix}$$

The inverse of a matrix is obtained by dividing the matrix of cofactors for *E* by $|E|$, the determinant of *E*.

$$E^{-1} = \begin{pmatrix} \frac{122}{6053} & \frac{-13}{6053} \\ \frac{-13}{6053} & \frac{51}{6053} \end{pmatrix} = \begin{pmatrix} 0.0202 & -0.0021 \\ -0.0021 & 0.0084 \end{pmatrix}$$

Calculation of $HE^{-1}$

$$HE^{-1} = \begin{pmatrix} 10.47 & -7.53 \\ -7.53 & 19.47 \end{pmatrix} \begin{pmatrix} 0.0202 & -0.0021 \\ -0.0021 & 0.0084 \end{pmatrix}$$

$$= \begin{pmatrix} [(10.47 \times 0.0202) + (-7.53 \times -0.0021)] & [(10.47 \times -0.0021) + (-7.53 \times 0.0084)] \\ [(-7.53 \times 0.0202) + (19.47 \times -0.0021)] & [(-7.53 \times -0.0021) + (19.47 \times 0.0084)] \end{pmatrix}$$

$$= \begin{pmatrix} 0.2273 & -0.0852 \\ -0.1930 & 0.1794 \end{pmatrix}$$

## Calculation of Eigenvalues

The eigenvalues or roots of any square matrix are the solutions to the determinantal equation $|A - \lambda I| = 0$, in which *A* is the square matrix in question and *I* is an identity matrix of the same size as *A*. The number of eigenvalues will equal the number of rows (or columns) of the square matrix. In this case the square matrix of interest is $\boldsymbol{HE^{-1}}$.

$$|HE^{-1} - \lambda I| = \left| \begin{pmatrix} 0.2273 & -0.0852 \\ -0.1930 & 0.1794 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right|$$

$$= \left| \begin{pmatrix} (0.2273 - \lambda) & -0.0852 \\ -0.1930 & (0.1794 - \lambda) \end{pmatrix} \right|$$

$$= [(0.2273 - \lambda)(0.1794 - \lambda) - (-0.1930 \times -0.0852)]$$

$$= \lambda^2 - 0.2273\lambda - 0.1794\lambda + 0.0407 - 0.0164$$

$$= \lambda^2 - 0.4067\lambda + 0.0243$$

Therefore the equation $|\boldsymbol{HE^{-1}} - \lambda I| = 0$ can be expressed as:

$$\lambda^2 - 0.4067\lambda + 0.0243 = 0$$

To solve the roots of any quadratic equation of the general form $a\lambda^2 + b\lambda + c = 0$ we can apply the following formula:

$$\lambda_i = \frac{-b \pm \sqrt{\left(b^2 - 4ac\right)}}{2a}$$

For the quadratic equation obtained, *a* = 1, *b* = –0.4067, *c* = 0.0243. If we replace these values into the formula for discovering roots, we get

$$\begin{aligned}
\lambda_i &= \frac{-b \pm \sqrt{\left(b^2 - 4ac\right)}}{2a} \\
&= \frac{0.4067 \pm \sqrt{\left[(-0.4067)^2 - 0.0972\right]}}{2} \\
&= \frac{0.4067 \pm 0.2612}{2} \\
&= \frac{0.6679}{2} \text{ or } \frac{0.1455}{2} \\
&= 0.334 \text{ or } 0.073
\end{aligned}$$

Hence, the eigenvalues are 0.334 and 0.073.