CHAPTER SEVEN

Rate-of-Return Analysis

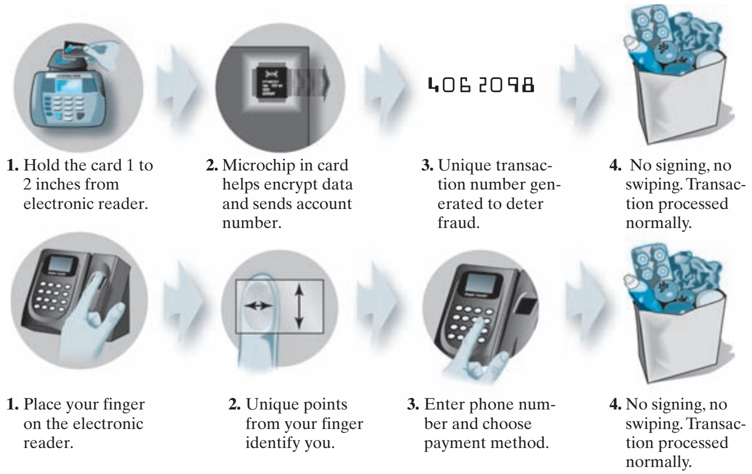
Will That be Cash, Credit—or Fingertip?[1] Have you ever found yourself short of cash or without a wallet when you want to buy something? Consider the following two types of technologies available in retail stores to speed up checkouts:

- **Pay By Touch** takes fingerprints when customers enroll in the program. The image is then converted to about 40 unique points of the finger. Those points are stored in a computer system with "military-level encryption." They want this to be your cash replacement because of the time savings, and a lot of customers who are paying cash will find it more convenient now to use these cards.
- **A contactless card** allows the shopper to pay in seconds by waving his or her contactless card in front of a reader, which lights up and beeps to tell the shopper the transaction is done.
  A contactless payment is twice as fast as a no-signature credit card purchase and three times as fast as using cash. That's why it's catching on at fast-food restaurants and convenience stores.

These stores' profits depend, in part, on how quickly they get customers—typically with small purchases—through the line. These new technologies being rolled out at convenience stores, supermarkets, and gas stations could some day make it passé to carry bulky wallets. Without the need to dig for cash and checks at the register, the quick stop-and-go payments promise speedier transactions for consumers—and perhaps fatter profits for retailers. The appeal is that there's no need to run them through a machine. A contactless-card transaction is usually more expensive for a retailer to process than a cash payment. But retailers that adopt contactless payments hope they'll bring in more customers, offsetting higher costs. If that turns out to be false, then some could turn their backs on the new technology.

[1] "Will that be cash, credit—or fingertip?" Kathy Chu, *USA Today*, Section B1, Friday, December 2, 2005.

Getting Through the Checkout Line Faster
Contactless Payment



One retailer who just installed a Pay By Touch™ system hopes to increase its customer traffic so that a 10% return on investment can be attained. The Pay By Touch™ scanners cost about $50 each, the monthly service fee ranges between $38 and $45, and each transaction fee costs 10 cents. In a society driven by convenience, anything that speeds up the payment process attracts consumers. But technology providers will need to convince consumers of the safety of their information before the technologies can become a staple in the checkout line.

What does the 10% rate of return for the retailer really represent? How do we compute the figure from the projected additional retail revenues? And once computed, how do we use the figure when evaluating an investment project? Our consideration of the concept of rate of return in this chapter will answer these and other questions.

Along with the NPW and the AE criteria, the third primary measure of investment worth is **rate of return**. As shown in Chapter 5, the NPW measure is easy to calculate and apply. Nevertheless, many engineers and financial managers prefer rate-of-return analysis to the NPW method because they find it intuitively more appealing to analyze investments in terms of percentage rates of return rather than dollars of NPW. Consider the following statements regarding an investment's profitability:

- This project will bring in a 15% rate of return on the investment.
- This project will result in a net surplus of $10,000 in the NPW.

Neither statement describes the nature of the investment project in any complete sense. However, the rate of return is somewhat easier to understand because many of us are so familiar with savings-and-loan interest rates, which are in fact rates of return.

In this chapter, we will examine four aspects of rate-of-return analysis: (1) the concept of return on investment, (2) the calculation of a rate of return, (3) the development of an internal rate-of-return criterion, and (4) the comparison of mutually exclusive alternatives based on a rate of return.

CHAPTER LEARNING OBJECTIVES

After completing this chapter, you should understand the following concepts:

- The meaning of the rate of return.
- The various methods to compute the rate of return.
- How you make an accept and reject decision with the rate of return.
- How to resolve the multiple rates of return problem.
- How you conduct an incremental analysis with the rate of return.

7.1 Rate of Return

Yield: The annual rate of return on an investment, expressed as a percentage.

Many different terms are used to refer to **rate of return**, including **yield** (i.e., the yield to maturity, commonly used in bond valuation), **internal rate of return**, and **marginal efficiency of capital**. First we will review three common definitions of rate of return. Then we will use the definition of internal rate of return as a measure of profitability for a single investment project throughout the text.

7.1.1 Return on Investment

There are several ways of defining the concept of a rate of return on investment. The first is based on a typical loan transaction, the second on the mathematical expression of the present-worth function, and the third on the project cash flow series.

Definition 1

The rate of return is the interest rate earned on the unpaid balance of an amortized loan.

Suppose that a bank lends $10,000 and is repaid $4,021 at the end of each year for three years. How would you determine the interest rate that the bank charges on this transaction? As we learned in Chapter 3, you would set up the equivalence equation

$$\$10{,}000 = \$4{,}021(P/A, i, 3)$$

and solve for i. It turns out that $i = 10\%$. In this situation, the bank will earn a return of 10% on its investment of $10,000. The bank calculates the balances over the life of the loan as follows:

Year	Unpaid Balance at Beginning of Year	Return on Unpaid Balance (10%)	Payment Received	Unpaid Balance at End of Year
0	−$10,000	$0	$0	−$10,000
1	−10,000	−1,000	+4,021	−6,979
2	−6,979	−698	+4,021	−3,656
3	−3,656	−366	+4,021	0

A negative balance indicates an unpaid balance. In other words, the customer still owes money to the bank.

Observe that, for the repayment schedule shown, the 10% interest is calculated only on each year's outstanding balance. In this situation, only part of the $4,021 annual payment represents interest; the remainder goes toward repaying the principal. Thus, the three annual payments repay the loan itself and additionally provide a return of 10% on the *amount still outstanding each year*.

Note that when the last payment is made, the outstanding principal is eventually reduced to zero.[2] If we calculate the NPW of the loan transaction at its rate of return (10%), we see that

$$\mathrm{PW}(10\%) = -\$10{,}000 + \$4{,}021(P/A, 10\%, 3) = 0,$$

which indicates that the bank can break even at a 10% rate of interest. In other words, the rate of return becomes the rate of interest that equates the present value of future cash repayments to the amount of the loan. This observation prompts the second definition of rate of return.

Definition 2

The rate of return is the break-even interest rate i that equates the present worth of a project's cash outflows to the present worth of its cash inflows, or*

$$\begin{aligned}\mathrm{PW}(i^*) &= \mathrm{PW}_{\text{Cash inflows}} - \mathrm{PW}_{\text{Cash outflows}} \\ &= 0.\end{aligned}$$

Note that the expression for the NPW is equivalent to

$$\mathrm{PW}(i^*) = \frac{A_0}{(1+i^*)^0} + \frac{A_1}{(1+i^*)^1} + \ldots + \frac{A_N}{(1+i^*)^N} = 0. \tag{7.1}$$

Here we know the value of A_n for each period, but not the value of i^*. Since it is the only unknown, however, we can solve for i^*. (Inevitably, there will be N values of i^* that satisfy this equation. In most project cash flows, you would be able to find a unique positive i^* that satisfies Eq. (7.1). However, you may encounter some cash flows that cannot be solved for a single rate of return greater than 100%. By the nature of the NPW function in

[2] As we learned in Section 5.3.2, this terminal balance is equivalent to the net future worth of the investment. If the net future worth of the investment is zero, its NPW should also be zero.

Eq. (7.1), it is possible to have more than one rate of return for certain types of cash flows. For some cash flows, we may not find a specific rate of return at all.)[3]

Note that the formula in Eq. (7.1) is simply the NPW formula solved for the particular interest rate (i^*) at which PW(i) is equal to zero. By multiplying both sides of Eq. (7.1) by $(1 + i^*)^N$, we obtain

$$\text{PW}(i^*)(1 + i^*)^N = \text{FW}(i^*) = 0.$$

If we multiply both sides of Eq. (7.1) by the capital recovery factor $(A/P, i^*, N)$, we obtain the relationship $\text{AE}(i^*) = 0$. Therefore, the i^* of a project may be defined as the rate of interest that equates the present worth, future worth, and annual equivalent worth of the entire series of cash flows to zero.

7.1.2 Return on Invested Capital

Investment projects can be viewed as analogous to bank loans. We will now introduce the concept of rate of return based on the return on invested capital in terms of a project investment. A project's return is referred to as the internal rate of return (IRR) or the **yield** promised by an **investment project** over its **useful life**.

Definition 3

The internal rate of return is the interest rate charged on the unrecovered project balance of the investment such that, when the project terminates, the unrecovered project balance will be zero.

Suppose a company invests \$10,000 in a computer with a three-year useful life and equivalent annual labor savings of \$4,021. Here, we may view the investing firm as the lender and the project as the borrower. The cash flow transaction between them would be identical to the amortized loan transaction described under Definition 1:

n	Beginning Project Balance	Return on Invested Capital	Ending Cash Payment	Project Balance
0	\$0	\$0	−\$10,000	−\$10,000
1	−10,000	−1,000	4,021	6,979
2	−6,979	−697	4,021	3,656
3	−3,656	−365	4,021	0

Internal rate of return: This is the return that a company would earn if it invested in itself, rather than investing that money elsewhere.

In our project balance calculation, we see that 10% is earned (or charged) on \$10,000 during year 1, 10% is earned on \$6,979 during year 2, and 10% is earned on \$3,656 during year 3. This indicates that the firm earns a 10% rate of return on funds that remain *internally* invested in the project. Since it is a return that is *internal* to the project, we refer to it as the **internal rate of return**, or IRR. This means that the computer project under consideration brings in enough cash to pay for itself in three years and also to provide the firm with a return

[3] You will always have N rates of return. The issue is whether they are real or imaginary. If they are real, the question "Are they in the $(-100\%, \infty)$ interval?" should be asked. A **negative rate of return** implies that you never recover your initial investment.

of 10% on its invested capital. Put differently, if the computer is financed with funds costing 10% annually, the cash generated by the investment will be exactly sufficient to repay the principal and the annual interest charge on the fund in three years.

Notice that only one cash outflow occurs at time 0, and the present worth of this outflow is simply \$10,000. There are three equal receipts, and the present worth of these inflows is $\$4{,}021(P/A, 10\%, 3) = \$10{,}000$. Since the $\text{NPW} = \text{PW}_{\text{Inflow}} - \text{PW}_{\text{Outflow}} = \$10{,}000 - \$10{,}000 = 0$, 10% also satisfies Definition 2 of the rate of return. Even though the preceding simple example implies that i^* coincides with IRR, only Definitions 1 and 3 correctly describe the true meaning of the internal rate of return. As we will see later, if the cash expenditures of an investment are not restricted to the initial period, several break-even interest rates (i^*'s) may exist that satisfy Eq. (7.1). However, there may not be a rate of return that is *internal* to the project.

7.2 Methods for Finding the Rate of Return

We may find i^* by several procedures, each of which has its advantages and disadvantages. To facilitate the process of finding the rate of return for an investment project, we will first classify various types of investment cash flow.

7.2.1 Simple versus Nonsimple Investments

We can classify an investment project by counting the number of sign changes in its net cash flow sequence. A change from either "+" to "−" or "−" to "+" is counted as one sign change. (We ignore a zero cash flow.) Then,

- A **simple investment** is an investment in which the initial cash flows are negative and only one sign change occurs in the remaining net cash flow series. If the initial flows are positive and only one sign change occurs in the subsequent net cash flows, they are referred to as **simple borrowing** cash flows.
- A **nonsimple investment** is an investment in which more than one sign change occurs in the cash flow series.

Simple investment: The project with only one sign change in the net cash flow series.

Multiple i^*'s, as we will see later, occur only in nonsimple investments. Three different types of investment possibilities are illustrated in Example 7.1.

EXAMPLE 7.1 Investment Classification

Consider the following three cash flow series and classify them into either simple or nonsimple investments:

Period	Net Cash Flow		
n	Project A	Project B	Project C
0	−\$1,000	−\$1,000	\$1,000
1	−500	3,900	−450
2	800	−5,030	−450
3	1,500	2,145	−450
4	2,000		

SOLUTION

Given: Preceding cash flow sequences.

Find: Classify the investments shown into either simple and nonsimple investments.

- Project A represents many common simple investments. This situation reveals the NPW profile shown in Figure 7.1(a). The curve crosses the *i*-axis only once.
- Project B represents a nonsimple investment. The NPW profile for this investment has the shape shown in Figure 7.1(b). The *i*-axis is crossed at 10%, 30%, and 50%.
- Project C represents neither a simple nor a nonsimple investment, even though only one sign change occurs in the cash flow sequence. Since the first cash flow is positive, this is a **simple borrowing** cash flow, not an investment flow. Figure 7.1(c) depicts the NPW profile for this type of investment.

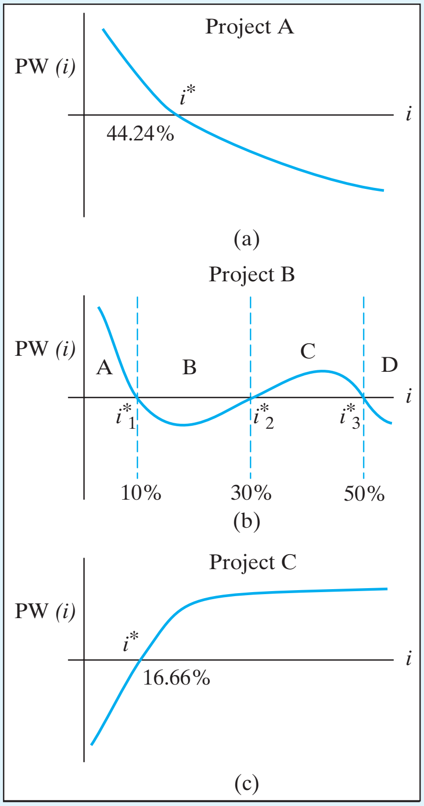


Figure 7.1 Present-worth profiles: (a) Simple investment, (b) nonsimple investment with multiple rates of return, and (c) simple borrowing cash flows.

COMMENTS: Not all NPW profiles for nonsimple investments have multiple crossings of the *i*-axis. Clearly, then, we should place a high priority on discovering this situation early in our analysis of a project's cash flows. The quickest way to predict multiple i^*'s is to generate an NPW profile on a computer and check whether it crosses the horizontal axis more than once. In the next section, we illustrate when to expect such multiple crossings by examining types of cash flows.

7.2.2 Predicting Multiple i^*'s

As hinted at in Example 7.1, for certain series of project cash flows, we may uncover the complication of multiple i^* values that satisfy Eq. (7.1). By analyzing and classifying cash flows, we may anticipate this difficulty and adjust our analysis approach later. Here we will focus on the initial problem of whether we can predict a unique i^* for a project by examining its cash flow pattern. Two useful rules allow us to focus on sign changes (1) in net cash flows and (2) in accounting net profit (accumulated net cash flows).

Net Cash Flow Rule of Signs

One useful method for predicting an upper limit on the number of positive i^*'s of a cash flow stream is to apply the rule of signs: *The number of real* i^**'s that are greater than −100% for a project with N periods is never greater than the number of sign changes in the sequence of the* A_n*'s. A zero cash flow is ignored.*

An example is

Period	A_n	Sign Change
0	−\$100	
1	−20	
2	+50	1
3	0	
4	+60	
5	−30	1
6	+100	1

Three sign changes occur in the cash flow sequence, so three or fewer real positive i^*'s exist.

It must be emphasized that the rule of signs provides an indication only of the *possibility* of multiple rates of return: The rule predicts only the *maximum* number of possible i^*'s. Many projects have multiple sign changes in their cash flow sequence, but still possess a unique real i^* in the $(-100\%, \infty)$ range.

Accumulated Cash Flow Sign Test

The accumulated cash flow is the sum of the net cash flows up to and including a given time. If the rule of cash flow signs indicates multiple i^*'s, we should proceed to the **accumulated cash flow sign test** to eliminate some possibility of multiple rates of return.

If we let A_n represent the net cash flow in period n and S_n represent the accumulated cash flow (the accounting sum) up to period n, we have the following:

Period (n)	Cash Flow (A_n)	Accumulated Cash Flow (S_n)
0	A_0	$S_0 = A_0$
1	A_1	$S_1 = S_0 + A_1$
2	A_2	$S_2 = S_1 + A_2$
$\vdots$	$\vdots$	$\vdots$
N	A_N	$S_N = S_{N-1} + A_N$

We then examine the sequence of accumulated cash flows $(S_0, S_1, S_2, S_3, \ldots, S_N)$ to determine the number of sign changes. *If the series S_n starts negatively and changes sign only once, then a unique positive i^* exists.* This cumulative cash flow sign rule is a more discriminating test for identifying the uniqueness of i^* than the previously described method.

EXAMPLE 7.2 Predicting the Number of i^*'s

Predict the number of real positive rates of return for each of the following cash flow series:

Period	A	B	C	D
0	−$100	−$100	$ 0	−$100
1	−200	+50	−50	+50
2	+200	−100	+115	0
3	+200	+60	−66	+200
4	+200	−100		−50

SOLUTION

Given: Four cash flow series and cumulative flow series.

Find: The upper limit on number of i^*'s for each series.

The cash flow rule of signs indicates the following possibilities for the positive values of i^*:

Project	Number of Sign Changes in Net Cash Flows	Possible Number of Positive Values of i^*
A	1	1 or 0
B	4	4, 3, 2, 1, or 0
C	2	2, 1, or 0
D	2	2, 1, or 0

For cash flows B, C, and D, we would like to apply the more discriminating cumulative cash flow test to see if we can specify a smaller number of possible values of i^*. Accordingly, we write

	Project B		Project C		Project D	
n	A_n	S_n	A_n	S_n	A_n	S_n
0	−$100	−$100	$ 0	$ 0	−$100	−$100
1	+50	−50	−50	−50	+50	−50
2	−100	−150	+115	+65	0	−50
3	+60	−90	−66	−1	+200	+150
4	−100	−190			−50	+100

Recall the test: If the series starts *negatively* and changes sign only once, a unique positive i^* exists.

- Only project D begins negatively and passes the test; therefore, we may predict a unique i^* value, rather than 2, 1, or 0 as predicted by the cash flow rule of signs. ($i_1^* = -75.16\%$ and $i_2^* = 35.05\%$)
- Project B, with no sign change in the cumulative cash flow series, has no rate of return.
- Project C fails the test, and we cannot eliminate the possibility of multiple i^*'s. ($i_1^* = 10\%$ and $i_2^* = 20\%$)

7.2.3 Computational Methods

Once we identify the type of an investment cash flow, several ways to determine its rate of return are available. Some of the most practical methods are as follows:

- Direct solution method,
- Trial-and-error method, and
- Computer solution method.

Direct Solution Method

For the special case of a project with only a two-flow transaction (an investment followed by a single future payment) or a project with a service life of two years of return, we can seek a direct mathematical solution for determining the rate of return. These two cases are examined in Example 7.3.

EXAMPLE 7.3 Finding i^* by Direct Solution: Two Flows and Two Periods

Consider two investment projects with the following cash flow transactions:

n	Project 1	Project 2
0	−$2,000	−$2,000
1	0	1,300
2	0	1,500
3	0	
4	3,500	

Compute the rate of return for each project.

SOLUTION

Given: Cash flows for two projects.
Find: i^* for each project.

Project 1: Solving for i^* in $\text{PW}(i^*) = 0$ is identical to solving $\text{FW}(i^*) = 0$, because FW equals PW times a constant. We could do either here, but we will set $\text{FW}(i^*) = 0$ to demonstrate the latter. Using the single-payment future-worth relationship, we obtain

$$\text{FW}(i^*) = -\$2{,}000(F/P, i^*, 4) + \$3{,}500 = 0,$$

$$\$3{,}500 = \$2{,}000(F/P, i^*, 4) = \$2{,}000(1 + i^*)^4,$$

$$1.75 = (1 + i^*)^4.$$

Solving for i^* yields

$$i^* = \sqrt[4]{1.75} - 1$$
$$= 0.1502 \text{ or } 15.02\%.$$

Project 2: We may write the NPW expression for this project as

$$\text{PW}(i) = -\$2{,}000 + \frac{\$1{,}300}{(1 + i)} + \frac{\$1{,}500}{(1 + i)^2} = 0.$$

Let $X = 1/(1 + i)$. We may then rewrite PW(i) as a function of X as follows:

$$\text{PW}(X) = -\$2{,}000 + \$1{,}300X + \$1{,}500X^2 = 0.$$

This is a quadratic equation that has the following solution:[4]

[4] The solution of the quadratic equation $aX^2 + bX + c = 0$ is $X = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$X = \frac{-1{,}300 \pm \sqrt{1{,}300^2 - 4(1{,}500)(-2{,}000)}}{2(1{,}500)}$$

$$= \frac{-1{,}300 \pm 3{,}700}{3{,}000}$$

$$= 0.8 \text{ or } -1.667.$$

Replacing X values and solving for i gives us

$$0.8 = \frac{1}{(1+i)} \rightarrow i = 25\%,$$

$$-1.667 = \frac{1}{(1+i)} \rightarrow i = -160\%.$$

Since an interest rate less than -100% has no economic significance, we find that the project's i^* is 25%.

COMMENTS: In both projects, one sign change occurred in the net cash flow series, so we expected a unique i^*. Also, these projects had very simple cash flows. When cash flows are more complicated, generally we use a trial-and-error method or a computer to find i^*.

Trial-and-Error Method

The first step in the trial-and-error method is to make an estimated ***guess***[5] at the value of i^*. For a simple investment, we use the "guessed" interest rate to compute the present worth of net cash flows and observe whether it is positive, negative, or zero. Suppose, then, that PW(i) is negative.

Since we are aiming for a value of i that makes $\text{PW}(i) = 0$, we must raise the present worth of the cash flow. To do this, we lower the interest rate and repeat the process. If PW(i) is positive, however, we raise the interest rate in order to lower PW(i). The process is continued until PW(i) is approximately equal to zero. Whenever we reach the point where PW(i) is bounded by one negative and one positive value, we use **linear interpolation** to approximate i^*. This process is somewhat tedious and inefficient. (The trial-and-error method does not work for nonsimple investments in which the NPW function is not, in general, a monotonically decreasing function of the interest rate.)

EXAMPLE 7.4 Finding i^* by Trial and Error

The Imperial Chemical Company is considering purchasing a chemical analysis machine worth \$13,000. Although the purchase of this machine will not produce any

[5] As we shall see later in this chapter, the ultimate objective of finding i^* is to compare it against the MARR. Therefore, it is a good idea to use the MARR as the initial guess.

increase in sales revenues, it will result in a reduction of labor costs. In order to operate the machine properly, it must be calibrated each year. The machine has an expected life of six years, after which it will have no salvage value. The following table summarizes the annual savings in labor cost and the annual maintenance cost in calibration over six years:

Year (n)	Costs ($)	Savings ($)	Net Cash Flow ($)
0	13,000		−13,000
1	2,300	6,000	3,700
2	2,300	7,000	4,700
3	2,300	9,000	6,700
4	2,300	9,000	6,700
5	2,300	9,000	6,700
6	2,300	9,000	6,700

Find the rate of return for this project.

SOLUTION

Given: Cash flows over six years as shown in Figure 7.2.
Find: i^*.

We start with a guessed interest rate of 25%. The present worth of the cash flows is

$$\begin{aligned} \text{PW}(25\%) &= -\$13{,}000 + \$3{,}700(P/F, 25\%, 1) + \$4{,}700(P/F, 25\%, 2) \\ &\quad + \$6{,}700(P/A, 25\%, 4)(P/F, 25\%, 2) \\ &= \$3{,}095. \end{aligned}$$

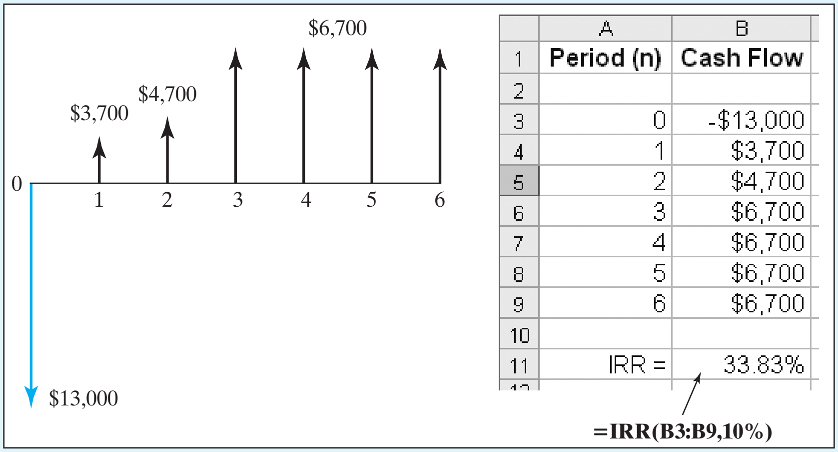


Figure 7.2 Cash flow diagram for a simple investment (Example 7.4).

Since this present worth is positive, we must raise the interest rate to bring PW toward zero. When we use an interest rate of 35%, we find that

$$\begin{aligned} PW(35\%) &= -\$13{,}000 + \$3{,}700(P/F, 35\%, 1) + \$4{,}700(P/F, 35\%, 2) \\ &\quad + \$6{,}700(P/A, 35\%, 4)(P/F, 35\%, 2) \\ &= -\$339. \end{aligned}$$

We have now bracketed the solution: PW(i) will be zero at i somewhere between 25% and 35%. Using straight-line interpolation, we approximate

$$\begin{aligned} i^* &\cong 25\% + (35\% - 25\%)\left[\frac{3{,}095 - 0}{3{,}095 - (-339)}\right] \\ &= 25\% + 10\%(0.9013) \\ &= 34.01\%. \end{aligned}$$

Now we will check to see how close this value is to the precise value of i^*. If we compute the present worth at this interpolated value, we obtain

$$\begin{aligned} PW(34\%) &= -\$13{,}000 + \$3{,}700(P/F, 34\%, 1) + \$4{,}700(P/F, 34\%, 2) \\ &\quad + \$6{,}700(P/A, 34\%, 4)(P/F, 34\%, 2) \\ &= -\$50.58. \end{aligned}$$

As this is not zero, we may recompute i^* at a lower interest rate, say, 33%:

$$\begin{aligned} PW(33\%) &= -\$13{,}000 + \$3{,}700(P/F, 33\%, 1) + \$4{,}700(P/F, 33\%, 2) \\ &\quad + \$6{,}700(P/A, 33\%, 4)(P/F, 33\%, 2) \\ &= \$248.56. \end{aligned}$$

With another round of linear interpolation, we approximate

$$\begin{aligned} i^* &\cong 33\% + (34\% - 33\%)\left[\frac{248.56 - 0}{248.56 - (-50.58)}\right] \\ &= 33\% + 1\%(0.8309) \\ &= 33.83\%. \end{aligned}$$

At this interest rate,

$$\begin{aligned} PW(33.83\%) &= -\$13{,}000 + \$3{,}700(P/F, 33.83\%, 1) \\ &\quad + \$4{,}700(P/F, 33.83\%, 2) \\ &\quad + \$6{,}700(P/A, 33.83\%, 4)(P/F, 33.83\%, 2) \\ &= -\$0.49, \end{aligned}$$

which is practically zero, so we may stop here. In fact, there is no need to be more precise about these interpolations, because the final result can be no more accurate than the basic data, which ordinarily are only rough estimates.

COMMENT: With Excel, you can evaluate the IRR for the project as =IRR (range,guess), where you specify the cell range for the cash flow (e.g., B3:B9) and the initial guess, such as 25%. Computing i^* for this problem in Excel, incidentally, gives us 33.8283%. Instead of using the factor notations, you may attempt use a tabular approach as follows:

Internal Rate of Return: What It Looks Like

		Discount Rate: 25%		Discount Rate: 35%	
Year	**Cash Flow**	**Factor**	**Amount**	**Factor**	**Amount**
0	−\$13,000	1.0000	−\$13,000	1.0000	−\$13,000
1	3,700	0.8000	2,960	0.7407	2,741
2	4,700	0.6400	3,008	0.5487	2,579
3	6,700	0.5120	3,430	0.4064	2,723
4	6,700	0.4096	2,744	0.3011	2,017
5	6,700	0.3277	2,196	0.2230	1,494
6	6,700	0.2621	1,756	0.1652	1,107
Total	+\$22,200	**NPW =**	**+\$3,095**	**NPW =**	**−\$339**
			IRR = close to 34%		

Graphical Method

We don't need to do laborious manual calculations to find i^*. Many financial calculators have built-in functions for calculating i^*. It is worth noting that many online financial calculators or spreadsheet packages have i^* functions, which solve Eq. (7.1) very rapidly,[6] usually with the user entering the cash flows via a computer keyboard or by reading a cash flow data file. (For example, Microsoft Excel has an IRR financial function that analyzes investment cash flows, as illustrated in Example 7.4.)

The most easily generated and understandable graphic method of solving for i^* is to create the **NPW profile** on a computer. On the graph, the horizontal axis indicates the interest rate and the vertical axis indicates the NPW. For a given project's cash flows, the NPW is calculated at an interest rate of zero (which gives the vertical-axis intercept) and several other interest rates. Points are plotted and a curve is sketched. Since i^* is defined as the interest rate at which $PW(i^*) = 0$, the point at which the curve crosses the horizontal axis closely approximates i^*. The graphical approach works for both simple and nonsimple investments.

[6] An alternative method of solving for i^* is to use a computer-aided economic analysis program. Cash Flow Analyzer (CFA) finds i^* visually by specifying the lower and upper bounds of the interest search limit and generates NPW profiles when given a cash flow series. In addition to the savings in calculation time, the advantage of computer-generated profiles is their precision. CFA can be found from the book's website.

EXAMPLE 7.5 Graphical Approach to Estimate i^*

Consider the cash flow series shown in Figure 7.3(a). Estimate the rate of return by generating the NPW profile on a computer.

SOLUTION

Given: Cash flow series in Figure 7.3.

Find: (a) i^* by plotting the NPW profile and (b) i^* by using Excel.

	A	B
1	Period	Cash Flow
2		
3	0	-10000
4	1	20000
5	2	20000
6	3	-25000
7		
8	IRR =	140%
9		
10	Interest (%)	NPW
11	0	$5,000
12	10	$5,928
13	20	$6,088
14	30	$5,840
15	40	$5,379
16	50	$4,815
17	60	$4,209
18	70	$3,597
19	80	$2,997
20	90	$2,422
21	100	$1,875
22	110	$1,359
23	120	$875
24	130	$422
25	140	($3)
26	150	($400)
27	160	($772)
28	170	($1,119)
29	180	($1,445)
30	190	($1,750)
31	200	($2,037)
32		

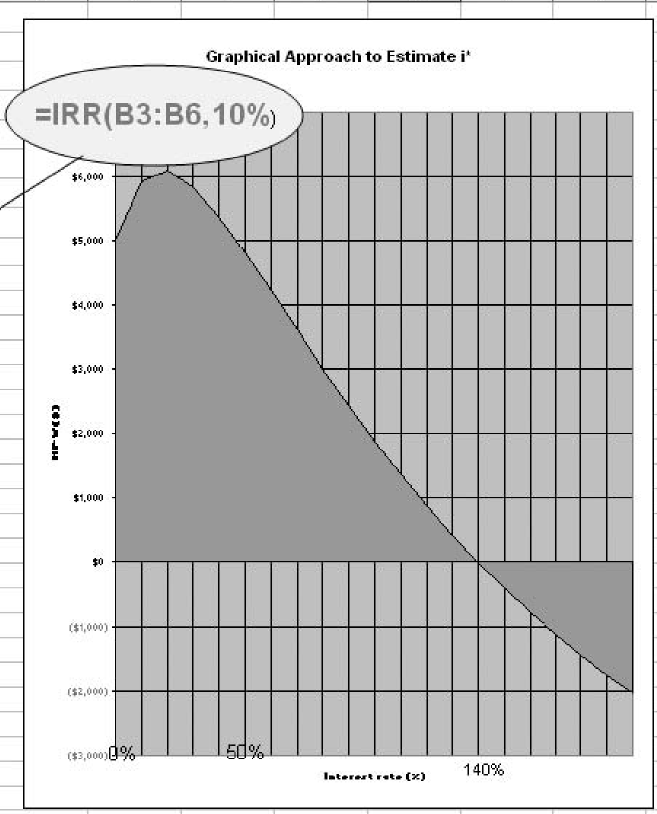


Figure 7.3 Graphical solution to rate-of-return problem for a typical nonsimple investment (Example 7.5).

(a) The present-worth function for the project cash flow series is

$$\text{PW}(i) = -\$10{,}000 + \$20{,}000(P/A, i, 2) - \$25{,}000(P/F, i, 3).$$

First we use $i = 0$ in this equation to obtain NPW $=$ \$5,000, which is the vertical-axis intercept. Then we substitute several other interest rates—10%, 20%, . . . , 140%—and plot these values of PW(i) as well. The result is Figure 7.3, which shows the curve crossing the horizontal axis at roughly 140%. This value can be verified by other methods if we desire to do so. Note that, in addition to establishing the interest rate that makes NPW $= 0$, the NPW profile indicates where positive and negative NPW values fall, thus giving us a broad picture of those interest rates for which the project is acceptable or unacceptable. (Note also that a trial-and-error method would lead to some confusion: As you increase the interest rate from 0% to 20%, the NPW value also keeps increasing, instead of decreasing.) Even though the project is a nonsimple investment, the curve crosses the horizontal axis only once. As mentioned in the previous section, however, most nonsimple projects have more than one value of i^* that makes NPW $= 0$ (i.e., more than one i^* per project). In such a case, the NPW profile would cross the horizontal axis more than once.[7]

(b) With Excel, you can evaluate the IRR for the project with the function

=IRR(range,guess)

in which you specify the cell range for the cash flow and the initial guess, such as 10%.

7.3 Internal-Rate-of-Return Criterion

Now that we have classified investment projects and learned methods for determining the i^* value for a given project's cash flows, our objective is to develop an accept–reject decision rule that gives results consistent with those obtained from NPW analysis.

7.3.1 Relationship to PW Analysis

As we already observed in Chapter 5, NPW analysis depends on the rate of interest used for the computation of NPW. A different rate may change a project from being considered acceptable to being unacceptable, or it may change the ranking of several projects:

- Consider again the NPW profile as drawn for the simple project in Figure 7.1(a). For interest rates below i^*, this project should be accepted because NPW > 0; for interest rates above i^*, it should be rejected.
- By contrast, for certain nonsimple projects, the NPW may look like the one shown in Figure 7.1(b). NPW analysis would lead you to accept the projects in regions **A** and **C**, but reject those in regions **B** and **D**. Of course, this result goes against intuition: A higher interest rate would change an unacceptable project into an acceptable one. The situation graphed in Figure 7.1(b) is one of the cases of multiple i^*'s mentioned in Definition 2.

[7] In Section 7.2.2, we discuss methods of predicting the number of i^* values by looking at cash flows. However, generating an NPW profile to discover multiple i^*'s is as practical and informative as any other method.

Therefore, for the simple investment situation in Figure 7.1(a), i^* can serve as an appropriate index for either accepting or rejecting the investment. However, for the nonsimple investment of Figure 7.1(b), it is not clear which i^* to use to make an accept–reject decision. Therefore, the i^* value fails to provide an appropriate measure of profitability for an investment project with multiple rates of return.

7.3.2 Net-Investment Test: Pure versus Mixed Investments

To develop a consistent accept–reject decision rule with the NPW, we need to further classify a project into either a pure or a mixed investment:

- A project is said to be a **net investment** when the project balances computed at the project's i^* values, $\text{PB}(i^*)_n$, are either less than or equal to zero throughout the life of the investment, with the first cash flow being negative $(A_0 < 0)$. The investment is *net* in the sense that the firm does not overdraw on its return at any point and hence investment is *not indebted* to the project. This type of project is called a **pure investment**. In contrast, **pure borrowing** is defined as the situation in which $\text{PB}(i^*)_n$ values are positive or zero throughout the life of the loan, with $A_0 > 0$. *Simple investments will always be pure investments*.
- If any of the project balances calculated at the project's i^* is positive, the project is not a pure investment. A positive project balance indicates that, at some time during the project life, the firm acts as a borrower $[\text{PB}(i^*)_n > 0]$ rather than an investor in the project $[\text{PB}(i^*)_n < 0]$. This type of investment is called a **mixed investment**.

Net investment test: A process to determine whether or not a firm borrows money from a project during the investment period.

Pure investment: An investment in which a firm never borrows money from the project.

Mixed investment: An investment in which a firm borrows money from the project during the investment period.

EXAMPLE 7.6 Pure versus Mixed Investments

Consider the following four investment projects with known i^* values:

	Project Cash Flows			
n	**A**	**B**	**C**	**D**
0	−\$1,000	−\$1,000	−\$1,000	−\$1,000
1	1,000	1,600	500	3,900
2	2,000	−300	−500	−5,030
3	1,500	−200	2,000	2,145
i^*	33.64%	21.95%	29.95%	(10%, 30%, 50%)

Determine which projects are pure investments.

SOLUTION

Given: Four projects with cash flows and i^*'s as shown.

Find: Which projects are pure investments?

We will first compute the project balances at the projects' respective i^*'s. If multiple rates of return exist, we may use the largest value of i^* greater than zero.[8]

Project A:

$$PB(33.64\%)_0 = -\$1{,}000,$$
$$PB(33.64\%)_1 = -\$1{,}000(1 + 0.3364) + (-\$1{,}000) = -\$2{,}336.40,$$
$$PB(33.64\%)_2 = -\$2{,}336.40(1 + 0.3364) + \$2{,}000 = -\$1{,}122.36,$$
$$PB(33.64\%)_3 = -\$1{,}122.36(1 + 0.3364) + \$1{,}500 = 0.$$

$(-, -, -, 0)$: passes the net-investment test (pure investment).

Project B:

$$PB(21.95\%)_0 = -\$1{,}000,$$
$$PB(21.95\%)_1 = -\$1{,}000(1 + 0.2195) + \$1{,}600 = \$380.50,$$
$$PB(21.95\%)_2 = +\$380.50(1 + 0.2195) - \$300 = \$164.02,$$
$$PB(21.95\%)_3 = +\$164.02(1 + 0.2195) - \$200 = 0.$$

$(-, +, +, 0)$: fails the net-investment test (mixed investment).

Project C:

$$PB(29.95\%)_0 = -\$1{,}000,$$
$$PB(29.95\%)_1 = -\$1{,}000(1 + 0.2995) + \$500 = -\$799.50,$$
$$PB(29.95\%)_2 = -\$799.50(1 + 0.2995) - \$500 = -\$1{,}538.95,$$
$$PB(29.95\%)_3 = -\$1{,}538.95(1 + 0.2995) + \$2{,}000 = 0.$$

$(-, -, -, 0)$: passes the net-investment test (pure investment).

Project D: There are three rates of return. We can use any of them for the net investment test. Thus,

$$PB(50\%)_0 = -\$1{,}000,$$
$$PB(50\%)_1 = -\$1{,}000(1 + 0.50) + \$3{,}900 = \$2{,}400,$$
$$PB(50\%)_2 = +\$2{,}400(1 + 0.50) - \$5{,}030 = -\$1{,}430,$$
$$PB(50\%)_3 = -\$1{,}430(1 + 0.50) + \$2{,}145 = 0.$$

$(-, +, -, 0)$: fails the net-investment test (mixed investment).

COMMENTS: As shown in Figure 7.4, projects A and C are the only pure investments. Project B demonstrates that the existence of a unique i^* is a necessary but not sufficient condition for a pure investment.

[8] In fact, it does not matter which rate we use in applying the net-investment test. If one value passes the test, they will all pass. If one value fails, they will all fail.

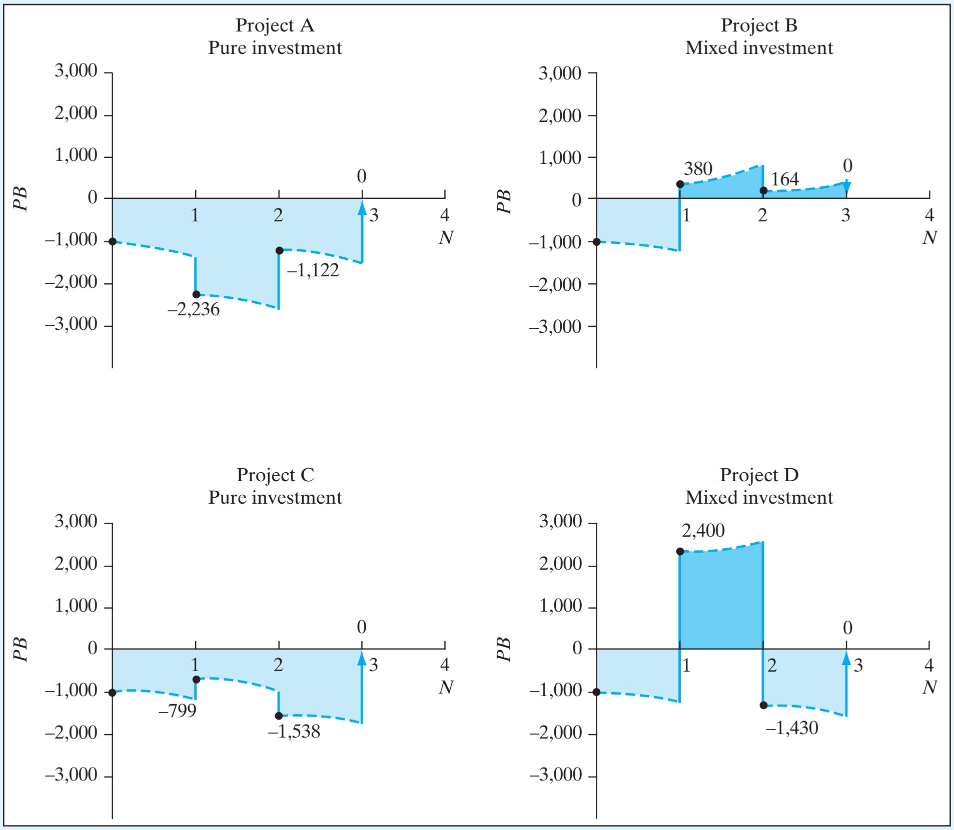


Figure 7.4 Net-investment test (Example 7.6).

7.3.3 Decision Rule for Pure Investments

Suppose we have a pure investment. (Recall that all simple investments are pure investments as well.) Why are we interested in finding the particular interest rate that equates a project's cost with the present worth of its receipts? Again, we may easily answer this question by examining Figure 7.1(a). In this figure, we notice two important characteristics of the NPW profile. First, as we compute the project's PW(i) at a varying interest rate i, we see that the NPW is positive for $i < i^*$, indicating that the project would be acceptable under PW analysis for those values of i. Second, the NPW is negative for $i > i^*$, indicating that the project is unacceptable for those values of i. Therefore, i^* serves as a **benchmark** interest rate, knowledge of which will enable us to make an accept–reject decision consistent with NPW analysis.

Note that, for a pure investment, i^* is indeed the IRR of the investment. (See Section 7.1.2.) Merely knowing i^*, however, is not enough to apply this method. Because firms typically wish to do better than break even (recall that at NPW $= 0$ we were indifferent to the project), a minimum acceptable rate of return (MARR) is indicated by company policy, management, or the project decision maker. If the IRR exceeds this MARR, we are assured that the company will more than break even. Thus, the IRR becomes a useful gauge against which to judge a project's acceptability, and the decision rule for a pure project is as follows:

$$\text{If IRR} > \text{MARR, accept the project.}$$
$$\text{If IRR} = \text{MARR, remain indifferent.}$$
$$\text{If IRR} < \text{MARR, reject the project.}$$

Note that this decision rule is designed to be applied for a single project evaluation. When we have to compare mutually exclusive investment projects, we need to apply the **incremental analysis approach**, as we shall see in Section 7.4.2.

EXAMPLE 7.7 Investment Decision for a Pure Investment

Merco, Inc., a machinery builder in Louisville, Kentucky, is considering investing \$1,250,000 in a complete structural beam-fabrication system. The increased productivity resulting from the installation of the drilling system is central to the project's justification. Merco estimates the following figures as a basis for calculating productivity:

- Increased fabricated steel production: 2,000 tons/year.
- Average sales price/ton fabricated steel: \$2,566.50/ton.
- Labor rate: \$10.50/hour.
- Tons of steel produced in a year: 15,000 tons.
- Cost of steel per ton (2,205 lb): \$1,950/ton.
- Number of workers on layout, hole making, sawing, and material handling: 17.
- Additional maintenance cost: \$128,500/year.

With the cost of steel at \$1,950 per ton and the direct labor cost of fabricating 1 lb at 10 cents, the cost of producing a ton of fabricated steel is about \$2,170.50. With a selling price of \$2,566.50 per ton, the resulting contribution to overhead and profit becomes \$396 per ton. Assuming that Merco will be able to sustain an increased production of 2,000 tons per year by purchasing the system, engineers have estimated the projected additional contribution to be 2,000 tons $\times$ \$396 $=$ \$792,000.

Since the drilling system has the capacity to fabricate the full range of structural steel, two workers can run the system, one on the saw and the other on the drill. A third operator is required to operate a crane for loading and unloading materials. Merco estimates that, to do the equivalent work of these three workers with conventional manufacturing techniques would require, on the average, an additional 14 people for center punching, hole making with a radial or magnetic drill, and material handling. This translates into a labor savings in the amount of \$294,000 per year (14 $\times$ \$10.50 $\times$ 40 hours/week $\times$ 50 weeks/year). The system can last for 15 years, with an estimated after-tax salvage value of \$80,000. However, after an annual deduction of \$226,000 in corporate income taxes, the net investment costs, as well as savings, are as follows:

- Project investment cost: \$1,250,000.
- Projected annual net savings:
  (\$792,000 $+$ \$294,000) $-$ \$128,500 $-$ \$226,000 $=$ \$731,500.

- Projected after-tax salvage value at the end of year 15: \$80,000.
  (a) What is the projected IRR on this fabrication investment?
  (b) If Merco's MARR is known to be 18%, is this investment justifiable?

SOLUTION

Given: Projected cash flows as shown in Figure 7.5 and MARR = 18%.
Find: (a) The IRR and (b) whether to accept or reject the investment.

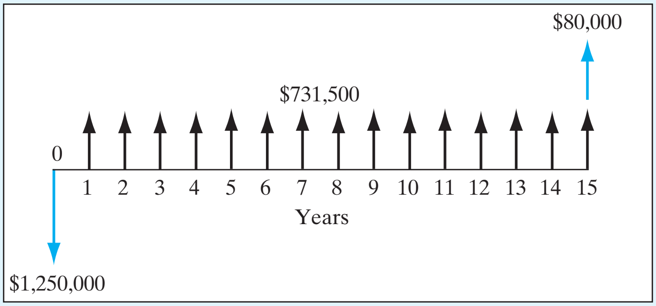


Figure 7.5 Cash flow diagram (Example 7.7).

(a) Since only one sign change occurs in the net cash flow series, the fabrication project is a simple investment. This indicates that there will be a unique rate of return that is internal to the project:

$$\begin{aligned} PW(i) &= -\$1{,}250{,}000 + \$731{,}500(P/A, i, 15) \\ &\quad + \$80{,}000(P/F, i, 15) \\ &= 0 \\ i^* &= 58.71\%. \end{aligned}$$

With Excel, you will also find that the IRR is about 58.71% for the net investment of \$1,250,000.

(b) The IRR figure far exceeds Merco's MARR, indicating that the fabrication system project is an economically attractive one. Merco's management believes that, over a broad base of structural products, there is no doubt that the installation of the fabricating system would result in a significant savings, even after considering some potential deviations from the estimates used in the analysis.

7.3.4 Decision Rule for Mixed Investments

Applied to pure projects, i^* provides an unambiguous criterion for measuring profitability. However, when multiple rates of return occur, none of them is an accurate portrayal of a project's acceptability or profitability. However, there is a correct method, which uses an **external interest rate**, for refining our analysis when we do discover multiple i^*'s. An external rate of return allows us to calculate a single accurate rate of return; if you choose to avoid these more complicated applications of rate-of-return techniques, you must be able to predict multiple i^*'s via the NPW profile and, when they occur, select an alternative method such as NPW or AE analysis for determining the project's acceptability.

Need for an External Interest Rate for Mixed Investments

In the case of a mixed investment, we can extend the economic interpretation of the IRR to the return-on-invested-capital measure if we are willing to make an assumption about what happens to the extra cash that the investor gets from the project during the intermediate years.

Project balance: The amount of money committed to a project at a specific period.

First, the **project balance** (PB), or investment balance, can also be interpreted from the viewpoint of a financial institution that borrows money from an investor and then pays interest on the PB. Thus, a negative PB means that the investor has money in a bank account; a positive PB means that the investor has borrowed money from the bank. Negative PBs represent interest paid by the bank to the investor; positive PBs represent interest paid by the investor to the bank.

Now, can we assume that the interest paid by the bank and the interest received from the investor are the same for the same amount of balance? In our banking experience, we know that is not the case. Normally, the borrowing rate (interest paid by the investor) is higher than the interest rate on your deposit (interest paid by the bank).

However, when we calculate the project balance at an i^* for mixed investments, we notice an important point: Cash borrowed (released) from the project is assumed to earn the same interest rate through external investment as money that remains internally invested. In other words, in solving a cash flow for an unknown interest rate, it is assumed that money released from a project can be reinvested to yield a rate of return equal to that received from the project. In fact, we have been making this assumption regardless of whether a cash flow does or does not produce a unique positive i^*. Note that money is borrowed only when $PB(i^*) > 0$, and the magnitude of the borrowed amount is the project balance. When $PB(i^*) < 0$, no money is borrowed, even though the cash flow may be positive at that time.

In reality, it is not always possible for cash borrowed (released) from a project to be reinvested to yield a rate of return equal to that received from the project. Instead, it is likely that the rate of return available on a capital investment in the business is much different—usually higher—from the rate of return available on other external investments. Thus, it may be necessary to compute the project balances for a project's cash flow at two rates of interest—one on the internal investment and one on the external investments. As we will see later, by separating the interest rates, we can measure the **true rate of return** of any internal portion of an investment project.

Calculation of Return on Invested Capital for Mixed Investments

For a mixed investment, we must calculate a rate of return on the portion of capital that remains invested internally. This rate is defined as the **true IRR** for the mixed investment and is commonly known as the **return on invested capital** (**RIC**). Then, what interest rate should we assume for the portion of external investment? Insofar as a project is not a net investment, one or more periods when the project has a net outflow of money (a positive project balance) must later be returned to the project. This money can be put into the firm's investment pool until such time as it is needed in the project. The interest rate of this investment pool is the interest rate at which the money can in fact be invested outside the project.

Return on invested capital (RIC): The amount that a company earns on the total investment it has made in its project.

Recall that the NPW method assumed that the interest rate charged to any funds withdrawn from a firm's investment pool would be equal to the MARR. In this book, *we will use the MARR as an established external interest rate* (i.e., the rate earned by money invested outside of the project). We can then compute the RIC as a function of the MARR by finding the value of the RIC that will make the terminal project balance equal to zero. (This implies that the firm wants to fully recover any investment made in the project and pays off any borrowed funds at the end of the project life.) This way of computing the rate of return is an accurate measure of the profitability of the project as represented by the cash flow. The following procedure outlines the steps for determining the IRR for a mixed investment:

Step 1. Identify the MARR (or external interest rate).

Step 2. Calculate $\text{PB}(i, \text{MARR})_n$ (or simply PB_n) according to the rule

$$\text{PB}(i, \text{MARR})_0 = A_0.$$

$$\text{PB}(i, \text{MARR})_1 = \begin{cases} \text{PB}_0(1 + i) + A_1, \text{ if } \text{PB}_0 < 0 \\ \text{PB}_0(1 + \text{MARR}) + A_1, \text{ if } \text{PB}_0 > 0, \end{cases}$$

$$\vdots$$

$$\text{PB}(i, \text{MARR})_n = \begin{cases} \text{PB}_{n-1}(1 + i) + A_n, \text{ if } \text{PB}_{n-1} < 0 \\ \text{PB}_{n-1}(1 + \text{MARR}) + A_n, \text{ if } \text{PB}_{n-1} > 0. \end{cases}$$

(As defined in the text, A_n stands for the net cash flow at the end of period n. Note that the terminal project balance must be zero.)

Step 3. Determine the value of i by solving the terminal project balance equation

$$\text{PB}(i, \text{MARR})_N = 0.$$

The interest rate i is the RIC (or IRR) for the mixed investment.

Using the MARR as an external interest rate, we may accept a project if the IRR exceeds the MARR, and we should reject the project otherwise. Figure 7.6 summarizes the IRR computation for a mixed investment.

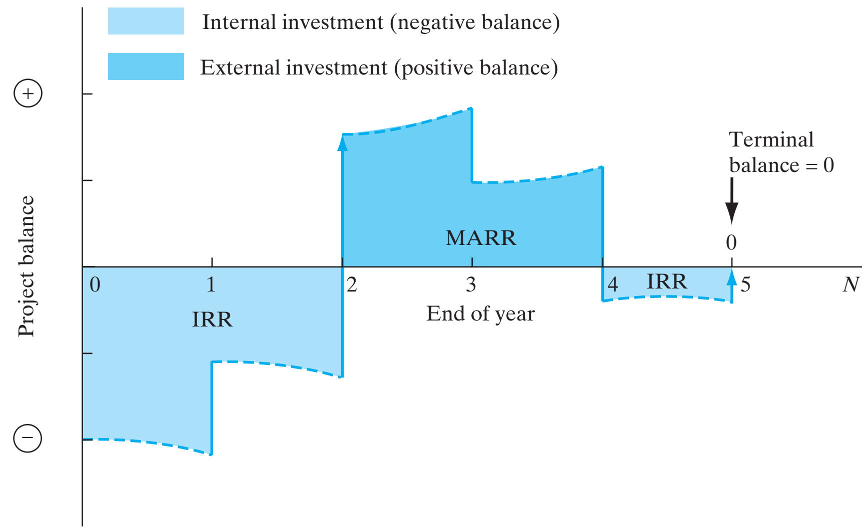


Figure 7.6 Computational logic for IRR (mixed investment).

EXAMPLE 7.8 IRR for a Mixed Investment

By outbidding its competitors, Trane Image Processing (TIP), a defense contractor, received a contract worth \$7,300,000 to build navy flight simulators for U.S. Navy pilot training over two years. With some defense contracts, the U.S. government makes an advance payment when the contract is signed, but in this case the government will make two progressive payments: \$4,300,000 at the end of the first year and the \$3,000,000 balance at the end of the second year. The expected cash outflows required to produce the simulators are estimated to be \$1,000,000 now, \$2,000,000 during the first year, and \$4,320,000 during the second year. The expected net cash flows from this project are summarized as follows:

Year	Cash Inflow	Cash Outflow	Net Cash Flow
0		\$1,000,000	−\$1,000,000
1	\$4,300,000	2,000,000	2,300,000
2	3,000,000	4,320,000	−1,320,000

In normal situations, TIP would not even consider a marginal project such as this one. However, hoping that the company can establish itself as a technology leader in the field, management felt that it was worth outbidding its competitors. Financially, what is the economic worth of outbidding the competitors for this project? That is,

(a) Compute the values of i^*'s for this project.

(b) Make an accept–reject decision based on the results in part (a). Assume that the contractor's MARR is 15%.

SOLUTION

Given: Cash flow shown and MARR = 15%.

Find: (a) Compute the NPW, (b) i^*, and (c) RIC at MARR = 15%, and determine whether to accept the project.

(a)

$$\begin{aligned} PW(15\%) &= -\$1{,}000{,}000 + \$2{,}300{,}000(P/F, 15\%, 1) \\ &= -\$1{,}320{,}000(P/F, 15\%, 2) \\ &= \$1{,}890 > 0. \end{aligned}$$

(b) Since the project has a two-year life, we may solve the net-present-worth equation directly via the quadratic formula:

$$-\$1{,}000{,}000 + \$2{,}300{,}000/(1 + i^*) - \$1{,}320{,}000/(1 + i^*)^2 = 0.$$

If we let $X = 1/(1 + i^*)$, we can rewrite the preceding expression as

$$-1{,}000{,}000 + 2{,}300{,}000X - 1{,}320{,}000X^2 = 0.$$

Solving for X gives $X = 10/11$ and $10/12$, or $i^* = 10\%$ and 20%. As shown in Figure 7.7, the NPW profile intersects the horizontal axis twice, once at 10% and again at 20%. The investment is obviously not a simple one; thus, neither 10% nor 20% represents the true internal rate of return of this government project.

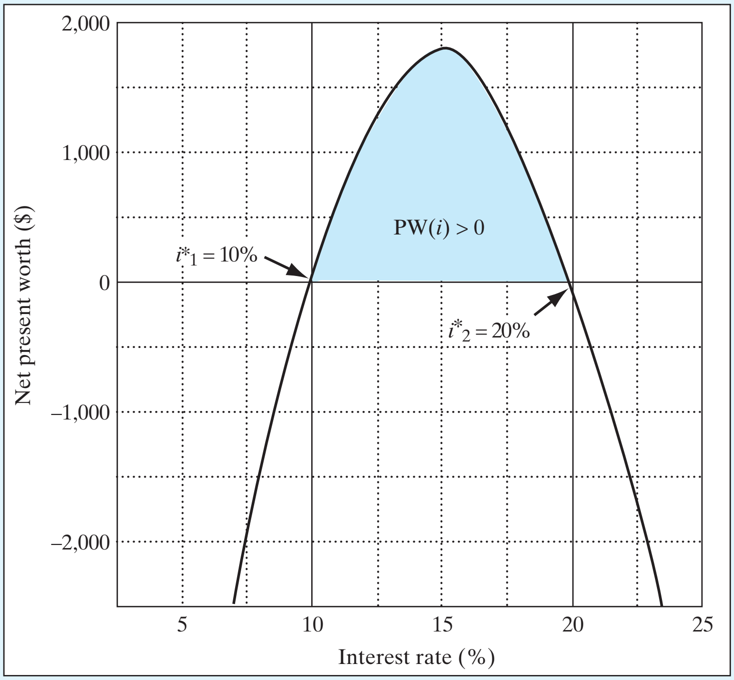


Figure 7.7 NPW plot for a nonsimple investment with multiple rates of return (Example 7.8).

(c) As calculated in (b), the project has multiple rates of return. This is obviously not a net investment, as the following table shows:

Net Investment Test	Using $i^* = 10\%$			Using $i^* = 20\%$		
N	0	1	2	0	1	2
Beginning balance	\$0	−\$1,000	\$1,200	\$0	−\$1,000	\$1,100
Return on investment	0	−100	120	0	−200	220
Payment	−1,000	2,300	−1,320	−1,000	2,300	−1,320
Ending balance	−\$1,000	\$1,200	0	−\$1,000	\$1,100	0

(Unit: \$1,000)

At $n = 0$, there is a net investment to the firm, so the project balance expression becomes

$$PB(i, 15\%)_0 = -\$1{,}000{,}000.$$

The net investment of \$1,000,000 that remains invested internally grows at the interest rate i for the next period. With the receipt of \$2,300,000 in year 1, the project balance becomes

$$\begin{aligned} PB(i, 15\%)_1 &= -\$1{,}000{,}000(1 + i) + \$2{,}300{,}000 \\ &= \$1{,}300{,}000 - \$1{,}000{,}000i \\ &= \$1{,}000{,}000(1.3 - i). \end{aligned}$$

At this point, we do not know whether $PB(i, 15\%)_1$ is positive or negative; we want to know this in order to test for net investment and the presence of a unique i^*. It depends on the value of i, which we want to determine. Therefore, we need to consider two situations: (1) $i < 1.3$ and (2) $i > 1.3$.

- **Case 1:** $i < 1.3 \rightarrow PB(i, 15\%)_1 > 0$.
  Since this indicates a positive balance, the cash released from the project would be returned to the firm's investment pool to grow at the MARR until it is required back in the project. By the end of year 2, the cash placed in the investment pool would have grown at the rate of 15% [to $\$1{,}000{,}000(1.3 - i)(1 + 0.15)$] and must equal the investment into the project of \$1,320,000 required at that time. Then the terminal balance must be

$$\begin{aligned} PB(i, 15\%)_2 &= \$1{,}000{,}000(1.3 - i)(1 + 0.15) - \$1{,}320{,}000 \\ &= \$175{,}000 - \$1{,}150{,}000i \\ &= 0. \end{aligned}$$

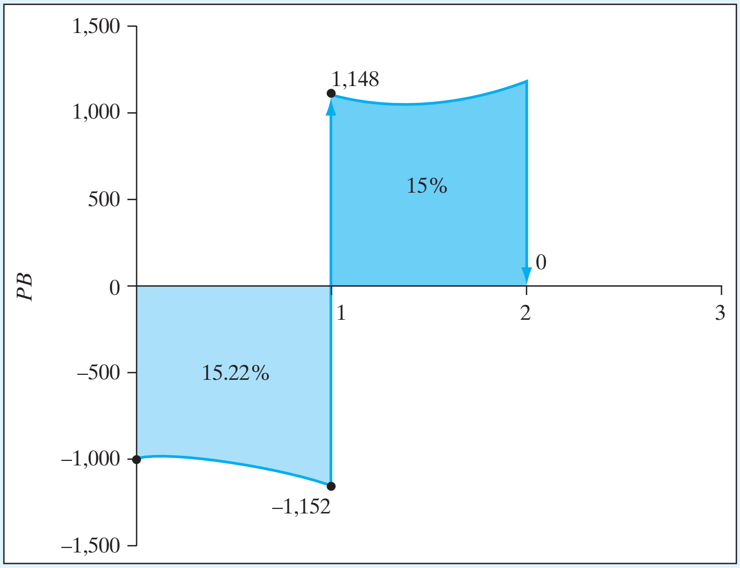


Figure 7.8 Calculation of the IRR for a mixed investment (Example 7.8).

Solving for i yields

$$\text{RIC} = \text{IRR} = 0.1522, \text{ or } 15.22\% > 15\%.$$

The computational process is shown graphically in Figure 7.8.

- **Case 2:** $i > 1.3 \rightarrow \text{PB}(i, 15\%)_1 < 0.$

The firm is still in an investment mode. Therefore, the balance at the end of year 1 that remains invested will grow at the rate i for the next period. With the investment of $1,320,000 required in year 2 and the fact that the net investment must be zero at the end of the project life, the balance at the end of year 2 should be

$$\begin{aligned} \text{PB}(i, 15\%)_2 &= \$1{,}000{,}000(1.3 - i)(1 + i) - \$1{,}320{,}000 \\ &= -\$20{,}000 + \$300{,}000i - \$1{,}000{,}000i^2 \\ &= 0. \end{aligned}$$

Solving for i gives

$$\text{IRR} = 0.1 \text{ or } 0.2 < 1.3,$$

which violates the initial assumption that $i > 1.3$. Therefore, Case 1 is the only correct situation. Since it indicates that IRR > MARR, the project is acceptable, resulting in the same decision as obtained in (a) by applying the NPW criterion.

COMMENTS: In this example, we could have seen by inspection that Case 1 was correct. Since the project required an investment as the final cash flow, the project balance at the end of the previous period (year 1) had to be positive in order for the final balance to equal zero. Inspection does not generally work with more complicated cash flows.

Trial-and-Error Method for Computing IRR for Mixed Investments

The trial-and-error approach to finding the IRR (RIC) for a mixed investment is similar to the trial-and-error approach to finding i^*. We begin with a given MARR and a guess for IRR and solve for the project balance. (A value of IRR close to the MARR is a good starting point for most problems.) Since we desire the project balance to approach zero, we can adjust the value of IRR as needed after seeing the result of the initial guess. For example, for a given pair of interest rates (IRRguess, MARR), if the terminal project balance is positive, the IRRguess value is too low, so we raise it and recalculate. We can continue adjusting our IRR guesses in this way until we obtain a project balance equal or close to zero.

EXAMPLE 7.9 IRR for a Mixed Investment by Trial and Error

Consider project D in Example 7.6. The project has the following cash flow:

n	A_n
0	−\$1,000
1	3,900
2	−5,030
3	2,145

We know from an earlier calculation that this is a mixed investment. Compute the IRR for this project. Assume that MARR = 6%.

SOLUTION

Given: Cash flow as stated for mixed investment and MARR = 6%.

Find: IRR.

For MARR = 6%, we must compute i by trial and error. Suppose we guess $i = 8\%$:

$$\begin{aligned}
PB(8\%, 6\%)_0 &= -\$1{,}000,\\
PB(8\%, 6\%)_1 &= -\$1{,}000(1 + 0.08) + \$3{,}900 = \$2{,}820.\\
PB(8\%, 6\%)_2 &= +\$2{,}820(1 + 0.06) - \$5{,}030 = -\$2{,}040.80,\\
PB(8\%, 6\%)_3 &= -\$2{,}040.80(1 + 0.08) + \$2{,}145 = -\$59.06.
\end{aligned}$$

The net investment is negative at the end of the project, indicating that our trial $i = 8\%$ is in error. After several trials, we conclude that, for MARR = 6%, the IRR is approximately 6.13%. To verify the results, we write

$$\begin{aligned}
PB(6.13\%, 6\%)_0 &= -\$1{,}000,\\
PB(6.13\%, 6\%)_1 &= -\$1{,}000.00(1 + 0.0613) + \$3{,}900 = \$2{,}838.66.\\
PB(6.13\%, 6\%)_2 &= +\$2{,}838.66(1 + 0.0600) - \$5{,}030 = -\$2{,}021.02,\\
PB(6.13\%, 6\%)_3 &= -\$2{,}021.02(1 + 0.0613) + \$2{,}145 = 0.
\end{aligned}$$

The positive balance at the end of year 1 indicates the need to borrow from the project during year 2. However, note that the net investment becomes zero at the end of the project life, confirming that 6.13% is the IRR for the cash flow. Since IRR $>$ MARR, the investment is acceptable.

COMMENTS: On the basis of the NPW criterion, the investment would be acceptable if the MARR was between zero and 10% or between 30% and 50%. The rejection region is $10\% < i < 30\%$ and $i > 50\%$. This can be verified in Figure 7.1(b). Note that the project also would be marginally accepted under the NPW analysis at $\text{MARR} = i = 6\%$:

$$\begin{aligned} PW(6\%) &= -\$1{,}000 + 3{,}900(P/F, 6\%, 1) \\ &= -\$5{,}030(P/F, 6\%, 2) + 2{,}145(P/F, 6\%, 3) \\ &= \$3.55 > 0. \end{aligned}$$

The flowchart in Figure 7.9 summarizes how you should proceed to apply the net cash flow sign test, accumulated cash flow sign test, and net-investment test to calculate an IRR and make an accept–reject decision for a single project. Given the complications

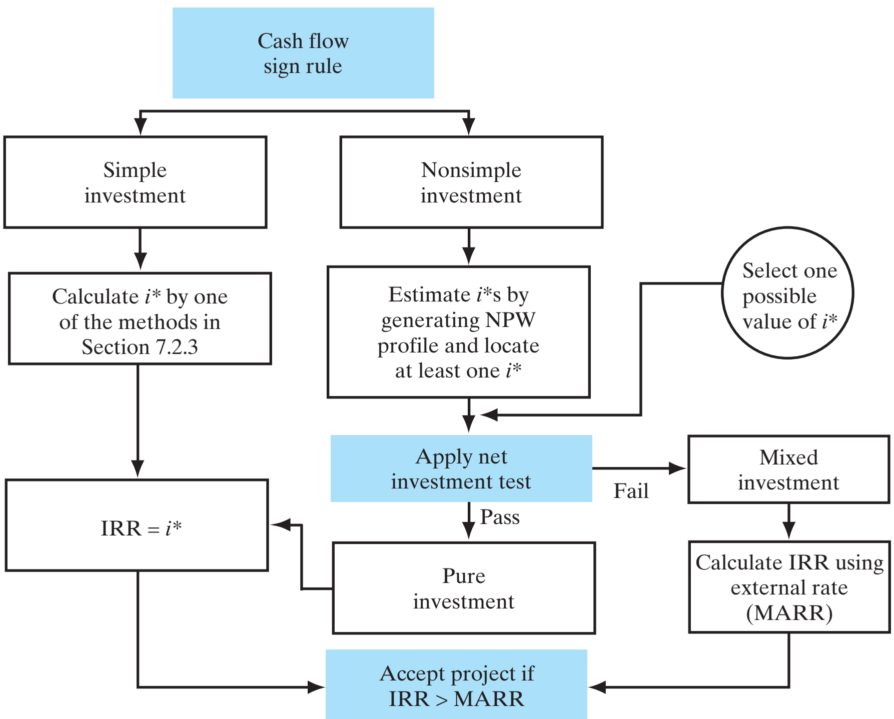


Figure 7.9 Summary of IRR criterion: A flowchart that summarizes how you may proceed to apply the net cash flow sign rule and net-investment test to calculate IRR for a pure as well as a mixed investment.

involved in using IRR analysis to compare alternative projects, it is usually more desirable to use one of the other equivalence techniques for this purpose. As an engineering manager, you should keep in mind the intuitive appeal of the rate-of-return measure. Once you have selected a project on the basis of NPW or AE analysis, you may also wish to express its worth as a rate of return, for the benefit of your associates.

7.4 Mutually Exclusive Alternatives

In this section, we present the decision procedures that should be used in comparing two or more mutually exclusive projects on the basis of the rate-of-return measure. We will consider two situations: (1) alternatives that have the same economic service life and (2) alternatives that have unequal service lives.

7.4.1 Flaws in Project Ranking by IRR

Under NPW or AE analysis, the mutually exclusive project with the highest worth was preferred. (This is known as the "total investment approach.") Unfortunately, the analogy does not carry over to IRR analysis: The project with the highest IRR may *not* be the preferred alternative. To illustrate the flaws inherent in comparing IRRs in order to choose from mutually exclusive projects, suppose you have two mutually exclusive alternatives, each with a 1-year service life: One requires an investment of \$1,000 with a return of \$2,000, and the other requires \$5,000 with a return of \$7,000. You already obtained the IRRs and NPWs at MARR = 10% as follows:

n	A1	A2
0	−\$1,000	−\$5,000
1	2,000	7,000
IRR	100%	40%
PW(10%)	\$818	\$1,364

Assuming that you have enough money in your investment pool to select either alternative, would you prefer the first project simply because you expect a higher rate of return?

On the one hand, we can see that A2 is preferred over A1 by the NPW measure. On the other hand, the IRR measure gives a numerically higher rating for A1. This inconsistency in ranking occurs because the NPW, NFW, and AE are **absolute (dollar)** measures of investment worth, whereas the IRR is a **relative (percentage)** measure and cannot be applied in the same way. That is, the IRR measure ignores the **scale** of the investment. Therefore, the answer to our question in the previous paragraph is no; instead, you would prefer the second project, with the lower rate of return, but higher NPW. Either the NPW or the AE measure would lead to that choice, but a comparison of IRRs would rank the smaller project higher. Another approach, referred to as **incremental analysis**, is needed.

7.4.2 Incremental Investment Analysis

In the previous example, the more costly option requires an incremental investment of \$4,000 at an incremental return of \$5,000. Let's assume that you have exactly \$5,000 in your investment pool.

- If you decide to invest in option A1, you will need to withdraw only \$1,000 from your investment pool. The remaining \$4,000 will continue to earn 10% interest. One year later, you will have \$2,000 from the outside investment and \$4,400 from the investment pool. With an investment of \$5,000, in one year you will have \$6,400. The equivalent present worth of this change in wealth is $\text{PW}(10\%) = -\$5{,}000 + \$6{,}400(P/F, 10\%, 1) = \$818$.
- If you decide to invest in option A2, you will need to withdraw \$5,000 from your investment pool, leaving no money in the pool, but you will have \$7,000 from your outside investment. Your total wealth changes from \$5,000 to \$7,000 in a year. The equivalent present worth of this change in wealth is $\text{PW}(10\%) = -\$5{,}000 + \$7{,}000(P/F, 10\%, 1) = \$1{,}364$.

In other words, if you decide to take the more costly option, certainly you would be interested in knowing that this additional investment can be justified at the MARR. The 10%-of-MARR value implies that you can always earn that rate from other investment sources (i.e., \$4,400 at the end of 1 year for a \$4,000 investment). However, in the second option, by investing the additional \$4,000, you would make an additional \$5,000, which is equivalent to earning at the rate of 25%. Therefore, the incremental investment can be justified.

Now we can generalize the decision rule for comparing mutually exclusive projects. For a pair of mutually exclusive projects (A and B, with B defined as the more costly option), we may rewrite B as

$$B = A + (B - A).$$

In other words, B has two cash flow components: (1) the same cash flow as A and (2) the incremental component $(B - A)$. Therefore, the only situation in which B is preferred to A is when the rate of return on the incremental component $(B - A)$ exceeds the MARR. Therefore, for two mutually exclusive projects, rate-of-return analysis is done by computing the *internal rate of return on the incremental investment* $(IRR\Delta)$ between the projects. Since we want to consider increments of investment, we compute the cash flow for the difference between the projects by subtracting the cash flow for the lower investment-cost project (A) from that of the higher investment-cost project (B). Then the decision rule is

Incremental IRR: IRR on the incremental investment from choosing a large project instead of a smaller project.

$$\text{If IRR}_{B-A} > \text{MARR, select } B,$$

$$\text{If IRR}_{B-A} = \text{MARR, select either project},$$

$$\text{If IRR}_{B-A} < \text{MARR, select } A,$$

where $B - A$ is an investment increment (negative cash flow). *If a "do-nothing" alternative is allowed, the smaller cost option must be profitable (its IRR must be greater than the MARR) at first.* This means that you compute the rate of return for each alternative in the mutually exclusive group and then eliminate the alternatives whose IRRs are less than the MARR before applying the incremental analysis.

It may seem odd to you how this simple rule allows us to select the right project. Example 7.10 illustrates the incremental investment decision rule.

EXAMPLE 7.10 IRR on Incremental Investment: Two Alternatives

John Covington, a college student, wants to start a small-scale painting business during his off-school hours. To economize the start-up business, he decides to purchase some used painting equipment. He has two mutually exclusive options: Do most of the painting by himself by limiting his business to only residential painting jobs (B1) or purchase more painting equipment and hire some helpers to do both residential and commercial painting jobs that he expects will have a higher equipment cost, but provide higher revenues as well (B2). In either case, John expects to fold up the business in three years, when he graduates from college.

The cash flows for the two mutually exclusive alternatives are as follows:

n	B1	B2	B2 − B1
0	−$3,000	−$12,000	−$9,000
1	1,350	4,200	2,850
2	1,800	6,225	4,425
3	1,500	6,330	4,830
IRR	25%	17.43%	

Knowing that both alternatives are revenue projects, which project would John select at MARR = 10%? (Note that both projects are also profitable at 10%.)

SOLUTION

Given: Incremental cash flow between two alternatives and MARR = 10%.
Find: (a) IRR on the increment and (b) which alternative is preferable.

(a) To choose the best project, we compute the incremental cash flow B2 − B1. Then we compute the IRR on this increment of investment by solving the equation

$$-\$9{,}000 + \$2{,}850(P/F, i, 1) + \$4{,}425(P/F, i, 2) + \$4{,}830(P/F, i, 3) = 0.$$

(b) We obtain $i^*_{B2-B1} = 15\%$, as plotted in Figure 7.10. By inspection of the incremental cash flow, we know that it is a simple investment, so $IRR_{B2-B1} = i^*_{B2-B1}$. Since $IRR_{B2-B1} > MARR$, we select B2, which is consistent with the NPW analysis. Note that, at MARR > 25%, neither project would be acceptable.

COMMENTS: Why did we choose to look at the increment B2 − B1 instead of B1 − B2? Because we want the first flow of the incremental cash flow series to be negative (an investment flow), so that we can calculate an IRR. By subtracting the lower initial investment project from the higher, we guarantee that the first increment will be an investment flow. If we ignore the investment ranking, we might end up

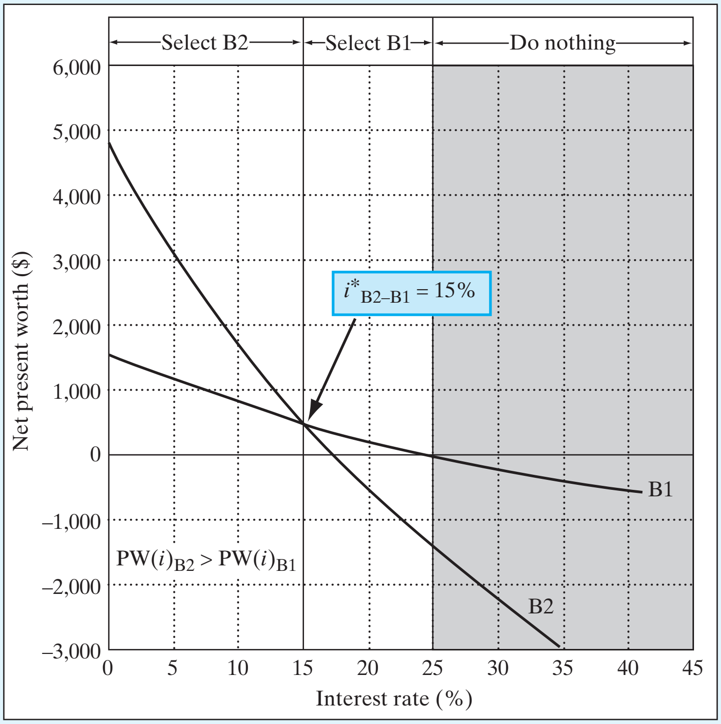


Figure 7.10 NPW profiles for B1 and B2 (Example 7. 10).

with an increment that involves *borrowing cash flow* and has no internal rate of return. This is indeed the case for B1 − B2. (i^*_{B1-B2} is also 15%, not −15%, but it has a different meaning: it is a borrowing rate, not a rate of return on your investment.) If, erroneously, we had compared this i^* with the MARR, we might have accepted project B1 over B2. This undoubtedly would have damaged our credibility with management!

The next example indicates that the inconsistency in ranking between NPW and IRR can also occur when differences in the timing of a project's future cash flows exist, even if their initial investments are the same.

EXAMPLE 7.11 IRR on Incremental Investment When Initial Flows Are Equal

Consider the following two mutually exclusive investment projects that require the same amount of investment:

Which project would you select on the basis of the rate of return on incremental investment, assuming that MARR = 12%? (Once again, both projects are profitable at 12%.)

n	C1	C2
0	−$9,000	−$9,000
1	480	5,800
2	3,700	3,250
3	6,550	2,000
4	3,780	1,561
IRR	18%	20%

SOLUTION

Given: Cash flows for two mutually exclusive alternatives as shown and MARR = 12%.

Find: (a) IRR on incremental investment and (b) which alternative is preferable.

(a) When the initial investments are equal, we progress through the cash flows until we find the first difference and then set up the increment so that this first nonzero flow is negative (i.e., an investment). Thus, we set up the incremental investment by taking $(C1 - C2)$:

n	C1 − C2
0	$ 0
1	−5,320
2	450
3	4,550
4	2,219

We next set the PW equation equal to zero:

$$-\$5{,}320 + \$450(P/F, i, 1) + \$4{,}550(P/F, i, 2) + \$2{,}219(P/F, i, 3) = 0.$$

(b) Solving for i yields $i^* = 14.71\%$, which is also an IRR, since the increment is a simple investment. Since $\text{IRR}_{C1-C2} = 14.71\% > \text{MARR}$, we would select C1. If we used NPW analysis, we would obtain $\text{PW}(12\%)_{C1} = \$1{,}443$ and $\text{PW}(12\%)_{C2} = \$1{,}185$, indicating that C1 is preferred over C2.

When you have more than two mutually exclusive alternatives, they can be compared in pairs by successive examination. Example 7.12 illustrates how to compare three mutually exclusive alternatives. (In Chapter 15, we will examine some multiple-alternative problems in the context of capital budgeting.)

EXAMPLE 7.12 IRR on Incremental Investment: Three Alternatives

Consider the following three sets of mutually exclusive alternatives:

n	D1	D2	D3
0	−\$2,000	−\$1,000	−\$3,000
1	1,500	800	1,500
2	1,000	500	2,000
3	800	500	1,000
IRR	34.37%	40.76%	24.81%

Which project would you select on the basis of the rate of return on incremental investment, assuming that MARR = 15%?

SOLUTION

Given: Preceding cash flows and MARR = 15%.

Find: IRR on incremental investment and which alternative is preferable.

Step 1: Examine the IRR of each alternative. At this point, we can eliminate any alternative that fails to meet the MARR. In this example, all three alternatives exceed the MARR.

Step 2: Compare D1 and D2 in pairs.[9] Because D2 has a lower initial cost, compute the rate of return on the increment (D1−D2), which represents an increment of investment.

n	D1 − D2
0	−\$1,000
1	700
2	500
3	300

The incremental cash flow represents a simple investment. To find the incremental rate of return, we write

$$-\$1{,}000 + \$700(P/F, i, 1) + \$500(P/F, i, 2) + \$300(P/F, i, 3) = 0.$$

Solving for i^*_{D1-D2} yields 27.61%, which exceeds the MARR; therefore, D1 is preferred over D2. Now you eliminate D2 from further consideration.

Step 3: Compare D1 and D3. Once again, D1 has a lower initial cost. Examine the increment $(D3 - D1)$:

[9] When faced with many alternatives, you may arrange them in order of increasing initial cost. This is not a required step, but it makes the comparison more tractable.

n	D3 − D1
0	−$1,000
1	0
2	1,000
3	200

Here, the incremental cash flow represents another simple investment. The increment (D3 − D1) has an unsatisfactory 8.8% rate of return; therefore, D1 is preferred over D3. Accordingly, we conclude that D1 is the best alternative.

EXAMPLE 7.13 Incremental Analysis for Cost-Only Projects

Falk Corporation is considering two types of manufacturing systems to produce its shaft couplings over six years: (1) a cellular manufacturing system (CMS) and (2) a flexible manufacturing system (FMS). The average number of pieces to be produced with either system would be 544,000 per year. The operating cost, initial investment, and salvage value for each alternative are estimated as follows:

Items	CMS Option	FMS Option
Annual O&M costs:		
Annual labor cost	$1,169,600	$707,200
Annual material cost	832,320	598,400
Annual overhead cost	3,150,000	1,950,000
Annual tooling cost	470,000	300,000
Annual inventory cost	141,000	31,500
Annual income taxes	1,650,000	1,917,000
Total annual costs	$7,412,920	$5,504,100
Investment	$4,500,000	$12,500,000
Net salvage value	$500,000	$1,000,000

Figure 7.11 illustrates the cash flows associated with each alternative. The firm's MARR is 15%. On the basis of the IRR criterion, which alternative would be a better choice?

DISCUSSION: Since we can assume that both manufacturing systems would provide the same level of revenues over the analysis period, we can compare the two alternatives on the basis of cost only. (These are service projects.) Although we cannot compute the IRR for each option without knowing the revenue figures, we can still

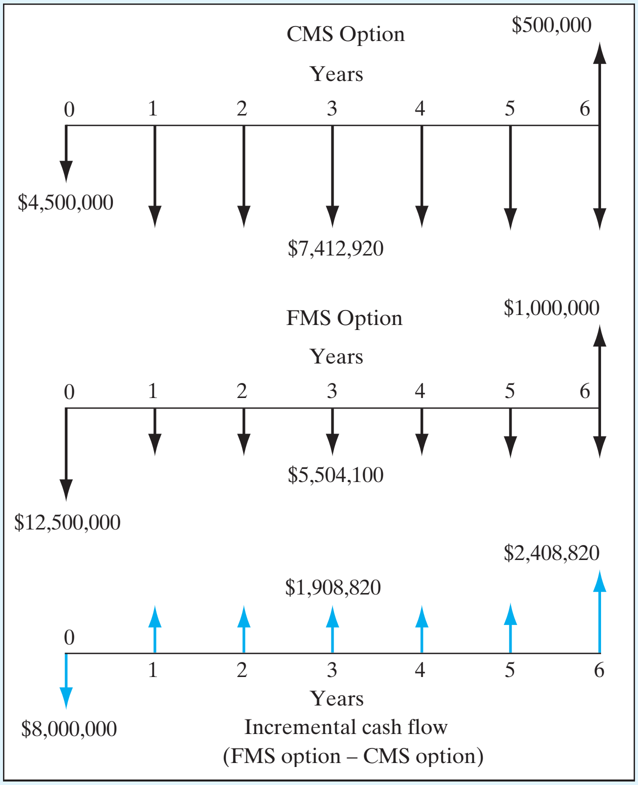


Figure 7.11 Comparison of mutually exclusive alternatives with equal revenues (cost only) (Example 7.13).

calculate the IRR on incremental cash flows. Since the FMS option requires a higher initial investment than that for the CMS, the incremental cash flow is the difference (FMS − CMS).

n	CMS Option	FMS Option	Incremental (FMS − CMS)
0	−\$4,500,000	−\$12,500,000	−\$8,000,000
1	−7,412,920	−5,504,100	1,908,820
2	−7,412,920	−5,504,100	1,908,820
3	−7,412,920	−5,504,100	1,908,820
4	−7,412,920	−5,504,100	1,908,820
5	−7,412,920	−5,504,100	1,908,820
6	−7,412,920	−5,504,100	
	+ $\boxed{\$500,000}$	+ $\boxed{\$1,000,000}$	\$2,408,820

SOLUTION

Given: Cash flows shown in Figure 7.11 and $i = 15\%$ per year.

Find: Incremental cash flows, and select the better alternative on the basis of the IRR criterion.

First, we have

$$
\begin{aligned}
\text{PW}(i)_{\text{FMS}-\text{CMS}} &= -\$8{,}000{,}000 + \$1{,}908{,}820(P/A, i, 5) \\
&= +\$2{,}408{,}820(P/F, i, 6) \\
&= 0.
\end{aligned}
$$

Solving for i by trial and error yields 12.43%. Since $\text{IRR}_{\text{FMS}-\text{CMS}} = 12.43\% < 15\%$, we would select CMS. Although the FMS would provide an incremental annual savings of \$1,908,820 in operating costs, the savings are not large enough to justify the incremental investment of \$8,000,000.

COMMENTS: Note that the CMS option is marginally preferred to the FMS option. However, there are dangers in relying solely on the easily quantified savings in input factors—such as labor, energy, and materials—from the FMS and in not considering gains from improved manufacturing performance that are more difficult and subjective to quantify. Factors such as improved product quality, increased manufacturing flexibility (rapid response to customer demand), reduced inventory levels, and increased capacity for product innovation are frequently ignored because we have inadequate means for quantifying their benefits. If these intangible benefits were considered, the FMS option could come out better than the CMS option.

7.4.3 Handling Unequal Service Lives

In Chapters 5 and 6, we discussed the use of the NPW and AE criteria as bases for comparing projects with unequal lives. The IRR measure can also be used to compare projects with unequal lives, as long as we can establish a common analysis period. The decision procedure is then exactly the same as for projects with equal lives. It is likely, however, that we will have a multiple-root problem, which creates a substantial computational burden. For example, suppose we apply the IRR measure to a case in which one project has a 5-year life and the other project has an 8-year life, resulting in a least common multiple of 40 years. Then when we determine the incremental cash flows over the analysis period, we are bound to observe many sign changes. This leads to the possibility of having many i^*'s. Example 7.14 uses i^* to compare mutually exclusive projects, in which the incremental cash flows reveal several sign changes. (Our purpose is not to encourage you to use the IRR approach to compare projects with unequal lives; rather, it is to show the correct way to compare them if the IRR approach *must* be used.)

EXAMPLE 7.14 IRR Analysis for Projects with Different Lives in Which the Increment is a Nonsimple Investment

Consider Example 5.14, in which a mail-order firm wants to install an automatic mailing system to handle product announcements and invoices. The firm had a choice between two different types of machines. Using the IRR as a decision criterion, select the best machine. Assume a MARR of 15%, as before.

SOLUTION

Given: Cash flows for two projects with unequal lives, as shown in Figure 5.11, and MARR = 15%.

Find: The alternative that is preferable.

Since the analysis period is equal to the least common multiple of 12 years, we may compute the incremental cash flow over this 12-year period. As shown in Figure 7.12, we subtract the cash flows of model A from those of model B to form the increment of investment. (Recall that we want the first cash flow difference to be a negative value.) We can then compute the IRR on this incremental cash flow.

Five sign changes occur in the incremental cash flows, indicating a nonsimple incremental investment:

n	Model A		Model B		Model B − Model A
0	−$12,500		−$15,000		−$2,500
1		−5,000		−4,000	1,000
2		−5,000		−4,000	1,000
3	−12,500	−3,000		−4,000	11,500
4		−5,000	−15,000	−2,500	−12,500
5		−5,000		−4,000	1,000
6	−12,500	−3,000		−4,000	11,500
7		−5,000		−4,000	1,000
8		−5,000	−15,000	−2,500	−12,500
9	−12,500	−3,000		−4,000	11,500
10		−5,000		−4,000	1,000
11		−5,000		−4,000	1,000
12		−3,000		−2,500	500
	Four replacement cycles		Three replacement cycles		Incremental cash flows

Even though there are five sign changes in the cash flow, there is only one positive i^* for this problem: 63.12%. Unfortunately, however, the investment is not a pure

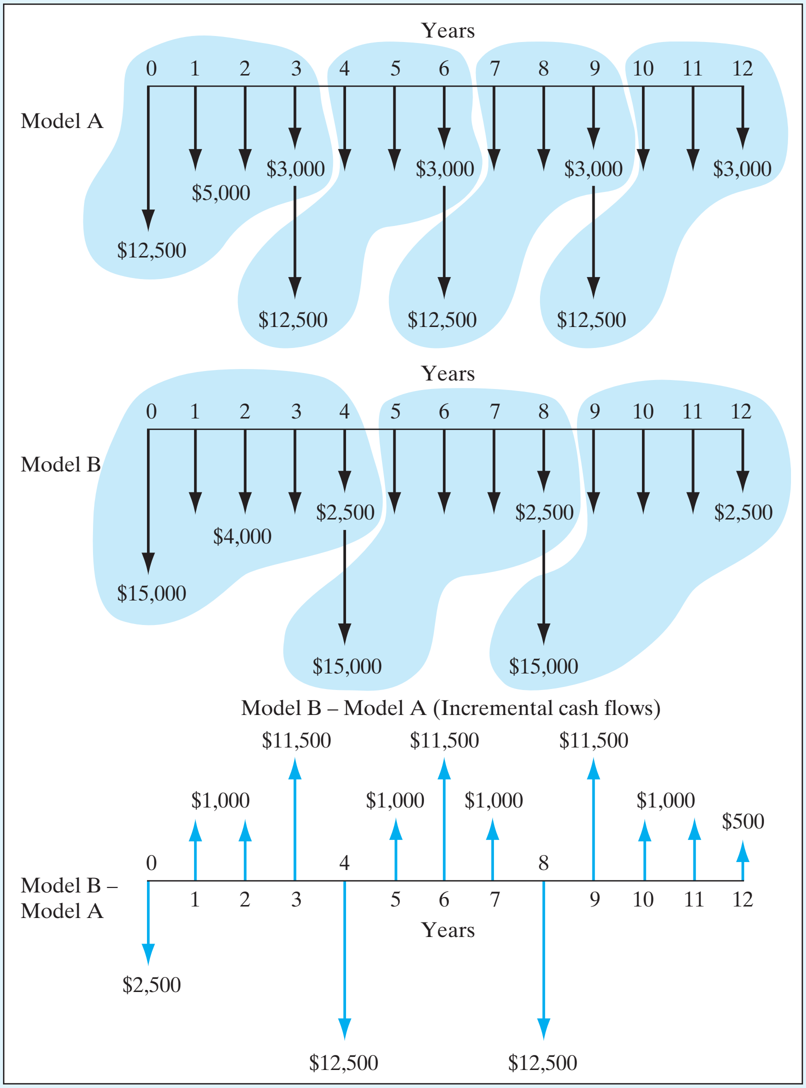


Figure 7.12 Comparison of projects with unequal lives (Example 7.14).

investment. We need to employ an external rate to compute the IRR in order to make a proper accept–reject decision. Assuming that the firm's MARR is 15%, we will use a trial-and-error approach. Try $i = 20\%$:

$$PB(20\%, 15\%)_0 = -\$2{,}500,$$

$$PB(20\%, 15\%)_1 = -\$2{,}500(1.20) + \$1{,}000 = -\$2{,}000.$$

$$PB(20\%, 15\%)_2 = -\$2{,}000(1.20) + \$1{,}000 = -\$1{,}400,$$

$$PB(20\%, 15\%)_3 = -\$1{,}400(1.20) + \$11{,}500 = \$9{,}820.$$

$$PB(20\%, 15\%)_4 = \$9{,}820(1.15) - \$12{,}500 = -\$1{,}207,$$

$$PB(20\%, 15\%)_5 = -\$1{,}207(1.20) + \$1{,}000 = -\$448.40.$$

$$PB(20\%, 15\%)_6 = -\$448.40(1.20) + \$11{,}500 = \$10{,}961.92,$$

$$PB(20\%, 15\%)_7 = \$10{,}961.92(1.15) + \$1{,}000 = \$13{,}606.21.$$

$$PB(20\%, 15\%)_8 = \$13{,}606.21(1.15) - \$12{,}500 = \$3{,}147.14,$$

$$PB(20\%, 15\%)_9 = \$3{,}147.14(1.15) + \$11{,}500 = \$15{,}119.21.$$

$$PB(20\%, 15\%)_{10} = \$15{,}119.21(1.15) + \$1{,}000 = \$18{,}387.09,$$

$$PB(20\%, 15\%)_{11} = \$18{,}387.09(1.15) + \$1{,}000 = \$22{,}145.16,$$

$$PB(20\%, 15\%)_{12} = \$22{,}145.16(1.15) + \$500 = \$25{,}966.93.$$

Since $PB(20\%, 15\%)_{12} > 0$, the guessed 20% is not the RIC. We may increase the value of i and repeat the calculations. After several trials, we find that the RIC is 50.68%.[10] Since $IRR_{B-A} > MARR$, model B would be selected, which is consistent with the NPW analysis. In other words, the additional investment over the years to obtain model B ($-\$2{,}500$ at $n = 0$, $-\$12{,}500$ at $n = 4$, and $-\$12{,}500$ at $n = 8$) yields a satisfactory rate of return.

COMMENTS: Given the complications inherent in IRR analysis in comparing alternative projects, it is usually more desirable to employ one of the other equivalence techniques for this purpose. As an engineering manager, you should keep in mind the intuitive appeal of the rate-of-return measure. Once you have selected a project on the basis of NPW or AE analysis, you may also wish to express its worth as a rate of return, for the benefit of your associates.

SUMMARY

- The **rate of return (ROR)** is the interest rate earned on unrecovered project balances such that an investment's cash receipts make the terminal project balance equal to zero. The rate of return is an intuitively familiar and understandable measure of project profitability that many managers prefer to NPW or other equivalence measures.
- Mathematically, we can determine the rate of return for a given project cash flow series by locating an interest rate that equates the net present worth of the project's cash flows to zero. This break-even interest rate is denoted by the symbol i^*.
- The **internal rate of return (IRR)** is another term for ROR which stresses the fact that we are concerned with the interest earned on the portion of the project that is internally invested, not those portions released by (borrowed from) the project.

[10] It is tedious to solve this type of problem by a trial-and-error method on your calculator. The problem can be solved quickly by using the Cash Flow Analyzer, which can be found from the book's website.

- To apply rate-of-return analysis correctly, we need to classify an investment as either simple or nonsimple. A **simple investment** is defined as an investment in which the initial cash flows are negative and only one sign change in the net cash flow occurs, whereas a **nonsimple investment** is an investment for which more than one sign change in the cash flow series occurs. Multiple i^*'s occur only in nonsimple investments. However, not all nonsimple investments will have multiple i^*'s. In this regard,

  1. The possible presence of multiple i^*'s (rates of return) can be predicted by

  - The net cash flow sign test.
  - The accumulated cash flow sign test.

  When multiple rates of return cannot be ruled out by the two methods, it is useful to generate an NPW profile to approximate the value of i^*.

  2. All i^* values should be exposed to the **net investment test**. Passing this test indicates that i^* is an internal rate of return and is therefore a suitable measure of project profitability. Failing to pass the test indicates project borrowing, a situation that requires further analysis with the use of an **external interest rate**.

  3. **Return-on-invested-capital** analysis uses one rate (the firm's MARR) on externally invested balances and solves for another rate (i^*) on internally invested balances.

- For a pure investment, i^* is the rate of return that is internal to the project. For a mixed investment, the RIC calculated with the use of the external interest rate (or MARR) is the true IRR; so the decision rule is as follows:

$$\text{If IRR} > \text{MARR, accept the project.}$$

$$\text{If IRR} = \text{MARR, remain indifferent.}$$

$$\text{If IRR} < \text{MARR, reject the project.}$$

  IRR analysis yields results consistent with NPW and other equivalence methods.

- In properly selecting among alternative projects by IRR analysis, **incremental investment** must be used. In creating an incremental investment, we always subtract the lower cost investment from the higher cost one. Basically, you want to know that the extra investment required can be justified on the basis of the extra benefits generated in the future.

PROBLEMS

Note: *The symbol i^* represents the interest rate that makes the net present value of the project in question equal to zero. The symbol IRR represents the internal rate of return of the investment. For a simple investment, $IRR = i^*$. For a nonsimple investment, i^* is generally not equal to IRR.*

Concept of Rate of Return

7.1 You are going to buy a new car worth $14,500. The dealer computes your monthly payment to be $267 for 72 months' financing. What is the dealer's rate of return on this loan transaction?

7.2 You wish to sell a bond that has a face value of \$1,000. The bond bears an interest rate of 6%, payable semiannually. Four years ago, the bond was purchased at \$900. At least an 8% annual return on the investment is desired. What must be the minimum selling price of the bond now in order to make the desired return on the investment?

7.3 In 1970, Wal-Mart offered 300,000 shares of its common stock to the public at a price of \$16.50 per share. Since that time, Wal-Mart has had 11 two-for-one stock splits. On a purchase of 100 shares at \$16.50 per share on the company's first offering, the number of shares has grown to 204,800 shares worth \$10,649,600 on January 2006. What is the return on investment for investors who purchased the stock in 1970 (say, over a 36-year ownership period)? Assume that the dividends received during that period were not reinvested.

7.4 Johnson Controls spent more than \$2.5 million retrofitting a government complex and installing a computerized energy-management system for the State of Massachusetts. As a result, the state's energy bill dropped from an average of \$6 million a year to \$3.5 million. Moreover, both parties will benefit from the 10-year life of the contract. Johnson recovers half the money it saved in reduced utility costs (about \$1.2 million a year over 10 years); Massachusetts has its half to spend on other things. What is the rate of return realized by Johnson Controls in this energy-control system?

7.5 Pablo Picasso's 1905 portrait *Boy with a Pipe* sold for \$104.2 million in an auction at Sotheby's Holdings, Inc., on June 24, 2004, shattering the existing record for art and ushering in a new era in pricing for 20th-century paintings. The Picasso, sold by the philanthropic Greentree Foundation, cost Mr. Whitney about \$30,000 in 1950. Determine the annual rate of appreciation of the artwork over 54 years.

Investment Classification and Calculation of i^*

7.6 Consider four investments with the following sequences of cash flows:

	Net Cash Flow			
n	Project A	Project B	Project C	Project D
0	−\$18,000	−\$30,000	\$34,578	−\$56,500
1	30,000	32,000	−18,000	2,500
2	20,000	32,000	−18,000	6,459
3	10,000	−22,000	−18,000	−78,345

(a) Identify all the simple investments.
(b) Identify all the nonsimple investments.
(c) Compute i^* for each investment.
(d) Which project has no rate of return?

7.7 Consider the following infinite cash flow series with repeated cash flow patterns:

n	A_n
0	−$1,000
1	400
2	800
3	500
4	500
5	400
6	800
7	500
8	500
⋮	⋮

Determine i^* for this infinite cash flow series.

7.8 Consider the following investment projects:

	Project Cash Flow				
n	A	B	C	D	E
0	−$100	−$100	−$200	−$50	−$50
1	60	70	$20	120	−100
2	150	70	10	40	−50
3		40	5	40	0
4		40	−180	−20	150
5			60	40	150
6			50	30	100
7			400		100

(a) Classify each project as either simple or nonsimple.
(b) Use the quadratic equation to compute i^* for project A.
(c) Obtain the rate(s) of return for each project by plotting the NPW as a function of the interest rate.

7.9 Consider the projects in Table P7.9.
(a) Classify each project as either simple or nonsimple.
(b) Identify all positive i^*'s for each project.
(c) For each project, plot the present worth as a function of the interest rate (i).

TABLE P7.9 **Net Cash Flow for Four Projects**

	Net Cash Flow			
n	A	B	C	D
0	−$2,000	−$1,500	−$1,800	−$1,500
1	500	800	5,600	−360
2	100	600	4,900	4,675
3	100	500	−3,500	2,288
4	2,000	700	7,000	
5			−1,400	
6			2,100	
7			900	

7.10 Consider the following financial data for a project:

Initial investment	$50,000
Project life	8 years
Salvage value	$10,000
Annual revenue	$25,000
Annual expenses (including income taxes)	$ 9,000

(a) What is i^* for this project?
(b) If the annual expense increases at a 7% rate over the previous year's expenses, but the annual income is unchanged, what is the new i^*?
(c) In part (b), at what annual rate will the annual income have to increase to maintain the same i^* obtained in part (a)?

7.11 Consider two investments, A and B, with the following sequences of cash flows:

	Net Cash Flow	
n	Project A	Project B
0	−$25,000	−$25,000
1	2,000	10,000
2	6,000	10,000
3	12,000	10,000
4	24,000	10,000
5	28,000	5,000

(a) Compute i^* for each investment.

(b) Plot the present-worth curve for each project on the same chart, and find the interest rate that makes the two projects equivalent.

7.12 Consider the following investment projects:

	Project Cash Flows					
N	A	B	C	D	E	F
0	−\$100	−\$100	−\$100	−\$100	−\$100	−\$100
1	200	470	−200	0	300	300
2	300	720	200	0	250	100
3	400	360	250	500	−40	400

(a) For each project, apply the sign rule to predict the number of possible i^*'s.

(b) For each project, plot the NPW profile as a function of i between 0 and 200%.

(c) For each project, compute the value(s) of i^*.

7.13 Consider an investment project with the following cash flows:

n	Net Cash Flow
0	−\$120,000
1	94,000
2	144,000
3	72,000

(a) Find the IRR for this investment.

(b) Plot the present worth of the cash flow as a function of i.

(c) On the basis of the IRR criterion, should the project be accepted at MARR = 15%?

Mixed Investments

7.14 Consider the following investment projects:

	Net Cash Flow		
n	Project 1	Project 2	Project 3
0	−\$1,000	−\$2,000	−\$1,000
1	500	1,560	1,400
2	840	944	−100
IRR	?	?	?

Assume that MARR = 12% in the following questions:

(a) Compute i^* for each investment. If the problem has more than one i^*, identify all of them.

(b) Compute IRR(true) for each project.

(c) Determine the acceptability of each investment.

7.15 Consider the following investment projects:

	Project Cash Flow				
n	A	B	C	D	E
0	−$100	−$100	−$5	−$100	$200
1	100	30	10	30	100
2	24	30	30	30	−500
3		70	−40	30	−500
4		70		30	200
5				30	600

(a) Use the quadratic equation to compute i^* for A.

(b) Classify each project as either simple or nonsimple.

(c) Apply the cash flow sign rules to each project, and determine the number of possible positive i^*'s. Identify all projects having a unique i^*.

(d) Compute the IRRs for projects B through E.

(e) Apply the net-investment test to each project.

7.16 Consider the following investment projects:

	Net Cash Flow		
n	Project 1	Project 2	Project 3
0	−$1,600	−$5,000	−$1,000
1	10,000	10,000	4,000
2	10,000	30,000	−4,000
3		−40,000	

Assume that MARR = 12% in the following questions:

(a) Identify the i^*('s) for each investment. If the project has more than one i^*, identify all of them.

(b) Which project(s) is (are) a mixed investment?

(c) Compute the IRR for each project.
(d) Determine the acceptability of each project.

7.17 Consider the following investment projects:

	Net Cash Flow		
n	Project A	Project B	Project C
0	−\$100	−\$150	−\$100
1	30	50	410
2	50	50	−558
3	80	50	252
4		100	
IRR	(23.24%)	(21.11%)	(20%, 40%, 50%)

Assume that MARR = 12% for the following questions:
(a) Identify the pure investment(s).
(b) Identify the mixed investment(s).
(c) Determine the IRR for each investment.
(d) Which project would be acceptable?

7.18 The Boeing Company has received a NASA contract worth \$460 million to build rocket boosters for future space missions. NASA will pay \$50 million when the contract is signed, another \$360 million at the end of the first year, and the \$50 million balance at the end of second year. The expected cash outflows required to produce these rocket boosters are estimated to be \$150 million now, \$100 million during the first year, and \$218 million during the second year. The firm's MARR is 12%. The cash flow is as follows:

n	Outflow	Inflow	Net Cash Flow
0	\$150	\$50	−\$100
1	100	360	260
2	218	50	−168

(a) Show whether this project is or is not a mixed investment.
(b) Compute the IRR for this investment.
(c) Should Boeing accept the project?

7.19 Consider the following investment projects:

	Net Cash Flow		
n	Project A	Project B	Project C
0	−\$100		−\$100
1	216	−150	50
2	−116	100	−50
3		50	200
4		40	
i^*	?	15.51%	29.95%

(a) Compute i^* for project A. If there is more than one i^*, identify all of them.
(b) Identify the mixed investment(s).
(c) Assuming that MARR = 10%, determine the acceptability of each project on the basis of the IRR criterion.

7.20 Consider the following investment projects:

	Net Cash Flow				
n	A	B	C	D	E
0	−\$1,000	−\$5,000	−\$2,000	−\$2,000	−\$1,000
1	3,100	20,000	1,560	2,800	3,600
2	−2,200	12,000	944	−200	−5,700
3		−3,000			3,600
i^*	?	?	18%	32.45%	35.39%

Assume that MARR = 12% in the following questions:
(a) Compute i^* for projects A and B. If the project has more than one i^*, identify all of them.
(b) Classify each project as either a pure or a mixed investment.
(c) Compute the IRR for each investment.
(d) Determine the acceptability of each project.

7.21 Consider an investment project whose cash flows are as follows:

n	Net Cash Flow
0	−\$5,000
1	10,000
2	30,000
3	−40,000

(a) Plot the present-worth curve by varying i from 0% to 250%.
(b) Is this a mixed investment?
(c) Should the investment be accepted at MARR = 18%?

7.22 Consider the following two mutually exclusive investment projects:

	Net Cash Flow	
n	Project A	Project B
0	−$300	−$800
1	0	1,150
2	690	40
i^*	51.66%	46.31%

Assume that MARR = 15%.
(a) According to the IRR criterion, which project would be selected?
(b) Sketch the PW(i) function on the incremental investment (B − A).

7.23 Consider the following cash flows of a certain project:

n	Net Cash Flow
0	−$100,000
1	310,000
2	−220,000

The project's i^*'s are computed as 10% and 100%, respectively. The firm's MARR is 8%.
(a) Show why this investment project fails the net-investment test.
(b) Compute the IRR, and determine the acceptability of this project.

7.24 Consider the following investment projects:

	Net Cash Flow		
n	Project 1	Project 2	Project 3
0	−$1,000	−$1,000	−$1,000
1	−1,000	1,600	1,500
2	2,000	−300	−500
3	3,000	−200	2,000

Which of the following statements is correct?
(a) All projects are nonsimple investments.
(b) Project 3 should have three real rates of return.
(c) All projects will have a unique positive real rate of return.
(d) None of the above.

IRR Analysis

7.25 Agdist Corporation distributes agricultural equipment. The board of directors is considering a proposal to establish a facility to manufacture an electronically controlled "intelligent" crop sprayer invented by a professor at a local university. This crop sprayer project would require an investment of \$10 million in assets and would produce an annual after-tax net benefit of \$1.8 million over a service life of eight years. All costs and benefits are included in these figures. When the project terminates, the net proceeds from the sale of the assets will be \$1 million. Compute the rate of return of this project. Is this a good project at MARR = 10%?

7.26 Consider an investment project with the following cash flows:

n	Cash Flow
0	−\$5,000
1	0
2	4,840
3	1,331

Compute the IRR for this investment. Is the project acceptable at MARR = 10%?

7.27 Consider the following cash flow of a certain project:

n	Net Cash Flow
0	−\$2,000
1	800
2	900
3	X

If the project's IRR is 10%,

(a) Find the value of X.

(b) Is this project acceptable at MARR = 8%?

7.28 You are considering a luxury apartment building project that requires an investment of \$12,500,000. The building has 50 units. You expect the maintenance cost for the apartment building to be \$250,000 the first year and \$300,000 the second year. The maintenance cost will continue to increase by \$50,000 in subsequent years. The cost to hire a manager for the building is estimated to be \$80,000 per year. After five years of operation, the apartment building can be sold for \$14,000,000. What is the annual rent per apartment unit that will provide a return on investment of 15%? Assume that the building will remain fully occupied during its five years of operation.

7.29 A machine costing \$25,000 to buy and \$3,000 per year to operate will save mainly labor expenses in packaging over six years. The anticipated salvage value of the machine at the end of the six years is \$5,000. To receive a 10% return on investment (rate of return), what is the minimum required annual savings in labor from this machine?

7.30 Champion Chemical Corporation is planning to expand one of its propylene-manufacturing facilities. At $n = 0$, a piece of property costing \$1.5 million must be purchased to build a plant. The building, which needs to be expanded during the first year, costs \$3 million. At the end of the first year, the company needs to spend about \$4 million on equipment and other start-up costs. Once the building becomes operational, it will generate revenue in the amount of \$3.5 million during the first operating year. This will increase at the annual rate of 5% over the previous year's revenue for the next 9 years. After 10 years, the sales revenue will stay constant for another 3 years before the operation is phased out. (It will have a project life of 13 years after construction.) The expected salvage value of the land at the end of the project's life would be about \$2 million, the building about \$1.4 million, and the equipment about \$500,000. The annual operating and maintenance costs are estimated to be approximately 40% of the sales revenue each year. What is the IRR for this investment? If the company's MARR is 15%, determine whether the investment is a good one. (Assume that all figures represent the effect of the income tax.)

7.31 Recent technology has made possible a computerized vending machine that can grind coffee beans and brew fresh coffee on demand. The computer also makes possible such complicated functions as changing \$5 and \$10 bills, tracking the age of an item, and moving the oldest stock to the front of the line, thus cutting down on spoilage. With a price tag of \$4,500 for each unit, Easy Snack has estimated the cash flows in millions of dollars over the product's six-year useful life, including the initial investment, as follows:

n	Net Cash Flow
0	−\$20
1	8
2	17
3	19
4	18
5	10
6	3

(a) On the basis of the IRR criterion, if the firm's MARR is 18%, is this product worth marketing?

(b) If the required investment remains unchanged, but the future cash flows are expected to be 10% higher than the original estimates, how much of an increase in IRR do you expect?

(c) If the required investment has increased from \$20 million to \$22 million, but the expected future cash flows are projected to be 10% smaller than the original estimates, how much of a decrease in IRR do you expect?

Comparing Alternatives

7.32 Consider two investments A and B with the following sequences of cash flows:

n	Net Cash Flow Project A	Net Cash Flow Project B
0	−\$120,000	−\$100,000
1	20,000	15,000
2	20,000	15,000
3	120,000	130,000

(a) Compute the IRR for each investment.

(b) At MARR = 15%, determine the acceptability of each project.

(c) If A and B are mutually exclusive projects, which project would you select, based on the rate of return on incremental investment?

7.33 With \$10,000 available, you have two investment options. The first is to buy a certificate of deposit from a bank at an interest rate of 10% annually for five years. The second choice is to purchase a bond for \$10,000 and invest the bond's interest in the bank at an interest rate of 9%. The bond pays 10% interest annually and will mature to its face value of \$10,000 in five years. Which option is better? Assume that your MARR is 9% per year.

7.34 A manufacturing firm is considering the following mutually exclusive alternatives:

n	Net Cash Flow Project A1	Net Cash Flow Project A2
0	−\$2,000	−\$3,000
1	1,400	2,400
2	1,640	2,000

Determine which project is a better choice at a MARR = 15%, based on the IRR criterion.

7.35 Consider the following two mutually exclusive alternatives:

n	Net Cash Flow Project A1	Net Cash Flow Project A2
0	−\$10,000	−\$12,000
1	5,000	6,100
2	5,000	6,100
3	5,000	6,100

(a) Determine the IRR on the incremental investment in the amount of \$2,000.

(b) If the firm's MARR is 10%, which alternative is the better choice?

7.36 Consider the following two mutually exclusive investment alternatives:

	Net Cash Flow	
n	Project A1	Project A2
0	−\$15,000	−\$20,000
1	7,500	8,000
2	7,500	15,000
3	7,500	5,000
IRR	23.5%	20%

(a) Determine the IRR on the incremental investment in the amount of \$5,000. (Assume that MARR = 10%.)

(b) If the firm's MARR is 10%, which alternative is the better choice?

7.37 You are considering two types of automobiles. Model A costs \$18,000 and model B costs \$15,624. Although the two models are essentially the same, after four years of use model A can be sold for \$9,000, while model B can be sold for \$6,500. Model A commands a better resale value because its styling is popular among young college students. Determine the rate of return on the incremental investment of \$2,376. For what range of values of your MARR is model A preferable?

7.38 A plant engineer is considering two types of solar water heating system:

	Model A	Model B
Initial cost	\$7,000	\$10,000
Annual savings	\$700	\$1,000
Annual maintenance	\$100	\$50
Expected life	20 years	20 years
Salvage value	\$400	\$500

The firm's MARR is 12%. On the basis of the IRR criterion, which system is the better choice?

7.39 Consider the following investment projects:

	Net Cash Flow					
n	A	B	C	D	E	F
0	−\$100	−\$200	−\$4,000	−\$2,000	−\$2,000	−\$3,000
1	60	120	2,410	1,400	3,700	2,500
2	50	150	2,930	1,720	1,640	1,500
3	50					
i^*	28.89%	21.65%	21.86%	31.10%	121.95%	23.74%

Assume that MARR = 15%.

(a) Projects A and B are mutually exclusive. Assuming that both projects can be repeated for an indefinite period, which one would you select on the basis of the IRR criterion?

(b) Suppose projects C and D are mutually exclusive. According to the IRR criterion, which project would be selected?

(c) Suppose projects E and F are mutually exclusive. Which project is better according to the IRR criterion?

7.40 Fulton National Hospital is reviewing ways of cutting the cost of stocking medical supplies. Two new stockless systems are being considered, to lower the hospital's holding and handling costs. The hospital's industrial engineer has compiled the relevant financial data for each system as follows (dollar values are in millions):

	Current Practice	Just-in-Time System	Stockless Supply System
Start-up cost	\$0	\$2.5	\$5
Annual stock holding cost	\$3	\$1.4	\$0.2
Annual operating cost	\$2	\$1.5	\$1.2
System life	8 years	8 years	8 years

The system life of eight years represents the period that the contract with the medical suppliers is in force. If the hospital's MARR is 10%, which system is more economical?

7.41 Consider the cash flows for the following investment projects:

	Project Cash Flow				
n	A	B	C	D	E
0	−\$1,000	−\$1,000	−\$2,000	\$1,000	−\$1,200
1	900	600	900	−300	400
2	500	500	900	−300	400
3	100	500	900	−300	400
4	50	100	900	−300	400

Assume that the MARR = 12%.

(a) Suppose A, B, and C are mutually exclusive projects. Which project would be selected on the basis of the IRR criterion?

(b) What is the borrowing rate of return (BRR) for project D?

(c) Would you accept project D at MARR $= 20\%$?

(d) Assume that projects C and E are mutually exclusive. Using the IRR criterion, which project would you select?

7.42 Consider the following investment projects:

	Net Cash Flow		
n	Project 1	Project 2	Project 3
0	−$1,000	−$5,000	−$2,000
1	500	7,500	1,500
2	2,500	600	2,000

Assume that MARR $= 15\%$.

(a) Compute the IRR for each project.

(b) On the basis of the IRR criterion, if the three projects are mutually exclusive investments, which project should be selected?

7.43 Consider the following two investment alternatives:

	Net Cash Flow	
N	Project A	Project B
0	−$10,000	−$20,000
1	5,500	0
2	5,500	0
3	5,500	40,000
IRR	30%	?
PW(15%)	?	6300

The firm's MARR is known to be 15%.

(a) Compute the IRR of project B.

(b) Compute the NPW of project A.

(c) Suppose that projects A and B are mutually exclusive. Using the IRR, which project would you select?

7.44 The E. F. Fedele Company is considering acquiring an automatic screwing machine for its assembly operation of a personal computer. Three different models with varying automatic features are under consideration. The required investments are $360,000 for model A, $380,000 for model B, and $405,000 for model C. All three models are expected to have the same service life of eight years. The following financial information, in which model $(B - A)$ represents the incremental cash flow determined by subtracting model A's cash flow from model B's, is available:

Model	IRR (%)
A	30%
B	15
C	25
Model	**Incremental IRR (%)**
(B − A)	5%
(C − B)	40
(C − A)	15

If the firm's MARR is known to be 12%, which model should be selected?

7.45 The GeoStar Company, a leading manufacturer of wireless communication devices, is considering three cost-reduction proposals in its batch job-shop manufacturing operations. The company has already calculated rates of return for the three projects, along with some incremental rates of return:

Incremental Investment	Incremental Rate of Return (%)
$A_1 - A_0$	18%
$A_2 - A_0$	20
$A_3 - A_0$	25
$A_2 - A_1$	10
$A_3 - A_1$	18
$A_3 - A_2$	23

A_0 denotes the do-nothing alternative. The required investments are \$420,000 for A_1, \$550,000 for A_2, and \$720,000 for A_3. If the MARR is 15%, what system should be selected?

7.46 A manufacturer of electronic circuit boards is considering six mutually exclusive cost-reduction projects for its PC-board manufacturing plant. All have lives of 10 years and zero salvage values. The required investment and the estimated after-tax reduction in annual disbursements for each alternative are as follows, along with computed rates of return on incremental investments:

Proposal	Required	After-Tax	Rate of
A_j	Investment	Savings	Return (%)
A_1	\$60,000	\$22,000	35.0%
A_2	100,000	28,200	25.2
A_3	110,000	32,600	27.0
A_4	120,000	33,600	25.0
A_5	140,000	38,400	24.0
A_6	150,000	42,200	25.1

Incremental Investment	Incremental Rate of Return (%)
$A_2 - A_1$	9.0%
$A_3 - A_2$	42.8
$A_4 - A_3$	0.0
$A_5 - A_4$	20.2
$A_6 - A_5$	36.3

If the MARR is 15%, which project would you select, based on the rate of return on incremental investment?

7.47 Baby Doll Shop manufactures wooden parts for dollhouses. The worker is paid $8.10 an hour and, using a handsaw, can produce a year's required production (1,600 parts) in just eight 40-hour weeks. That is, the worker averages five parts per hour when working by hand. The shop is considering purchasing of a power band saw with associated fixtures, to improve the productivity of this operation. Three models of power saw could be purchased: Model A (the economy version), model B (the high-powered version), and model C (the deluxe high-end version). The major operating difference between these models is their speed of operation. The investment costs, including the required fixtures and other operating characteristics, are summarized as follows:

Category	By Hand	Model A	Model B	Model C
Production rate (parts/hour)	5	10	15	20
Labor hours required (hours/year)	320	160	107	80
Annual labor cost (@ $8.10/hour)	2,592	1,296	867	648
Annual power cost ($)		400	420	480
Initial investment ($)		4,000	6,000	7,000
Salvage value ($)		400	600	700
Service life (years)		20	20	20

Assume that MARR = 10%. Are there enough savings to purchase any of the power band saws? Which model is most economical, based on the rate-of-return principle? (Assume that any effect of income tax has been already considered in the dollar estimates.) (*Source*: This problem is adapted with the permission of Professor Peter Jackson of Cornell University.)

Unequal Service Lives

7.48 Consider the following two mutually exclusive investment projects for which MARR = 15%:

	Net Cash Flow	
n	Project A	Project B
0	−\$100	−\$200
1	60	120
2	50	150
3	50	
IRR	28.89%	21.65%

On the basis of the IRR criterion, which project would be selected under an infinite planning horizon with project repeatability likely?

7.49 Consider the following two mutually exclusive investment projects:

	Net Cash Flow	
n	Project A1	Project A2
0	−\$10,000	−\$15,000
1	5,000	20,000
2	5,000	
3	5,000	

(a) To use the IRR criterion, what assumption must be made in comparing a set of mutually exclusive investments with unequal service lives?

(b) With the assumption defined in part (a), determine the range of MARRs which will indicate that project A1 should be selected.

Short Case Studies

ST7.1 Critics have charged that, in carrying out an economic analysis, the commercial nuclear power industry does not consider the cost of decommissioning, or "mothballing," a nuclear power plant and that the analysis is therefore unduly optimistic. As an example, consider the Tennessee Valley Authority's Bellefont twin nuclear generating facility under construction at Scottsboro, in northern Alabama: The initial cost is \$1.5 billion (present worth at the start of operations), the estimated life is 40 years, the annual operating and maintenance costs the first year are assumed to be 4.6% of the initial cost and are expected to increase at the fixed rate of 0.05%

of the initial cost each year, and annual revenues are estimated to be three times the annual operating and maintenance costs throughout the life of the plant.

(a) The criticism that the economic analysis is overoptimistic because it omits "mothballing" costs is not justified, since the addition of a cost of 50% of the initial cost to "mothball" the plant decreases the 10% rate of return only to approximately 9.9%.

(b) If the estimated life of the plants is more realistically taken to be 25 years instead of 40 years, then the criticism is justified. By reducing the life to 25 years, the rate of return of approximately 9% without a "mothballing" cost drops to approximately 7.7% when a cost to "mothball" the plant equal to 50% of the initial cost is added to the analysis.

Comment on these statements.

ST7.2 The B&E Cooling Technology Company, a maker of automobile air-conditioners, faces an uncertain, but impending, deadline to phase out the traditional chilling technique, which uses chlorofluorocarbons (CFCs), a family of refrigerant chemicals believed to attack the earth's protective ozone layer. B&E has been pursuing other means of cooling and refrigeration. As a near-term solution, the engineers recommend a cold technology known as absorption chiller, which uses plain water as a refrigerant and semiconductors that cool down when charged with electricity. B&E is considering two options:

- **Option 1.** Retrofit the plant now to adapt the absorption chiller and continue to be a market leader in cooling technology. Because of untested technology on a large scale, it may cost more to operate the new facility while personnel are learning the new system.
- **Option 2.** Defer the retrofitting until the federal deadline, which is three years away. With expected improvement in cooling technology and technical know-how, the retrofitting cost will be cheaper, but there will be tough market competition, and the revenue would be less than that of Option 1.

The financial data for the two options are as follows:

	Option 1	Option 2
Investment timing	Now	3 years from now
Initial investment	\$6 million	\$5 million
System life	8 years	8 years
Salvage value	\$1 million	\$2 million
Annual revenue	\$15 million	\$11million
Annual O&M costs	\$6 million	\$7 million

(a) What assumptions must be made to compare these two options?

(b) If B&E's MARR is 15%, which option is the better choice, based on the IRR criterion?

ST7.3 An oil company is considering changing the size of a small pump that is currently operational in wells in an oil field. If this pump is kept, it will extract 50% of the known crude-oil reserve in the first year of its operation and the remaining 50% in the second year. A pump larger than the current pump will cost \$1.6 million, but it will extract 100% of the known reserve in the first year. The total oil revenues over the two years are the same for both pumps, namely, \$20 million. The advantage of the large pump is that it allows 50% of the revenues to be realized a year earlier than with the small pump.

	Current Pump	Larger Pump
Investment, year 0	0	\$1.6 million
Revenue, year 1	\$10 million	\$20 million
Revenue, year 2	\$10 million	0

If the firm's MARR is known to be 20%, what do you recommend, based on the IRR criterion?

ST7.4 You have been asked by the president of the company you work for to evaluate the proposed acquisition of a new injection molding machine for the firm's manufacturing plant. Two types of injection molding machines have been identified, with the following estimated cash flows:

	Net Cash Flow	
n	Project 1	Project 2
0	−\$30,000	−\$40,000
1	20,000	43,000
2	18,200	5,000
IRR	18.1%	18.1%

You return to your office, quickly retrieve your old engineering economics text, and then begin to smile: Aha—this is a classic rate-of-return problem! Now, using a calculator, you find out that both projects have about the same rate of return: 18.1%. This figure seems to be high enough to justify accepting the project, but you recall that the ultimate justification should be done with reference to the firm's MARR. You call the accounting department to find out the current MARR the firm should use in justifying a project. "Oh boy, I wish I could tell you, but my boss will be back next week, and he can tell you what to use," says the accounting clerk.

A fellow engineer approaches you and says, "I couldn't help overhearing you talking to the clerk. I think I can help you. You see, both projects have the same IRR, and on top of that, project 1 requires less investment, but returns more cash flows ($-\$30{,}000 + \$20{,}000 + \$18{,}200 = \$8{,}200$, and $-\$40{,}000 + \$43{,}000 + \$5{,}000 = \$8{,}000$); thus, project 1 dominates project 2. For this type of decision problem, you don't need to know a MARR!"

(a) Comment on your fellow engineer's statement.

(b) At what range of MARRs would you recommend the selection of project 2?