CHAPTER 1

MAKING SENSE OF DATA AND FUNCTIONS

OVERVIEW

How can you describe patterns in data? In this chapter we explore how to use graphs to visualize the shape of single-variable data and to show changes in two-variable data. Functions, a fundamental concept in mathematics, are introduced and used to model change.

After reading this chapter, you should be able to

- describe patterns in single- and two-variable data
- construct a "60-second summary"
- define a function and represent it in multiple ways
- identify properties of functions
- use the language of functions to describe and create graphs

1.1 *Describing Single-Variable Data*

This course starts with you. How would you describe yourself to others? Are you a 5-foot 6-inch, black, 26-year-old female studying biology? Or perhaps you are a 5-foot 10-inch, Chinese, 18-year-old male English major. In statistical terms, characteristics such as height, race, age, and major that vary from person to person are called *variables*. Information collected about a variable is called *data*.[1]

Some variables, such as age, height, or number of people in your household, can be represented by a number and a unit of measure (such as 18 years, 6 feet, or 3 people). These are called *quantitative variables*. For other variables, such as gender or college major, we use categories (such as male and female or biology and English) to classify information. These are called *categorical* (or *qualitative*) data. The dividing line between classifying a variable as categorical or quantitative is not always clear-cut. For example, you could ask individuals to list their years of education (making education a quantitative variable) or ask for their highest educational category, such as college or graduate school (making education a categorical variable).

Exploration 1.1 provides an opportunity to collect your own data and to think about issues related to classifying and interpreting data.

Many of the controversies in the social sciences have centered on how particular variables are defined and measured. For nearly two centuries, the categories used by the U.S. Census Bureau to classify race and ethnicity have been the subject of debate. For example, Hispanic used to be considered a racial classification. It is now considered an ethnic classification, since Hispanics can be black, or white, or any other race.

Visualizing Single-Variable Data

Humans are visual creatures. Converting data to an image can make it much easier to recognize patterns.

Bar charts: How well educated are Americans?

Categorical data are usually displayed with a bar chart. Typically the categories are listed on the horizontal axis. The height of the bar above a single category tells you either the *frequency count* (the number of observations that fall into that category) or the *relative frequency* (the percentage of total observations). Since the relative size of the bars is the same using either frequency or relative frequency counts, we often put the two scales on different vertical axes of the same chart. For example, look at the vertical scales on the left- and right-hand sides of Figure 1.1, a bar chart of the educational attainment of Americans age 25 or older in 2004.

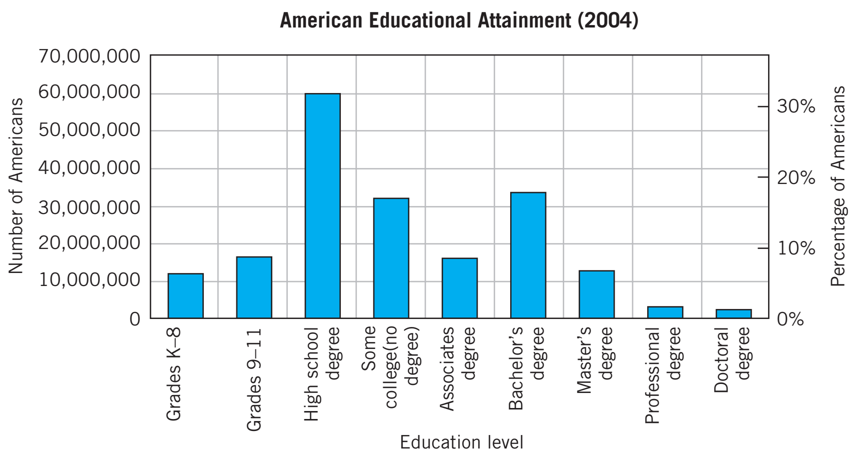


Figure 1.1 Bar chart showing the education levels for Americans age 25 or older.
Source: U.S. Bureau of the Census, *www.census.gov.*

[1]*Data* is the plural of the Latin word *datum* (meaning "something given")—hence one datum, two data.

The vertical scale on the left tells us the number (the frequency count) of Americans who fell into each educational category. For example, in 2004 approximately 60 million Americans age 25 or older had a high school degree but never went on to college.

It's often more useful to know the percentage (the relative frequency) of all Americans who have only a high school degree. Given that in 2004 the number of people 25 years or older was approximately 186,877,000 and the number who had only a high school degree was approximately 59,810,000, then the percentage of those with only a high school degree was

$$\frac{\text{Number with only a high school degree}}{\text{Total number of people age 25 or older}} = \frac{59{,}810{,}000}{186{,}877{,}000}$$

$$\approx 0.32 \text{ (in decimal form) or } 32\%$$

The vertical scale on the right tells us the percentage (relative frequency). Using this scale, the percentage of Americans with only a high school degree was about 32%, which is consistent with our calculation.

EXAMPLE 1 **What does the bar chart tell us?**

a. Using Figure 1.1, estimate the number and percentage of people age 25 or older who have bachelor's degrees, but no further advanced education.

b. Estimate the total number of people and the percentage of the total population age 25 or older who have at least a high school education.

c. What doesn't the bar chart tell us?

d. Write a brief summary of educational attainment in the United States.

SOLUTION

a. Those with bachelor's degrees but no further education number about 34 million, or 18%.

b. Those who have completed a high school education include everyone with a high school degree up to a Ph.D. We could add up all the numbers (or percentages) for each of those seven categories. But it's easier to subtract from the whole those who do not meet the conditions, that is, subtract those with either a grade school or only some high school education from the total population (people age 25 or older) of about 187 million.

	Grade School + Some High School	= Total without High School Degree
Number (approx.)	12 million + 16 million	= 28 million
Percentage (approx.)	6% + 9%	= 15%

The number of Americans (age 25 or over) with a high school degree is about 187 million − 28 million = 159 million. The corresponding percentage is about 100% − 15% = 85%. So more than four out of five Americans 25 years or older have completed high school.

c. The bar chart does not tell us the total size of the population or the total number (or percentage) of Americans who have a high school degree. For example, if we include younger Americans between age 18 and age 25, we would expect the percentage with a high school degree to be higher.

d. About 85% of adult Americans (age 25 or older) have at least a high school education. The breakdown for the 85% includes 32% who completed high school but did not go on, 43% who have some college (up to a bachelor's degree), and about 10% who have graduate degrees. This is not surprising, since the United States population ranks among the mostly highly educated in the world.

An important aside: What a good graph should contain

When you encounter a graph in an article or you produce one for a class, there are three elements that should always be present:

1. An informative title that succinctly describes the graph
2. Clearly labeled axes (or a legend) including the units of measurement (e.g., indicating whether age is measured in months or years)
3. The source of the data cited in the data table, in the text, or on the graph

Histograms: What is the distribution of ages in the U.S. population?

See the program "F1: Histograms."

A histogram is a specialized form of a bar chart that is used to visualize single-variable quantitative data. Typically, the horizontal axis on a histogram is a subset of the real numbers with the unit (representing, for example, number of years) and the size of each interval marked. The intervals are usually evenly spaced to facilitate comparisons (e.g., placed every 10 years). The size of the interval can reveal or obscure patterns in the data. As with a bar chart, the vertical axis can be labeled with a frequency or a relative frequency count. For example, the histogram in Figure 1.2 shows the distribution of ages in the United States in 2005.

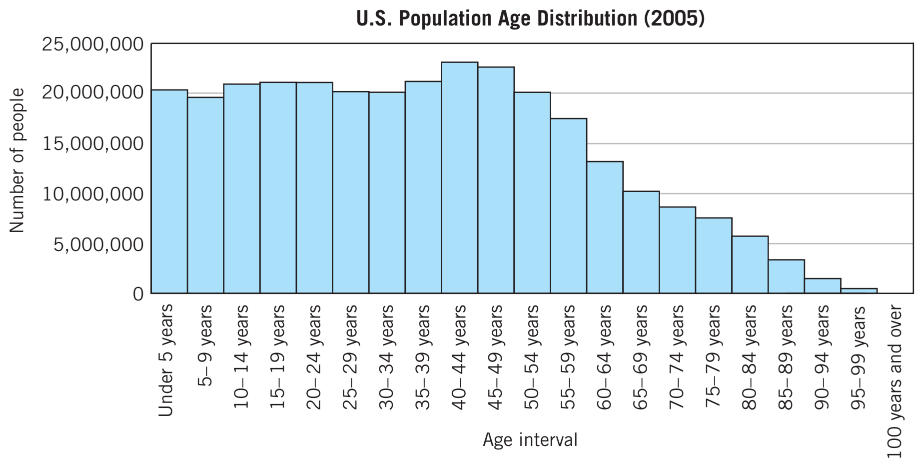


Figure 1.2 Age distribution of the U.S. population in 5-year intervals.
Source: U.S. Bureau of the Census, *www.census.gov*.

EXAMPLE 2 **What does the histogram tell us?**

a. What 5-year age interval contains the most Americans? Roughly how many are in that interval? (Refer to Figure 1.2.)

b. Estimate the number of people under age 20.

c. Construct a topic sentence for a report about the U.S. population.

SOLUTION

a. The interval from 40 to 44 years contains the largest number of Americans, about 23 million.

b. The sum of the frequency counts for the four intervals below age 20 is about 80 million.

c. According to the U.S. Census Bureau 2005 data, the number of Americans in each 5-year age interval remained fairly flat up to age 40, peaked between ages 40 to 50, then fell in a gradual decline.

EXAMPLE 3 Describe the age distribution for Tanzania, one of the poorest countries in the world (Figure 1.3).

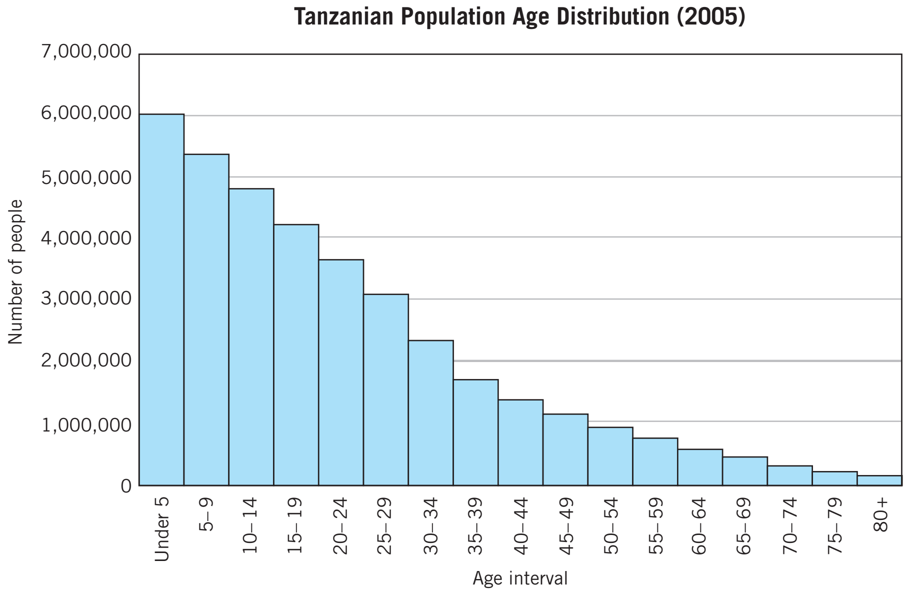


Figure 1.3 The age distribution in 2005 of the Tanzanian population in 5-year intervals.
Source: U.S. Bureau of the Census, International Data Base, April 2005.

SOLUTION The age distributions in Tanzania and the United States are quite different. Tanzania is a much smaller country and has a profile typical of a developing country; that is, each subsequent 5-year interval has fewer people. For example, there are about 6 million children 0 to 4 years old, but only about 5.3 million children age 5–9 years, a drop of over 10%. For ages 35 to 39 years, there are only about 1.7 million people, less than a third of the number of children between 0 and 4 years. Although the histogram gives a static picture of the Tanzanian population, the shape suggests that mortality rates are much higher than in the United States.

SOMETHING TO THINK ABOUT

What are some trade-offs in using pie charts versus histograms?

Pie charts: Who gets the biggest piece?

Both histograms and bar charts can be transformed into pie charts. For example, Figure 1.4 shows two pie charts of the U.S. and Tanzanian age distributions (both now divided into 20-year intervals). One advantage of using a pie chart is that it clearly shows the size of each piece relative to the whole. Hence, they are usually labeled with percentages rather than frequency counts.

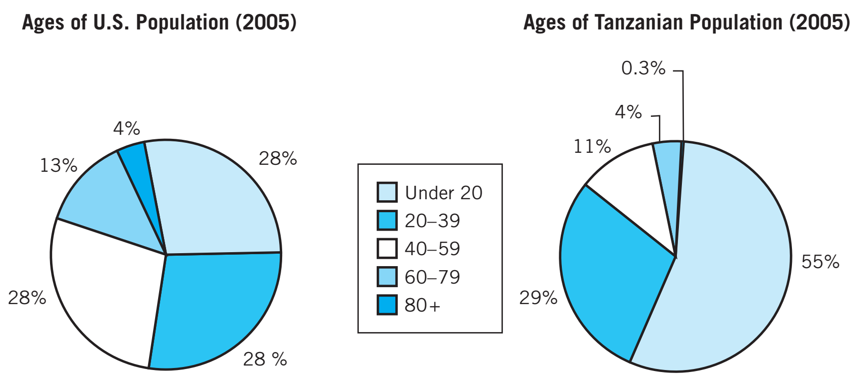


Figure 1.4 Two pie charts displaying information about the U.S. and Tanzanian age distributions.
Source: U.S. Bureau of the Census, *www.census.gov*.

In the United States the first three 20-year age intervals (under 20, 20–39, and 40–59 years) are all approximately equal in size and together make up about 84% of the population. Those 60 and older represent 17% of the population. Note that the percentages add up to more than 100% due to rounding.

In Tanzania, the proportions are entirely different. Over half of the population are under 20 years and more than 80% are under 40 years old. Those 60 and older make up less than 5% of Tanzania's population.

Mean and Median: What Is "Average" Anyway?

In 2005 the U.S. Bureau of the Census reported that the mean age for Americans was 37.2 and the median age was 36.7 years.

The Mean and Median

The *mean* is the sum of a list of numbers divided by the number of terms in the list.

The *median* is the middle value of an ordered numerical list; half the numbers lie at or below the median and half at or above it.

The mean age of 37.2 represents the sum of the ages of every American divided by the total number of Americans. The median age of 36.7 means that if you placed all the ages in order, 36.7 would lie right in the middle; that is, half of Americans are younger than or equal to 36.7 and half are 36.7 or older.

In the press you will most likely encounter the word "average" rather than the term "mean" or "median."[2] The term "average" is used very loosely. It usually represents the mean, but it could also represent the median or something much more vague, such as the "average" American household. For example, the media reported that:

- The *average* American home now has more television sets than people. . . . There are 2.73 TV sets in the typical home and 2.55 people.[3]
- The *average* American family now owes more than $9,000 in credit debt. . . . and is averaging about seven cards.[4]

SOMETHING TO THINK ABOUT

If someone tells you that in his town "all of the children are above average," you might be skeptical. (This is called the "Lake Wobegon effect.") But could most (more than half) of the children be above average? Explain.

The significance of the mean and median

The median divides the number of entries in a data set into two equal halves. If the median age in a large urban housing project is 17, then half the population is 17 or under. Hence, issues such as day care, recreation, and education should be high priorities with the management. If the median age is 55, then issues such as health care and wheelchair accessibility might dominate the management's concerns.

The median is unchanged by changes in values above and below it. For example, as long as the median income is larger than the poverty level, it will remain the same even if all poor people suddenly increase their incomes up to that level and everyone else's income remains the same.

See the reading "The Median Isn't the Message" to find out how an understanding of the median gave renewed hope to the renowned scientist Stephen J. Gould when he was diagnosed with cancer.

The mean is the most commonly cited statistic in the news media. One advantage of the mean is that it can be used for calculations relating to the whole data set. Suppose a corporation wants to open a new factory similar to its other factories. If the managers know the mean cost of wages and benefits for employees,

[2]The word "average" has an interesting derivation according to Klein's etymological dictionary. It comes from the Arabic word *awariyan,* which means "merchandise damaged by seawater." The idea being debated was that if your ships arrived with water-damaged merchandise, should you have to bear all the losses yourself or should they be spread around, or "averaged," among all the other merchants? The words *averia* in Spanish, *avaria* in Italian, and *avarie* in French still mean "damage."

[3]*Source:* USA Today, *www.usatoday.com/life/television/news/2006.*

[4]*Source:* Newsweek, *www.msnbc.msn.com/id/14366431/site/newsweek/.*

they can make an estimate of what it will cost to employ the number of workers needed to run the new factory:

$$\text{total employee cost} = (\text{mean cost for employees}) \cdot (\text{number of employees})$$

The mean, unlike the median, can be affected by a few extreme values called *outliers*. For example, suppose Bill Gates, founder of Microsoft and the richest man in the world, were to move into a town of 10,000 people, all of whom earned nothing. The median income would be $0, but the mean income would be in the millions. That's why income studies usually use the median.

EXAMPLE 4 **"Million-dollar Manhattan apartment? Just about average"**

According to a report cited on *money.cnn.com*, in 2006 the median price of purchasing an apartment in Manhattan was $880,000 and the mean price was $1.4 million. How could there be such a difference in price? Which value do you think better represents apartment prices in Manhattan?

SOLUTION Apartments that sold for exorbitant prices (in the millions) could raise the mean above the median. If you want to buy an apartment in Manhattan, the median price is probably more important because it tells you that half of the apartments cost $880,000 or less.

An Introduction to Algebra Aerobics

In each section of the text there are "Algebra Aerobics" with answers in the back of the book. They are intended to give you practice in the algebraic skills introduced in the section and to review skills we assume you have learned in other courses. These skills should provide a good foundation for doing the exercises at the end of each chapter. The exercises include more complex and challenging problems and have answers for only the odd-numbered ones. We recommend you work out these Algebra Aerobics practice problems and then check your solutions in the back of this book. The Algebra Aerobics are numbered according to the section of the book in which they occur.

Algebra Aerobics 1.1

1. Fill in Table 1.1. Round decimals to the nearest thousandth.

Fraction	Decimal	Percent
$\frac{7}{12}$		
	0.025	
		2%
$\frac{1}{200}$		
	0.35	
		0.8%

Table 1.1

2. Calculate the following:

a. A survey reported that 80 people, or 16% of the group, were smokers. How many people were surveyed?

b. Of the 236 students who took a test, 16.5% received a B grade. How many students received a B grade?

c. Six of the 16 people present were from foreign countries. What percent were foreigners?

3. When looking through the classified ads, you found that 16 jobs had a starting salary of $20,000, 8 had a starting salary of $32,000, and 1 had a starting salary of $50,000. Find the mean and median starting salary for these jobs.[5]

[5]Recall that given the list of numbers 9, 2, −2, 6, 5, the *mean* = the sum 9 + 2 + (−2) + 6 + 5 divided by 5 (the number of items in the list) = 20/5 = 4; the *median* = the middle number of the list in ascending order −2, 2, 5, 6, 9, which is 5. If the list had an even number of elements—for example, −2, 2, 5, 6—the median would be the mean of the middle two numbers on the ordered list, in this case (2 + 5)/2 = 7/2 = 3.5.

4. Find the mean and median grade point average (GPA) from the data given in Table 1.2.

GPA	Frequency Count
1.0	56
2.0	102
3.0	46
4.0	12

Table 1.2

5. Figure 1.5 presents information about the Hispanic population in the United States from 2000 to 2005.

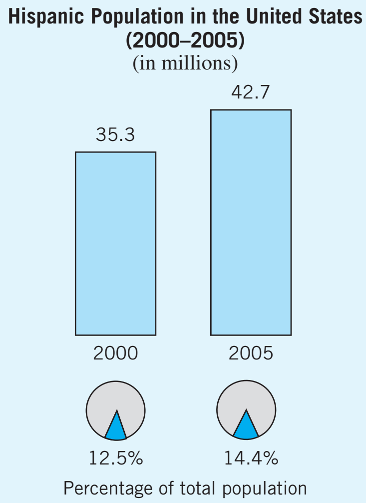


Figure 1.5 Change in the Hispanic population in the United States.
Source: U.S. Bureau of the Census, *www.census.gov*.

a. What does the bar chart tell you that the pie chart does not?

b. Using the bar and pie charts, what was the U.S. population in 2000? In 2005?

6. a. Fill in Table 1.3. Round your answers to the nearest whole number.

Age	Frequency Count	Relative Frequency (%)
1–20		38
21–40	35	
41–60	28	
61–80		
Total	137	

Table 1.3

b. Calculate the percentage of the population who are over 40 years old.

7. Use Table 1.3 to create a histogram and pie chart.

8. From the histogram in Figure 1.6, create a frequency distribution table. Assume that the total number of people represented by the histogram is 1352. (*Hint:* Estimate the relative frequencies from the graph and then calculate the frequency count in each interval.)

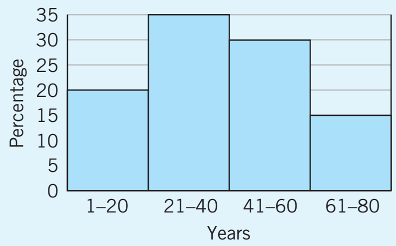


Figure 1.6 Distribution of ages (in years).

9. Calculate the mean and median for the following data:

a. \$475, \$250, \$300, \$450, \$275, \$300, \$6000, \$400, \$300

b. 0.4, 0.3, 0.3, 0.7, 1.2, 0.5, 0.9, 0.4

10. Explain why the mean may be a misleading numerical summary of the data in Problem 9(a).

Exercises for Section 1.1

1. Internet use as reported by teenagers in 2006 in the United States is shown in the accompanying graph.

a. What percentage of 13- to 17-year-old females spend at least 3 hours per day on the internet outside of school?

b. What percentage of 13- to 17-year-old males spend at least 3 hours per day on the internet outside of school?

c. What additional information would you need in order to find out the percentage of 13 to 17 year olds who spend at least 3 hours per day on the internet?

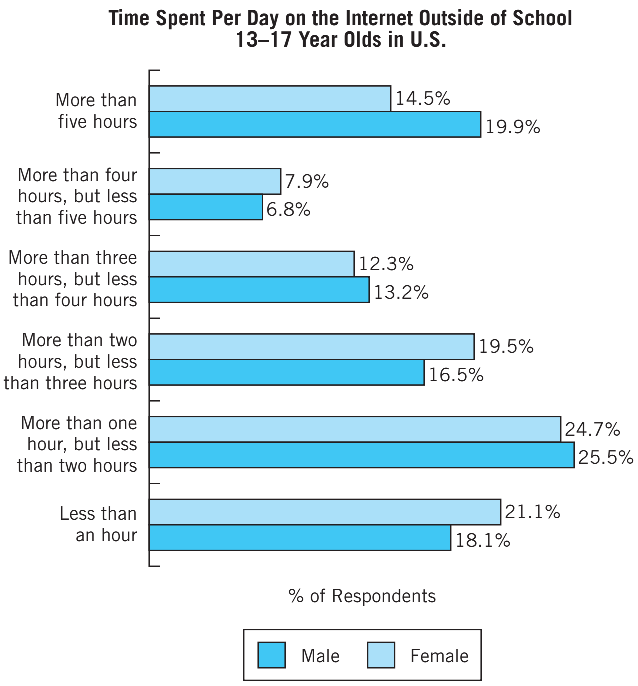
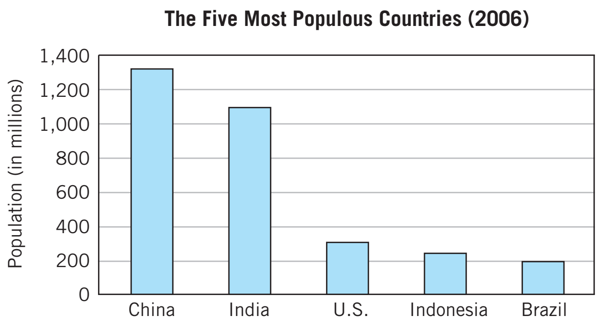


Chart 1 – Time Spent Online Outside of School
Source: BURST Research, May 2006

2. The accompanying bar chart shows the five countries with the largest populations in 2006.



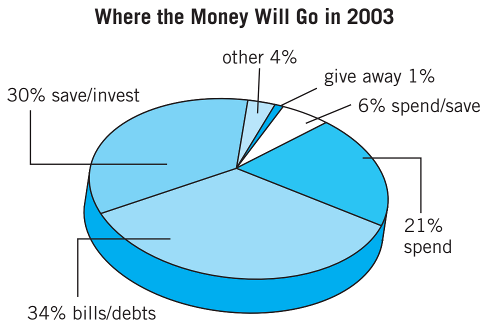
Source: CIA Factbook, *www.cia.gov/cia.*

a. What country has the largest population, and approximately what is its population, size?

b. The population of India is projected in the near future to exceed the population of China. Given the current data, what is the minimum number of additional persons needed to make India's population larger than China's?

c. The world population in 2006 was estimated to be about 6.5 billion. Approximately what percentage of the world's population live in China? In India? In the United States?

3. In 2003 some taxpayers received $300–600 tax rebates. Congress approved this spending as a means to stimulate the economy. According to a May 2003 ABC News/*Washington Post* poll, the accompanying pie chart shows how people would use the money.



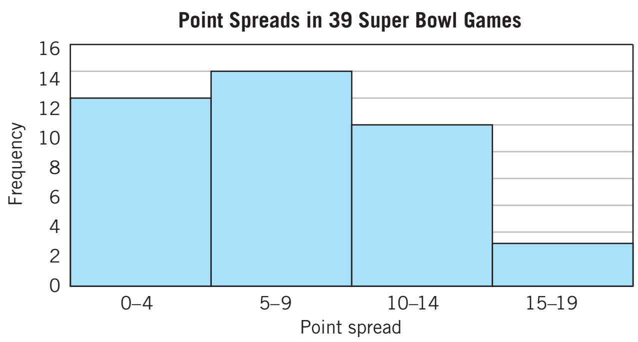
Source: ABCNEWS/*Washington Post* poll. May, 2003.

a. What is the largest category on which people say they will spend their rebates? Why does the category look so much larger than its actual relative size?

b. What might make you suspicious about the numbers in this pie chart?

4. The point spread in a football game is the difference between the winning team's score and the losing team's score. For example, in the 2004 Super Bowl game, the Patriots won with 32 points versus the Carolina Panthers' 29 points. So the point spread was 3 points.

a. In the accompanying bar chart, what is the interval with the most likely point spread in a Super Bowl? The least likely?



Source: www.docsports.com/point-spreads-for-every-super-bowl.html.

b. What percentage of these Super Bowl games had a point spread of 9 or less? Of 14 or less?

5. Given here is a table of salaries taken from a survey of recent graduates (with bachelor degrees) from a well-known university in Pittsburgh.

Salary (in thousands)	Number of Graduates Receiving Salary
21–25	2
26–30	3
31–35	10
36–40	20
41–45	9
46–50	1

a. How many graduates were surveyed?

b. Is this quantitative or qualitative data? Explain.

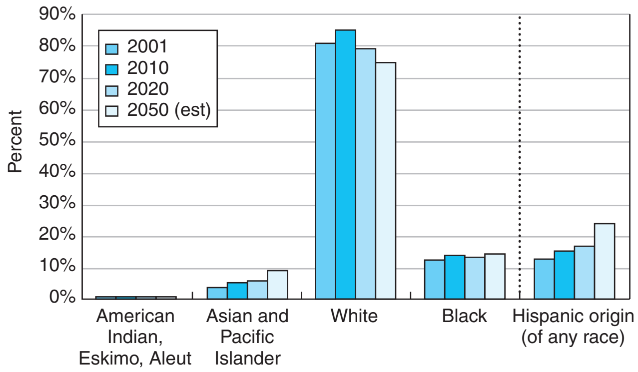
5. (continued)

c. What is the relative frequency of people having a salary between $26,000 and $30,000?

d. Create a histogram of the data.

6. The accompanying bar chart shows the predictions of the U.S. Census Bureau about the future racial composition of American society. Hispanic origin may be of any race, so the other categories may include people of Hispanic origin.

Percentage of U.S. Population by Race and Ethnicity (2001–2050 est)



Sources: U.S. Bureau of the Census, *Statistical Abstract of the United States: 2002.*

a. Estimate the following percentages:

i. Asian and Pacific Islanders in the year 2050

ii. Combined white and black population in the year 2020

iii. Non-Hispanic population in the year 2001

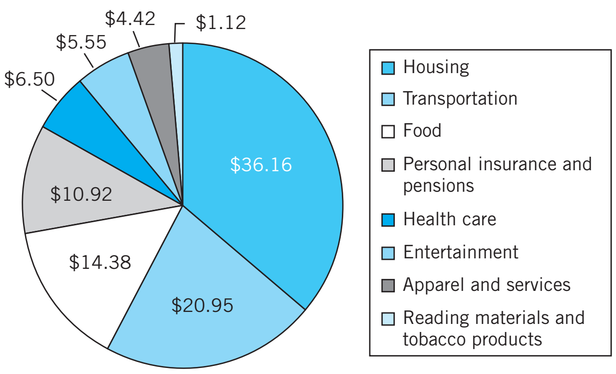
b. The U.S. Bureau of the Census has projected that there will be approximately 392,031,000 people in the United States in the year 2050. Approximately how many people will be of Hispanic origin in the year 2050?

c. Write a topic sentence describing the overall trend.

7. Shown is a pie chart of America's spending patterns at the end of 2006.

How Americans Spend Their Money

How the average American spends $100, as measured in late 2006. The Labor Department uses this survey—along with price samples—to calculate the inflation rate.



Sources: U.S. Bureau of the Census, *Statistical Abstract of the United States: 2006.*

a. In what single category did Americans spend the largest percentage of their income? Estimate this percentage.

b. According to this chart, if an American family has an income of $35,000, how much of it would be spent on food?

c. If you were to write a newspaper article to accompany this pie chart, what would your opening topic sentence be?

8. Attendance at a stadium for the last 30 games of a college baseball team is listed as follows:

5072	3582	2504	4834	2456	3956
2341	2478	3602	5435	3903	4535
1980	1784	1493	3674	4593	5108
1376	978	2035	1239	2456	5189
3654	3845	673	2745	3768	5227

Create a histogram to display these data. Decide how large the intervals should be to illustrate the data well without being overly detailed.

9. a. Compute the mean and median for the list: 5, 18, 22, 46, 80, 105, 110.

b. Change one of the entries in the list in part (a) so that the median stays the same but the mean increases.

10. Suppose that a church congregation has 100 members, each of whom donates 10% of his or her income to the church. The church collected $250,000 last year from its members.

a. What was the mean contribution of its members?

b. What was the mean income of its members?

c. Can you predict the median income of its members? Explain your answer.

11. Suppose that annual salaries in a certain corporation are as follows:

Level I (30 employees)	$18,000
Level II (8 employees)	$36,000
Level III (2 employees)	$80,000

Find the mean and median annual salary. Suppose that an advertisement is placed in the newspaper giving the average annual salary of employees in this corporation as a way to attract applicants. Why would this be a misleading indicator of salary expectations?

12. Suppose the grades on your first four exams were 78%, 92%, 60%, and 85%. What would be the lowest possible average that your last two exams could have so that your grade in the class, based on the average of the six exams, is at least 82%?

13. Read Stephen Jay Gould's article "The Median Isn't the Message" and explain how an understanding of statistics brought hope to a cancer victim.

14. a. On the first quiz (worth 25 points) given in a section of college algebra, one person received a score of 16, two people got 18, one got 21, three got 22, one got 23, and one got 25. What were the mean and median of the quiz scores for this group of students?

14. (continued)

b. On the second quiz (again worth 25 points), the scores for eight students were 16, 17, 18, 20, 22, 23, 25, and 25.

i. If the mean of the scores for the nine students was 21, then what was the missing score?

ii. If the median of the scores was 22, then what are possible scores for the missing ninth student?

15. Why is the mean age larger than the median age in the United States? What prediction would you make for your State? What predictions would you make for other countries? Check your predictions with data from the U.S. Census Bureau at *www.census.gov*.

16. Up to and including George W. Bush, the ages of the last 15 presidents when they first took office[6] were 56, 55, 51, 54, 51, 60, 62, 43, 55, 56, 52, 69, 64, 46, 54.

a. Find the mean and median ages of the past 15 presidents when they took office.

b. If the mean age of the past 16 presidents is 54.94, at what age did the missing president take office?

c. Beginning with age 40 and using 5-year intervals, find the frequency count for each age interval.

d. Create a frequency histogram using your results from part (c).

17. Herb Caen, a Pulitzer Prize–winning columnist for the *San Francisco Chronicle,* remarked that a person moving from state A to state B could raise the average IQ in both states. Is he right? Explain.

18. Why do you think most researchers use median rather than mean income when studying "typical" households?

19. According to the 2000 U.S. Census, the median net worth of American families was \$55,000 and the mean net worth was \$282,500. How could there be such a wide discrepancy?

20. Read the *CHANCE News* article and explain why the author was concerned.

21. The Greek letter Σ (called sigma) is used to represent the sum of all of the terms of a certain group. Thus, $a_1 + a_2 + a_3 + \cdots + a_n$ can be written as

$$\sum_{i=1}^{n} a_i$$

which means to add together all of the values of a_i from a_1 to a_n.

a. Using Σ notation, write an algebraic expression for the mean of the five numbers x_1, x_2, x_3, x_4, x_5.

b. Using Σ notation, write an algebraic expression for the mean of the n numbers $t_1, t_2, t_3, \ldots, t_n$.

c. Evaluate the following sum:

$$\sum_{k=1}^{5} 2k$$

[6] *http://www.campvishus.org/PresAgeDadLeft.htm#AgeOffice.*

22. The accompanying table gives the ages of students in a mathematics class.

Ages of Students

Age Interval	Frequency Count
15–19	2
20–24	8
25–29	4
30–34	3
35–39	2
40–44	1
45–49	1
Total	21

a. Use this information to estimate the mean age of the students in the class. Show your work. (*Hint:* Use the mean age of each interval.)

b. What is the largest value the actual mean could have? The smallest? Why?

23. (Use of calculator or other technology recommended.) Use the following table to generate an estimate of the mean age of the U.S. population. Show your work. (*Hint:* Replace each age interval with an age approximately in the middle of the interval.)

Ages of U.S. Population in 2004

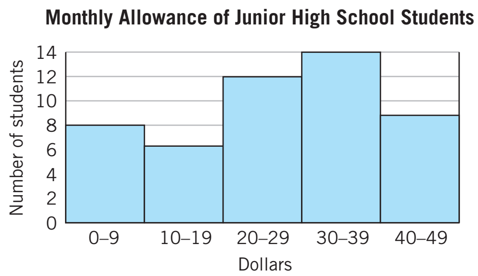
DATA USPOPAGE

Age (years)	Population (thousands)
Under 10	39,677
10–19	41,875
20–29	40,532
30–39	41,532
40–49	45,179
50–59	35,986
60–74	31,052
75–84	12,971
85 and over	4,860
Total	293,655

Source: U.S. Bureau of the Census, *Statistical Abstract of the United States: 2006.*

24. An article titled "Venerable Elders" (*The Economist,* July 24, 1999) reported that "both Democratic and Republican images are selective snapshots of a reality in which the median net worth of households headed by Americans aged 65 or over is around double the national average—but in which a tenth of such households are also living in poverty." What additional statistics would be useful in forming an opinion on whether elderly Americans are wealthy or poor compared with Americans as a whole?

25. Estimate the mean and median from the given histogram. (See hint in Exercise 23.) The program "F4: Measures of Central Tendency" in *FAM1000 Census Graphs* can help you understand the mean and median and their relationship to histograms.

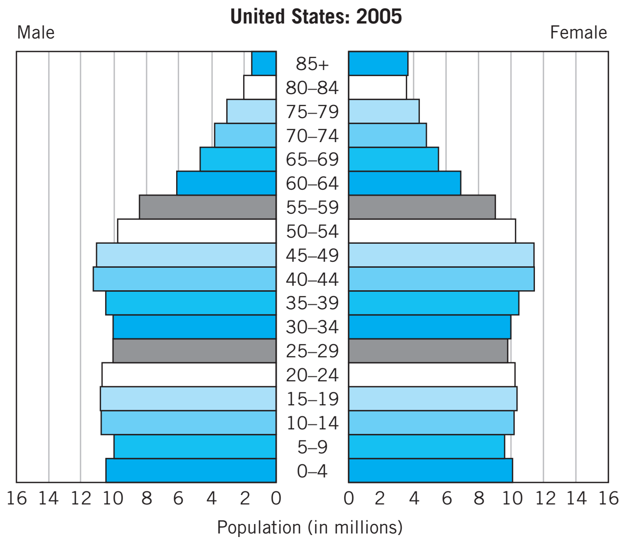


26. Choose a paragraph of text from any source and construct a histogram of word lengths (the number of letters in the word). If the same word appears more than once, count it as many times as it appears. You will have to make some reasonable decisions about what to do with numbers, abbreviations, and contractions. Compute the mean and median word lengths from your graph. Indicate how you would expect the graph to be different if you used:

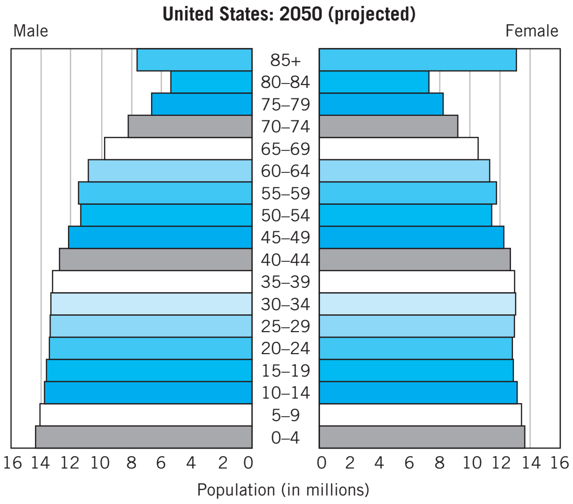
a. A children's book
b. A work of literature
c. A medical textbook

27. (Computer and course software required.) Open up the program "F1: Histograms" in *FAM1000 Census Graphs* in the course software. The 2006 U.S. Census data on 1000 randomly selected U.S. individuals and their families are imbedded in this program. You can use it to create histograms for education, age, and different measures of income. Try using different interval sizes to see what patterns emerge. Decide on one variable (say education) and compare the histograms of this variable for different groups of people. For example, you could compare education histograms for men and women or for people living in two different regions of the country. Pick a comparison that you think is interesting. Create a possible headline for these data. Describe three key features that support your headline.

28. Population pyramids are a type of chart used to depict the overall age structure of a society. Use the accompanying population pyramids for the United States to answer the following questions.



Source: U.S. Bureau of Census, International Data Base, *www.census.gov.*



Source: U.S. Bureau of the Census, International Data Base, *www.census.gov.*

a. Estimate the number of:

i. Males who were between the ages 35 and 39 years in 2005.

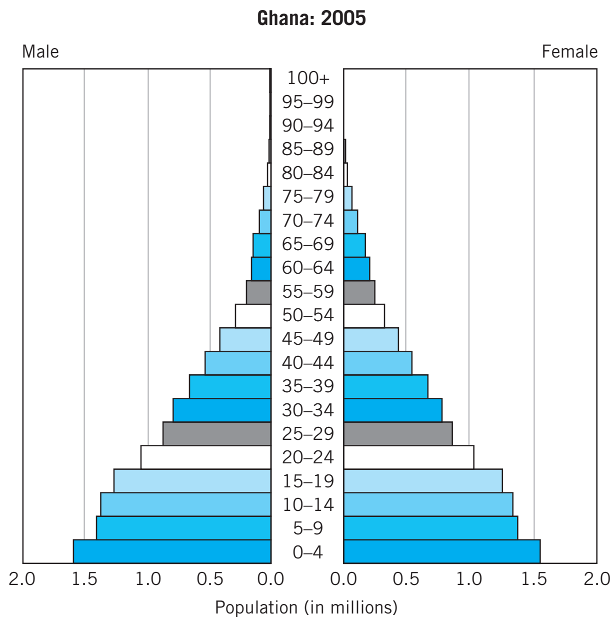
ii. Females who were between the ages 55 and 59 years in 2005.

iii. Males 85 years and older in the year 2050; females 85 years and older in the year 2050.

iv. All males and females between the ages of 0 and 9 years in the year 2050.

b. Describe two changes in the distribution of ages from the year 2005 to the predictions for 2050.

29. The accompanying population pyramid shows the age structure in Ghana, a developing country in Africa, for 2005. The previous exercise contains a population pyramid for the United States, an industrialized nation, for 2005. Describe three major differences in the distribution of ages in these two countries in 2005.



Source: U.S. Census Bureau, International Data Base, *www.census.gov.*

1.2 *Describing Relationships between Two Variables*

By looking at two-variable data, we can learn how change in one variable affects change in another. How does the weight of a child determine the amount of medication prescribed by a pediatrician? How does median age or income change over time? In this section we examine how to describe these changes with graphs, data tables, written descriptions, and equations.

Visualizing Two-Variable Data

EXAMPLE 1 **Scatter Plots**

Table 1.4 shows data for two variables, the year and the median age of the U.S. population. Plot the data in Table 1.4 and then use your graph to describe the changes in the U.S. median age over time.

DATA
MEDAGE
Excel and graph link files for the median age data are called MEDAGE.

Median Age of the U.S. Population, 1850–2050*

Year	Median Age	Year	Median Age
1850	18.9	1950	30.2
1860	19.4	1960	29.5
1870	20.2	1970	28.0
1880	20.9	1980	30.0
1890	22.0	1990	32.8
1900	22.9	2000	35.3
1910	24.1	2005	36.7
1920	25.3	2010	36.0
1930	26.4	2025	37.5
1940	29.0	2050	38.1

Table 1.4
*Data for 2010–2050 are projected.
Source: U.S., Bureau of the Census, *Statistical Abstract of the United States:* 1, 2006.

SOLUTION In Table 1.4 we can think of a year and its associated median age as an ordered pair of the form (year, median age). For example, the first row corresponds to the ordered pair (1850, 18.9) and the second row corresponds to (1860, 19.4). Figure 1.7 shows a scatter plot of the data. The graph is called a *time series* because it shows changes over time. In newspapers and magazines, the time series is the most frequently used form of data graphic.[7]

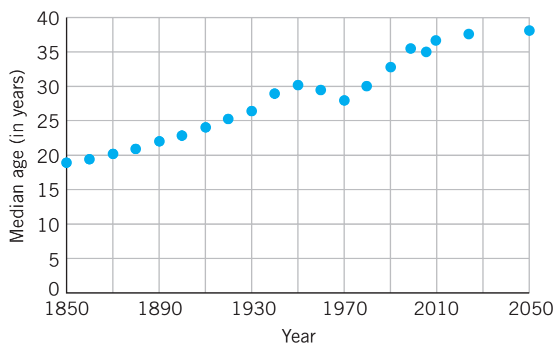


Figure 1.7 Median age of U.S. population over time.

[7]Edward Tufte in *The Visual Display of Information* (Cheshire, Conn.: Graphics Press, 2001, p. 28) reported on a study that found that more than 75% of all graphics published were time series.

SOMETHING TO THINK ABOUT

? *What are some of the trade-offs in using the median instead of the mean age to describe changes over time?*

Our graph shows that the median age of the U.S. population grew quite steadily for one hundred years, from 1850 to 1950. Although the median age decreased between 1950 and 1970, since 1970 it has continued to increase. From 1850 to the present, the median age nearly doubled, and projections for 2025 and 2050 indicate continued increases, though at a slower pace.

Constructing a "60-Second Summary"

To communicate effectively, you need to describe your ideas succinctly and clearly. One tool for doing this is a "60-second summary"—a brief synthesis of your thoughts that could be presented in one minute. Quantitative summaries strive to be straightforward and concise. They often start with a topic sentence that summarizes the key idea, followed by supporting quantitative evidence.

After you have identified a topic you wish to write about or present orally, some recommended steps for constructing a 60-second summary are:

- Collect relevant information (possibly from multiple sources, including the Internet).
- Search for patterns, taking notes.
- Identify a key idea (out of possibly many) that could provide a topic sentence.
- Select evidence and arguments that support your key idea.
- Examine counterevidence and arguments and decide if they should be included.
- Construct a 60-second summary, starting with your topic sentence.

You will probably weave back and forth among the steps in order to refine or modify your ideas. You can help your ideas take shape by putting them down on paper. Quantitative reports should not be written in the first person. For example, you might say something like "The data suggest that . . ." rather than "I found that the data . . ."

EXAMPLE 2 **A 60-Second Summary**

The annual federal surplus (+) or deficit (−) since World War II is shown in Table 1.5 and Figure 1.8 (a scatter plot where the points have been connected). Construct a 60-second summary describing the changes over time.

Federal Budget: Surplus (+) or Deficit (–)

Year	Billions of Dollars	Year	Billions of Dollars	Year	Billions of Dollars
1945	−$48	1979	−$41	1993	−$255
1950	−$3	1980	−$74	1994	−$203
1955	−$3	1981	−$79	1995	−$164
1960	$0	1982	−$128	1996	−$108
1965	−$1	1983	−$208	1997	−$22
1970	−$3	1984	−$185	1998	+$69
1971	−$23	1985	−$212	1999	+$126
1972	−$23	1986	−$221	2000	+$236
1973	−$15	1987	−$150	2001	+$127
1974	−$6	1988	−$155	2002	−$159
1975	−$53	1989	−$152	2003	−$378
1976	−$74	1990	−$221	2004	−$412
1977	−$54	1991	−$269	2005	−$427
1978	−$59	1992	−$290		

Table 1.5
Source: U.S. office of Management and Budget.

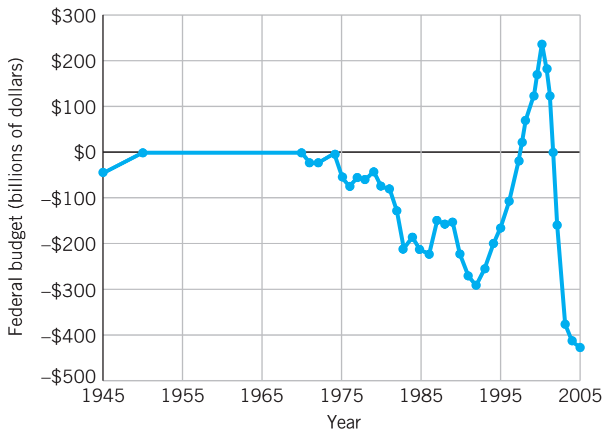


Figure 1.8 Annual federal budget surplus or deficit in billions of dollars.

SOLUTION Between 1945 and 2005 the annual U.S. federal deficit moved from a 30-year stable period, with as little as $0 deficit, to a period of oscillations, leading in 2005 to the largest deficit ever recorded. From 1971 to 1992, the federal budget ran an annual deficit, which generally was getting larger until it reached almost $300 billion in 1992. From 1992 to 1997, the deficit steadily decreased, and from 1998 to 2001 there were relatively large surpluses. The maximum surplus occurred in 2000, when it reached $236 billion. But by 2002 the federal government was again running large deficits. In 2005 the deficit reached $427 billion, the largest recorded up to that time.

Algebra Aerobics 1.2a

1. The net worth of a household at any given time is the difference between assets (what you *own*) and liabilities (what you *owe*). Table 1.6 and Figure 1.9 show the median net worth of U.S. households, adjusted for inflation.[8]

Median Net Worth of Households (adjusted for inflation using year 2000 dollars)

Year	Median Net Worth ($)
1984	50,018
1988	49,855
1991	44,615
1993	43,567
1995	44,578
1998	49,932
2000	55,000

Table 1.6
Source: U.S. Bureau of the Census, *www.census.gov.*

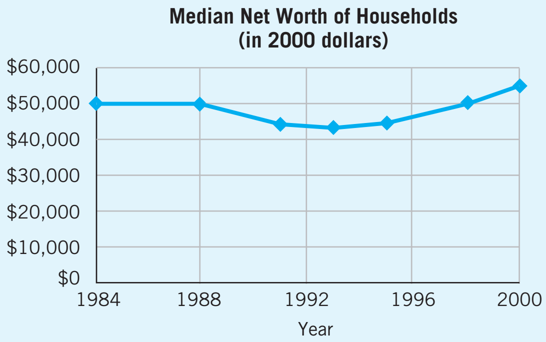


Figure 1.9

a. Write a few sentences about the trend in U.S. median household net worth.

b. What additional information might be useful in describing the trend in median net worth?

2. Use Figure 1.10 to estimate:

a. The year when the world population reached 4 billion.

b. The year that it is projected to reach 8 billion.

c. The number of years it will take to grow from 4 to 8 billion.

3. Use Figure 1.10 to estimate the following projections for the year 2150.

a. The total world population.

b. The total populations of all the more developed countries.

c. The total populations of all the less developed countries.

d. Write a topic sentence about the estimated world population in 2150.

4. Use Figure 1.10 to answer the following:

a. The world population in 2000 was how many times greater than the world population in 1900? What was the difference in population size?

b. The world population in 2100 is projected to be how many times greater than the world population in 2000? What is the difference in population size?

c. Describe the difference in the growth in world population in the twentieth century (1900–2000) versus the projected growth in the twenty-first century (2000–2150).

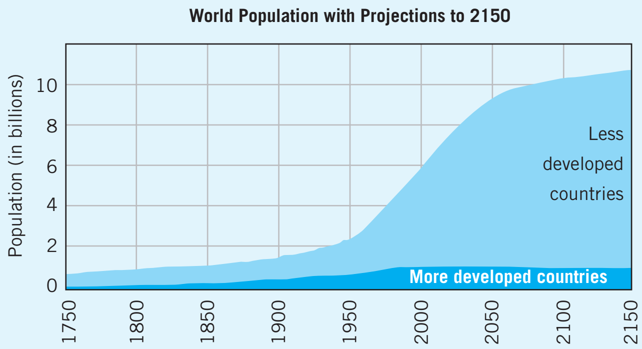


Figure 1.10 World population growth, 1750–2150 (est.).
Source: Population Reference Bureau, *www.prb.org.*

[8] "Constant dollars" is a measure used by economists to compare incomes and other variables in terms of purchasing power, eliminating the effects of inflation. To say the median income in 1986 was $37,546 in "constant 2000 dollars" means that the median income in 1986 could buy an amount of goods and services that would cost $37,546 to buy in 2000. The actual median income in 1986 (measured in what economists call "current dollars") was much lower. Income corrected for inflation is sometimes called "real" income.

Using Equations to Describe Change

Sometimes the relationship between two variables can also be described with an equation. An equation gives a rule on how change in the value of one variable affects change in the value of the other. If the variable n represents the number of years of education beyond grammar school and e represents yearly median earnings (in dollars) for people living in the United States, then the following equation models the relationship between e and n:

$$e = 3780 + 4320n$$

This equation provides a powerful tool for describing how earnings and education are linked and for making predictions.[9] For example, to predict the median earnings, e, for those with a high school education, we replace n with 4 (representing 4 years beyond grammar school, or a high school education) in our equation to get

$$\begin{aligned} e &= 3780 + 4320 \cdot 4 \\ &= \$21{,}060 \end{aligned}$$

Thus our equation predicts that for those with a high school education, median earnings will be about \$21,060.

An equation that is used to describe a real-world situation is called a *mathematical model*. Such models offer compact, often simplified descriptions of what may be a complex situation. Thus, the accuracy of the predictions made with such models can be questioned and disciplines outside of mathematics may be needed to help answer such questions. Yet these models are valuable guides in our quest to understand social and physical phenomena in our world.

Describing the relationship between abstract variables

Variables can represent quantities that are not associated with real objects or events. The following equation or mathematical sentence defines a relationship between two quantities, which are named by the abstract variables x and y:

$$y = x^2 + 2x - 3$$

By substituting various values for x and finding the associated values for y, we can generate pairs of values for x and y, called *solutions to the equation*, that make the sentence true. By convention, we express these solutions as ordered pairs of the form (x, y). Thus, $(1, 0)$ would be a solution to $y = x^2 + 2x - 3$, since $0 = 1^2 + 2(1) - 3$, whereas $(0, 1)$ would not be a solution, since $1 \neq 0^2 + 2(0) - 3$.

There are infinitely many possible solutions to the equation $y = x^2 + 2x - 3$, since we could substitute any real number for x and find a corresponding y. Table 1.7 lists a few solutions.

We can use technology to graph the equation (see Figure 1.11). All the points on the graph represent solutions to the equation, and every solution is a point on the graph of the equation.

[9] In "Extended Exploration: Looking for Links between Education and Earnings," which follows Chapter 2, we show how such equations are derived and how they are used to analyze the relationship between education and earnings.

x	y
–4	5
–3	0
–2	–3
–1	–4
0	–3
1	0
2	5
3	12

Table 1.7

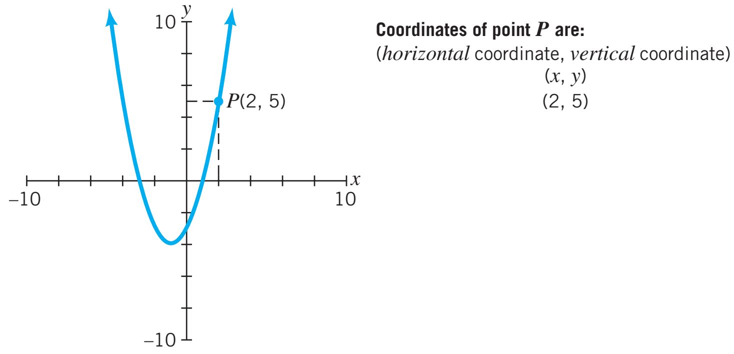


Figure 1.11 Graph of $y = x^2 + 2x - 3$ where one solution of infinitely many solutions is labeled.

Note that sometimes an arrow is used to show that a graph extends indefinitely in the indicated direction. In Figure 1.11, the arrows show that both arms of the graph extend indefinitely upward.

Solutions of an Equation

The *solutions* of an equation in two variables x and y are the ordered pairs (x, y) that make the equation a true statement.

Graph of an Equation

The *graph* of an equation in two variables displays the set of points that are solutions to the equation.

EXAMPLE 3 **Solutions for equations in one or two variables**

Describe how the solutions for the following equations are similar and how they differ.

$$3x + 5 = 11$$
$$x + 2 = x + 2$$
$$3 + x = y + 5$$

SOLUTION The solutions are similar in the sense that each solution for each particular statement makes the statement true. They are different because:

There is only one solution ($x = 2$) of the single-variable equation $3x + 5 = 11$.

There are an infinite number of solutions for x of the single-variable equation $x + 2 = x + 2$, since any real number will make the statement a true statement.

There are infinitely many solutions, in the form of ordered pairs (x, y), of the two-variable equation $3 + x = y + 5$.

EXAMPLE 4 **Estimating solutions from a graph**

The graph of the equation $x^2 + 4y^2 = 4$ is shown in Figure 1.12.

a. From the graph, estimate three solutions of the equation.

b. Check your solutions using the equation.

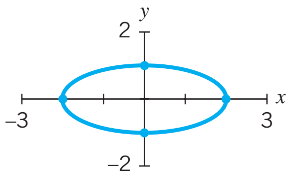


Figure 1.12

SOLUTION

a. The coordinates (0, 1), (−2, 0), and (1, 0.8) appear to lie on the ellipse, which is the graph of the equation $x^2 + 4y^2 = 4$.

b. If substituting the ordered pair (0, 1) into the equation makes it a true statement, then (0, 1) is a solution.

Given	$x^2 + 4y^2 = 4$
substitute $x = 0$ and $y = 1$	$(0)^2 + 4(1)^2 = 4$
evaluate	$4 = 4$

We get a true statement, so (0, 1) is a solution to the equation.

For the ordered pair (−2, 0):

Given	$x^2 + 4y^2 = 4$
substitute $x = -2$ and $y = 0$	$(-2)^2 + 4(0)^2 = 4$
evaluate	$4 + 0 = 4$
	$4 = 4$

Again we get a true statement, so (−2, 0) is a solution to the equation.

For the ordered pair (1, 0.8):

Given	$x^2 + 4y^2 = 4$
substitute $x = 1$ and $y = 0.8$	$(1)^2 + 4(0.8)^2 = 4$
evaluate	$1 + 4(0.64) = 4$
	$3.56 \neq 4$

We get a false statement, so (1, 0.8) is not a solution, although it is close to a solution.

Algebra Aerobics 1.2b

Problem 4(c) requires a graphing program.

1. **a.** Describe in your own words how to compute the value for y, given a value for x, using the following equation:

$$y = 3x^2 - x + 1$$

   b. Which of the following ordered pairs represent solutions to the equation?

(0, 0), (0, 1), (1, 0), (−1, 2), (−2, 3), (−1, 0)

   c. Use $x = 0, \pm 1, \pm 2, \pm 3$ to generate a small table of values that represent solutions to the equation.

2. Repeat the directions in Problem 1(a), (b), and (c) using the equation $y = (x - 1)^2$.

3. Given the equations $y_1 = 4 - 3x$ and $y_2 = -2x^2 - 3x + 5$, fill in Table 1.8.

x	−4	−2	−1	0	1	2	4
y_1							
y_2							

Table 1.8

   a. Use the table to create two scatter plots, one for the ordered pairs (x, y_1) and the other for (x, y_2).

   b. Draw a smooth curve through the points on each graph.

c. Is (1, 1) a solution for equation y_1? For y_2?

d. Is (-1, 6) a solution for equation y_1? For y_2?

e. Look at the graphs. Is the ordered pair (-3, 2) a solution for either equation? Verify your answer by substituting the values into each equation.

4. Given the equation $y = x^2 - 3x + 2$,

a. If $x = 3/2$, find y.

b. Find two points that are *not* solutions to this equation.

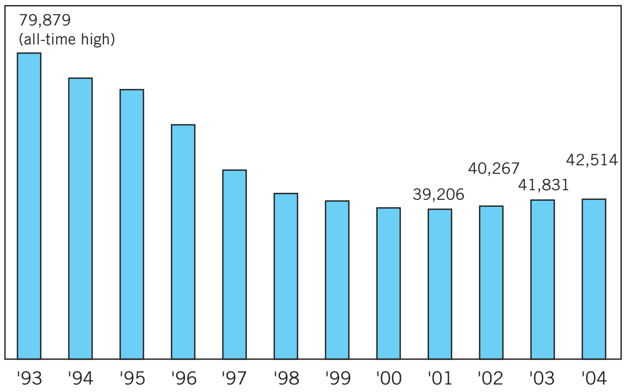
c. If available, use technology to graph the equation and then confirm your results for parts (a) and (b).

Exercises for Section 1.2

Course software recommended for Exercise 20.

1. Assume you work for a newspaper and are asked to report on the following data.

AIDS Cases in U.S.



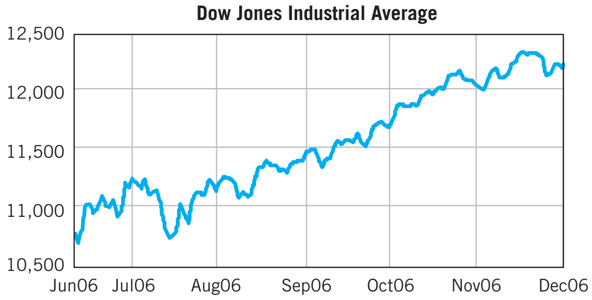
Source: Centers for Disease Control and Prevention, *www.cdc.gov.*

a. What are three important facts that emerge from this graph?

b. Construct a 60-second summary that could accompany the graph in the newspaper article.

2. The following graph shows changes in the Dow Jones Industrial Average, which is based on 30 stocks that trade on the New York Stock Exchange and is the best-known index of U.S. stocks.

Dow Jones Industrial Average



Source: http://finance.yahoo.com.

a. What time period does the graph cover?

b. Estimate the lowest Dow Jones Industrial Average. During what month did it occur?

c. Estimate the highest value for the Dow Jones during that period. When did it occur?

d. Write a topic sentence describing the change in the Dow Jones over the given time period.

3. The accompanying table shows the number of personal and property crimes in the United States from 1995 to 2003.

Year	Personal Crimes (in thousands)	Property Crimes (in thousands)
1995	1,799	12,064
1998	1,534	10,952
2000	1,425	10,183
2003	1,381	10,436

Source: U.S. Bureau of the Census, Statistical *Abstract, 2006.*

a. Create a scatter plot of the personal crimes over time. Connect the points with line segments.

b. Approximately how many *times* more property crimes than personal crimes were committed in 1995? In 2003?

c. Write a topic sentence that compares property and personal crime from 1995 to 2003.

4. The National Cancer Institute now estimates that after 70 years of age, 1 woman in 8 will have gotten breast cancer. Fortunately, they also estimate that 95% of breast cancer can be cured, especially if caught early. The data in the accompanying table show how many women in different age groups are likely to get breast cancer.

Lifetime Risk of Developing Breast Cancer

Age Group	Chance of Developing Cancer	Chance in 1000 Women
30–39	1 in 229	4 per 1000
40–49	1 in 68	15 per 1000
50–59	1 in 37	27 per 1000
60–69	1 in 26	38 per 1000
70+	1 in 8	125 per 1000

(*Note:* Men may get breast cancer too, but less than 1% of all breast cancer cases occur in men.)

a. What is the overall relationship between age and breast cancer?

b. Make a bar chart using the chance of breast cancer in 1000 women for the age groups given.

c. Using the "chance in 1000 women" data, estimate how much more likely that women in their 40s would have had breast cancer than women in their 30s. How much more likely for women in their 50s than women in their 40s?

d. It is common for women to have yearly mammograms to detect breast cancer after they turn 50, and health insurance companies routinely pay for them. Looking at these data, would you recommend an earlier start for yearly mammograms? Explain your answer in terms of the interests of the patient and the insurance company. (*Note:* Some research says that mammograms are not that good at detection.)

5. The National Center for Chronic Disease Prevention and Health Promotion published the following data on the chances that a man has had prostate cancer at different ages.

Lifetime Risk of Developing Prostate Cancer

Age	Risk	Percent Risk
45	1 in 25,000	0.004%
50	1 in 476	0.21%
55	1 in 120	0.83%
60	1 in 43	2.3%
65	1 in 21	4.8%
70	1 in 13	7.7%
75	1 in 9	11.1%
80	1 in 6	16.7%

a. What is the relationship between age and getting prostate cancer?

b. Make a scatter plot of the percent risk for men of the ages given.

c. Using the "percent risk" data, how much more likely are men 50 years old to have had prostate cancer than men who are 45? How much more likely are men 55 years old to have had prostate cancer than men who are 50?

d. Looking at these data, when would you recommend annual prostate checkups to begin for men? Explain your answer in terms of the interests of the patient and the insurance company.

6. Birth rate data in the United States are given as the number of live births per 1000 women in each age category.

Age	1950	2000
10–14	1.0	0.9
15–19	81.6	48.5
20–24	196.6	112.3
25–29	166.1	121.4
30–34	103.7	94.1
35–39	52.9	40.4
40–44	15.1	7.9
45–49	1.2	0.5

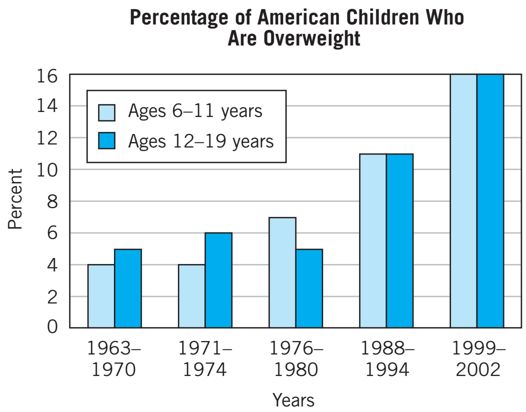
Source: National Center for Health Statistics, U.S. Dept. of Health and Human Services.

a. Construct a bar chart showing the birth rates for the year 1950. Which mother's age category had the highest rate of live births? What percentage of women in that category delivered live babies? In which age category was the lowest rate of babies born? What percentage of women in that category delivered live babies?

b. Construct a bar chart showing the birth rates for the year 2000. Which mother's age category had the highest rate of live births? What percentage of women in that category delivered live babies? In which age category was the lowest rate of babies born? What percentage of women in that category delivered live babies?

c. Write a paragraph comparing and contrasting the birth rates in 1950 and in 2000. Bear in mind that since 1950 there have been considerable medical advances in saving premature babies and in increasing the fertility of couples.

7. The National Center for Health Statistics published the accompanying chart on childhood obesity.



Source: National Center for Health Statistics, *www.cdc.gov/nchs.*

a. How would you describe the overall trend in the weights of American children?

b. Over which years did the percentage of overweight children age 6 to 11 increase?

c. Over which time period was there no change in the percentage of overweight children age 6 to 11?

d. During which time period were there relatively more overweight 6- to 11-year-olds than 12- to 19-year-olds?

e. One of the national health objectives for the year 2010 is to reduce the prevalence of obesity among children to less than 15%. Does this seem like a reasonable goal?

8. Some years are more severe for influenza- and pneumonia-related deaths than others. The table at the top of the next page shows data from Centers for Disease Control figures for the U.S. for selected years from 1950 to 2000.

Age-Adjusted Death Rate for Influenza and Pneumonia

	Death Rate per 100,000	
Year	Males	Females
1950	55	42
1960	66	44
1970	54	33
1980	42	25
1990	48	31
2000	29	21

Source: Centers for Disease Control and Prevention, *www.cdc.gov.*

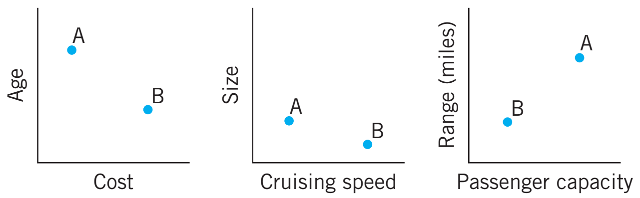
a. Create a double bar chart showing the death rates both for men and for women who died of influenza and pneumonia between 1950 and 2000.

b. In which year were death rates highest for both men and women?

c. Were there any decades in which there was an increase in male deaths but a decrease for women?

d. Write a 60-second summary about deaths due to influenza and pneumonia over the years 1950 to 2000.

9. The following three graphs describe two cars, A and B.



For parts (a)–(d), decide whether the statement is true or false. Explain your reasoning.

a. The newer car is more expensive.

b. The slower car is larger.

c. The larger car is newer.

d. The less expensive car carries more passengers.

e. State two other facts you can derive from the graphs.

f. Which car would you buy? Why?

10. a. Which (if any) of the following ordered pairs (x, y) is a solution to the equation $y = x^2 - 2x + 1$? Show how you came to your conclusion.

$$(-2, 7), \quad (1, 0), \quad (2, 1)$$

b. Find one additional ordered pair that is a solution to the equation above. Show how you found your solution.

11. Consider the equation $R = 2 - 5T$.

a. Determine which, if any, of the following points (T, R) satisfy this equation.

$$(0, 4), \quad (1, -3), \quad (2, 0)$$

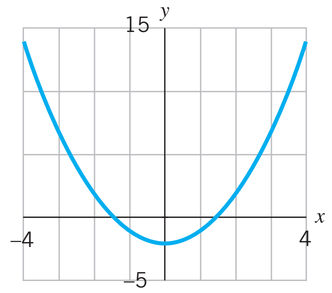
b. Find two additional ordered pairs that are solutions to the equation.

c. Make a scatter plot of the solution points found.

d. What does the scatter plot suggest about where more solutions could be found? Check your predictions.

12. Use the accompanying graph to estimate the missing values for x or y in the table.

x	y
-3	
	2
-1	
	-2
	-1
2	
	7



13. For parts (a)–(d) use the following equation: $y = \dfrac{x + 1}{x - 1}$.

a. Describe in words how to find the value for y given a value for x.

b. Find the ordered pair that represents a solution to the equation when the value of x is 5.

c. Find the ordered pair that represents a solution to the equation when the value of y is 3.

d. Is there an ordered-pair solution to the equation when the value of x is 1? If so, find it; if not, explain why.

14. For parts (a)–(d) use the following equation: $y = \dfrac{1}{x + 1}$.

a. Describe in words how to find the value for y given a value for x.

b. Find the ordered pair that represents a solution to the equation when the value of x is 0.

c. Find the ordered pair that represents a solution to the equation when the value of y is 4.

d. Is there an ordered-pair solution to the equation when the value of x is -1? If so, find it; if not, explain why.

15. For parts (a)–(d) use the following equation: $y = -2x^2$.

a. If $x = 0$, find the value of y.

b. If x is greater than zero, what can you say about the value of y?

c. If x is a negative number, what can you say about the value of y?

d. Can you find an ordered pair that represents a solution to the equation when y is greater than zero? If so, find it; if not, explain why.

16. Find the ordered pairs that represent solutions to each of the following equations when $x = 0$, when $x = 3$, and when $x = -2$.

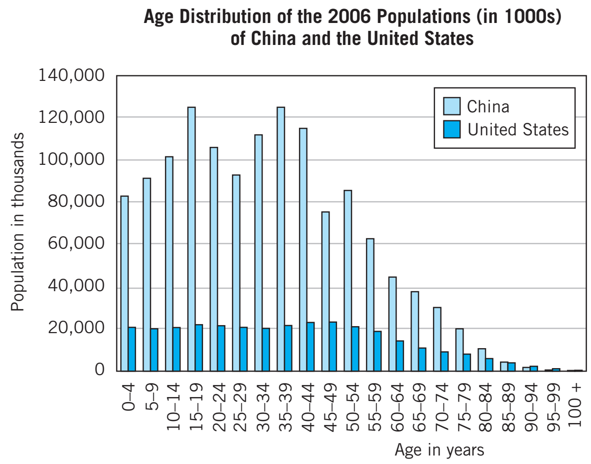
a. $y = 2x^2 + 5x$ **c.** $y = x^3 + x^2$

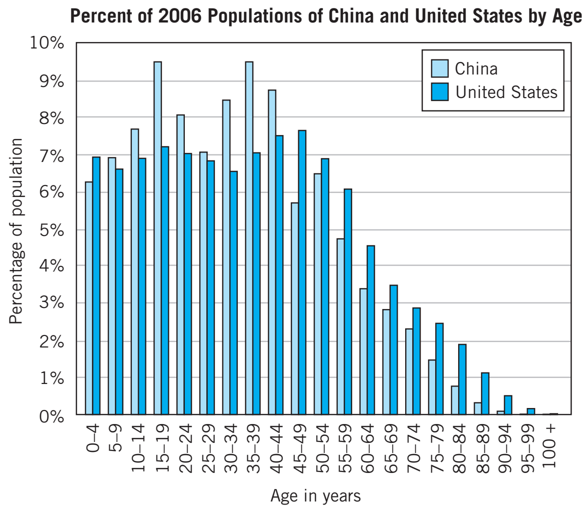
b. $y = -x^2 + 1$ **d.** $y = 3(x - 2)(x - 1)$

17. Given the four ordered pairs $(-1, 3)$, $(1, 0)$, $(2, 3)$, and $(1, 2)$, for each of the following equations, identify which points (if any) are solutions for that equation.

a. $y = 2x + 5$ **c.** $y = x^2 - x + 1$

b. $y = x^2 - 1$ **d.** $y = \dfrac{4}{x + 1}$





Source: U.S. Bureau of the Census, *www.census.gov.*

18. The accompanying graphs (and related Excel and graph link file USCHINA) contain information about the populations of the United States and China. Write a 60-second summary comparing the two populations.

19. Read *The New York Times* op-ed article "A Fragmented War on Cancer" by Hamilton Jordan, who was President Jimmy Carter's chief of staff. Jordan claims that we are on the verge of a cancer epidemic.

a. Use what you know about the distribution of ages over time in the United States to refute his claim. (See Sec.1.1 Exercise 28 for some ideas.) Are there other arguments that refute his claim?

b. Read *The New York Times* letter to the editor by William M. London, Director of Public Health, American Council on Science and Health. London argues that Hamilton Jordan's assertions are misleading. What questions are raised by the arguments of William London? What additional data would you need to evaluate his arguments?

c. Write a paragraph refuting Hamilton Jordan's claim that we are on the verge of a cancer epidemic.

20. (Computer required.) Make a prediction about the distribution of income for males and females in the United States. Check your predictions using the course software "F1: Histograms" in *FAM1000 Census Graphs* and/or using data from the U.S. Census Bureau at *www.census.gov*. Write a 60-second summary describing your results.

1.3 *An Introduction to Functions*

What Is a Function?

When we speak informally of one quantity being a function of some other quantity, we mean that one depends on the other. For example, someone may say that what they wear is a function of where they are going, or what they weigh is a function of what they eat, or how well a car runs is a function of how well it is maintained.

In mathematics, the word "function" has a precise meaning. A function is a special relationship between two quantities. If the value of one quantity uniquely determines the value of a second quantity, then the second quantity is a *function* of the first.

Median age and the federal deficit are functions of time since each year determines a unique (one and only one) value of median age or the federal deficit. The equation $y = x^2 + 2x - 3$ defines y as a function of x since each value of x we substitute in the equation determines a unique value of y.

> **Definition of a Function**
>
> A variable y is a *function* of a variable x if each value of x determines a unique value of y.

Representing Functions in Multiple Ways

We can think of a function as a "rule" that takes certain inputs and assigns to each input value exactly one output value. The rule can be described using words, data tables, graphs, or equations.

EXAMPLE 1 **Sales tax**

Eleven states have a sales tax of 6%; that is, for each dollar spent in a store in these states, the law says that you must pay a tax of 6 cents, or \$0.06. Represent the sales tax as a function of purchase price using an equation, table, and graph.[10]

SOLUTION **Using an Equation**

We can write this relationship as an equation where T represents the amount of sales tax and P represents the price of the purchase (both measured in dollars):

$$\text{Amount of sales tax} = 0.06 \cdot \text{price of purchase}$$

$$T = 0.06P$$

Our function rule says: "Take the given value of P and multiply it by 0.06; the result is the corresponding value of T." The equation represents T as a function of P, since for each value of P the equation determines a unique (one and only one) value of T. The purchase price, P, is restricted to dollar amounts greater than or equal to zero.

Using a Table

We can use this formula to make a table of values for T determined by the different values of P (see Table 1.9). Such tables were once posted next to many cash registers.

P (purchase price in \$)	0	1	2	3	4	5	6	7	8	9	10
T (sales tax in \$)	0.00	0.06	0.12	0.18	0.24	0.30	0.36	0.42	0.48	0.54	0.60

Table 1.9

Using a Graph

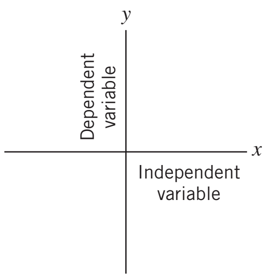
The points in Table 1.9 were used to create a graph of the function (Figure 1.13). The table shows the sales tax only for selected purchase prices, but we could have used any positive dollar amount for P. We connected the points on the scatter plot to suggest the many possible intermediate values for price. For example, if $P = \$2.50$, then $T = \$0.15$.

Figure 1.13 Graph of 6% sales tax.

[10]In 2007, a sales tax of 6% was the most common rate for a sales tax in the United States. See *www.taxadmin.org* for a listing of the sales tax rates for all of the states.

Independent and Dependent Variables

Since a function is a rule that assigns to each input a unique output, we think of the output as being dependent on the input. We call the input of a function the *independent variable* and the output the *dependent variable*. When a set of ordered pairs represents a function, then each ordered pair is written in the form



(independent variable, dependent variable)

or equivalently, (input, output)

If x is the independent and y the dependent variable, then the ordered pairs would be of the form

$$(x, y)$$

The mathematical convention is for the first variable, or input of a function, to be represented on the horizontal axis and the second variable, or output, on the vertical axis.

Sometimes the choice of the independent variable is arbitrary or not obvious. For example, economists argue as to whether wealth is a function of education or education is a function of wealth. As seen in the next example, there may be more than one correct choice.

EXAMPLE 2 **Identifying Independent and dependent variables**

In the sales tax example, the equation $T = 0.06P$ gives the sales tax, T, as a function of purchase price, P. In this case T is the dependent variable, or output, and P is the independent variable, or input. But for this equation we can see that P is also a function of T; that is, each value of T corresponds to one and only one value of P. It is easier to see the relationship if we solve for P in terms of T, to get

$$P = \frac{T}{0.06}$$

Now we are thinking of the purchase price, P, as the dependent variable, or output, and the sales tax, T, as the independent variable, or input. So, if you tell me how much tax you paid, I can find the purchase price.

When Is a Relationship Not a Function?

Not all relationships define functions. A function is a special type of relationship, one where for each input, the rule specifies one and only one output. Examine the following examples.

Function		Not a Function		Function	
Input	Output	Input	Output	Input	Output
1 →	6	1 →	6, 7	1 →	6
2 →	7	2 →	8	2 →	6
3 →	8	3 →	9	3 →	6
Each input has only one output.		The input of 1 gives *two different outputs*, 6 and 7, so this relationship is *not* a function.		Each input has only one output. Note that a function may have identical outputs for different inputs.	

EXAMPLE 3 **Does the table represent a function?**

Consider the set of data in Table 1.10. The first column shows the year, T, of the Olympics. The second column shows the winning distance, D (in feet), for that year for the men's Olympic 16-pound shot put.

a. Is D a function of T?

b. Is T a function of D?

c. What should be your choice for the dependent and independent variables?

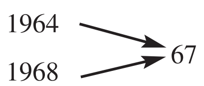
Olympic Shot Put

Year, T	Winning Distance in Feet Thrown, D
1960	65
1964	67
1968	67
1972	70
1976	70
1980	70
1984	70
1988	74
1992	71
1996	71
2000	70
2004	69

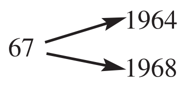
Table 1.10
Source: The World Almanac and Book of Facts, 2006.

SOLUTION

a. *D* is a function of *T*.
To determine if D is a function of T, we need to find out if each value of T (the input) determines one and only one value for D (the output). So, the ordered pair representing the relationship would be of the form (T, D). Using Table 1.10, we can verify that for each T, there is one and only one D. So D, the winning shot put distance, is a function of the year of the Olympics, T. Note that different inputs (such as 1964 and 1968) can have the same output (67 feet), and the relationship can still be a function. There are even 5 different years that have 70 feet as their output.



b. *T* is *not* a function of *D*.
To determine if T is a function of D, we need to find out if each value of D (now the input) determines one and only one value for T (the output). Now the ordered pairs representing the relationship would be of the form (D, T). Table 1.11 shows this new pairing, where D is thought of as the input and T as the output.



The year, T, is *not* a function of D, the winning distance, since some values of D give more than one value for T. For example, when $D = 67$ there are two corresponding values for T, 1964 and 1968, and this violates the condition of a unique (one and only one) output for each input.

Olympic Shot Put

Winning Distance in Feet Thrown, D	Year, T
65	1960
67	1964
67	1968
70	1972
70	1976
70	1980
70	1984
74	1988
71	1992
71	1996
70	2000
69	2004

Table 1.11

c. D, the winning distance, is a function of the year, T, but T is *not* a function of D. The distance, D, depends on the year, T, but T does not depend on D. To construct a function relating T and D, we must choose T as the independent variable and D as the dependent variable. The ordered pairs that represent the function would be written as (T, D).

EXAMPLE 4 How would the axes be labeled for each graph of the following functions?

a. Density of water is a function of temperature.

b. Radiation intensity is a function of wavelength.

c. A quantity Q is a function of time t.

SOLUTION

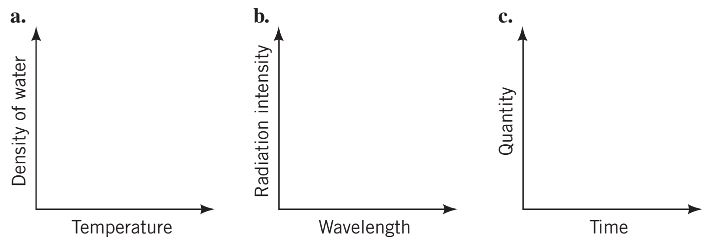


Figure 1.14 Various labels for axes, depending on the context.

How to tell if a graph represents a function: The vertical line test

For a graph to represent a function, each value of the input on the horizontal axis must be associated with one and only one value of the output on the vertical axis. If you can draw a vertical line that intersects a graph in more than one point, then at least one input is associated with two or more outputs, and the graph does not represent a function.

The graph in Figure 1.15 represents y as a function of x. For each value of x, there is only one corresponding value of y. No vertical line intersects the curve in more than one point. The graph in Figure 1.16 does *not* represent a function. One can draw a vertical line (an infinite number, in fact) that intersects the graph in more than one point. Figure 1.16 shows a vertical line that intersects the graph at both (4, 2) and (4, −2). That means that the value $x = 4$ does not determine one and only one value of y. It corresponds to y values of both 2 and −2.

Exploration 1.2 will help you develop an intuitive sense of functions.

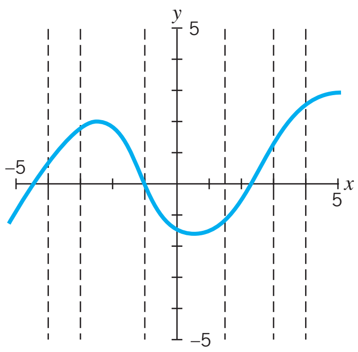


Figure 1.15 The graph represents y as a function of x since there is no vertical line that intersects the curve at more than one point.

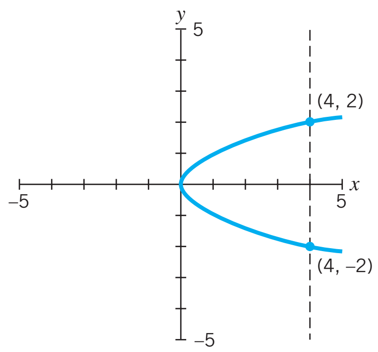


Figure 1.16 The graph does not represent y as a function of x since there is at least one vertical line that intersects this curve at more than one point.

> **Vertical Line Test**
>
> If there is a vertical line that intersects a graph more than once, the graph does not represent a function.

Algebra Aerobics 1.3

1. Which of the following tables represent functions? Justify your answer.

Input	Output
1	5
2	8
3	8
4	10

Table A

Input	Output
1	5
2	7
2	8
4	10

Table B

2. Does the following table represent a function? If so, why? If not, how could you change the values in the table so it represents a function?

Input	Output
1	5
1	7
3	8
4	10

3. Refer to the graph in Figure 1.17. Is y a function of x?

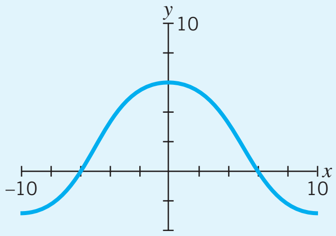


Figure 1.17 Graph of an abstract relationship between x and y.

4. Which of the graphs in Figure 1.18 represent functions and which do not? Why?

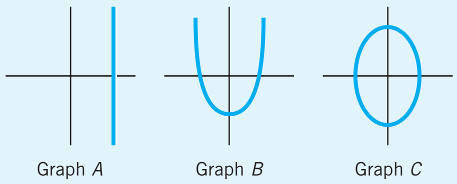


Figure 1.18

5. Consider the scatter plot in Figure 1.19. Is weight a function of height? Is height a function of weight? Explain your answer.

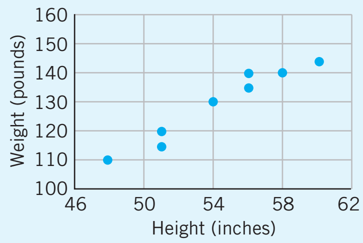


Figure 1.19 Graph of weight versus height.

6. Consider Table 1.12.
   a. Is D a function of Y?
   b. Is Y a function of D?

Y	1992	1993	1994	1995	1996	1997
D	\$2.50	\$2.70	\$2.40	−\$0.50	\$0.70	\$2.70

Table 1.12

7. Plot the following points with x on the horizontal and y on the vertical axis. Draw a line through the points and determine if the line represents a function.

a.

x	y
−1	3
0	3
2	3
3	3

b.

x	y
−2	−1
−2	0
−2	1
−2	4

8. **a.** Write an equation for computing a 15% tip in a restaurant. Does your equation represent a function? If so, what are your choices for the independent and dependent variables?
   b. How much would the equation suggest you tip for an \$8 meal?
   c. Compute a 15% tip on a total check of \$26.42.

Exercises for Section 1.3

A graphing program is required for Exercise 12.

1. The following table gives the high temperature in Rome, Italy, for each of five days in October 2006.
   a. Is the temperature a function of the date?
   b. Is the date a function of the temperature?

Date	Rome High Temperature
Oct. 26	27°C
Oct. 27	27°C
Oct. 28	25°C
Oct. 29	26°C
Oct. 30	22°C

2. Which of the following tables describe functions? Explain your answer.

a.

Input value	−2	−1	0	1	2
Output value	−8	−1	0	1	8

b.

Input value	0	1	2	1	0
Output value	−4	−2	0	2	4

c.

Input value	10	7	4	7	10
Output value	3	6	9	12	15

d.

Input value	0	3	9	12	15
Output value	3	3	3	3	3

3. Determine whether each set of points represents a function. (*Hint:* It may be helpful to plot the points.)

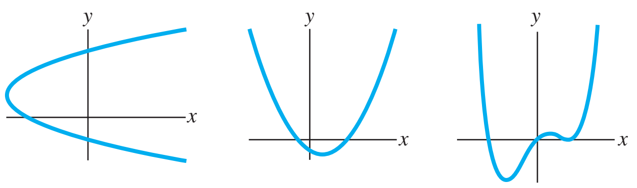
a. (2, 6), (−4, 6), (1, −3), (4, −3)

b. (2, −2), (3, −2), (4, −2), (6, −2)

c. (2, −3), (2, 3), (–2, −3), (−2, 3)

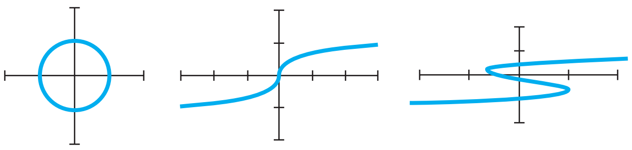
d. (−1, 2), (−1, 0), (−1, −1), (−1, −2)

4. Which of the accompanying graphs describe functions? Explain your answer.



Graph *A* Graph *B* Graph *C*

5. Which of the accompanying graphs describe functions? Explain your answer.



Graph *A* Graph *B* Graph *C*

6. Consider the accompanying table, listing the weights (W) and heights (H) of five individuals. Based on this table, is height a function of weight? Is weight a function of height? Justify your answers.

Weight W (lb)	Height H (in)
120	54
120	55
125	58
130	60
135	56

7. a. Find an equation that represents the relationship between x and y in each of the accompanying tables.

i.

x	y
0	5
1	6
2	7
3	8
4	9

ii.

x	y
0	1
1	2
2	5
3	10
4	17

iii.

x	y
0	3
1	3
2	3
3	3
4	3

b. Which of your equations represents y as function of x? Justify your answer.

8. For each of the accompanying tables find a function formula that takes the x values and produces the given y values.

a.

x	y
0	0
1	3
2	6
3	9
4	12

b.

x	y
0	–2
1	1
2	4
3	7
4	10

c.

x	y
0	0
1	–1
2	–4
3	–9
4	–16

9. The basement of a large department store features discounted merchandise. Their policy is to reduce the previous month's price of the item by 10% each month for 5 months, and then give the unsold items to charity.

a. Let S_1 be the sale price for the first month and P the original price. Express S_1 as a function of P. What is the price of a $100 garment on sale for the first month?

b. Let S_2 be the sale price for the second month and P the original price. Express S_2 as a function of P. What is the price of a $100 garment on sale for the second month?

c. Let S_3 be the sale price for the third month and P the original price. Express S_3 as a function of P. What is the price of a $100 garment on sale for the third month?

d. Let S_5 be the sale price for the fifth month and P the original price. Express S_5 as a function of P. What is the final price of a $100 garment on sale for the fifth month? By what total percentage has the garment now been reduced from its original price?

10. Write a formula to express each of the following sentences:

a. The sale price is 20% off the original price. Use S for sale price and P for original price to express S as a function of P.

b. The time in Paris is 6 hours ahead of New York. Use P for Paris time and N for New York time to express P as a function of N. (Represent your answer in terms of a 12-hour clock.) How would you adjust your formula if P comes out greater than 12?

c. For temperatures above 0°F the wind chill effect can be estimated by subtracting two-thirds of the wind speed (in miles per hour) from the outdoor temperature. Use C for the effective wind chill temperature, W for wind speed, and T for the actual outdoor temperature to write an equation expressing C in terms of W and T.

11. Determine whether y is a function of x in each of the following equations. If the equation does not define a function, find a value of x that is associated with two different y values.

a. $y = x^2 + 1$
b. $y = 3x - 2$
c. $y = 5$
d. $y^2 = x$

12. (Graphing program required.) For each equation below, write an equivalent equation that expresses z in terms of t. Use technology to sketch the graph of each equation. Is z a function of t? Why or why not?

a. $3t - 5z = 10$
b. $12t^2 - 4z = 0$
c. $2(t - 4) - (z + 1) = 0$

13. If we let D stand for ampicillin dosage expressed in milligrams and W stand for a child's weight in kilograms, then the equation

$$D = 50W$$

gives a rule for finding the safe maximum daily drug dosage of ampicillin (used to treat respiratory infections) for children who weigh less than 10 kilograms (about 22 pounds).[11]

a. What are logical choices for the independent and dependent variables?
b. Does the equation represent a function? Why?
c. Generate a small table and graph of the function.
d. Think of the function $D = 50W$ for ampicillin dosage as an abstract mathematical equation. How will the table and graph change?

1.4 *The Language of Functions*

Not all equations represent functions. (See page 17, Example 4 where the graph of the equation does not pass the vertical line test.) But functions have important qualities, so it is useful to have a way to indicate when a relationship is a function.

Function Notation

When a quantity y is a function of x, we can write

y is a function of x

or in abbreviated form,

y equals "f of x"

or using function notation,

$$y = f(x)$$

The expression $y = f(x)$ means that the rule f is applied to the input value x to give the output value, $f(x)$:

$$\text{output} = f(\text{input})$$

or

$$\text{dependent variable} = f(\text{independent variable})$$

The letter f is often used to denote the function, but we could use any letter, not just f.

Understanding the symbols

Suppose we have a function that triples the input. We could write this function as

$$y = 3x \qquad (1)$$

or with function notation as

$$T(x) = 3x \qquad \text{where } y = T(x) \qquad (2)$$

Equations (1) and (2) represent the same function, but with function notation we name the function—in this case T—and identify the input, x, and output, $3x$.

Function notation can provide considerable economy in writing and reading. For example, throughout a discussion we can use $T(x)$ instead of the full expression to represent the function.

[11]Information extracted from Anna M. Curren and Laurie D. Muntlay, *Math for Meds; Dosages and Solutions*, 6th ed. (San Diego: W. I. Publications, 1990), p. 198.

The Language of Functions

If y is a function, f, of x, then

$$y = f(x)$$

where f is the name of the function,
y is the output or *dependent* variable,
x is the input or *independent* variable.

output = f(input)
dependent = f(independent)

Finding output values: Evaluating a function

Function notation is particularly useful when a function is being evaluated at a specific input value. Suppose we want to find the value of the previous function $T(x)$ when our input value is 10. Using equation (1) we would say, "find the value of y when $x = 10$." With function notation, we simply write $T(10)$. To evaluate the function T at 10 means calculating the value of the output when the value of the input is 10:

Given $\quad T(x) = 3x$

Substitute 10 for x $\quad T(10) = 3(10) = 30$

So, applying the function rule T to the input value of 10 gives an output value of 30.

Common Error

The expression $f(x)$ does not mean "f times x." It means the function f evaluated at x.

EXAMPLE 1 **Using function notation with equations**

a. Given $f(x) = 2x^2 + 3$, evaluate $f(5)$, $f(0)$, and $f(-2)$

b. Evaluate $g(0)$, $g(2)$, and $g(-2)$ for the function

$$g(x) = \frac{1}{x - 1}$$

SOLUTION **a.** To evaluate $f(5)$, we replace every x in the formula with 5.

Given $\quad f(x) = 2x^2 + 3$

Substitute 5 for x $\quad f(5) = 2(5)^2 + 3 = 2(5)(5) + 3 = 53$

Similarly,

$$f(0) = 2(0)^2 + 3 = 3$$

$$f(-2) = 2(-2)^2 + 3 = (2 \cdot 4) + 3 = 11$$

b. $g(0) = \dfrac{1}{0 - 1} = -1, \quad g(2) = \dfrac{1}{2 - 1} = 1, \quad g(-2) = \dfrac{1}{-2 - 1} = \dfrac{1}{-3} = -\dfrac{1}{3}$

EXAMPLE 2 **Using function notation with data tables**

Use Table 1.13 to fill in the missing values:

a. $S(0) = ?$

b. $S(-1) = ?$

c. $S(?) = 4$

Input, x	-1	-2	0	1	2
Output, $S(x)$	1	4	0	1	4

Table 1.13

SOLUTION

a. $S(0)$ means to evaluate S when the input $x = 0$. The table says that the corresponding output is also 0, so $S(0) = 0$.

b. $S(-1) = 1$.

c. $S(?) = 4$ means to find the input when the output is 4. When the output is 4, the input is -2 or 2, so $S(-2) = 4$ and $S(2) = 4$.

EXAMPLE 3 **Using function notation with graphs**

Use the graph in Figure 1.20 to estimate the missing values:

a. $f(0) = ?$

b. $f(-5) = ?$

c. $f(?) = 0$

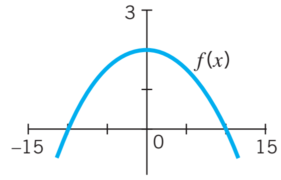


Figure 1.20 Graph of a function.

SOLUTION Remember that by convention the horizontal axis represents the input (or independent variable) and the vertical axis represents the output (or dependent variable).

a. $f(0) = 2$

b. $f(-5) = 1.5$

c. $f(10) = 0$ and $f(-10) = 0$

Rewriting equations using function notation

In order to use function notation, an equation needs to be in the form

$$\text{output} = \text{some rule applied to input}$$

or equivalently

$$\text{dependent variable} = \text{some rule applied to independent variable}$$

Translating an equation into this format is called putting the equation in *function form*. Many graphing calculators and computer graphing programs accept only equations in function form as input.

To put an equation into function form, we first need to identify the independent and the dependent variables. The choice is sometimes obvious, at other times arbitrary. If we use the mathematical convention that x represents the input or independent variable and y the output or dependent variable, when we put equations into function form, we want

$$y = \text{some rule applied to } x$$

EXAMPLE 4 Analyze the equation $4x - 3y = 6$. Decide whether or not the equation represents a function. If it does, write the relationship using function notation.

SOLUTION First, put the equation into function form. Assume y is the output.

Given the equation	$4x - 3y = 6$
subtract $4x$ from both sides	$-3y = 6 - 4x$
divide both sides by -3	$\dfrac{-3y}{-3} = \dfrac{6 - 4x}{-3}$

simplify $\quad y = \dfrac{6}{-3} + \dfrac{-4x}{-3}$

simplify and rearrange terms $\quad y = \dfrac{4}{3}x - 2$

We now have an expression for y in terms of x.

Using technology or by hand, we can generate a graph of the equation (see Figure 1.21). Since the graph passes the vertical line test, y is a function of x.

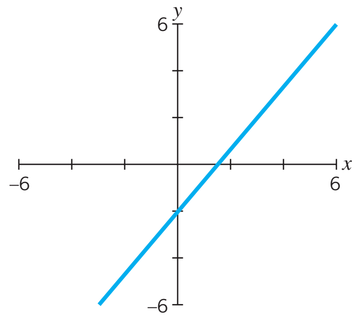


Figure 1.21 Graph of $y = \frac{4}{3}x - 2$.

If we name our function f, then using function notation, we have

$$y = f(x) \qquad \text{where } f(x) = \tfrac{4}{3}x - 2$$

EXAMPLE 5 Analyze the equation $y^2 - x = 0$. Generate a graph of the equation. Decide whether or not the equation represents a function. If the equation represents a function, write the relationship using function notation. Assume y is the output.

SOLUTION Put the equation in function form:

Given the equation $\quad y^2 - x = 0$

add x to both sides $\quad y^2 = x$

To solve this equation, we take the *square root* of both sides of the equation and we get

$$y = \pm\sqrt{x}$$

This gives us two solutions for any value of $x > 0$ as shown in Table 1.14. For example, if $x = 4$, then y can either be 2 or -2 since both $2^2 = 4$ and $(-2)^2 = 4$.

x	y
0	0
1	1 or -1
2	$\sqrt{2}$ or $-\sqrt{2}$
4	2 or -2
9	3 or -3

Table 1.14

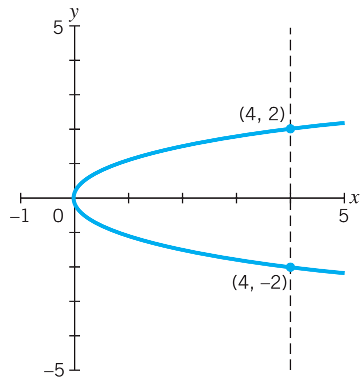


Figure 1.22 Graph of the equation $y^2 = x$.

The graph of the equation in Figure 1.22 does not pass the vertical line test. In particular, the solutions $(4, -2)$ and $(4, 2)$ lie on the same vertical line. So y is not a function of x and we cannot use function notation to represent this relationship.

Algebra Aerobics 1.4a

1. Given $g(x) = 3x$, evaluate $g(0)$, $g(-1)$, $g(1)$, $g(20)$, and $g(100)$.
2. Consider the function $f(x) = x^2 - 5x + 6$. Find $f(0)$, $f(1)$, and $f(-3)$.
3. Given the function $f(x) = \dfrac{2}{x-1}$, evaluate $f(0)$, $f(-1)$, and $f(-3)$.
4. Determine the value of t for which each of the functions has a value of 3.

   $r(t) = 5 - 2t \qquad p(t) = 3t - 9 \qquad m(t) = 5t - 12$

In Problems 5–7 solve for y in terms of x. Determine if y is a function of x. If it is, rewrite using $f(x)$ notation.

5. $2(x - 1) - 3(y + 5) = 10$
6. $x^2 + 2x - y + 4 = 0$
7. $7x - 2y = 5$
8. From the graph in Figure 1.23, estimate $f(-4)$, $f(-1)$, $f(0)$, and $f(3)$. Find two approximate values for x such that $f(x) = 0$.

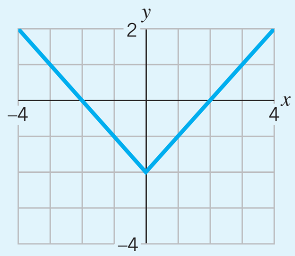


Figure 1.23 Graph of $f(x)$.

9. From Table 1.15 find $f(0)$ and $f(20)$. Find two values of x for which $f(x) = 10$. Explain why $f(x)$ is a function.

x	$f(x)$
0	20
10	10
20	0
30	10
40	20

Table 1.15

Domain and Range

A function is often defined only for certain values of the input (or independent variable). The set of all possible values for the input is called the *domain* of the function. The set of corresponding values of the output (or dependent variable) is called the *range* of the function.

Domain and Range of a Function

The *domain* of a function is the set of possible values of the input.
The *range* is the set of corresponding values of the output.

EXAMPLE 6 **Finding a reasonable domain and range**

In the sales tax example at the beginning of this section, we used the equation

$$T = 0.06P$$

to represent the sales tax, T, as a function of the purchase price, P (where all units are in dollars). What are the domain and range of this function?

SOLUTION Since negative values for P are meaningless, P is restricted to dollar amounts greater than or equal to zero. In theory there is no upper limit on prices, so we assume P has no maximum amount. In this example,

the domain is all dollar values of P greater than or equal to 0

We can express this more compactly with the symbol $\geq$, which means "greater than or equal to." So,

the domain is all dollar values of P such that $P \geq 0$

or abbreviated to the domain is $P \geq 0$

What are the corresponding values for the tax T? The values for T in our model will also always be nonnegative. As long as there is no maximum value for P, there will be no maximum value for T. So,

the range is all dollar values of T greater than or equal to 0

or we can shorten this to the range is $T \geq 0$

Representing the Domain and Range with Interval Notation

Interval notation is often used to represent the domain and range of a function.

Interval Notation

A *closed interval* $[a, b]$ indicates all real numbers x for which $a \leq x \leq b$. Closed intervals include their endpoints.

An *open interval* (a, b) indicates all real numbers x for which $a < x < b$. Open intervals exclude their endpoints.

Half-open (or equivalently half-closed) intervals are represented by
$[a, b)$ which indicates all real numbers x for which $a \leq x < b$ or
$(a, b]$ which indicates all real numbers x for which $a < x \leq b$.

For example, if the domain is values of n greater than or equal to 50 and less than or equal to 100, then

$$\begin{aligned} \text{domain} &= \text{all } n \text{ values with } 50 \leq n \leq 100 \\ &= [50, 100] \end{aligned}$$

If the domain is values of n greater than 50 and less than 100, then

$$\begin{aligned} \text{domain} &= \text{all } n \text{ values with } 50 < n < 100 \\ &= \text{interval } (50, 100) \end{aligned}$$

If we want to exclude 50 but include 100 as part of the domain, we would represent the interval as $(50, 100]$. The interval can be displayed on the real number line as:

In general, a hollow dot indicates exclusion and a solid dot inclusion.
Note: Since the notation (a, b) can also mean the coordinates of a point, we will say the *interval* (a, b) when we want to refer to an interval.

EXAMPLE 7 **Finding the domain and range from a graph**

The graph in Figure 1.24 shows the water level of the tides in Pensacola, Florida, over a 24-hour period. Are the Pensacola tides a function of the time of day? If so, identify the independent and dependent variables. Use interval notation to describe the domain and range of this function.

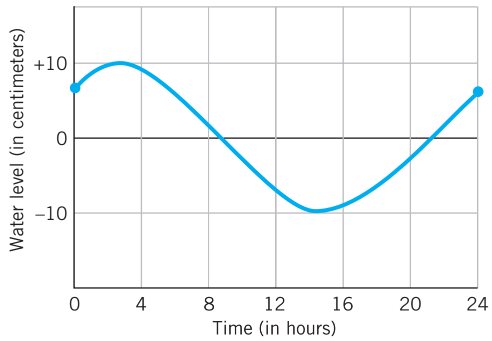


Figure 1.24 Diurnal tides in a 24-hour period in Pensacola, Florida.
Source: Adapted from Fig. 8.2 in *Oceanography: An Introduction to the Planet Oceanus,* by Paul R. Pinet. Copyright © 1992 by West Publishing Company, St. Paul, MN. All rights reserved.

SOLUTION The Pensacola tides are a function of the time of day since the graph passes the vertical line test. The independent variable is time, and the dependent variable is water level. The domain is from 0 to 24 hours, and the range is from about −10 to +10 centimeters. Using interval notation:

$$\text{domain} = [0, 24]$$
$$\text{range} = [-10, 10]$$

When are there restrictions on the domain and range?

When specifying the domain and range of a function, we need to consider whether the function is undefined for any values. For example, for the function

$$y = \frac{1}{x}$$

the expression $1/x$ is undefined when $x = 0$. For any other value for x, the function is defined. Thus,

the domain is all real numbers except 0

> **SOMETHING TO THINK ABOUT**
>
> *What happens to the value of $y = \frac{1}{x}$ as $x \to 0$?*

To find the range, we need to determine the possible output values for y. Sometimes it is easier to find the y values that are not possible. In this case, y can't equal zero. Why? Our rule says to take 1 and divide by x, but it is impossible to divide 1 by a real number in our domain and get zero as a result. Thus,

the range is all real numbers except 0

The interval (−infinity, +infinity) or $(-\infty, \infty)$ represents all real numbers. To represent all real numbers except 0 using interval notation, we use the union symbol, $\cup$, of two intervals:

$$(-\infty, 0) \cup (0, \infty)$$

Using the graph of the function in Figure 1.25 we can check to see if our domain and range are reasonable. The graph suggests that x comes very close to 0, but does not equal 0. The same is true for y.

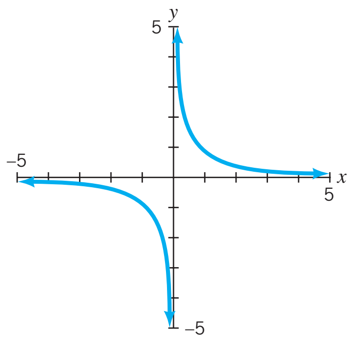


Figure 1.25 Graph of $y = \frac{1}{x}$.

Another expression that is undefined is $\sqrt{\text{negative number}}$, if we restrict ourselves to the real number system.

EXAMPLE 8 **Finding restrictions on the domain and range**

Specify the domain and range for each of the following functions:

a. $y = \sqrt{x - 4}$ **b.** $y = \frac{1}{x - 2}$

SOLUTION

a. First, we ask ourselves, "Is the domain restricted?" In this case, the answer is yes. The expression $\sqrt{x - 4}$ is not defined for negative values, so we must have the expression $x - 4 \geq 0 \Rightarrow x \geq 4$. Corresponding y values must be ≥ 0. So,

Domain: all real numbers ≥ 4
Range: all real numbers ≥ 0.

Using interval notation, we get

Domain: $[4, +\infty)$
Range: $[0, +\infty)$

b. The domain of $y = \frac{1}{x - 2}$ is also restricted. In this case, the denominator cannot equal 0 since $\frac{1}{0}$ is not defined. This means the expression $x - 2 \neq 0 \Rightarrow x \neq 2$. The range is also restricted. It is not possible for $y = 0$, since 1 divided by any real number is never 0. So,

Domain: all real numbers except 2
Range: all real numbers except 0

Using interval notation, we get

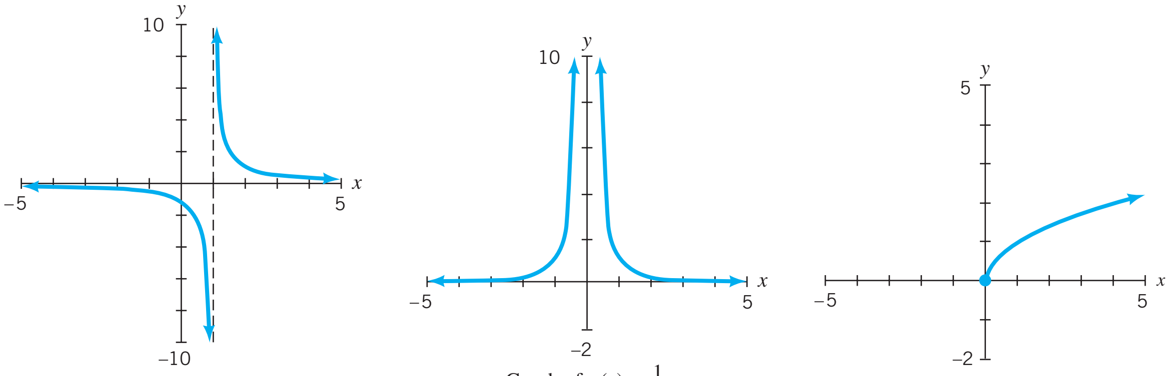
Domain: $(-\infty, 2) \cup (2, +\infty)$
Range: $(-\infty, 0) \cup (0, +\infty)$

Two Cases Where the Domain and Range Are Restricted

$y = \frac{1}{x}$	Domain: Range:	all real numbers except 0. all real numbers except 0
$y = \sqrt{x}$	Domain: Range:	all real numbers ≥ 0 all real numbers ≥ 0

EXAMPLE 9 **When is a function undefined?**

Match each function graph in Figure 1.26 with the appropriate domain and range listed in parts (a) to (e). [*Note:* The dotted line in the graph of $f(x)$ is not part of the function.]



Graph of $f(x) = \frac{1}{x-1}$ Graph of $g(x) = \frac{1}{x^2}$ Graph of $h(x) = \sqrt{x}$

Figure 1.26 Graphs for three functions.

a. Domain: $[0, \infty)$ Range: $[0, \infty)$

b. Domain: $(0, \infty)$ Range: $(0, \infty)$

c. Domain: $(-\infty, 1) \cup (1, \infty)$ Range: $(-\infty, 1) \cup (1, \infty)$

d. Domain: $(-\infty, 1) \cup (1, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$

e. Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(0, \infty)$

SOLUTION $f(x) = \frac{1}{x-1}$ matches with (d)

$g(x) = \frac{1}{x^2}$ matches with (e)

$h(x) = \sqrt{x}$ matches with (a)

Algebra Aerobics 1.4b

1. Express each of the following using interval notation.

a. $x > 2$

b. $4 \le x < 20$

c. $t \le 0$ or $t > 500$

2. Express the given interval as an inequality.

a. $[-3, 10)$

b. $(-2.5, 6.8]$

c. $(-\infty, 5] \cup [12, \infty)$

3. Express each of the following statements in interval notation.

a. Harry's GPA is at least 2.5 but at most 3.6.

b. A good hitter has a batting average of at least 0.333.

c. Starting annual salary at a position is anything from \$35,000 to \$50,000 depending upon experience.

In Problems 4–8 solve for y in terms of x. Determine if y is a function of x. If it is, rewrite using $f(x)$ notation and determine the domain and range.

4. $2(x + 1) + 3y = 5$

5. $x + 2y = 3x - 4$

6. $y = \sqrt{x}$

7. $2xy = 6$

8. $\frac{x}{2} + \frac{y}{3} = 1$

9. Find values of x for which the function is undefined, and determine the domain and range.

$$f(x) = \frac{x+1}{x+5}, \quad g(x) = \frac{1}{x+1}, \quad h(x) = \sqrt{x - 10}$$

Exercises for Section 1.4

A graphing program is required for Exercise 6.

1. Given $T(x) = x^2 - 3x + 2$, evaluate $T(0)$, $T(-1)$, $T(1)$, and $T(-5)$.

2. Given $f(x) = \frac{x}{x-1}$, evaluate $f(0)$, $f(-1)$, $f(1)$, $f(20)$, and $f(100)$.

3. Assume that for persons who earn less than \$20,000 a year, income tax is 16% of their income.

a. Generate a formula that describes income tax in terms of income for people earning less than \$20,000 a year.

b. What are you treating as the independent variable? The dependent variable?

c. Does your formula represent a function? Explain.

d. If it is a function, what is the domain? The range?

4. Suppose that the price of gasoline is \$3.09 per gallon.

a. Generate a formula that describes the cost, C, of buying gas as a function of the number of gallons of gasoline, G, purchased.

b. What is the independent variable? The dependent variable?

c. Does your formula represent a function? Explain.

d. If it is a function, what is the domain? The range?

e. Generate a small table of values and a graph.

5. The cost of driving a car to work is estimated to be \$2.00 in tolls plus 32 cents per mile. Write an equation for computing the total cost C of driving M miles to work. Does your equation represent a function? What is the independent variable? What is the dependent variable? Generate a table of values and then graph the equation.

6. (Graphing program required.) For each equation, write the equivalent equation that expresses y in terms of x. Use technology to graph each function and then estimate its domain and range.

a. $3x + 5x - y = 3y$ **c.** $x(x - 1) + y = 2x - 5$

b. $3x(5 - x) = x - y$ **d.** $2(y - 1) = y + 5x(x + 1)$

7. If $f(x) = x^2 - x + 2$, find:

a. $f(2)$ **b.** $f(-1)$ **c.** $f(0)$ **d.** $f(-5)$

8. If $g(x) = 2x + 3$, evaluate $g(0)$, $g(1)$, and $g(-1)$.

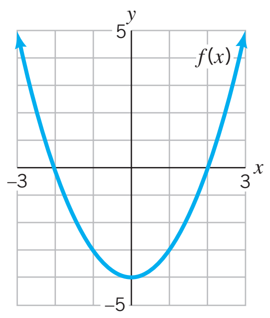
9. Look at the accompanying table.

a. Find $p(-4)$, $p(5)$, and $p(1)$.

b. For what value(s) of n does $p(n) = 2$?

n	−4	−3	−2	−1	0	1	2	3	4	5
$p(n)$	0.063	0.125	0.25	0.5	1	2	4	8	16	32

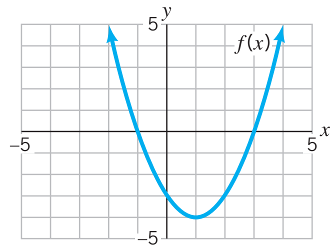
10. Consider the function $y = f(x)$ graphed in the accompanying figure.



a. Find $f(-3)$, $f(0)$, $f(1)$, and $f(2.5)$.

b. Find two values of x such that $f(x) = 0$.

11. From the accompanying graph of $y = f(x)$:



a. Find $f(-2)$, $f(-1)$, $f(0)$, and $f(1)$.

b. Find two values of x for which $f(x) = -3$.

c. Estimate the range of f. Assume that the arms of the graph extend upward indefinitely.

12. Find $f(3)$, if it exists, for each of the following functions:

a. $f(x) = (x - 3)^2$ **c.** $f(x) = \dfrac{x + 1}{x - 3}$

b. $f(x) = \dfrac{1}{x}$ **d.** $f(x) = \dfrac{2x}{x - 1}$

Determine the domain for each function.

13. If $f(x) = (2x - 1)^2$, evaluate $f(0)$, $f(1)$, and $f(-2)$.

14. Find the domain for each of the following functions:

a. $f(x) = 300.4 + 3.2x$ **d.** $k(x) = 3$

b. $g(x) = \dfrac{5 - 2x}{2}$ **e.** $f(x) = x^2 + 3$

c. $j(x) = \dfrac{1}{x + 1}$

15. Given $f(x) = 1 - 0.5x$ and $g(x) = x^2 + 1$, evaluate:

a. $f(0)$, $g(0)$ **c.** $f(2)$, $f(1)$

b. $f(-2)$, $g(-3)$ **d.** $f(3)$, $g(3)$

16. Each of the following functions has a restricted domain and range. Find the domain and range for each function and explain why the restrictions occur.

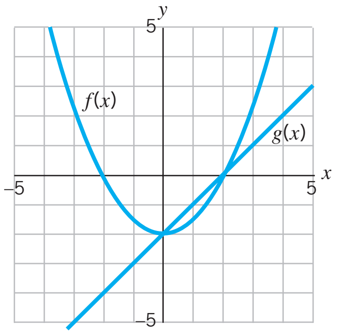
a. $f(x) = \dfrac{3}{x + 2}$

b. $g(x) = \sqrt{x - 5}$

c. $h(x) = -\dfrac{1}{2x - 3}$

d. $k(x) = \dfrac{1}{x^2 - 4}$

e. $l(x) = \sqrt{x + 3}$



17. For the functions $f(x)$ and $g(x)$ shown on the accompanying graph, find the values of x that make the following true.

a. $f(x) = 0$ **b.** $g(x) = 0$ **c.** $f(x) = g(x)$

18. Determine the domain of each of the following functions. Explain your answers.

$f(x) = 4$ $g(x) = 3x + 5$ $h(x) = \dfrac{x - 1}{x - 2}$

$F(x) = x^2 - 4$ $G(x) = \dfrac{x - 1}{x^2 - 4}$ $H(x) = \sqrt{x - 2}$

1.5 *Visualizing Functions*

In this section we return to the question: How does change in one variable affect change in another variable? Graphs are one of the easiest ways to recognize change. We start with three basic questions:

Is There a Maximum or Minimum Value?

If a function has a *maximum* (or *minimum*) value, then it appears as the highest point (or lowest point) on its graph.

EXAMPLE 1 Determine if each function in Figure 1.27 has a maximum or minimum, then estimate its value.

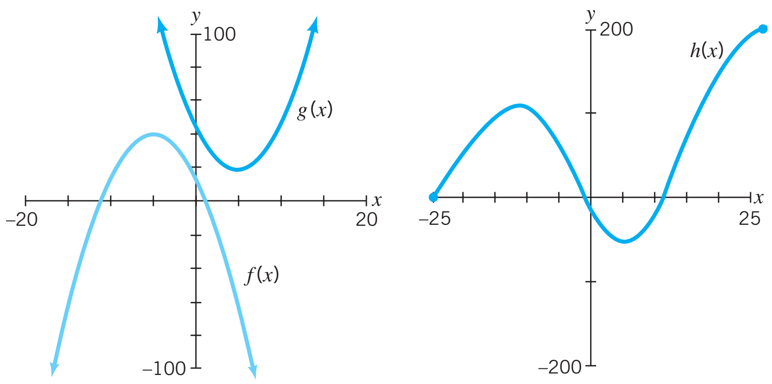


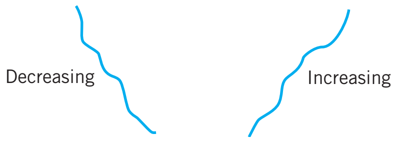
Figure 1.27 Graphs of $f(x)$, $g(x)$, and $h(x)$.

SOLUTION The function $f(x)$ in Figure 1.27 appears to have a maximum value of 40 when $x = -5$ but has no minimum value since both arms of the function extend indefinitely downward.

The function $g(x)$ appears to have a minimum value of 20 when $x = 5$, but no maximum value since both arms of the function extend indefinitely upward.

The function $h(x)$ appears to have a maximum value of 200, which occurs when $x = 25$, and a minimum value of -50, when $x = 5$.

Is the Function Increasing or Decreasing?



A function *f* is *decreasing* over a specified interval if the values of $f(x)$ decrease as x increases over the interval. A function *f* is *increasing* over a specified interval if the values of $f(x)$ increase as x increases over the interval.

The graph of an increasing function climbs as we move from left to right. The graph of a decreasing function falls as we move from left to right.

EXAMPLE 2

Increasing and decreasing production

Figure 1.28 shows a 75-year history of annual natural gas production in the United States. Create a 60-second summary about gas production from 1930 to 2005 in the U.S.

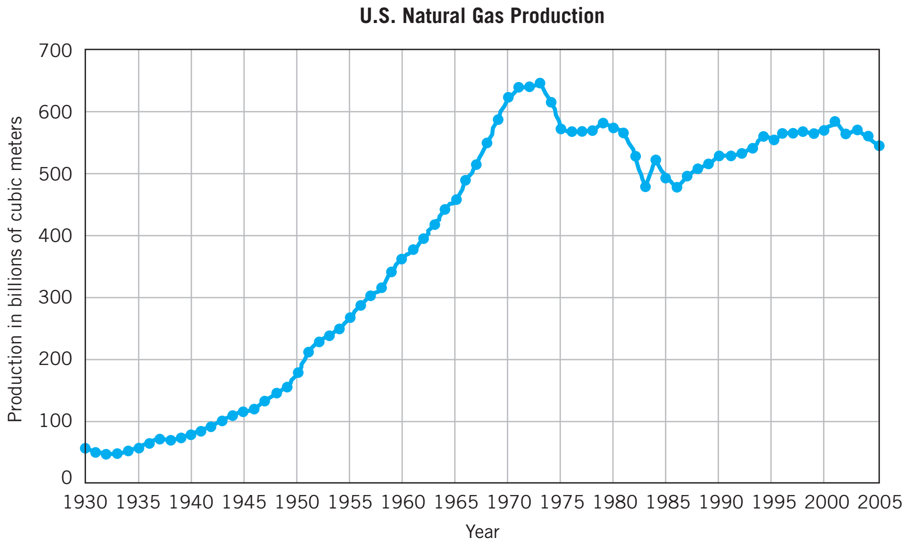


Figure 1.28

Source: Wikipedia. *http://www.answers.com/topic/natural-gas-prices.*

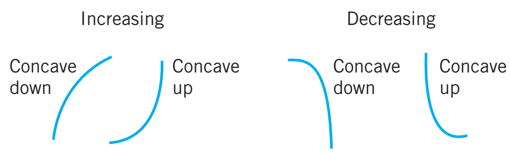
SOLUTION

Natural gas production in the United States increased from approximately 50 billion cubic meters in 1930 to a high of approximately 650 billion cubic meters in 1973. For the next 10 years production generally decreased to under 500 billion cubic meters in 1983. Between 1983 and 2005, annual production oscillated between approximately 500 and 600 billion cubic meters, with production at approximately 550 billion in 2005.

Is the Graph Concave Up or Concave Down?

What does the concavity of a graph mean? The graph of a function is *concave up* if it bends upward and it is *concave down* if it bends downward.

Concavity is independent of whether the function is increasing or decreasing.



EXAMPLE 3 **Graphs are not necessarily pictures of events**[12]

The graph in Figure 1.29 shows the speed of a roller coaster car as a function of time.

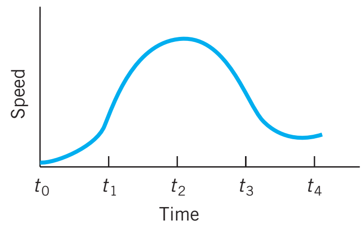


Figure 1.29

a. Describe how the speed of the roller coaster car changes over time. Describe the changes in the graph as the speed changes over time.

b. Draw a picture of a possible track for this roller coaster.

SOLUTION

a. The speed of the roller coaster car increases from t_0 to t_2, reaching a maximum for this part of the ride at t_2. The speed decreases from t_2 to t_4.

The graph of speed versus time is concave up and increasing from t_0 to t_1 and then concave down and increasing from t_1 to t_2. From t_2 to t_3 the graph is concave down and decreasing, and from t_3 to t_4 it is concave up and decreasing.

b. A picture for a possible track of the roller coaster is shown in Figure 1.30. Notice how the track is an upside-down picture of the graph of speed versus time. (When the roller coaster car goes down the speed increases, and when the roller coaster car goes up, the speed decreases.)

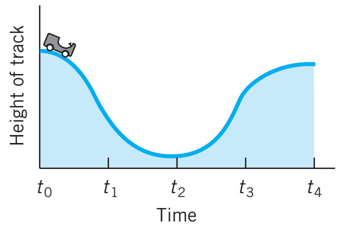


Figure 1.30

EXAMPLE 4 **Growth patterns**

Figure 1.31 shows the growth patterns for three areas in Virginia. Compare the differences in growth for these areas between 1900 and 2005.

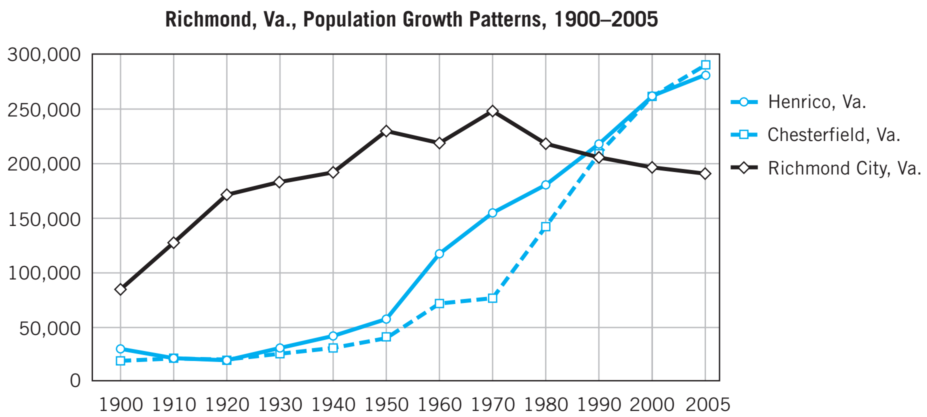


Figure 1.31

Source: www.savethebay.org/land/images.

[12]Example 3 is adapted from Shell Centre for Mathematical Education, *The Language of Functions and Graphs.* Manchester, England: University of Nottingham, 1985.

SOLUTION From 1900 to about 1988, Richmond City consistently had the largest population; however, after about 1950 Henrico and Chesterfield counties were growing faster than Richmond City. By 1990 all three areas had populations of about 200,000. After 1990, Henrico's and Chesterfield's populations continued to grow and Richmond's continued to decline.

Getting the Big Idea

We now have the basic vocabulary for describing a function's behavior. Think of a function as telling a story. We want to decipher not just the individual words, but the overall plot. In each situation we should ask, What is really happening here? What do the words tell us about the shape of the graph? What does the graph tell us about the underlying phenomenon?

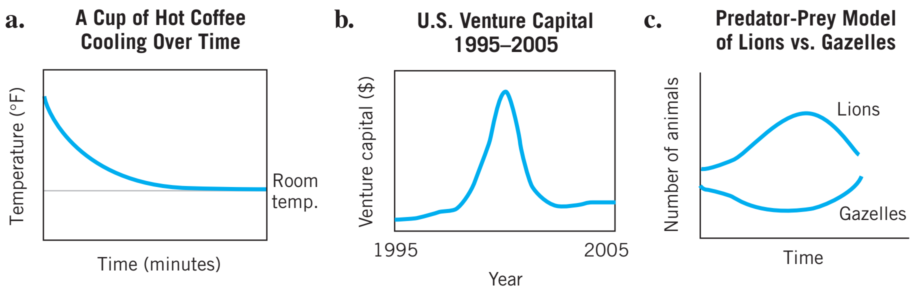
EXAMPLE 5 Generate a rough sketch of each of the following situations.

a. A cup of hot coffee cooling.

b. U.S. venture capital (money provided by investment companies to business start-ups) increased modestly but steadily in the mid-1990s, soared during the "dotcom bubble" (in the late 1990s), with a high in 2000, and then suffered a drastic decrease back to pre-dotcom levels.

c. Using a simple predator-prey model: initially as the number of lions (the predators) increases, the number of gazelles (their prey) decreases. When there are not enough gazelles to feed all the lions, the number of lions decreases and the number of gazelles starts to increase.

SOLUTION



EXAMPLE 6 You are a TV journalist. Summarize for your viewers the essence of each of the graphs in Figures 1.32 and 1.33.

a. *Note:* The vertical scale is used in two different ways on this graph from the U.S. Bureau of the Census.

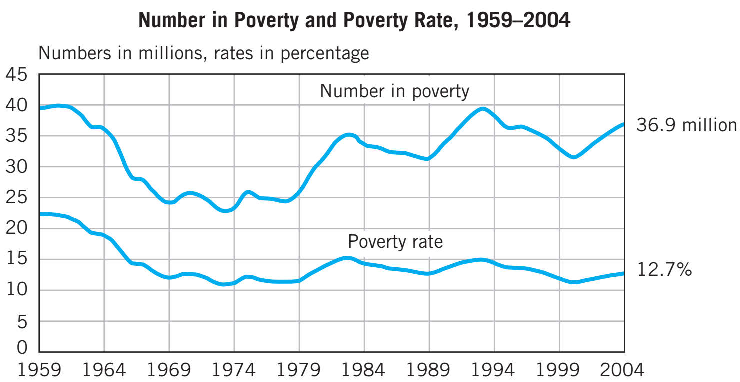


Figure 1.32

Source: U.S. Bureau of the Census *www.census.gov/compendia/statab/incom_expenditures_wealth/household_incom.*

b.

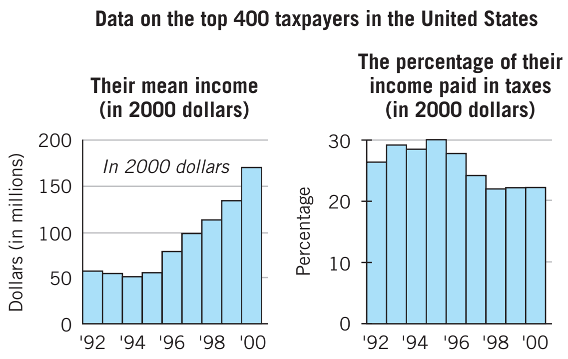


Figure 1.33
Source: The New York Times, June 26, 2003.

SOLUTION

a. In 1959, according to the U.S. Bureau of the Census, there were about 40 million Americans in poverty, representing almost 23% of the population. Between 1959 and 1974 both the number and the percentage of Americans in poverty decreased to a low of about 23 million and 12%, respectively. From 1974 to 2004, the percentage in poverty continued to hover between 12% and 15%. During the same time period the number in poverty vacillated, but the overall trend was an increase to about 37 million in 2004.

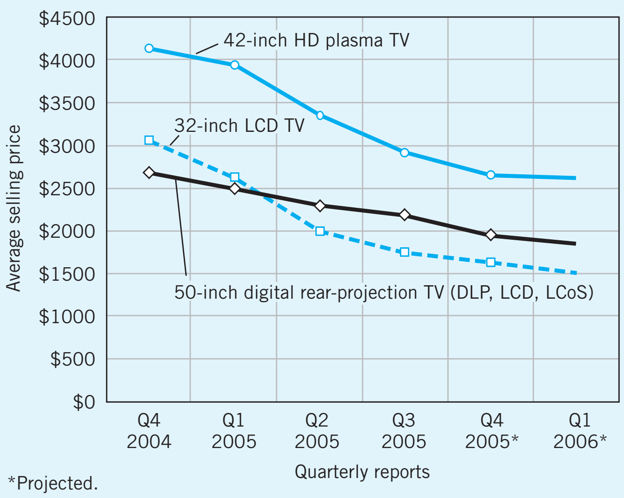
b. From 1994 to 2000 the mean income of the top 400 taxpayers grew steadily, but overall the percentage of their income that went toward taxes decreased.

We will spend the rest of this text examining patterns in functions and their graphs. We'll study "families of functions"—linear, exponential, logarithmic, power, and polynomial—that will provide mathematical tools for describing the world around us.

Algebra Aerobics 1.5

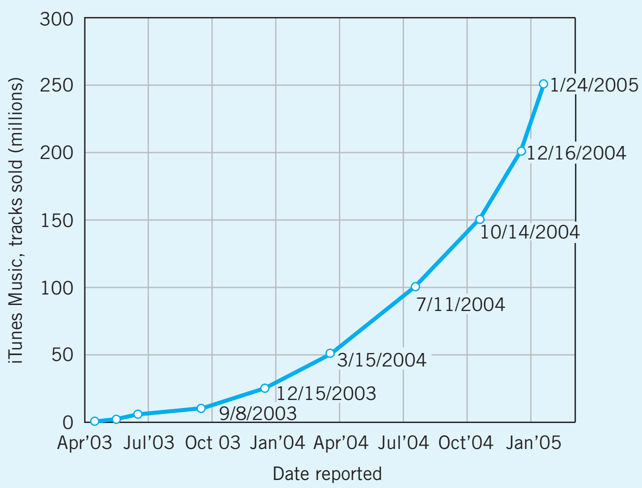
1. Create a title for each of the following graphs.

a.



*Projected.
Source: Displaysearch.

b.



Source: Displaysearch.

2. Use Figure 1.34 to answer the following questions.

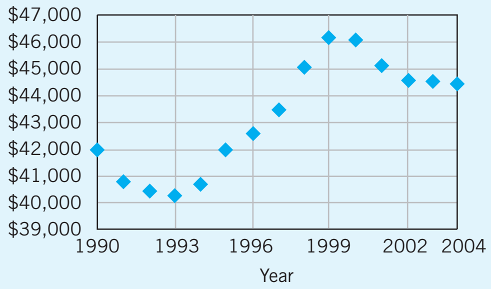


Figure 1.34 Median household income adjusted for inflation (in constant 2004 dollars).

a. Estimate the maximum value for median household income during the time period represented on the graph. In what year does the maximum occur? What are the approximate coordinates at the maximum point?

b. What is the minimum value for median household income? In what year does this occur? What are the coordinates of this point?

c. Describe the changes in median household income from 1990 to 2004.

3. Choose the "best" graph in Figure 1.35 to describe the following situation. Speed (S) is on the vertical axis and time (t) is on the horizontal axis.

A child in a playground tentatively climbs the steps of a large slide, first at a steady pace, then gradually slowing down until she reaches the top, where she stops to rest before sliding down.

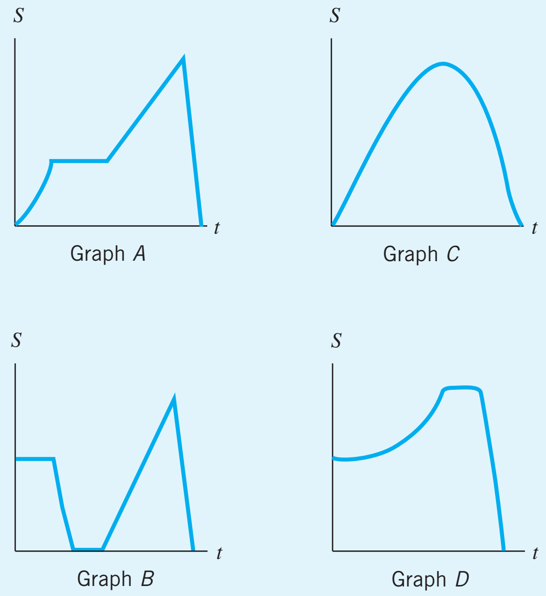


Figure 1.35

4. Generate a rough sketch of the following situation. U.S. AIDS cases increased dramatically, reaching an all-time high for a relatively short period, and then consistently decreased, until a recent small increase.

5. Sketch a graph for each of the following characteristics, and then indicate with an arrow which arm of the graph is increasing and which is decreasing.

a. Concave up with a minimum point at $(-2, 1)$.

b. Concave down with a maximum point at $(3, -2)$.

Exercises for Section 1.5

A graphing program is required for Exercises 12, 13, 20, 27, and 29.

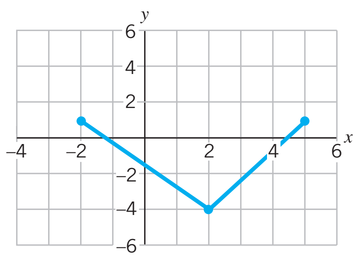
1. Identify the graph (A or B) that

a. Increases for $1 < x < 3$

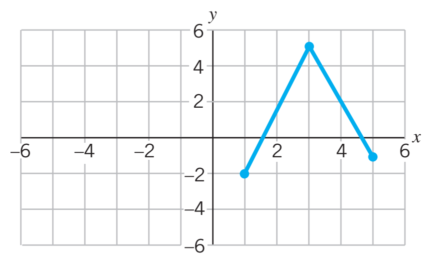
b. Increases for $2 < x < 5$

c. Decreases for $-2 < x < 2$

d. Decreases for $3 < x < 5$



Graph *A*



Graph *B*

2. The Federal Reserve is the central bank of the United States that sets monetary policy. The Federal Reserve oversees money supply, interest rates, and credit with the goal of keeping the U.S. economy and currency stable. The federal funds rate is the interest rate that banks with excess reserves at a Federal Reserve district bank charge other banks that need overnight loans. Look at the accompanying graphs for the time period between 2000 and 2003.

a. Describe the overall trends for this time period for the federal funds rate.

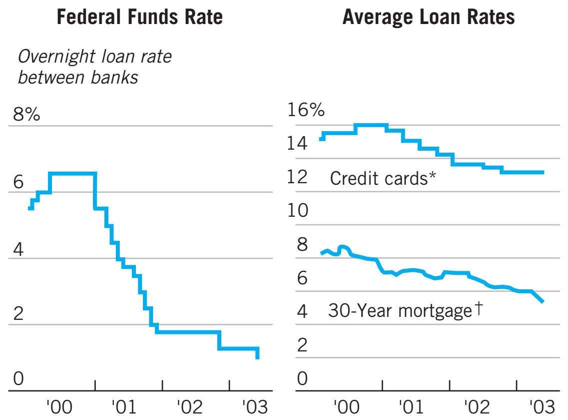
b. Describe the overall trends for this time period for credit card rates.

c. Describe the overall trends for this time period for the 30-year mortgage rates.

d. Estimate the maximum federal funds rate for this time period. When did it occur?

e. Approximate the minimum federal funds rate for this time period. When did it occur?

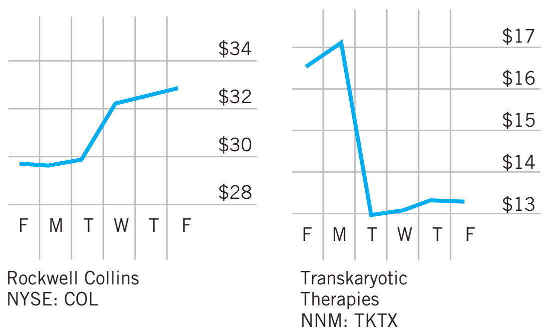
f. Write a topic sentence that compares the federal funds rate with the consumer loan rates for credit cards and mortgages for this time period.



* Average annual percentage rate for all credit card accounts in banks in the United States that respond to the Federal Reserve's Survey.
† National average for a 30-year fixed rate on a single-family home.

Source: www.federalreserve.gov

3. The accompanying graphs show the price of shares of stock of two companies over the one-week period January 9 to 16, 2004. Describe the changes in price over this time period.



Source: The New York Times, Jan. 17, 2004.

4. For each of the following functions,

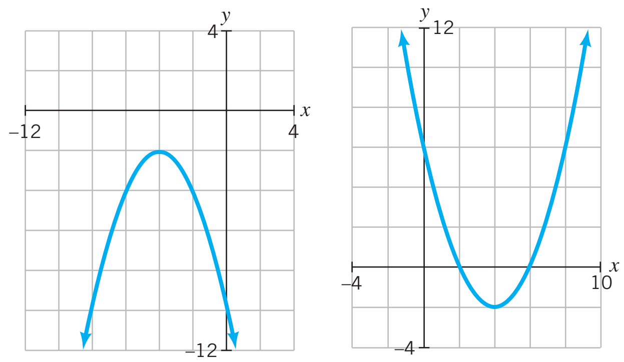
a. Over which interval is the function decreasing?

b. Over which interval is the function increasing?

c. Does the function appear to have a minimum? If so, where?

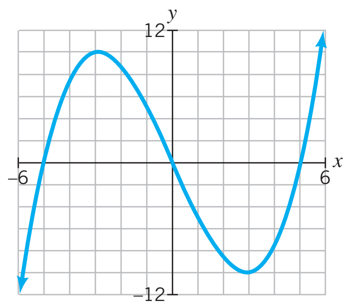
d. Does the function appear to have a maximum? If so, where?

e. Describe the concavity.



Graph *A* Graph *B*

5. For the following function,



a. Over which interval(s) is the function positive?

b. Over which interval(s) is the function negative?

c. Over which interval(s) is the function decreasing?

d. Over which interval(s) is the function increasing?

e. Does the function appear to have a minimum? If so, where?

f. Does the function appear to have a maximum? If so, where?

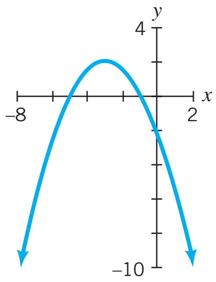
6. Choose which graph(s) at the top of the next page match the description: As x increases, the graph is:

a. Increasing and concave up

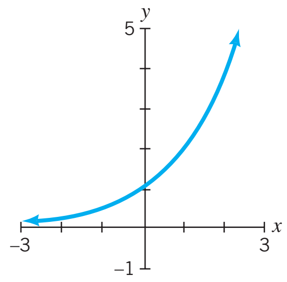
b. Increasing and concave down

c. Concave up and appears to have a minimum value at $(-3, 2)$

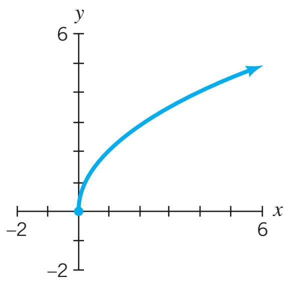
d. Concave down and appears to have a maximum value at $(-3, 2)$



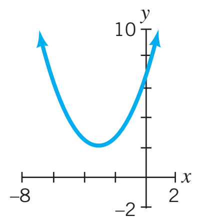
Graph *A*



Graph *C*



Graph *B*



Graph *D*

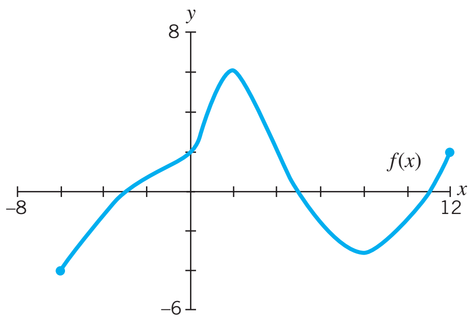
7. Examine each of the graphs in Exercise 6. Assume each graph describes a function $f(x)$. The arrows indicate that the graph extends indefinitely in the direction shown.

a. For each function estimate the domain and range.

b. For each function estimate the x interval(s) where $f(x) > 0$.

c. For each function estimate the x interval(s) where $f(x) < 0$.

8. Look at the graph of $y = f(x)$ in the accompanying figure.



a. Find $f(-6)$, $f(2)$, and $f(12)$.

b. Find $f(0)$.

c. For what values of x is $f(x) = 0$?

d. Is $f(8) > 0$ or is $f(8) < 0$?

e. How many times would the line $y = 1$ intersect the graph of $f(x)$?

f. What are the domain and range of $f(x)$?

g. What is the maximum? The minimum?

9. Use the graph of Exercise 8 to answer the following questions about $f(x)$.

a. Over which interval(s) is $f(x) < 0$?

b. Over which interval(s) is $f(x) > 0$?

c. Over which interval(s) is $f(x)$ increasing?

d. Over which interval(s) is $f(x)$ decreasing?

e. How would you describe the concavity of $f(x)$ over the interval (0, 5) for x? Over (5, 8) for x?

f. Find a value for x when $f(x) = 4$.

g. $f(-8) = ?$

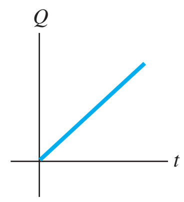
10. Match each graph with the best description of the function. Assume that the horizontal axis represents time, t.

i. The height of a ball thrown straight up is a function of time.

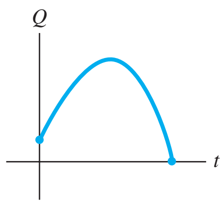
ii. The distance a truck travels at a constant speed is a function of time.

iii. The number of daylight hours is a function of the day of the year.

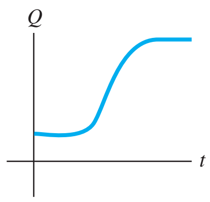
iv. The temperature of a pie baking in an oven is a function of time.



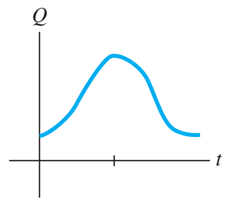
Graph *A*



Graph *C*



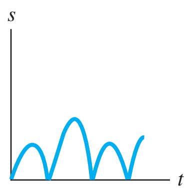
Graph *B*



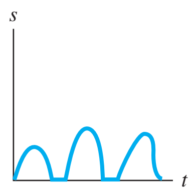
Graph *D*

11. Choose the best graph to describe the situation.

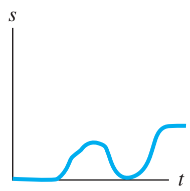
a. A student in a large urban area takes a local bus whose route ends at the college. Time, t, is on the horizontal axis and speed, s, is on the vertical axis.



Graph *A*

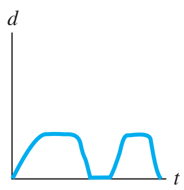


Graph *B*

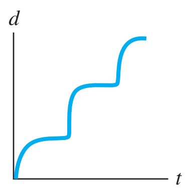


Graph *C*

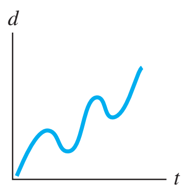
b. What graph depicts the total distance the student traveled in the bus? Time, t, is on the horizontal axis and distance, d, is on the vertical axis.



Graph *D*



Graph *E*



Graph *F*

12. (Graphing program required.) Use technology to graph each function. Then approximate the x intervals where the function is concave up, and then where it is concave down

a. $f(x) = x^3$ **b.** $g(x) = x^3 - 4x$

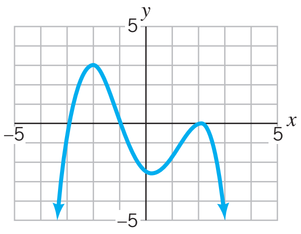
13. (Graphing program required.) Use technology to graph each function. Then approximate the x intervals where the function is concave up, and then where it is concave down.

a. $h(x) = x^4$

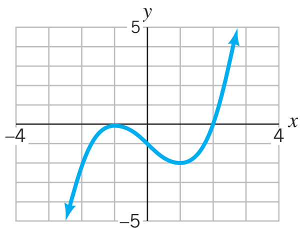
b. $k(x) = x^4 - 24x + 50$ (*Hint:* Use an interval of $[-5, 5]$ for x and $[0, 200]$ for y.)

14. a. In the accompanying graphs, estimate the coordinates of the maximum and minimum points (if any) of the function.

b. Specify the interval(s) over which each function is increasing.

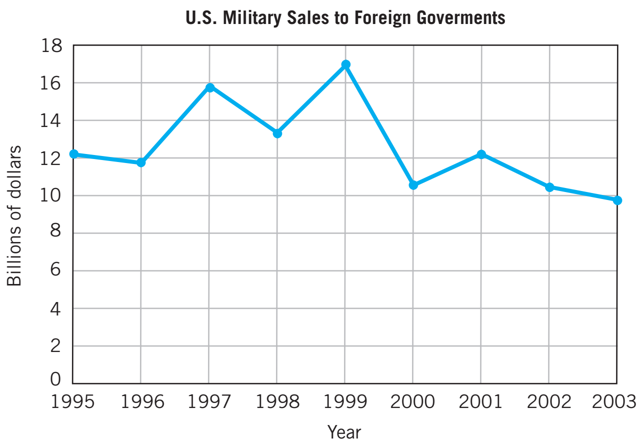


Graph *A*



Graph *B*

15. Consider the accompanying graph of U.S. military sales to foreign governments from 1995 to 2003.



Source: U.S. Department of Defense, Defense Security Cooperation Agency, *DSCA Data and Statistics.*

a. Between what years did sales increase?

b. Between what years did sales decrease?

c. Estimate the maximum value for sales.

d. Estimate the minimum value for sales.

16. Sketch a plausible graph for each of the following and label the axes.

a. The amount of snow in your backyard each day from December 1 to March 1.

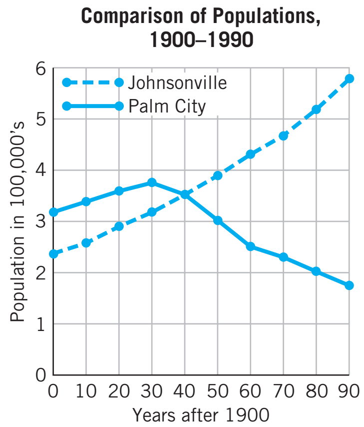
b. The temperature during a 24-hour period in your home town during one day in July.

c. The amount of water inside your fishing boat if your boat leaks a little and your fishing partner bails out water every once in a while.

d. The total hours of daylight each day of the year.

e. The temperature of an ice-cold drink left to stand.

17. Examine the accompanying graph, which shows the populations of two towns.



a. What is the range of population size for Johnsonville? For Palm City?

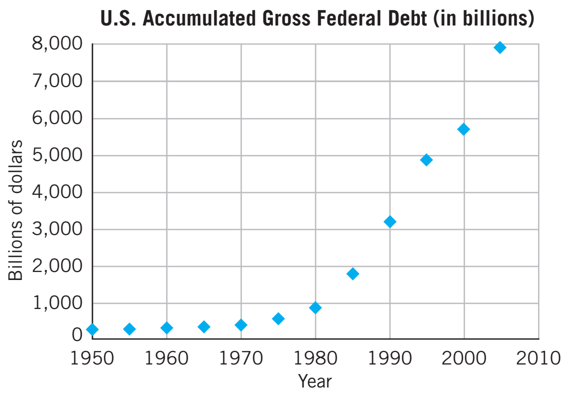
b. During what years did the population of Palm City increase?

c. During what years did the population of Palm City decrease?

d. When were the populations equal?

18. In Section 1.2 we examined the annual federal budget *surplus* or *deficit*. The federal *debt* takes into account the cumulative effect of all the deficits and surpluses for each year together with any interest or payback of principal. The accompanying graph shows the accumulated gross federal debt from 1950 to 2005. (See related Excel or graph link file FEDDEBT.) Create a topic sentence for this graph for a newspaper article.

DATA



Source: U.S. Dept. of Treasury, www.ustreas.gov

19. Consider the accompanying table.

Y	*P*
1990	\$1.4
1991	\$2.3
1992	\$0
1993	−\$0.5
1994	\$1.4
1995	\$1.2

a. Is *P* a function of *Y*?

b. What is the domain? What is the range?

c. What is the maximum value of *P*? In what year did this occur?

d. During what intervals was *P* increasing? Decreasing?

e. Now consider *P* as the independent variable and *Y* as the dependent variable. Is *Y* a function of *P*?

20. (Graphing program required.) Using technology, graph each function over the intervals $[-6, 6]$ for x and $[-20, 20]$ for y.

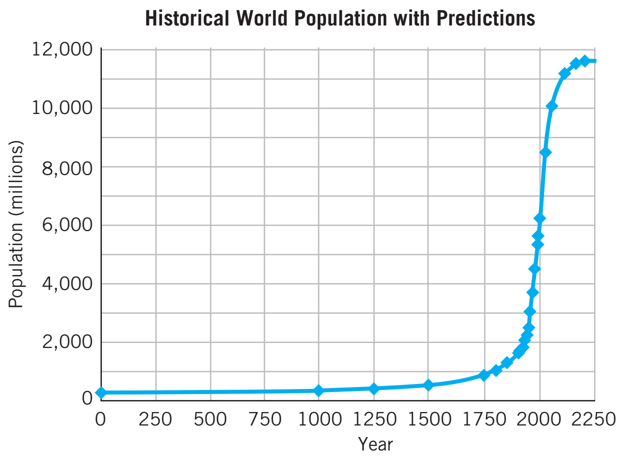
$$y_1 = x^2 - 3x + 2 \qquad y_2 = 0.5x^3 - 2x - 1$$

For each function,

a. Estimate the maximum value of y on each interval.

b. Estimate the minimum value of y on each interval.

21. The accompanying graph shows the world population over time and future predictions.



Source: www.worldonline.n1/invd/world/whist2.html

a. Over what interval does the world population show dramatic growth?

b. Does the dramatic growth slow down?

c. Write a topic sentence for a report for the United Nations.

22. Make a graph showing what you expect would be the relative ups and downs throughout the year of sales (in dollars) of

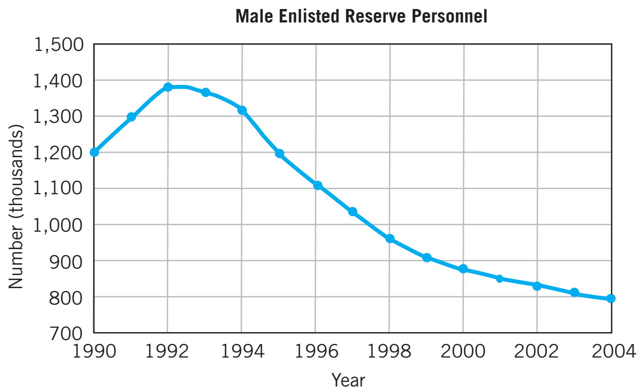
a. Turkeys

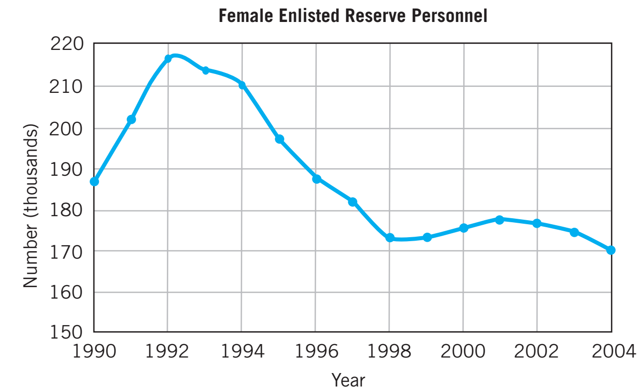
b. Candy

c. Bathing suits in your state

d. Textbooks at your school bookstore

23. Examine the graphs of military reserve enlisted personnel over the years 1990 to 2004.





Source: U.S. Dept. of Defense, *Official Guard and Reserve Manpower Strengths and Statistics*, annual.

a. In what year(s) did female and male enlisted personnel reach a maximum?

b. What was the maximum and minimum for both men and women over the time interval [1990, 2004]?

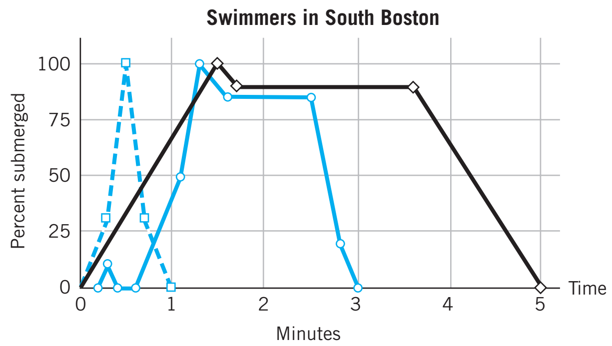
c. Describe the trends in number of female and male reserve enlisted personnel.

d. Given the current state of world affairs, what would you expect would happen to the enlistment numbers for 2004 to the present?

24. Generate a rough sketch of a graph of internal pressure vs. time for the following situation: When a soda is removed from the fridge, the internal pressure is slightly above the surrounding air pressure. With the can unopened, the internal pressure soon more than doubles, stabilizing at a level three times the surrounding air pressure.

25. A student breaks her ankle and is taken to a doctor, who puts a cast on her leg and tells her to keep the foot off the ground altogether. After 2 weeks she is given crutches and can begin to walk around more freely, but at the beginning of the third week she falls and is resigned to keeping stationary again for a while. After 6 weeks from her first fall, she is given a walking cast in which she can begin to put her foot on the ground again. She is now able to limp around using crutches. Her walking speed slowly progresses. At 12 weeks she hits a plateau and, seeing no increase in her mobility, starts physical therapy. She rapidly improves. At 16 weeks the cast is removed and she can walk freely. Make a graph of the student's mobility level during her recovery.

26. Every January 1 a hardy group called the L Street Brownies celebrates the New Year by going for a swim at the L Street Beach in South Boston. The water is always very cold, and swimmers adopt a variety of strategies for getting into it. The graph shows the progress of three different friends who join in the event, with percentage of body submerged on the vertical axis and time on the horizontal axis. Match the graphs to the descriptions below of how each of the friends manages to get completely submerged in the icy ocean.



a. Ali has done this before and confidently walks in until his head is underwater; then he puts his head out and swims around a few minutes; then he walks out.

b. Ben dashes in until the water is up to his knee, trips over a hidden rock, and falls in completely; he stands up and, since he is now totally wet, runs back out of the water.

c. Cat puts one foot in, takes it out again, and shivers. She makes up her mind to get it over with, runs until she is up to her waist, dives in, swims back as close to the water's edge as she can get, stands up, and steps out of the water.

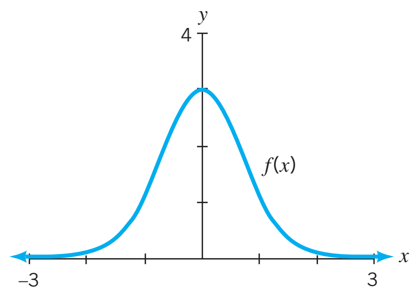
27. (Graphing program required.) Use technology to graph the following functions and then complete both sentences for each function.

$$y_1 = x^3, \qquad y_2 = x^2, \qquad y_3 = \frac{1}{x+3}, \qquad y_4 = \frac{1}{x} + 2$$

a. As x approaches positive infinity, y approaches __________.

b. As x approaches negative infinity, y approaches __________.

28. Describe the behavior of $f(x)$ in the accompanying figure over the interval $(-\infty, +\infty)$ for x, using such words as "increases," "decreases," "concavity," "maximum/minimum," and "approaches infinity."



29. (Graphing program required.) This exercise is to be done with a partner. Name the partners person #1 and person #2.

a. Person #1, using technology, graphs the function $f(x) = 0.5(x - 3)(x + 2)^2$, but does not show the graph to person #2.

b. Person #1 describes to person #2 the behavior of the graph of $f(x)$ so that he/she can sketch it on a piece of paper.

c. Switch roles; now person #2, using technology, graphs $g(x) = -0.5(x - 3)(x + 2)^2$, but does not show the graph to person #1.

d. Person #2 describes to person #1 the behavior of the graph of $g(x)$ so that he/she can sketch it on a piece of paper.

e. Compare the accuracy of the graphs and compare the shapes of the two graphs. What do $f(x)$ and $g(x)$ have in common? How do they differ?

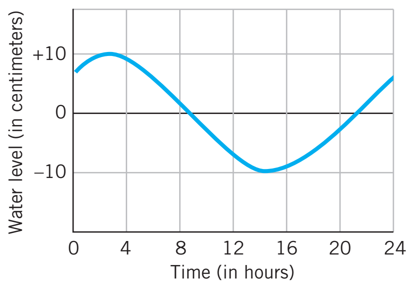
CHAPTER SUMMARY

Single-Variable Data

Single-variable data are often represented with bar charts, pie charts, and histograms.

Two-Variable Data

Graphs of two-variable data show how change in one variable affects change in the other. The accompanying graph is called a *time series* because it shows changes over time.



Equations in Two Variables

The *solutions of an equation* in two variables x and y are the ordered pairs (x, y) that make the equation a true statement. For example, $(3, -5)$ is one solution of the equation $2x + y = 1$.

The *graph of an equation* in two variables displays the set of points that are solutions to the equation.

Functions

A variable y is a *function* of a variable x if each value of x determines a unique (one and only one) value of y. Functions can be represented with words, graphs, equations, and tables.

When a set of ordered pairs represents a function, then each ordered pair is written in the form

(independent variable, dependent variable)

(input, output)

(x, y)

By convention, on the graph of a function, the independent variable is represented on the horizontal axis and the dependent variable on the vertical axis.

A graph does not represent a function if it fails the *vertical line test*. If you can draw a vertical line that crosses the graph two or more times, the graph does not represent a function.

The *domain* of a function is the set of all possible values of the independent variable.

The *range* is the set of corresponding values of the dependent variable.

Function Notation

The expression $f(x)$ means the rule f is applied to the input value x to give the output value, $f(x)$:

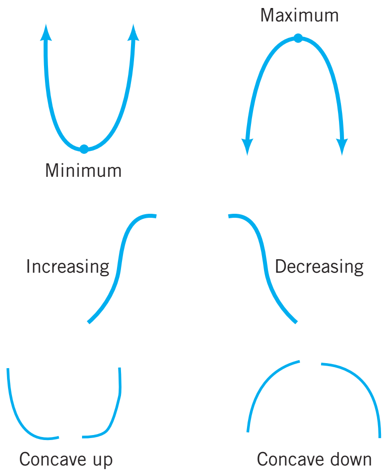
$$f(\text{input}) = \text{output}$$

For example, $f(x) = 2x + 5$ tells us f is the name of a function where the input is x, the output is $f(x)$, and the rule is to multiply the input by 2 and add 5. To evaluate this function when the input is 4, we write

$$f(4) = (2 \bullet 4) + 5 \Rightarrow f(4) = 13$$

Visualizing Functions

Graphs of functions may demonstrate the following properties:



CHECK YOUR UNDERSTANDING

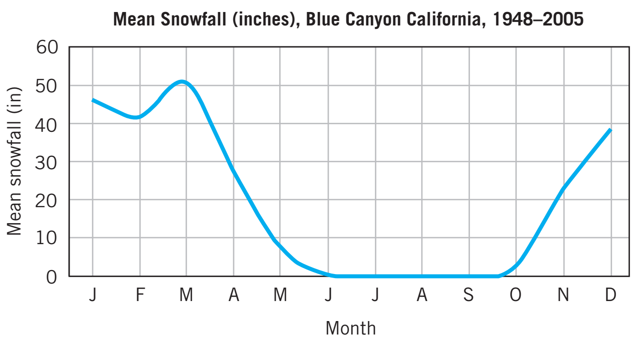
I. Is each of the statements in Problems 1–27 true or false?

1. Histograms, bar charts, and pie charts are used to graph single-variable data.

2. Means and medians, both measures of central tendency, can be used interchangeably.

3. A scatter plot is a plot of data points (x, y) for two-variable data.

Problems 4 and 5 refer to the accompanying figure.

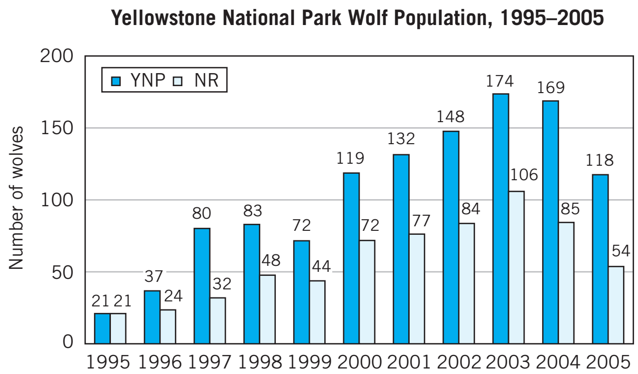


Source: Western Regional Climate Center, *www.wrcc.dri.edu.*

4. The graph shows the changes in mean snowfall for each month of the year over the years 1948–2005.

5. The maximum snowfall occurs in March, with mean snowfall of about 52 inches.

6. The accompanying figure, of the wolf population in Yellowstone National Park (YNP) and the Northern



Source: Yellowstone Wolf Project, Annual Report 2005, U.S. Department of the Interior.

Region (NR) of Yellowstone, shows that the wolf population in the Greater Yellowstone area is increasing.

7. The independent variable of a function is also called the output of the function.

8. The graph of the equation describing the sales revenue R (\$ million) as a function of the amount spent on advertising A (\$ thousand) is the set of ordered pairs (R, A) that satisfy the equation.

9. If sales revenue R is a function of advertising A, then R is the dependent variable and A is the independent variable.

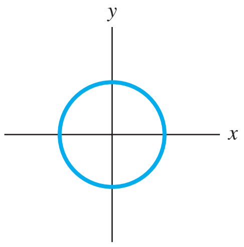
10. If $M = F(q)$, then F is a function of q.

Problems 11 and 12 refer to the function $F(q) = \dfrac{2}{3q}$.

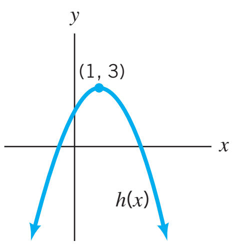
11. $F(-1) = \dfrac{-2}{3}$.

12. The domain of $F(q)$ is the set of all real numbers.

13. In the accompanying graph y is a function of x.



Problems 14–16 refer to the accompanying graph of $h(x)$.



14. The function $h(x)$ has a maximum value at (1, 3).

15. The function $h(x)$ is concave down.

16. The function $h(x)$ increases to the right of $x = 1$ and increases to the left of $x = 1$.

Problems 17 and 18 refer to the following table.

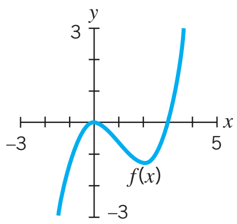
R	-1	2	0	3
S	10	8	6	4

17. The variable S is a function of R.

18. The variable S is decreasing over the domain for R.

19. A set of ordered pairs of the form (M, C) implies that M is a function of C.

Problems 20–22 refer to the accompanying graph of $f(x)$.

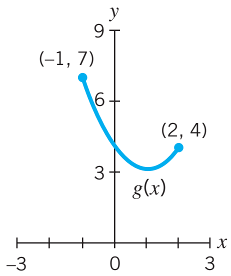


20. The function $f(x)$ has both a maximum and a minimum over the interval $[-1, 3)$.

21. The function $f(x)$ decreases over the interval (0, 2).

22. The function $f(x)$ is concave up for $x > 1$.

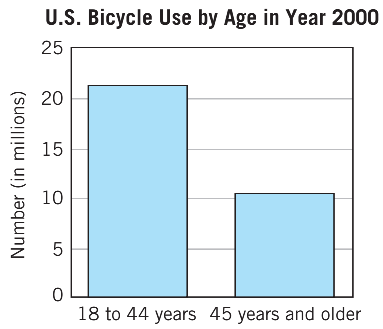
Problems 23 and 24 refer to the accompanying figure.



23. The domain of the function $g(x)$ is the interval $[-1, 2]$.

24. The range of the function $g(x)$ is the interval [4, 7].

Problems 25–27 refer to the accompanying figure.



Source: National Survey on Recreation and the Environment, *www.bts.gov*

25. The histogram describes the number of bicycle riders by age.

26. There are about twice as many bicycle riders who are 18 to 44 years old as there are riders who are 45 years or older.

27. The total number of bicycle riders is over 30 million.

II. In Problems 28–34, give an example of a graph, relationship, function, or functions with the specified properties.

28. A relationship between two variables w and z described with an equation where z is not a function of w.

29. A relationship between two variables w and z described as a table where w is a function of z but z is not a function of w.

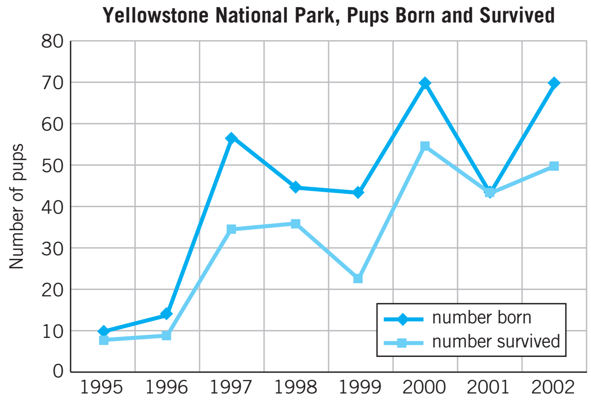
30. A graph of a function that is increasing and concave down.

31. A graph of a function that is decreasing and concave up.

32. A graph of a function that is concave up and has a minimum value at the point $(-2, 0)$.

33. A graph of a function where the domain is the set of real numbers and that has no maximum or minimum values.

34. A topic sentence describing the wolf pups that were born and survived in Yellowstone Park as shown in the accompanying graph.



Source: Yellowstone Wolf Project, Annual Report 2002, U.S. Department of the Interior.

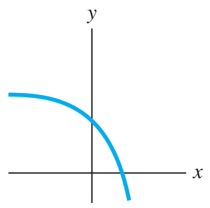
III. Is each of the statements in Problems 35–42 true or false? If a statement is true, explain how you know. If a statement is false, give a counterexample or explain why it is false.

35. A function can have either a maximum or a minimum but not both.

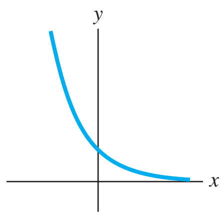
36. Neither horizontal nor vertical lines are functions.

37. A function is any relationship between two quantities.

38. The graph in the accompanying figure is decreasing and concave down.



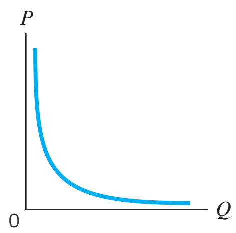
39. The graph in the accompanying figure is increasing and concave up.



40. All functions have at least one minimum value.

41. Sometimes the choice as to which variable will be the independent variable for a function is arbitrary.

42. The graph in the accompanying figure represents P as a function of Q.



CHAPTER 1 REVIEW: PUTTING IT ALL TOGETHER

1. A man weighed 160 pounds at age 20. Now, at age 60, he weighs 200 pounds.

a. What percent of his age 20 weight is his weight gain?

b. If he went on a diet and got back to 160 pounds, what percent of his age 60 weight would be his weight loss?

c. Explain why the weight gain is a different percentage than the weight loss even though it is the same number of pounds.

2. Data from the *World Health Organization Report* 2005 shows vast differences in average spending per person on health for different countries. Data on health spending from selected countries is shown here.

Country	As % of Gross Domestic Product (GDP)	$ per Capita
China	5.8	63
India	6.1	30
Iraq	1.5	11
North Korea	4.6	0.3
South Korea	5.0	532
United States	14.6	5274

a. North and South Korea spent close to the same percentage of their gross domestic product on health, but a very different amount per person (per capita) in 2005. How can this be?

b. In 2005, how many Iraqis could receive health care (at the Iraqi level of spending) for the amount the United States spent on one person? If Iraq spent the same percent of its GDP on health care as the United States, how much would they be spending per person?

c. China and India had the two largest populations in 2005, with China at 1.31 billion and India at 1.08 billion. Estimate how much each of these countries is spending on health care.

3. In April 2005 there was considerable debate in the media about whether "average" incomes had gone up or down in the United Kingdom (UK). The Institute for Fiscal Studies produced a report in which they stated that the mean household income in the UK in 2004 fell by 0.2% over the previous year and that median household income in the UK rose for the same period by 0.5%. Explain how the mean income could go down while the median income rose.

4. In 2001 there were 130,651 thousand metric tons of commercial fish caught worldwide. The chart below lists the world's 10 leading commercial fishing nations.

The 10 Leading Commercial Fishing Nations in 2001

	Fish caught (in thousands of metric tons)
China	42,579
Peru	7,996
India	5,897
Japan	5,515
United States	5,424
Indonesia	5,137
Chile	4,363
Russia	3,718
Thailand	3,657
Norway	3,198

Source: U.S. Bureau of the Census, *Statistical Abstract of the United States:* 2006.

a. What are the mean and median for this set of data?

b. China caught what percent of the commercial fish among the leading 10 nations? Of the world?

c. Will the mean and median for the 10 leading nations change if China substantially increases the amount of commercial fish it catches while other nations remain at the same level?

5. The following table shows the four leading causes of death around the world.

The Four Leading Causes of Death Worldwide

Cause	% of All Deaths
Heart disease	12.6
Stroke	9.7
Lower respiratory infections (e.g., pneumonia, emphysema, bronchitis)	6.8
HIV/AIDS	4.9

Source: World Health Organization (WHO), *The World Health Report*, 2003.

a. What is the leading cause of death worldwide?

b. Create a bar chart using these data.

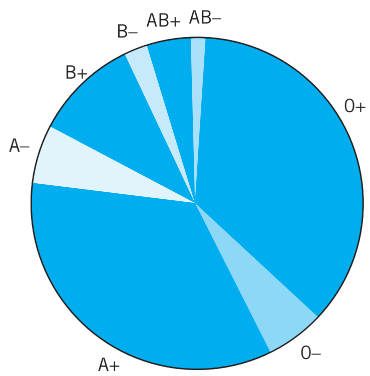
c. What percentage of worldwide deaths is not accounted for in the table? List at least two other diseases or conditions that can cause death. Could either of them account for more that 5% of all deaths worldwide?

6. Before doctors transfuse blood they must know the patient's blood type and which types of blood are compatible with that type.

a. On the following graph, types O+ and O− make up approximately 37% and 6%, respectively, of all U.S. blood types. Estimate the percentage of the population in the United States for each of the other blood types.

b. How could you determine the number of people with each blood type?

% U.S. Population by Blood Type in 2006

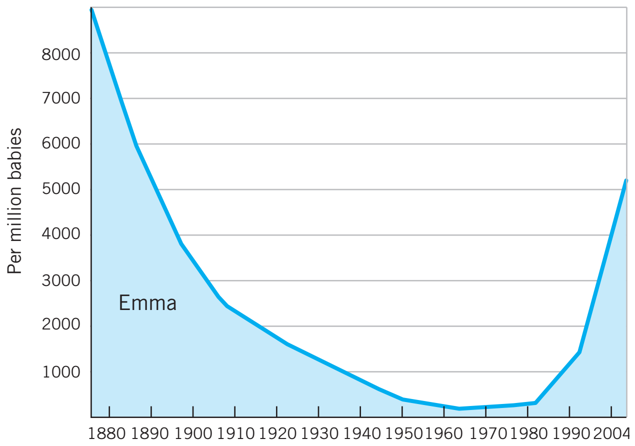


Source: http://bloodcenter.stanford.edu/about_blood/blood_types.html.

7. In a certain state, car buyers pay an excise tax of 2.5%. This means that someone who buys a car must pay the state a one-time fee of 2.5% of the car's value.
Use P to represent the price of the car and E to represent excise tax, then express E as a function of P.

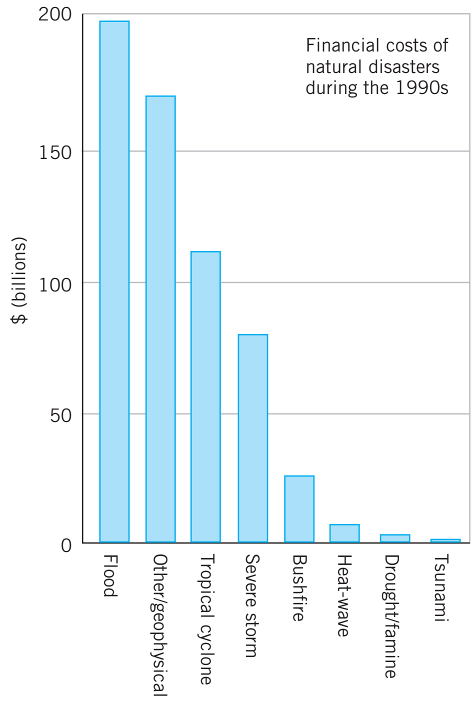
8. A waitress makes $2.50 an hour in wages and receives tips totaling 18% of the price of each meal she serves. Let H represent the hours she works, P represent the total cost of the meals she served, and E represent what she earns in a week before taxes. Write an equation expressing E in terms of H and P.

9. The following graph shows the number of babies per million babies that were named Emma in the United States between 1880 and 2004. Create a title for the graph and explain in a few sentences what is happening over time to the name Emma.



Source: thebabywizard.ivillage.com.

10. During the 1990s natural disasters worldwide took more than 666,000 lives and cost over $683 billion (US $). Using the accompanying chart, write a 60-second summary describing the worldwide financial costs of natural disasters during this period.



Source: Australian government, Department of Meteorology.

11. a. Determine whether each set of points represents a function.

i. $(6, -1), (5, -1), (8, 3), (-3, 2)$

ii. $(5, 3), (7, -1), (5, -2), (4, -3)$

iii. $(7, -3), (8, -3), (-2, -3), (9, -3)$

iv. $(4, 0), (4, -3), (4, 7), (4 - 1)$

b. Create a table of values for the variables A and B, where B is a function of A but A is not a function of B.

c. Create two graphs, one that represents a function and one that does not represent a function.

12. Find an equation that represents the relationship between x and y in each of the following tables. Specify in each case if y is a function of x.

a.

x	y
0	2
1	4
2	6
3	8
4	10

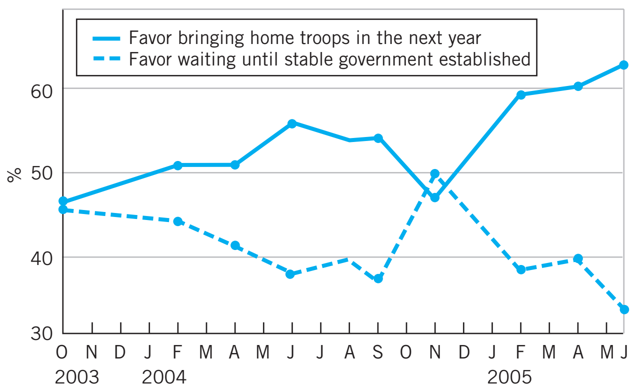
b.

x	y
0	−1
1	0
2	3
3	8
4	15

c.

x	y
0	1
1	2
2	9
3	28
4	65

13. The following graph shows changes in U.S. opinion regarding the war in Iraq over the period between October 2003 and June 2005.



Source: Harris Interactive.

a. In February 2004, what was the approximate percentage of Americans who favored bringing troops home in the next year?

b. In the period covered by this graph, when did more people favor waiting for the establishment of a stable government than favored a withdrawal in the next year?

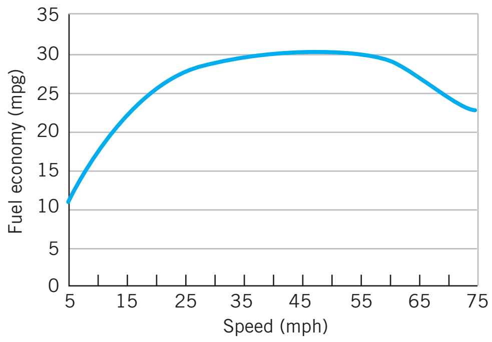
c. During which interval(s) did the amount of people in favor of waiting for a stable government increase?

d. On which date was the difference in the popularity of the two opinions the greatest? Estimate this difference.

14. The following graph shows fuel economy in relation to speed.

a. Does the graph represent a function? If not, why not? If so, what are the domain and range?

b. Describe in words how speed relates to fuel economy.



(mpg = miles per gallon of gas, mph = miles per hour)
Source: www.fueleconomy.gov.

15. In 2006 the exchange rate for the Chinese yuan and U.S. dollar was about 8 to 1; that is, you could exchange 8 yuan for 1 U.S. dollar.

a. Let y = number of Chinese yuan and d = number of U.S. dollars. Complete the following table.

d	1	2	3	4	10	20	100
y							

b. Find an equation that expresses y as a function of d. What is the independent variable? The dependent variable?

c. Complete the following table:

y	8	12	16	20	50	80	100
d							

d. Find an equation that expresses d as a function of y. What is the independent variable? The dependent variable?

16. The average number of calories burned with exercising varies by the weight of the person. The accompanying table gives the approximate calories burned per hour from disco dancing.

Number of Calories Burned/hr from Disco Dancing

Weight	100 lb. person	125 lb. person	150 lb. person	175 lb. person	200 lb. person
calories burned per hour	264	330	396	462	528

Source: http://www.fitresource.com/Fitness.

Let t be the number of hours spent exercising and C the total number of calories burned.

a. Write an equation expressing C_1 in terms of t for a 125 lb. person disco dancing and an equation for C_2 for a 175 lb. person disco dancing.

b. Do your equations in part (a) represent functions? If not, why not?

c. For the equations in part (a) that represent functions, what is the independent variable? The dependent variable? What is a reasonable domain? What is a reasonable range?

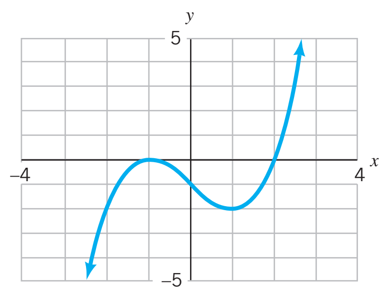
d. Generate small tables of values and graphs for C_1 and C_2. How do the two graphs compare?

e. How do you think these graphs would compare to the corresponding graphs for a 200 lb. person? Explain.

17. Sketch the graph of a function $f(t)$ with domain = [0, 10] and range = [0, 100] that satisfies all of the given conditions:

$f(0) = 100,\ f(5) = 60,\ f(10) = 0$
$f(t)$ is decreasing and concave down for interval (0, 5)
$f(t)$ is decreasing and concave up for interval (5, 10)

18. Below is the graph of $f(x)$.



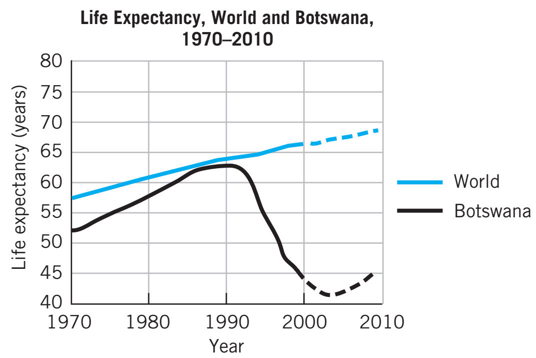
a. Determine the values for $f(-1)$, $f(0)$, $f(2)$.

b. For what value(s) of x does $f(x) = -2$?

c. Specify the interval (s) over which the function is

i. Increasing
ii. Decreasing
iii. Concave up
iv. Concave down

19. The following graph shows the life expectancy in Botswana compared with world life expectancy. The dotted lines on the graph represent projections.



Source: UN Population Division, 1999.

a. Does the graph for Botswana represent a function? If not, why not? If so, what are the domain and range?

b. Over the given time period, when did life expectancy in Botswana reach its maximum? Estimate this maximum value.

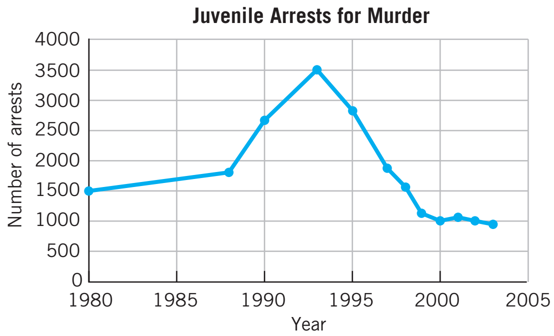
c. When was life expectancy in Botswana projected to reach its minimum? Estimate this minimum value. What might have caused the sudden drop in life expectancy?

d. Over what interval(s) was life expectancy (actual and projected) in Botswana increasing? Decreasing?

e. Over what interval(s) is Botswana's graph concave down? Concave up?

f. Write a short description of life expectancy in Botswana compared with world life expectancy.

20. Below is a graph of data collected by the FBI.



Source: U.S. Bureau of The Census, *Statistical Abstract of the United States: 2006.*

If we let $N(x)$ = number of juvenile arrests for murder in year x,

a. Estimate $N(1993)$. What are the coordinates of this point? What does this point represent?

b. Estimate the coordinates of the minimum point. What does this point represent?

c. Over what interval is the function increasing? Over what interval is the function decreasing?

d. What is the domain of $N(x)$? What is the range?

e. Write a brief summary about juvenile arrests between 1980 and 2003.

21. UNICEF has made children's education worldwide a top priority. According to UNICEF, 25 years ago only half of the world's children received a primary school education. Today, 86% receive a primary education. The following pie chart shows for each region the percentage of the *total* number of worldwide children out of school.

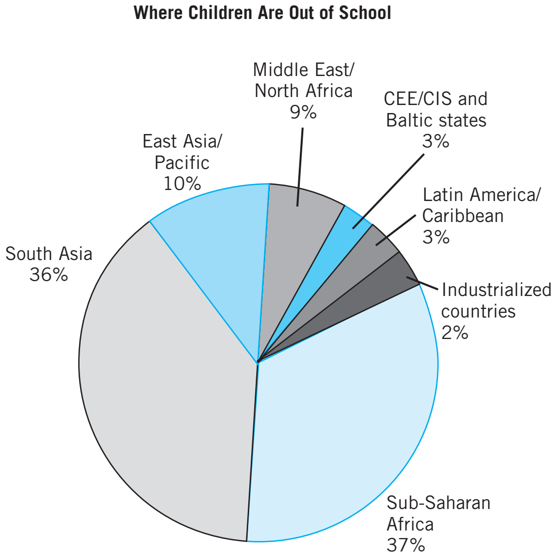
a. Explain what the label 3% means next to the region Latin America/Caribbean.

b. Does this graph tell us what percentage of children in Sub-Saharan Africa are out of school? Explain why or why not.

c. What does this graph not show? Why might this be confusing?

d. What is the region with the largest percentage of the world's children who are out of school? What is the total percentage of the two regions with the most children out of school? What is the region with the smallest percentage?

e. Write a 60-second summary about where you would suggest UNICEF focus its efforts in childhood education.



22. a. Given $g(x) = 2x^2 + 3x - 1$, evaluate $g(0)$ and $g(-1)$.

b. Find the domain of $g(x)$.

23. a. Given $f(x) = \dfrac{1}{x - 2}$, evaluate $f(0)$, $f(-1)$, and $f(2)$.

b. Find the domain of $f(x)$.

24. Find the domain and range of the following functions:

a. $h(x) = \sqrt{x}$ **b.** $f(x) = \sqrt{x - 6}$

25. The following table shows the cost each month in a small condominium in Maine for natural gas, which is used for cooking, heating, and hot water.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
$ gas	220	234	172	83	51	42	30	32	51	89	102	150

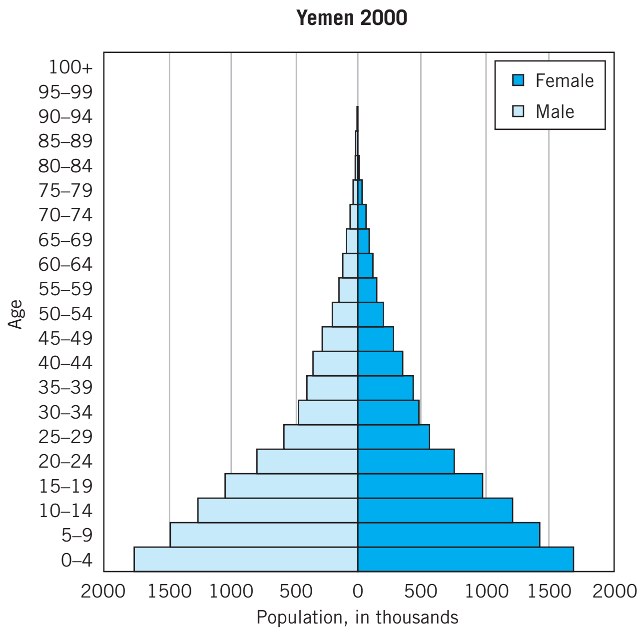
a. What is the month with the maximum cost for gas? The month with the minimum? Do these numbers seem reasonable? Why or why not?

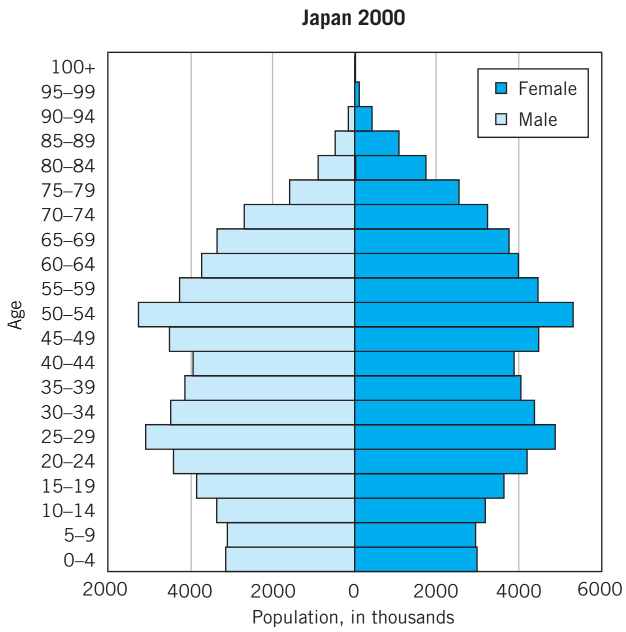
b. What is the mean monthly cost of gas for the year? What is the median monthly cost of gas? Describe in words what these descriptors tell you about the cost of gas in this condominium.

c. Create a table with intervals $0–49, $50–99, $100–149, and so on, and next to each interval show the number of months that fall within that cost interval. Use the table to construct a histogram of these data (with the cost intervals on the horizontal axis).

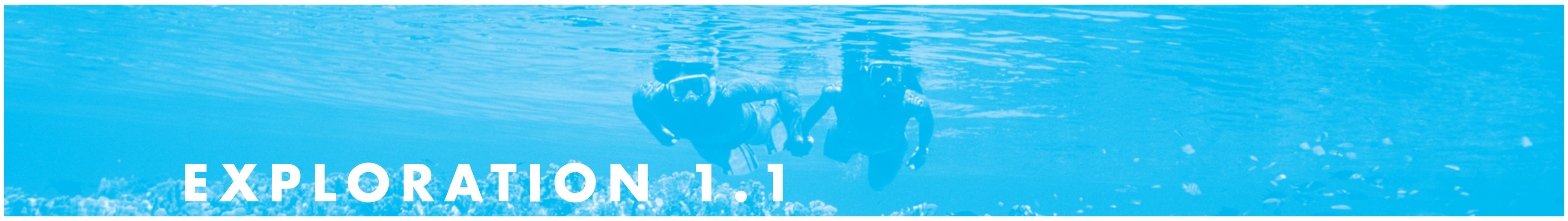
d. Describe two patterns in the cost of gas for this condominium.

26. The following graphs are population pyramids for Yemen (a developing country) and Japan (a highly developed country). Write a 60-second summary comparing and contrasting the populations in the year 2000 in these two countries.





Source. WHO World Population Prospects, 2002.



EXPLORATION 1.1

Collecting, Representing, and Analyzing Data

Objectives

- explore issues related to collecting data.
- learn techniques for organizing and graphing data using a computer (with a spreadsheet program) or a graphing calculator.
- describe and analyze the overall shape of single-variable data using frequency and relative frequency histograms.
- use the mean and median to represent single-variable data.

Material/Equipment

- class questionnaire
- measuring tapes in centimeters and inches
- optional measuring devices: eye chart, flexibility tester, measuring device for blood pressure
- computer with spreadsheet program or graphing calculator with statistical plotting features
- data from class questionnaire or other small data set formatted either as spreadsheet or graph link file
- overhead projector and projection panel for computer or graphing calculator
- transparencies for printing or drawing graphs for overhead projector (optional)

Related Readings

(On the web at *www.wiley.com/college/kimeclark*)
"U.S. Government Definitions of Census Terms"
"Health Measurements"

Related Software

"F1: Histograms," in *FAM 1000 Census Graphs*

Procedure

This exploration may take two class periods.

Day One

In a Small Group or with a Partner

1. Pick (or your instructor will assign you) one of the undefined variables on the questionnaire. Spend about 15 minutes coming up with a workable definition and a strategy for measuring that variable. Be sure there is a way in which a number or single letter can be used to record each individual's response on the questionnaire.
2. Consult the reading "Health Measurements" if you decide to collect health data.

Class Discussion

After all of the definitions are recorded on the board, discuss your definition with the class. Is it clear? Does everyone in the class fall into one of the categories of your definition? Can anyone think of someone who might not fit into any of the categories? Modify the definition until all can agree on some wording. As a class, decide on the final version of the questionnaire and record it in your class notebook.

In a Small Group or with a Partner

Help each other when necessary to take measurements and fill out the entire questionnaire. Questionnaires remain anonymous, and you can leave blank any question you can't or don't want to answer. Hand in your questionnaire to your instructor by the end of class.

Exploration-Linked Homework

Read "U.S. Government Definitions of Census Terms" for a glimpse into the federal government's definitions of the variables you defined in class. How do the "class" definitions and the "official" ones differ?

Day Two

Class Demonstration

1. If you haven't used a spreadsheet or graphing calculator before, you'll need a basic technical introduction. (*Note:* If you are using a TI-83 or TI-84 graphing calculator, there are basic instructions in the Graphing Calculator Manual on *www.wiley.com/college/kimeclark.*) Then you'll need an electronic version of the data set from which you will choose one variable for the whole class to study (e.g., age from the class data).

a. If you're using a spreadsheet:

- Copy the column with the data onto a new spreadsheet and graph the data. What does this graph tell you about the data?
- Sort and replot the data. Is this graph any better at conveying information about the data?

b. If you're using a graphing calculator:

- Discuss window sizes, changing interval sizes, and statistical plot procedures.

2. Select an interval size and then construct a frequency histogram and a relative frequency histogram. If possible, label one of these carefully and print it out. If you have access to a laser printer, you can print onto an overhead transparency.

3. Calculate the mean and median using spread sheet or graphing calculator functions.

In a Small Group or with a Partner

Choose another variable from your data. Pick an interval size, and then generate both a frequency histogram and a relative frequency histogram. If possible, make copies of the histograms for both your partner and yourself. Calculate the mean and median.

Discussion/Analysis

With your partner(s), analyze and jot down patterns that emerge from the data. How could you describe your results? What are some limitations of the data? What other questions are raised and how might they be resolved? Record your ideas for a 60-second summary.

Exploration-Linked Homework

Prepare a verbal 60-second summary to give to the class. If possible, use an overhead projector with a transparency of your histogram or a projector linked to your graphing calculator. If not, bring in a paper copy of your histogram. Construct a written 60-second summary. (See Section 1.2 for some writing suggestions.)

ALGEBRA CLASS QUESTIONNAIRE
(You may leave any category blank)

1. Age (in years)
2. Sex (female = 1, male = 2)
3. Your height (in inches)
4. Distance from your navel to the floor (in centimeters)
5. Estimate your average travel time to school (in minutes)
6. What is your most frequent mode of transportation to school? (F = by foot, C = car, P = public transportation, B = bike, O = other)

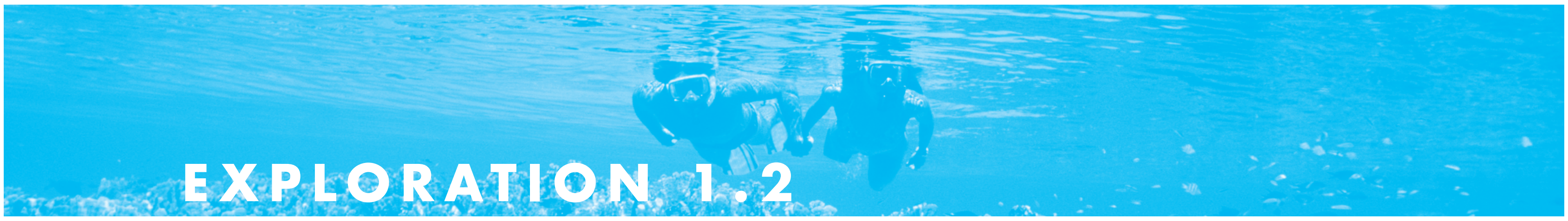
The following variables will be defined in class. We will discuss ways of coding possible responses and then use the results to record our personal data.

7. The number of people in your household
8. Your employment status
9. Your ethnic classification
10. Your attitude toward mathematics

Health Data

11. Your pulse rate before jumping (beats per minute)
12. Your pulse rate after jumping for 1 minute (beats per minute)
13. Blood pressure: systolic (mm Hg)
14. Blood pressure: diastolic (mm Hg)
15. Flexibility (in inches)
16. Vision, left eye
17. Vision, right eye

Other Data



EXPLORATION 1.2

Picturing Functions

Objective

- develop an intuitive understanding of functions.

Material/Equipment

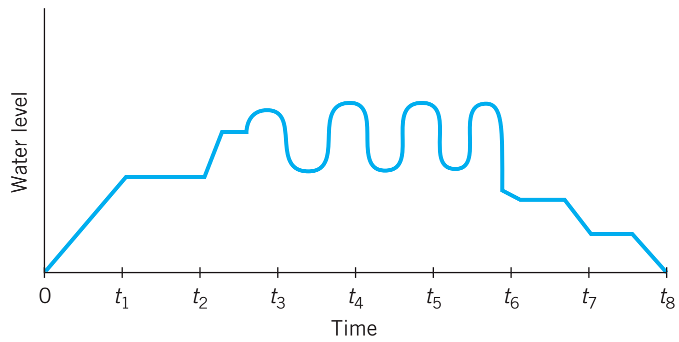
None required

Procedure

Part I

Class Discussion

Bridget, the 6-year-old daughter of a professor at the University of Pittsburgh, loves playing with her rubber duckie in the bath at night. Her mother drew the accompanying graph for her math class. It shows the water level (measured directly over the drain) in Bridget's tub as a function of time.



Pick out the time period during which:

- The tub is being filled
- Bridget is entering the tub
- She is playing with her rubber duckie
- She leaves the tub
- The tub is being drained

With a Partner

Create your own graph of a function that tells a story. Be as inventive as possible. Some students have drawn functions that showed the decibel levels during a phone conversation of a boyfriend and girlfriend, number of hours spent doing homework during one week, and amount of money in one's pocket during the week.

Class Discussion

Draw your graph on the blackboard and tell its story to the class.

Part II

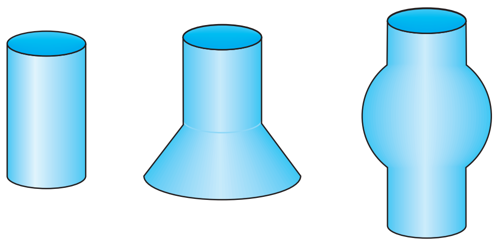
With a Partner

Generate a plausible graph for each of the following:

1. Time spent driving to work as a function of the amount of snow on the road. (*Note:* The first inch or so may not make any difference; the domain may be only up to about a foot of snow since after that you may not be able to get to work.)
2. The hours of nighttime as a function of the time of year.
3. The temperature of ice cream taken out of the freezer and left to stand.
4. The distance that a cannonball (or javelin or baseball) travels as a function of the angle of elevation at which is it is launched. (The maximum distance is attained for angles of around 45° from the horizontal.)
5. Assume that you leave your home walking at a normal pace, realize you have forgotten your homework and run back home, and then run even faster to school. You sit for a while in a classroom and then walk leisurely home. Now plot your distance from home as a function of time.

Bonus Question

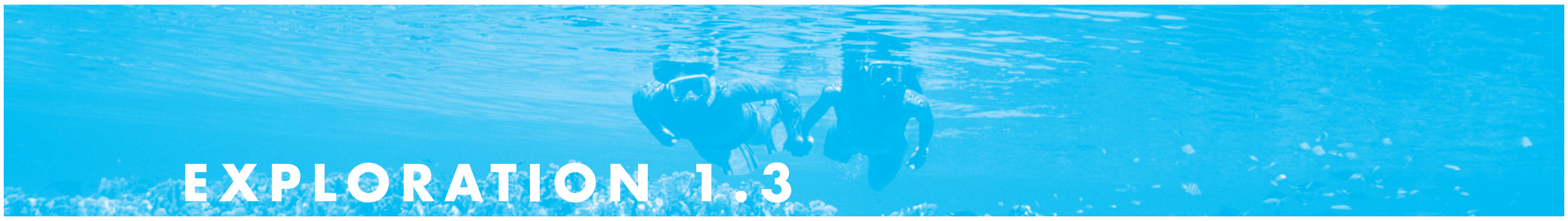
Assume that water is pouring at a constant rate into each of the containers shown. The height of water in the container is a function of the volume of liquid. Sketch a graph of this function for each container.



(*a*) (*b*) (*c*)

Discussion/Analysis

Are your graphs similar to those generated by the rest of the class? Can you agree as a class as to the basic shape of each of the graphs? Are there instances in which the graphs could look quite different?



EXPLORATION 1.3

Deducing Formulas to Describe Data

Objective

- find and describe patterns in data.
- deduce functional formulas from data tables.
- extend patterns using functional formulas.

Material/Equipment

None required

Procedure

Class Discussion

1. Examine data tables (a) and (b). In each table, look for a pattern in terms of how y changes when x changes. Explain in your own words how to find y in terms of x.

a.

x	y
0	0.0
1	0.5
2	1.0
3	1.5
4	2.0

b.

x	y
0	5
1	8
2	11
3	14
4	17

2. Assuming that the pattern continues indefinitely, use the rule you have found to extend the data table to include negative numbers for x.

3. Check your extended data tables. Did you find only one value for y given a particular value for x?

4. Use a formula to describe the pattern that you have found. Do you think this formula describes a function? Explain.

On Your Own

1. For each of the data tables (c) to (h), explain in your own words how to find y in terms of x. Then extend each table using the rules you have found.

c.

x	y
0	0
1	1
2	4
3	9
4	16

d.

x	y
0	0
1	1
2	8
3	27
4	64

e.

x	y
0	0
1	2
2	12
3	36
4	80

[*Hint:* For table (e) think about some combination of data in tables (c) and (d).]

f.

x	y
−2	0
0	10
5	35
10	60
100	510

g.

x	y
0	−1
1	0
2	3
3	8
4	15

h.

x	y
0	3
10	8
20	13
30	18
100	53

2. For each table, construct a formula to describe the pattern you have found.

Discussion/Analysis

With a Partner

Compare your results. Do the formulas that you have found describe functions? Explain.

Class Discussion

Does the rest of the class agree with your results? Remember that formulas that look different may give the same results.

Exploration-Linked Homework

1. a. For data tables (i) and (j), explain in your own words how to find y in terms of x. Using the rules you have found, extend the data tables to include negative numbers.

i.

x	y
−10	10.0
0	0.0
3	0.9
8	6.4
10	10.0

j.

x	y
0	−3
1	1
2	5
3	9
4	13

b. For each table, find a formula to describe the pattern you have found. Does your formula describe a function? Explain.

2. Make up a functional formula, generate a data table, and bring the data table on a separate piece of paper to class. The class will be asked to find your rule and express it as a formula.

k.

x	y