Chapter 13 Fluids

"When did science begin? Where did it begin? It began whenever and wherever men tried to solve the innumerable problems of life. The first solutions were mere expedients, but that must do for a beginning. Gradually the expedients would be compared, generalized, rationalized, simplified, interrelated, integrated; the texture of science would be slowly woven" George Sarton

13.1 Introduction

Matter is usually said to exist in three phases: solid, liquid, and gas. Solids are hard bodies that resist deformations, whereas liquids and gases have the characteristic of being able to flow. A liquid flows and takes the shape of whatever container in which it is placed. A gas also flows into a container and spreads out until it occupies the entire volume of the container. *A fluid is defined as any substance that can flow, and hence liquids and gases are both considered to be **fluids.***

Liquids and gases are made up of billions upon billions of molecules in motion and to properly describe their behavior, Newton's second law should be applied to each of these molecules. However, this would be a formidable task, if not outright impossible, even with the use of modern high-speed computers. Also, the actual motion of a particular molecule is sometimes not as important as the overall effect of all those molecules when they are combined into the substance that is called the fluid. Hence, instead of using the microscopic approach of dealing with each molecule, we will treat the fluid from a macroscopic approach. That is, we will analyze the fluid in terms of its large-scale characteristics, such as its mass, density, pressure, and its distribution in space.

The study of fluids will be treated from two different approaches. First, we will consider only fluids that are at rest. This portion of the study of fluids is called **fluid statics or hydrostatics.** Second, we will study the behavior of fluids when they are in motion. This part of the study is called **fluid dynamics or hydrodynamics.** Let us start the study of fluids by defining and analyzing the macroscopic variables.

13.2 Density

*The **density** of a substance is defined as the amount of mass in a unit volume of that substance.* We use the symbol ρ (the lower case Greek letter rho) to designate the density and write it as

$$\rho = \frac{m}{V} \tag{13.1}$$

A substance that has a large density has a great deal of mass in a unit volume, whereas a substance of low density has a small amount of mass in a unit volume. Density is expressed in SI units as kg/m^3, and occasionally in the laboratory as g/cm^3. Densities of solids and most liquids are very nearly constant but the densities of gases vary greatly with temperature and pressure. Table 13.1 is a list of densities for various materials. We observe from the table that in interstellar space the densities are extremely small, of the order of 10^{-18} to 10^{-21} kg/m^3. That is, interstellar space is almost empty space. The density of the proton and neutron is of the order of 10^{17} kg/m^3, which is an extremely large density. Hence, the nucleus of a chemical element is extremely dense. Because an atom of hydrogen has an approximate density of 2680 kg/m^3, whereas the proton in the nucleus of that hydrogen atom has a density of about 1.5×10^{17} kg/m^3, we see that the

Table 13.1
Densities of Various Materials

Substance	kg/m^3
Air (0 ^{0}C, 1 atm pressure)	1.29
Aluminum	2,700
Benzene	879
Blood	1.05×10^3
Bone	1.7×10^3
Brass	8,600
Copper	8,920
Critical density for universe to collapse under gravitation	5×10^{-27}
Planet Earth	5,520
Ethyl alcohol	810
Glycerine	1,260
Gold	19,300
Hydrogen atom	2,680
Ice	920
Interstellar space	10^{-18}-10^{-21}
Iron	7,860
Lead	11,340
Mercury	13,630
Nucleus	1×10^{17}
Proton	1.5×10^{17}
Silver	10,500
Sun (avg)	1,400
Water (pure)	1,000
Water (sea)	1,030
Wood (maple)	620-750

density of the nucleus is about 10^{13} times as great as the density of the atom. Hence, an atom consists almost entirely of empty space with the greatest portion of its mass residing in a very small nucleus.

Example 13.1

The density of an irregularly shaped object. In order to find the density of an irregularly shaped object, the object is placed in a beaker of water that is filled completely to the top. Since no two objects can occupy the same space at the same time, 25.0 cm^3 of the water, which is equal to the volume of the unknown object, overflows into an attached calibrated beaker. The object is placed on a balance scale and is found to have a mass of 262.5 g. Find the density of the material

Solution

The density, found from equation 13.1, is

$$\rho = \frac{m}{V} = \frac{262.5 \text{ g}}{25.0 \text{ cm}^3} = 10.5 \frac{\text{g}}{\text{cm}^3} = 10{,}500 \frac{\text{kg}}{\text{m}^3}$$

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Example 13.2

Your own water bed. A person would like to design a water bed for the home. If the size of the bed is to be 2.20 m long, 1.80 m wide, and 0.300 m deep, what mass of water is necessary to fill the bed?

Solution

The mass of the water, found from equation 13.1, is

$$m = \rho V \tag{13.2}$$

The density is found from table 13.1. Hence, the mass of water required is

$$m = \rho V = \left(1000 \ \frac{\text{kg}}{\text{m}^3}\right)(2.20 \text{ m})(1.80 \text{ m})(0.300 \text{ m})$$
$$= 1190 \text{ kg}$$

As a matter of curiosity let us compute the weight of this water. The weight of the water is given by

$$w = mg = (1190 \text{ kg})(9.80 \text{ m/s}^2) = 11{,}600 \text{ N}$$

To give you a "feel" for this weight of water, it is equivalent to 2620 lb. In some cases, it will be necessary to reinforce the floor underneath this water bed or the bed might end up in the basement below.

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13.3 Pressure

***Pressure** is defined as the magnitude of the normal force acting per unit surface area.* The pressure is thus a scalar quantity. We write this mathematically as

$$p = \frac{F}{A} \quad (13.3)$$

The SI unit for pressure is newton/meter^2, which is given the special name pascal, in honor of the French mathematician, physicist, and philosopher, Blaise Pascal (1623-1662) and is abbreviated Pa.[1] Hence, 1 Pa = 1 N/m^2. Pressures are not limited to fluids, as the following examples show.

Example 13.3

Pressure exerted by a man. A man has a mass of 90.0 kg. At one particular moment when he walks, his right heel is the only part of his body that touches the ground. If the heel of his shoe measures 9.00 cm by 8.30 cm, what pressure does the man exert on the ground?

Solution

The pressure that the man exerts on the ground, given by equation 13.3, is

$$p = \frac{F}{A}$$

$$= \frac{w}{A} = \frac{mg}{A} = \frac{(90.0 \text{ kg})(9.80 \text{ m/s}^2)}{(0.090 \text{ m})(0.083 \text{ m})}$$

$$= 1.18 \times 10^5 \text{ N/m}^2$$

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Example 13.4

Pressure exerted by a woman. A 45.0-kg woman is wearing "high-heel" shoes. The cross section of her high-heel shoe measures 1.27 cm by 1.80 cm. At a particular moment when she is walking, only one heel of her shoe makes contact with the ground. What is the pressure exerted on the ground by the woman?

Solution

The pressure exerted on the ground, found from equation 13.3, is

$$p = \frac{F}{A}$$

$$= \frac{w}{A} = \frac{mg}{A} = \frac{(45.0 \text{ kg})(9.80 \text{ m/s}^2)}{(0.0127 \text{ m})(0.0180 \text{ m})}$$

$$= 1.93 \times 10^6 \text{ N/m}^2$$

Thus, the 45.0-kg woman exerts a pressure through her high heel of 1.93×10^6 N/m^2, whereas the man, who has twice as much mass, exerts a pressure of only 1.18×10^5 N/m^2. That is, *the woman exerts about 16 times more pressure than the man.* The key to the great difference lies in the definition of pressure. Pressure is the force exerted per unit area. *Because the area of the woman's high heel is so very small, the pressure becomes very large. The area of the man's heel is relatively large, hence the pressure he exerts is relatively small.* When they are wearing high heels, women usually do not like to walk on soft ground because the large pressure causes the shoe to sink into the ground.

To go to this Interactive Example click on this sentence.

[1] In the British engineering system the units are lb/in.^{2}, which is sometimes denoted by psi.

A further example of the effect of the surface area on pressure is found in the application of snowshoes. Here, a person's weight is distributed over such a large area that the pressure exerted on the snow is very small. Hence, the person is capable of walking in deep snow, while another person, wearing ordinary shoes, would sink into the snow finding walking almost impossible.

Pressure exerted by a fluid is easily determined with the aid of figure 13.1, which represents a pool of water. We want to determine the pressure p at the bottom of the pool caused by the water in the pool. By our definition, equation 13.3, the pressure at the bottom of the pool is the magnitude of the force acting on a unit area of the bottom of the pool. But the force acting on the bottom of the pool is caused by the weight of all the water above it. Thus,

$$p = \frac{F}{A} = \frac{\text{weight of water}}{\text{area}} \quad (13.4)$$

$$p = \frac{w}{A} = \frac{mg}{A} \quad (13.5)$$

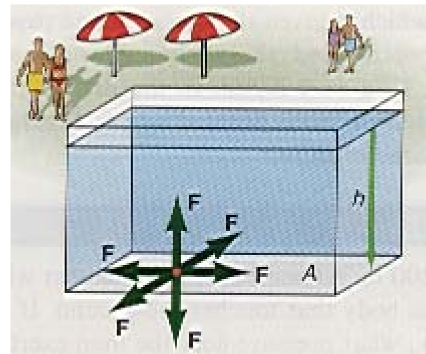

Figure 13.1 Pressure in a pool of water.

We have set the weight w of the water equal to mg in equation 13.5. The mass of the water in the pool, given by equation 13.2, is

$$m = \rho V$$

The volume of all the water in the pool is just equal to the area A of the bottom of the pool times the depth h of the water in the pool, that is,

$$V = Ah \quad (13.6)$$

Substituting equations 13.2 and 13.6 into equation 13.5 gives for the pressure at the bottom of the pool:

$$p = \frac{mg}{A} = \frac{\rho Vg}{A} = \frac{\rho Ahg}{A}$$

Thus,

$$p = \rho gh \quad (13.7)$$

(Although we derived equation 13.7 to determine the water pressure at the bottom of a pool of water, it is completely general and gives the water pressure at any depth h in the pool.) Equation 13.7 says that the water pressure at any depth h in any pool is given by the product of the density of the water in the pool, the acceleration due to gravity g, and the depth h in the pool. *Equation 13.7 is sometimes called **the hydrostatic equation.***

Example 13.5

Pressure in a swimming pool. Find the water pressure at a depth of 3.00 m in a swimming pool.

Solution

The density of water, found in table 13.1, is 1000 kg/m^3, and the water pressure, found from equation 13.7, is

$$p = \rho gh$$
$$= (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(3.00 \text{ m})$$
$$= 2.94 \times 10^4 \text{ N/m}^2 = 2.94 \times 10^4 \text{ Pa}$$

To go to this Interactive Example click on this sentence.

The pressure at the depth of 3 m in the pool in figure 13.1 is the same everywhere. Hence, the force exerted by the fluid is the same in all directions. That is, the force is the same in up-down, right-left, or in-out directions. If the force due to the fluid were not the same in all directions, then the fluid would flow in the direction away from the greatest pressure and would not be a fluid at rest. A fluid at rest is a fluid in equilibrium. Thus, in example 13.5, the pressure is 2.94×10^4 Pa at every point at a depth of 3 m in the pool and exerts the same force in every direction at that depth. You experience this pressure when swimming at a depth of 3.00 m as a pressure on your ears. As you swim up to the surface, the pressure on your ears decreases because h is decreasing. Or to look at it another way, the closer you swim up toward the surface, the smaller is the amount of water that is above you. Because the pressure is caused by the weight of that water above you, the smaller the amount of water, the smaller will be the pressure.

Just as there is a water pressure at the bottom of a swimming pool caused by the weight of all the water above the bottom, there is also an air pressure exerted on every object at the surface of the earth caused by the weight of all the air that is above us in the atmosphere. That is, there is an atmospheric pressure exerted on us, given by equation 13.3 as

$$p = \frac{F}{A} = \frac{\text{weight of air}}{\text{area}} \tag{13.8}$$

However we can not use the same result obtained for the pressure in the pool of water, the hydrostatic equation 13.7, because air is compressible and hence its density ρ is not constant with height throughout the vertical portion of the atmosphere. The pressure of air at any height in the atmosphere can be found by the use of calculus and the density variation in the atmosphere. However, since calculus is beyond the scope of this course, we will revert to the use of experimentation to determine the pressure of the atmosphere.

The pressure of the air in the atmosphere was first measured by Evangelista Torricelli (1608-1647), a student of Galileo, by the use of a mercury **barometer.** A long narrow tube is filled to the top with mercury, chemical symbol Hg. It is then placed upside down into a reservoir filled with mercury, as shown in figure 13.2.

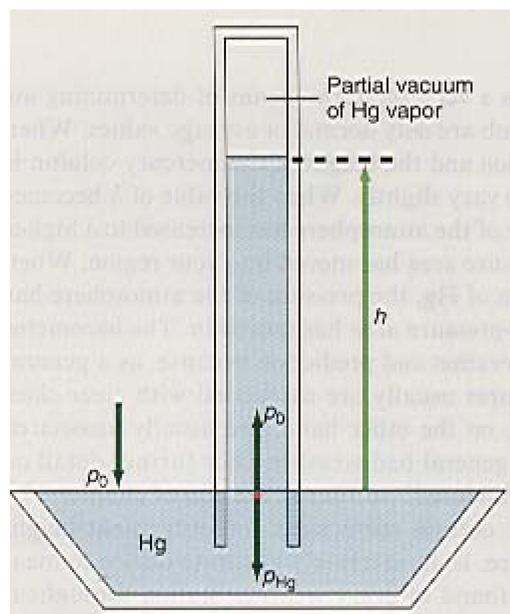

Figure 13.2 A mercury barometer.

The mercury in the tube starts to flow out into the reservoir, but it comes to a stop when the top of the mercury column is at a height h above the top of the mercury reservoir, as also shown in figure 13.2. The mercury does not empty completely because the normal pressure of the atmosphere p_0 pushes downward on the mercury reservoir. Because the force caused by the pressure of a fluid is the same in all directions, there is also a force acting upward inside the tube at the height of the mercury reservoir, and hence there is also a pressure p_0 acting upward as shown in figure 13.2. This force upward is capable of holding the weight of the mercury in the tube up to a height h. Thus, the pressure exerted by the mercury in the tube is exactly balanced by the normal atmospheric pressure on the reservoir, that is,

$$p_0 = p_{\mathbf{Hg}} \tag{13.9}$$

But the pressure of the mercury in the tube $p_{\mathbf{Hg}}$, given by equation 13.7, is

$$p_{\mathbf{Hg}} = \rho_{\mathbf{Hg}} g h \tag{13.10}$$

Substituting equation 13.10 back into equation 13.9, gives

$$p_0 = \rho_{\mathbf{Hg}} g h \tag{13.11}$$

Equation 13.11 says that normal atmospheric pressure can be determined by measuring the height h of the column of mercury in the tube. It is found experimentally, that on the average, normal atmospheric pressure can support a column of mercury 76.0 cm high, or 760 mm high. The unit of 1.00 mm of Hg is sometimes called a torr in honor of Torricelli. Hence, normal atmospheric pressure can also be given as 760 torr. Using the value of the density of mercury of 1.360 × 10^4 kg/m^3, found in table 13.1, normal atmospheric pressure, determined from equation 13.11, is

$$p_0 = \rho_{Hg} gh = \left(1.360 \times 10^4 \ \frac{\text{kg}}{\text{m}^3}\right)\left(9.80 \ \frac{\text{m}}{\text{s}^2}\right)(0.760 \text{ m})$$
$$= 1.013 \times 10^5 \text{ N/m}^2 = 1.013 \times 10^5 \text{ Pa}$$

Thus, the average or normal atmospheric pressure acting on us at the surface of the earth is 1.013 × 10^5 Pa, which is a rather large number as we will see presently. In the study of meteorology, the science of the weather, a different unit of pressure is usually employed, namely the millibar, abbreviated mb. The conversion factor between millibars and Pa (see appendix A) is

$$1 \text{ Pa} = 10^{-2} \text{ mb}$$

Using this conversion factor, normal atmospheric pressure[2] can also be expressed as

$$p_0 = \left(1.013 \times 10^5 \text{ Pa}\right)\left(\frac{10^{-2} \text{ mb}}{1 \text{ Pa}}\right)$$
$$= 1013 \text{ mb}$$

On all surface weather maps in a weather station, pressures are always expressed in terms of millibars.

The mercury barometer is thus a very accurate means of determining air pressure. The value of 76.0 cm or 1013 mb are only normal or average values. When the barometer is kept at the same location and the height of the mercury column is recorded daily, the value of h is found to vary slightly. When the value of h becomes greater than 76.0 cm of Hg, the pressure of the atmosphere has increased to a higher pressure. It is then said that a high-pressure area has moved into your region. When the value of h becomes less than 76.0 cm of Hg, the pressure of the atmosphere has decreased to a lower pressure and a low-pressure area has moved in. The barometer is extremely important in weather observation and prediction because, as a general rule of thumb, high atmospheric pressures usually are associated with clear skies and good weather. Low-pressure areas, on the other hand, are usually associated with cloudy skies, precipitation, and in general bad weather. (For further detail on the weather see the "Have You Ever Wondered" section at the end of chapter 17.)

The mercury barometer, after certain corrections for instrument height above sea level and ambient temperature, is an extremely accurate device to measure atmospheric pressure and can be found in every weather station throughout the world. Its chief limitation is its size. It must always remain vertical, and the glass tube and reservoir are somewhat fragile. Hence, another type of barometer is also used to measure atmospheric pressure. It is called an *aneroid barometer,* and is shown in figure 13.3. It is based on the principle of a partially evacuated, waferlike, metal cylinder called a Sylphon cell. When the

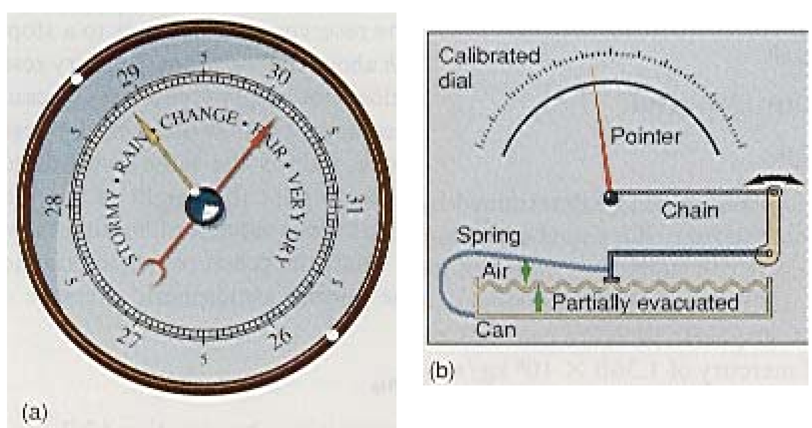

Figure 13.3 An aneroid barometer.

atmospheric pressure increases, the cell decreases in size. A combination of linkages and springs are connected to the cell and to a pointer needle that moves over a calibrated scale that indicates the pressure. The aneroid barometer is a more portable device that is rugged and easily used, although it is originally calibrated with a

[2]To express normal atmospheric pressure in the British engineering system, the conversion factor

$$1 \text{ Pa} = 1.45 \times 10^{-4} \text{ lb/in.}^2$$

found in appendix A, is used. Hence, normal atmospheric pressure can also be expressed as

$$p_0 = \left(1.013 \times 10^5 \text{ Pa}\right)\left(\frac{1.45 \times 10^{-4} \text{ lb/in}^2}{1 \text{ Pa}}\right)$$
$$= 14.7 \text{ lb/in.}^2$$

mercury barometer. The word *aneroid* means not containing fluid. The aneroid barometer is calibrated in both centimeters of Hg and inches of Hg. Using a conversion factor, we can easily see that a height of 29.92 in. of Hg also corresponds to normal atmospheric pressure. Hence, as seen in figure 13.3, the pressure can be measured in terms of inches of mercury. Also note that regions of high pressure (30 in. of Hg) are labeled to indicate fair weather, while regions of low pressure (29 in. of Hg) are labeled to indicate rain or poor weather.

As we go up into the atmosphere the pressure decreases, because there is less air above us. The aneroid barometer will read smaller and smaller pressures with altitude. Instead of calibrating the aneroid barometer in terms of centimeters of mercury or inches of mercury, we can also calibrate it in terms of feet or meters above the surface of the earth where this air pressure is found. An aneroid barometer so calibrated is called an *altimeter,* a device to measure the altitude or height of an airplane. The height of the plane is not really measured, the pressure is. But in the standard atmosphere, a particular pressure is found at a particular height above the ground. Hence, when the aneroid barometer measures this pressure, it corresponds to a fixed altitude above the ground. The pilot can read this height directly from the newly calibrated aneroid barometer, the altimeter.

Let us now look at some examples associated with atmospheric pressure.

Example 13.6

Why you get tired by the end of the day. The top of a student's head is approximately circular with a radius of 8.90 cm. What force is exerted on the top of the student's head by normal atmospheric pressure?

Solution

The area of the top of the student's head is found from

$$A = \pi r^2 = \pi(0.089 \text{ m})^2 = 0.0249 \text{ m}^2$$

We find the magnitude of the force exerted on the top of the student's head by rearranging equation 13.3 into the form

$$F = pA \tag{13.12}$$

Hence,

$$F = \left(1.013\times10^5 \ \frac{\text{N}}{\text{m}^2}\right)\left(0.0249 \text{ m}^2\right)$$
$$= 2520 \text{ N}$$

This is a rather large force (2520 N = 567 lb) to have exerted on our heads all day long. However, we do not notice this enormous force because when we breathe air into our nose or mouth that air is exerting the same force upward inside our head. Thus, the difference in force between the top of the head and the inside of the head is zero.

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Example 13.7

Atmospheric pressure on the walls of your house. Find the force on the outside wall of a ranch house, 3.05 m high and 10.7 m long, caused by normal atmospheric pressure.

Solution

The area of the wall of the house is given by

$$A = (\text{length})(\text{height})$$
$$= (10.7 \text{ m})(3.05 \text{ m})$$
$$= 32.6 \text{ m}^2$$

The force on the wall, given by equation 13.12, is

$$F = pA = \left(1.013 \times 10^5 \ \frac{\text{N}}{\text{m}^2}\right)\left(32.6 \text{ m}^2\right)$$
$$= 3.30 \times 10^6 \text{ N}$$

To go to this Interactive Example click on this sentence.

The force on the outside wall of the house in example 13.7 is thus 3.30×10^6 N = 743,000 lb. This is truly an enormous force. Why doesn't the wall collapse under this great force? The wall does not collapse because that same atmospheric air is also inside the house. Remember that air is a fluid and flows. Hence, in addition to being outside the house, the air also flows to the inside of the house. Because the force exerted by the pressure in the fluid is the same in all directions, the air inside the house exerts the same force of 3.30×10^6 N against the inside wall of the house, as shown in figure 13.4(a). The net force on the wall is therefore

$$\text{Net force} = (\text{force})_{\text{in}} - (\text{force})_{\text{out}}$$
$$= 3.30 \times 10^6 \text{ N} - 3.30 \times 10^6 \text{ N}$$
$$= 0$$

Figure 13.4 Pressure on the walls in a house.

A very interesting case occurs when this net force is not zero. Suppose a tornado, an extremely violent storm, were to move over your house, as shown in figure 13.4(b). The pressure inside the tornado is very low. No one knows for sure how low, because it is slightly difficult to run into a tornado with a barometer to measure it. In the very few cases on record where tornadoes actually went over a weather station, there was never anything left of the weather station, to say nothing of the barometer that was in that station. That is, neither the barometer nor the weather station were ever found again. The pressure can be estimated, however, from the very high winds associated with the tornado. A good estimate is that the pressure inside the tornado is at least 10% below the actual atmospheric pressure. Let us assume that the actual pressure is the normal atmospheric pressure of 1013 mb, then 10% of that is 101 mb. Thus, the pressure in the tornado is approximately

$$1013 \text{ mb} - 101 \text{ mb} = (912 \text{ mb})\left(\frac{1 \text{ N/m}^2}{10^{-2} \text{ mb}}\right) = 9.12 \times 10^4 \text{ N/m}^2$$

When the tornado goes over the house, the force on the outside wall is given by

$$F = pA = \left(9.12 \times 10^4 \ \frac{\text{N}}{\text{m}^2}\right)\left(32.6 \text{ m}^2\right)$$
$$= 2.97 \times 10^6 \text{ N}$$

The force on the outside wall is now 2.97×10^6 N (= 668,000 lb) while the original air inside the house is still there and is still exerting a force of 3.30×10^6 N outward on the walls. The net force on the house is now

$$\text{Net force} = 3.30 \times 10^6 \text{ N} - 2.97 \times 10^6 \text{ N}$$
$$= 3.30 \times 10^5 \text{ N}$$

There is now a net force acting outward on the wall of 3.30×10^5 N (about 75,000 lb), enough to literally explode the walls of the house outward. This pressure differential, with its accompanying winds, accounts for the enormous destruction associated with a tornado. Thus, the force exerted by atmospheric pressure can be extremely significant.

It has always been customary to open the doors and windows in a house whenever a tornado is in the vicinity in the hope that a great deal of the air inside the house will flow out through these open windows and doors. Hence, the pressure differential between the inside and the outside walls of the house will be minimized. However many victims of tornadoes do not follow this procedure, because tornadoes are spawned out of severe thunderstorms, which are usually accompanied by torrential rain. Usually the first thing one does in a house is to close the windows once the rain starts. A picture of a typical tornado is shown in figure 13.4(c).

Now that we have discussed atmospheric pressure, it is obvious that the total pressure exerted at a depth h in a pool of water must be greater than the value determined previously, because the air above the pool is exerting an atmospheric pressure on the top of the pool. This additional pressure is transmitted undiminished throughout the pool. Hence, the total or *absolute pressure* observed at the depth h in the pool is the sum of the atmospheric pressure plus the pressure of the water itself, that is,

$$p_{\mathbf{abs}} = p_0 + p_{\mathbf{w}} \tag{13.13}$$

Using equation 13.7, this becomes

$$p_{\mathbf{abs}} = p_0 + \rho g h \tag{13.14}$$

Example 13.8

Absolute pressure. What is the absolute pressure at a depth of 3.00 m in a swimming pool?

Solution

The water pressure at a depth of 3.00 m has already been found to be $p_{\mathbf{w}} = 2.94 \times 10^4$ Pa, the absolute pressure, found by equation 13.13, is

$$p_{\mathbf{abs}} = p_0 + p_{\mathbf{w}}$$
$$= 1.013 \times 10^5 \text{ Pa} + 2.94 \times 10^4 \text{ Pa}$$
$$= 1.31 \times 10^5 \text{ Pa}$$

To go to this Interactive Example click on this sentence.

When the pressure of the air in an automobile tire is measured, the actual pressure being measured is called the **gauge pressure,** that is, the pressure as indicated on the measuring device that is called a gauge. This measuring device, the gauge, reads zero when it is actually under normal atmospheric pressure. Thus, the total pressure or absolute pressure of the air inside the tire is the sum of the pressure recorded on the gauge plus normal atmospheric pressure. We can write this mathematically as

$$p_{\mathbf{abs}} = p_{\text{gauge}} + p_0 \tag{13.15}$$

Example 13.9

Gauge pressure and absolute pressure. A gauge placed on an automobile tire reads a pressure of 34.0 lb/in.^{2}. What is the absolute pressure of the air in the tire?

Solution

The absolute pressure of the air in the tire, found from equation 13.15, is

$$p_{abs} = p_{gauge} + p_0$$
$$= 34.0 \frac{\text{lb}}{\text{in.}^2} + 14.7 \frac{\text{lb}}{\text{in.}^2}$$
$$= 48.7 \text{ lb/in.}^2 = 3.36 \times 10^5 \text{ N/m}^2$$

To go to this Interactive Example click on this sentence.

13.4 Pascal's Principle

The pressure exerted on the bottom of a pool of water by the water itself is given by ρgh. However, there is also an atmosphere over the pool, and, as we saw in section 13.3, there is thus an additional pressure, normal atmospheric pressure p_0, exerted on the top of the pool. This pressure on the top of the pool is transmitted through the pool waters so that the total pressure at the bottom of the pool is the sum of the pressure of the water plus the pressure of the atmosphere, equations 13.13 and 13.14. The addition of both pressures is a special case of a principle, called ***Pascal's principle*** *and it states that if the pressure at any point in an enclosed fluid at rest is changed* (Δp), *the pressure changes by an equal amount* (Δp), *at all points in the fluid.* As an example of the use of Pascal's principle, let us consider the hydraulic lift shown in figure 13.5. A noncompressible fluid fills both cylinders and the connecting pipe. The smaller cylinder has a piston of cross-sectional area a, whereas the larger cylinder has a cross-sectional area A. As we can see in the figure, the cross-sectional area A of the larger cylinder is greater than the cross-sectional area a of the smaller cylinder. If a small force f is applied to the piston of the small cylinder, this creates a change in the pressure of the fluid given by

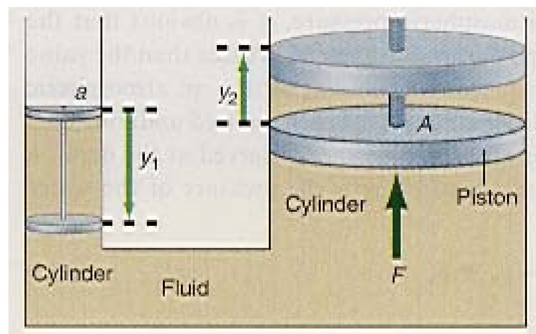

Figure 13.5 The hydraulic lift.

$$\Delta p = \frac{f}{a} \tag{13.16}$$

But by Pascal's principle, this pressure change occurs at all points in the fluid, and in particular at the large piston on the right. This same pressure change applied to the right piston gives

$$\Delta p = \frac{F}{A} \tag{13.17}$$

where F is the force that the fluid now exerts on the large piston of area A. Because these two pressure changes are equal by Pascal's principle, we can set equation 13.17 equal to equation 13.16. Thus,

$$\Delta p = \Delta p$$
$$\frac{F}{A} = \frac{f}{a}$$

The force F on the large piston is therefore

$$F = \frac{A}{a} f \tag{13.18}$$

Since the area A is greater than the area a, the force F will be greater than f. Thus, the hydraulic lift is a device that is capable of multiplying forces.

Example 13.10

Amplifying a force. The radius of the small piston in figure 13.5 is 5.00 cm, whereas the radius of the large piston is 30.0 cm. If a force of 2.00 N is applied to the small piston, what force will occur at the large piston?

Solution

The area of the small piston is

$$a = \pi r_1^2 = \pi(5.00 \text{ cm})^2 = 78.5 \text{ cm}^2$$

while the area of the large piston is

$$A = \pi r_2^2 = \pi(30.0 \text{ cm})^2 = 2830 \text{ cm}^2$$

The force exerted by the fluid on the large piston, found from equation 13.18, is

$$F = \frac{A f}{a}$$

$$= \left(\frac{2830 \text{ cm}^2}{78.5 \text{ cm}^2}\right)(2.00 \text{ N})$$

$$= 72.1 \text{ N}$$

Thus, the relatively small force of 2.00 N applied to the small piston produces the rather large force of 72.1 N at the large piston. The force has been magnified by a factor of 36.

To go to this Interactive Example click on this sentence.

It is interesting to compute the work that is done when the force f is applied to the small piston in figure 13.5. When the force f is applied, the piston moves through a displacement y_1, such that the work done is given by

$$W_1 = fy_1$$

But from equation 13.16

$$f = a\Delta p$$

Hence, the work done is

$$W_1 = a(\Delta p)y_1 \tag{13.19}$$

When the change in pressure is transmitted through the fluid, the force F is exerted against the large piston and the work done by the fluid on the large piston is

$$W_2 = Fy_2$$

where y_2 is the distance that the large piston moves and is shown in figure 13.5. But the force F, found from equation 13.17, is

$$F = A\Delta p$$

The work done on the large piston by the fluid becomes

$$W_2 = A(\Delta p)y_2 \tag{13.20}$$

Applying the law of conservation of energy to a frictionless hydraulic lift, the work done to the fluid at the small piston must equal the work done by the fluid at the large piston, hence

$$W_1 = W_2 \qquad (13.21)$$

Substituting equations 13.19 and 13.20 into equation 13.21, gives

$$a(\Delta p)y_1 = A(\Delta p)y_2 \qquad (13.22)$$

Because the pressure change Δp is the same throughout the fluid, it cancels out of equation 13.22, leaving

$$ay_1 = Ay_2$$

Solving for the distance y_1 that the small piston moves

$$y_1 = \frac{A}{a} y_2 \qquad (13.23)$$

Since A is much greater than a, it follows that y_1 must be much greater than y_2.

Example 13.11

You can never get something for nothing. The large piston of example 13.10 moves through a distance of 0.200 cm. By how much must the small piston be moved?

Solution

The areas of the pistons are given from example 13.10 as A = 2830 cm² and a = 78.5 cm², hence the distance that the small piston must move, given by equation 13.23, is

$$y_1 = \frac{A}{a} y_2$$
$$= \left(\frac{2830 \text{ cm}^2}{78.5 \text{ cm}^2}\right)(0.200 \text{ N})$$
$$= 7.21 \text{ cm}$$

Although a very large force is obtained at the large piston, the large piston is displaced by only a very small amount. Whereas the input force f, on the small piston is relatively small, the small piston must move through a relatively large displacement (36 times greater than the large piston). Usually there are a series of valves in the connecting pipe and the small cylinder is connected to a fluid reservoir also by valves. Hence, many displacements of the small piston can be made, each time adding additional fluid to the right cylinder. In this way the final displacement y_2 can be made as large as desired.

To go to this Interactive Example click on this sentence.

13.5 Archimedes' Principle

The variation of pressure with depth has a surprising consequence, it allows the fluid to exert buoyant forces on bodies immersed in the fluid. If this buoyant force is equal to the weight of the body, the body floats in the fluid. This result was first enunciated by Archimedes (287-212 BC) and is now called Archimedes' principle.

Archimedes' principle *states that a body immersed in a fluid is buoyed up by a force that is equal to the weight of the fluid displaced.* This principle can be verified with the help of figure 13.6.

If we submerge a cylindrical body into a fluid, such as water, then the bottom of the body is at some depth h_1 below the surface of the water and experiences a water pressure p_1 given by

$$p_1 = \rho g h_1 \qquad (13.24)$$

where ρ is the density of the water. Because the force due to the pressure acts equally in all directions, there is an upward force on the bottom of the body. The force upward on the body is given by

$$F_1 = p_1 A \quad (13.25)$$

where A is the cross-sectional area of the cylinder. Similarly, the top of the body is at a depth h_2 below the surface of the water, and experiences the water pressure p_2 given by

$$p_2 = \rho g h_2 \quad (13.26)$$

However, in this case the force due to the water pressure is acting downward on the body causing a force downward given by

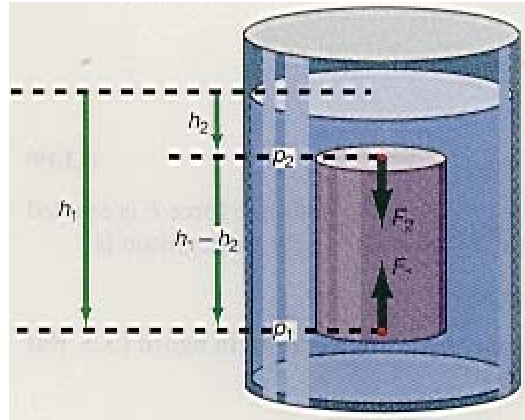

Figure 13.6 Archimedes' principle.

$$F_2 = p_2 A \quad (13.27)$$

Because of the difference in pressure at the two depths, h_1 and h_2, there is a different force on the bottom of the body than on the top of the body. Since the bottom of the submerged body is at the greater depth, it experiences the greater force. Hence, there is a net force upward on the submerged body given by

$$\text{Net force upward} = F_1 - F_2$$

Replacing the forces F_1 and F_2 by their values in equations 13.25 and 13.27, this becomes

$$\text{Net force upward} = p_1 A - p_2 A$$

Replacing the pressures p_1 and p_2 from equations 13.24 and 13.26, this becomes

$$\text{Net force upward} = \rho g h_1 A - \rho g h_2$$
$$A = \rho g A(h_1 - h_2) \quad (13.28)$$

But

$$A(h_1 - h_2) = V$$

the volume of the cylindrical body, and hence the volume of the water displaced. Equation 13.28 thus becomes

$$\text{Net force upward} = \rho g V \quad (13.29)$$

But ρ is the density of the water and from the definition of the density

$$\rho = \frac{m}{V} \quad (13.1)$$

Substituting equation 13.1 back into equation 13.29 gives

$$\text{Net force upward} = \frac{m}{V} gV$$
$$= mg$$

But $mg = w$, the weight of the water displaced. Hence,

$$\text{Net force upward} = \text{Weight of water displaced} \quad (13.30)$$

The net force upward on the body is called the *buoyant force* (BF). When the buoyant force on the body is equal to the weight of the body, the body does not sink in the water but rather floats, figure 13.7(b). Since the buoyant force is equal to the weight of the water displaced, *a body floats when the weight of the body is equal to the weight of the fluid displaced.*

Example 13.12

Wood floats. A block of oak wood 5.00 cm high, 5.00 cm wide, and 10.0 cm long is placed into a tub of water, figure 13.7(a). The density of the wood is 7.20×10^2 kg/m^3. How far will the block of wood sink before it floats?

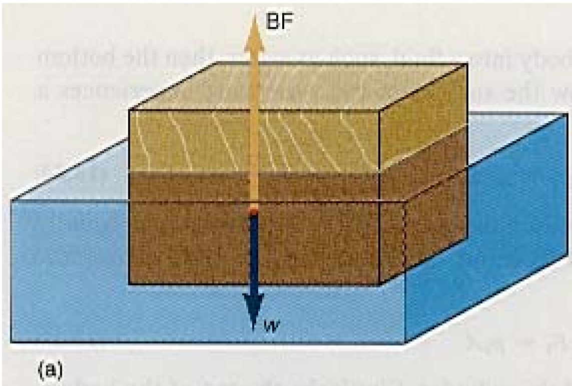

Figure 13.7 A body floats when the buoyant force is equal to the weight of the body.

Solution

The block of wood will float when the buoyant force (BF), which is the weight of the fluid displaced by the volume of the body submerged, is equal to the weight of the body. The weight of the block of wood is found from

$$w = mg = \rho Vg$$

The volume of the wooden block is $V = Ah$. Thus, the weight of the wooden block is

$$w = (7.20 \times 10^2 \text{ kg/m}^3)(0.0500 \text{ m})(0.0500 \text{ m})(0.100 \text{ m})(9.80 \text{ m/s}^2)$$
$$= 1.76 \text{ N}$$

The buoyant force is equal to the weight of the water displaced, and for the body to float, this buoyant force must also equal the weight of the block. Hence,

$$\text{BF} = w_{\text{water}} = w_{\text{wood}}$$
$$w_{\text{water}} = m_{\text{water}}\, g = \rho_{\text{water}} Vg = \rho_{\text{water}} Ahg \tag{13.31}$$

Thus,

$$\rho_{\text{water}} Ahg = w_{\text{wood}}$$
$$h = \frac{w_{\text{wood}}}{\rho_{\text{water}} Ag} \tag{13.32}$$
$$= \frac{1.76 \text{ N}}{(1.00 \times 10^3 \text{ kg/m}^3)(0.0500 \text{ m})(0.100 \text{ m})(9.80 \text{ m/s}^2)}$$
$$= 0.0359 \text{ m} = 3.59 \text{ cm}$$

Thus, the block sinks to a depth of 3.59 cm. At this point the buoyant force becomes equal to the weight of the wooden block and the wooden block floats.

To go to this Interactive Example click on this sentence.

Example 13.13

Iron sinks. Repeat example 13.12 for a block of iron of the same dimensions.

Solution

The density of iron, found from table 13.1, is 7860 kg/m³. The weight of the iron block is given by

$$w_{\text{iron}} = mg = \rho V g$$
$$= (7860 \text{ kg/m}^3)(0.0500 \text{ m})(0.0500 \text{ m})(0.100 \text{ m})(9.80 \text{ m/s}^2)$$
$$= 19.3 \text{ N}$$

The depth that the iron block would have to sink in order to displace its own weight, again found from equation 13.32, is

$$h = \frac{w_{\text{iron}}}{\rho_{\text{water}} A g}$$
$$= \frac{19.3 \text{ N}}{(1.00 \times 10^3 \text{ kg/m}^3)(0.0500 \text{ m})(0.100 \text{ m})(9.80 \text{ m/s}^2)}$$
$$= 39.4 \text{ cm}$$

But the block is only 10 cm high. Hence, the buoyant force is not great enough to lift an iron block of this size, and the iron block sinks to the bottom.

Another way to look at this problem is to calculate the buoyant force on this piece of iron. The buoyant force on the iron, given by equation 13.29, is

$$\text{Net force upward} = \rho g V$$
$$= (1 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.0500 \text{ m})(0.500 \text{ m})(0.100 \text{ m})$$
$$= 2.45 \text{ N}$$

Thus, the net force upward on a block of iron of this size is 2.45 N. But the block weighs 19.3 N. Hence, the weight of the iron is greater than the buoyant force and the iron block sinks to the bottom.

To go to this Interactive Example click on this sentence.

But ships are made of iron and they do not sink. Why should the block sink and not the ship? If this same weight of iron is made into thin slabs, these thin slabs could be welded together into a boat structure of some kind. By increasing the size and hence the volume of this iron boat, a greater volume of water can be displaced. An increase in the volume of water displaced increases the buoyant force. If this can be made equal to the weight of the iron boat, then the boat floats.

Example 13.14

An iron boat. The iron block of example 13.13 is cut into 16 slices, each 5.00 cm by 10.0 cm by 5/16 cm. They are now welded together to form a box 20.0 cm wide by 10.0 cm long by 10.0 cm high, as shown in figure 13.8. Will this iron body now float or will it sink?

Solution

In this new configuration the iron displaces a much greater volume of water, and since the buoyant force is equal to the weight of the water displaced it is possible that this new configuration will float. We assume that no mass of iron is lost in cutting the blocks into the 16 slabs, and that the weight of the welding material is negligible. Thus, the weight of the box is also equal to 19.3 N. This example is analyzed in the same way as the previous example. Let us solve for the depth that the iron box must sink in order that the buoyant force be equal to the weight of the box. Thus, the depth that the box sinks, again found from the modified equation 13.32, is

$$h = \frac{w_{box}}{\rho_{water} A g}$$

$$= \frac{19.3 \text{ N}}{(1.00 \times 10^3 \text{ kg})(0.200 \text{ m})(0.100 \text{ m})(9.80 \text{ m/s}^2)}$$

$$= 9.84 \times 10^{-2} \text{ m} = 9.84 \text{ cm}$$

Because the iron box is 10 cm high, it sinks to a depth of 9.84 cm and it then floats. Note that this is the same mass of iron that sank in example 13.13. *That same mass can now float because the new distribution of that mass results in a displacement of a much larger volume of water.* Since the buoyant force is equal to the weight of the water displaced, by increasing the volume taken up by the iron and the enclosed space, the amount of the water displaced has increased and so has the buoyant force.

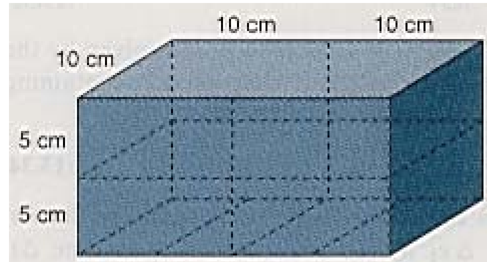

Figure 13.8 Iron can float.

To go to this Interactive Example click on this sentence.

Examples 13.12-13.14 dealt with bodies submerged in water, but remember that Archimedes' principle applies to all fluids.

13.6 The Equation of Continuity

Up to now, we have studied only fluids at rest. *Let us now study fluids in motion, the subject matter of hydrodynamics.* The study of fluids in motion is relatively complicated, but the analysis can be simplified by making a few assumptions. Let us assume that the fluid is incompressible and flows freely without any turbulence or friction between the various parts of the fluid itself and any boundary containing the fluid, such as the walls of a pipe. A fluid in which friction can be neglected is called a *nonviscous fluid.* A fluid, flowing steadily without turbulence, is usually referred to as being in *streamline flow.* The rather complicated analysis is further simplified by the use of two great conservation principles: the conservation of mass, and the conservation of energy. *The law of conservation of mass results in a mathematical equation, usually called the equation of continuity. The law of conservation of energy is the basis of Bernoulli's theorem,* the subject matter of section 13.7.

Let us consider an incompressible fluid flowing in the pipe of figure 13.9. At a particular instant of time the small mass of fluid Δm, shown in the left-hand portion of the pipe will be considered. This mass is given by a slight modification of equation 13.2, as

$$\Delta m = \rho \Delta V \qquad (13.33)$$

Because the pipe is cylindrical, the small portion of volume of fluid is given by the product of the cross-sectional area A_1 times the length of the pipe Δx_1 containing the mass Δm, that is,

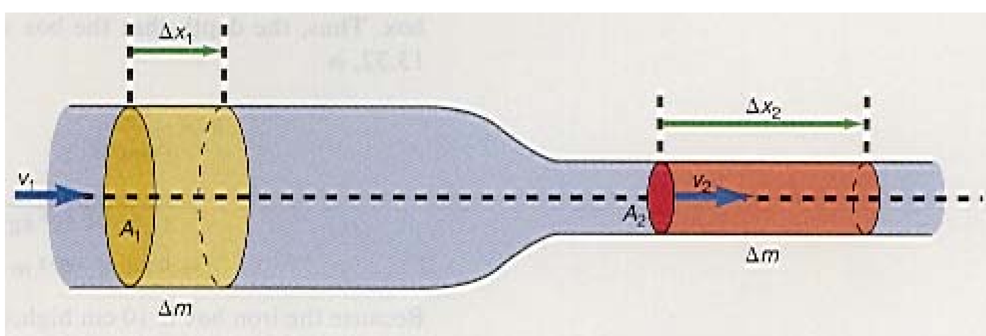

Figure 13.9 The law of conservation of mass and the equation of continuity.

$$\Delta V = A_1 \Delta x_1 \qquad (13.34)$$

The length Δx_1 of the fluid in the pipe is related to the velocity v_1 of the fluid in the left-hand pipe. Because the fluid in Δx_1 moves a distance Δx_1 in time Δt, $\Delta x_1 = v_1 \Delta t$. Thus,

$$\Delta x_1 = v_1 \Delta t \tag{13.35}$$

Substituting equation 13.35 into equation 13.34, we get for the volume of fluid,

$$\Delta V = A_1 v_1 \Delta t \tag{13.36}$$

Substituting equation 13.36 into equation 13.33 yields the mass of the fluid as

$$\Delta m = \rho A_1 v_1 \Delta t \tag{13.37}$$

We can also express this as the rate at which the mass is flowing in the left-hand portion of the pipe by dividing both sides of equation 13.37 by Δt, thus

$$\frac{\Delta m}{\Delta t} = \rho A_1 v_1 \tag{13.38}$$

Example 13.15

Flow rate. What is the mass flow rate of water in a pipe whose diameter d is 10.0 cm when the water is moving at a velocity of 0.322 m/s.

Solution

The cross-sectional area of the pipe is

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi (0.100 \text{ m})^2}{4}$$
$$= 7.85 \times 10^{-3} \text{ m}^2$$

The flow rate, found from equation 13.38, is

$$\frac{\Delta m}{\Delta t} = \rho A_1 v_1$$
$$= (1.00 \times 10^3 \text{ kg/m}^3)(7.85 \times 10^{-3} \text{ m}^2)(0.322 \text{ m/s})$$
$$= 2.53 \text{ kg/s}$$

Thus 2.53 kg of water flow through the pipe per second.

To go to this Interactive Example click on this sentence.

When this fluid reaches the narrow constricted portion of the pipe to the right in figure 13.9, the same amount of mass Δm is given by

$$\Delta m = \rho \Delta V \tag{13.39}$$

But since ρ is a constant, the same mass Δm must occupy the same volume ΔV. However, the right-hand pipe is constricted to the narrow cross-sectional area A_2. Thus, the length of the pipe holding this same volume must increase to a larger value Δx_2, as shown in figure 13.9. Hence, the volume of fluid is given by

$$\Delta V = A_2 \Delta x_2 \tag{13.40}$$

The length of pipe Δx_2 occupied by the fluid is related to the velocity of the fluid by

$$\Delta x_2 = v_2 \Delta t \tag{13.41}$$

Substituting equation 13.41 back into equation 13.40, we get for the volume of fluid,

$$\Delta V = A_2 v_2 \Delta t \tag{13.42}$$

It is immediately obvious that since A_2 has decreased, v_2 must have increased for the same volume of fluid to flow. Substituting equation 13.42 back into equation 13.39, the mass of the fluid flowing in the right-hand portion of the pipe becomes

$$\Delta m = \rho A_2 v_2 \Delta t \tag{13.43}$$

Dividing both sides of equation 13.43 by Δt yields the rate at which the mass of fluid flows through the right-hand side of the pipe, that is,

$$\frac{\Delta m}{\Delta t} = \rho A_2 v_2 \tag{13.44}$$

But *the **law of conservation of mass** states that mass is neither created nor destroyed in any ordinary mechanical or chemical process.* Hence, the law of conservation of mass can be written as

Mass flowing into the pipe = mass flowing out of the pipe

or

$$\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta t} \tag{13.45}$$

Thus, setting equation 13.38 equal to equation 13.44 yields

$$\rho A_1 v_1 = \rho A_2 v_2 \tag{13.46}$$

Equation 13.46 is called **the equation of continuity** and is an indirect statement of the law of conservation of mass. Since we have assumed an incompressible fluid, the densities on both sides of equation 13.46 are equal and can be canceled out leaving

$$A_1 v_1 = A_2 v_2 \tag{13.47}$$

Equation 13.47 is a special form of the equation of continuity for incompressible fluids (i.e., liquids).

Applying equation 13.47 to figure 13.9, we see that the velocity of the fluid v_2 in the narrow pipe to the right is given by

$$v_2 = \frac{A_1 v_1}{A_2} \tag{13.48}$$

Because the cross-sectional area A_1 is greater than the cross-sectional area A_2, the ratio A_1/A_2 is greater than one and thus the velocity v_2 must be greater than v_1.

Example 13.16

Applying the equation of continuity. In example 13.15 the cross-sectional area A_1 was 7.85×10^{-3} m² and the velocity v_1 was 0.322 m/s. If the diameter of the pipe to the right in figure 13.9 is 4.00 cm, find the velocity of the fluid in the right-hand pipe.

Solution

The cross-sectional area of the right-hand side of the pipe is

$$A_2 = \frac{\pi d_2^2}{4}$$
$$= \frac{\pi (0.0400 \text{ m})^2}{4}$$
$$= 1.26 \times 10^{-3} \text{ m}^2$$

The velocity of the fluid on the right-hand side v_2, found from equation 13.48, is

$$v_2 = \frac{A_1}{A_2}v_1 = \left(\frac{7.85\times10^{-3}\ \text{m}^2}{1.26\times10^{-3}\ \text{m}^2}\right)(0.322\ \text{m/s})$$
$$= 2.01\ \text{m/s}$$

The fluid velocity increased more than six times when it flowed through the constricted pipe.

To go to this Interactive Example click on this sentence.

Therefore, as a general rule, the equation of continuity for liquids, equation 13.47, says that when the cross-sectional area of a pipe gets smaller, the velocity of the fluid must become greater in order that the same amount of mass passes a given point in a given time. Conversely, when the cross-sectional area increases, the velocity of the fluid must decrease. Equation 13.47, the equation of continuity, is sometimes written in the equivalent form

$$Av = \text{constant} \tag{13.49}$$

Example 13.17

Flow rate revisited. What is the flow of mass per unit time for the example 13.16?

Solution

The rate of mass flow for the right-hand side of the pipe, given by equation 13.44, is

$$\frac{\Delta m}{\Delta t} = \rho A_2 v_2$$
$$= (1.0\times10^3\ \text{kg/m}^3)(1.26\times10^{-3}\ \text{m}^2)(2.01\ \text{m/s})$$
$$= 2.53\ \text{kg/s}$$

Note that this is the same rate of flow found earlier for the left-hand side of the pipe, as it must be by the law of conservation of mass.

A compressible fluid (i.e., a gas) can have a variable density, and requires an additional equation to specify the flow velocity.

To go to this Interactive Example click on this sentence.

13.7 Bernoulli's Theorem

Bernoulli's theorem is a fundamental theory of hydrodynamics that describes a fluid in motion. It is really the application of the law of conservation of energy to fluid flow. Let us consider the fluid flowing in the pipe of figure 13.10. The left-hand side of the pipe has a uniform cross-sectional area A_1, which eventually tapers to the uniform cross-sectional area A_2 of the right-hand side of the pipe. The pipe is filled with a nonviscous, incompressible fluid. A uniform pressure p_1 is applied, such as from a piston, to a small element of mass of the fluid Δm and causes this mass to move through a distance Δx_1 of the pipe. Because the fluid is incompressible, the fluid moves throughout the rest of the pipe. The same small mass Δm, at the right-hand side of the pipe, moves through a distance Δx_2. The work done on the system by moving the small mass through the distance Δx_1 is given by the definition of work as

$$W_1 = F_1\Delta x_1$$

Using equation 13.12, we can express the force F_1 moving the mass to the right in terms of the pressure exerted on the fluid as

$$F_1 = p_1A_1$$

Hence,

$$W_1 = p_1A_1\Delta x_1$$

But

$$A_1\Delta x_1 = \Delta V$$

the volume of the fluid moved through the pipe. Thus, we can write the work done on the system as

$$W_1 = p_1\Delta V_1 \qquad (13.50)$$

As this fluid moves through the system, the fluid itself does work by exerting a force F_2 on the mass Δm on the right side, moving it through the distance Δx_2. Hence, the work done by the fluid system is

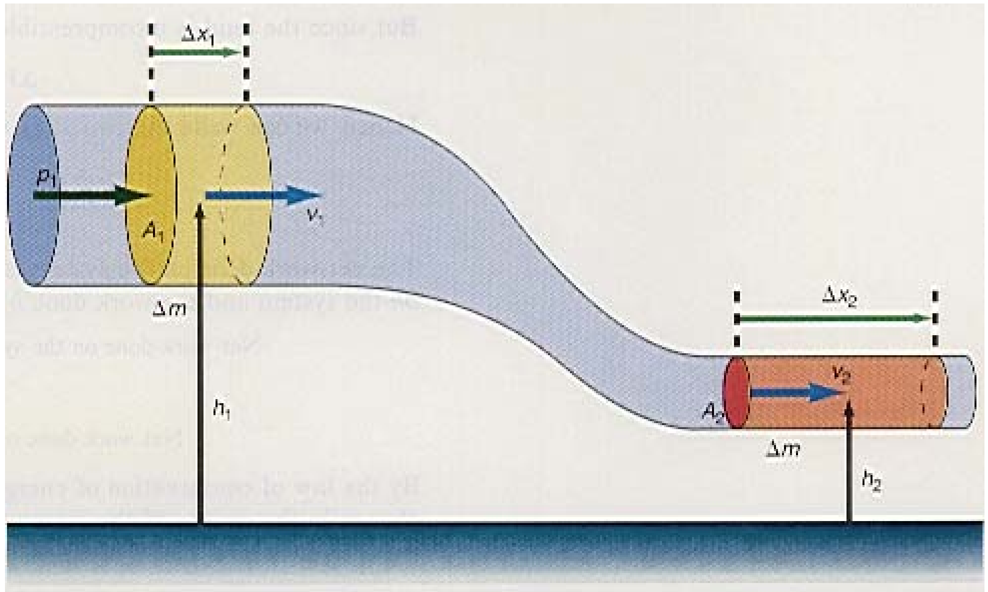

Figure 13.10 Bernoulli's theorem.

$$W_2 = F_2\Delta x_2$$

But we can express the force F_2 in terms of the pressure p_2 on the right side by

$$F_2 = p_2A_2$$

Therefore, the work done by the system is

$$W_2 = p_2A_2\Delta x_2$$

But

$$A_2\Delta x_2 = \Delta V_2$$

the volume moved through the right side of the pipe. Thus, the work done by the system becomes

$$W_2 = p_2\Delta V_2 \qquad (13.51)$$

But since the fluid is incompressible,

$$\Delta V_1 = \Delta V_2 = \Delta V$$

Hence, we can write the two work terms, equations 13.50 and 13.51, as

$$W_1 = p_1\Delta V$$
$$W_2 = p_2\Delta V$$

The net work done on the system is equal to the difference between the work done *on* the system and the work done *by* the system. Hence,

$$\text{Net work done on the system} = W_{\mathbf{on}} - W_{\mathbf{by}}$$
$$= W_1 - W_2 = p_1\Delta V - p_2\Delta V$$
$$\text{Net work done on the system} = (p_1 - p_2)\Delta V \qquad (13.52)$$

By the law of conservation of energy, the net work done on the system produces a change in the energy of the system. The fluid at position 1 is at a height h_1 above the reference level and therefore possesses a potential energy given by

$$\text{PE}_1 = (\Delta m)gh_1 \qquad (13.53)$$

Because this same fluid is in motion at a velocity v_1, it possesses a kinetic energy given by

$$KE_1 = \frac{1}{2}(\Delta m)v_1^2 \quad (13.54)$$

Similarly at position 2, the fluid possesses the potential energy

$$PE_2 = (\Delta m)gh_2 \quad (13.55)$$

and the kinetic energy

$$KE_2 = \frac{1}{2}(\Delta m)v_2^2 \quad (13.56)$$

Therefore, we can now write the law of conservation of energy as

$$\text{Net work done on the system} = \text{Change in energy of the system} \quad (13.57)$$
$$\text{Net work done on the system} = (E_{tot})_2 - (E_{tot})_1 \quad (13.58)$$
$$\text{Net work done on the system} = (PE_2 + KE_2) - (PE_1 + KE_1) \quad (13.59)$$

Substituting equations 13.52 through 13.56 into equation 13.59 we get

$$(p_1 - p_2)\Delta V = \left[(\Delta m)gh_2 + \frac{1}{2}(\Delta m)v_2^2\right] - \left[(\Delta m)gh_1 + \frac{1}{2}(\Delta m)v_1^2\right] \quad (13.60)$$

But the total mass of fluid moved Δm is given by

$$\Delta m = \rho\Delta V \quad (13.61)$$

Substituting equation 13.61 back into equation 13.60, gives

$$(p_1 - p_2)\Delta V = \rho(\Delta V)gh_2 + \frac{1}{2}\rho(\Delta V)v_2^2 - \rho(\Delta V)gh_1 - \frac{1}{2}\rho(\Delta V)v_1^2$$

Dividing each term by ΔV gives

$$(p_1 - p_2) = \rho gh_2 + \frac{1}{2}\rho v_2^2 - \rho gh_1 - \frac{1}{2}\rho v_1^2 \quad (13.62)$$

If we place all the terms associated with the fluid at position 1 on the left-hand side of the equation and all the terms associated with the fluid at position 2 on the right-hand side, we obtain

$$p_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2 \quad (13.63)$$

Equation 13.63 is the mathematical statement of

> ***Bernoulli's theorem.*** *It says that the sum of the pressure, the potential energy per unit volume, and the kinetic energy per unit volume at any one location of the fluid is equal to the sum of the pressure, the potential energy per unit volume, and the kinetic energy per unit volume at any other location in the fluid, for a nonviscous, incompressible fluid in streamlined flow.*

Since this sum is the same at any arbitrary point in the fluid, the sum itself must therefore be a constant. Thus, we sometimes write Bernoulli's equation in the equivalent form

$$p + \rho gh + \frac{1}{2}\rho v^2 = \text{constant} \quad (13.64)$$

Example 13.18

Applying Bernoulli's theorem. In figure 13.10, the pressure $p_1 = 2.94 \times 10^3$ N/m^2, whereas the velocity of the water is $v_1 = 0.322$ m/s. The diameter of the pipe at location 1 is 10.0 cm and it is 5.00 m above the ground. If the diameter of the pipe at location 2 is 4.00 cm, and the pipe is 2.00 m above the ground, find the velocity of the water v_2 at position 2, and the pressure p_2 of the water at position 2.

Solution

The area A_1 is

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi}{4}(0.100 \text{ m})^2 = 7.85 \times 10^{-3} \text{ m}^2$$

whereas the area A_2 is

$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi}{4}(0.0400 \text{ m})^2 = 1.26 \times 10^{-3} \text{ m}^2$$

The velocity at location 2 is found from the equation of continuity, equation 13.48, as

$$v_2 = \frac{A_1}{A_2} v_1 = \left(\frac{7.85 \times 10^{-3} \text{ m}^2}{1.26 \times 10^{-3} \text{ m}^2}\right)(0.322 \text{ m/s})$$
$$= 2.01 \text{ m/s}$$

The pressure at location 2 is found from rearranging Bernoulli's equation 13.63 as

$$p_2 = p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 - \rho g h_2 - \frac{1}{2}\rho v_2^2$$

$$= 2.94 \times 10^3 \frac{\text{N}}{\text{m}^2} + \left(1 \times 10^3 \frac{\text{kg}}{\text{m}^3}\right)\left(9.80 \frac{\text{m}}{\text{s}^2}\right)(5.00 \text{ m})$$
$$+ \tfrac{1}{2}\left(1 \times 10^3 \frac{\text{kg}}{\text{m}^3}\right)(0.322 \text{ m/s})^2 - \left(1 \times 10^3 \frac{\text{kg}}{\text{m}^3}\right)\left(9.80 \frac{\text{m}}{\text{s}^2}\right)(2.00 \text{ m})$$
$$- \tfrac{1}{2}\left(1 \times 10^3 \frac{\text{kg}}{\text{m}^3}\right)(2.01 \text{ m/s})^2$$

$$= 2.94 \times 10^3 \text{ N/m}^2 + 4.9 \times 10^4 \text{ N/m}^2 + 5.18 \times 10^1 \text{ N/m}^2$$
$$- 1.96 \times 10^4 \text{ N/m}^2 - 2.02 \times 10^3 \text{ N/m}^2$$
$$= 3.04 \times 10^4 \text{ N/m}^2$$

To go to this Interactive Example click on this sentence.

13.8 Application of Bernoulli's Theorem

Let us now consider some special cases of Bernoulli's theorem.

The Venturi Meter

Let us first consider the constricted tube studied in figure 13.9 and slightly modified and redrawn in figure 13.11(a). Since the tube is completely horizontal $h_1 = h_2$ and there is no difference in potential energy between the locations 1 and 2. Bernoulli's equation therefore reduces to

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \tag{13.65}$$

But by the equation of continuity,

$$v_2 = \frac{A_1}{A_2} v_1 \tag{13.48}$$

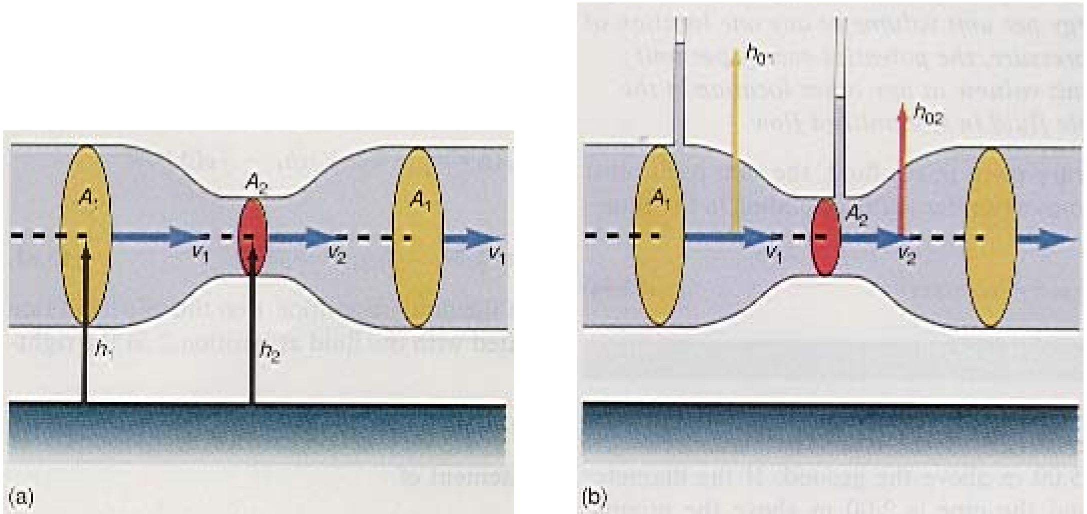

Figure 13.11 A Venturi meter.

Since A_1 is greater than A_2, v_2 must be greater than v_1, as shown before. Let us rewrite equation 13.65 as

$$p_2 = p_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$$

or

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) \qquad (13.66)$$

But since v_2 is greater than v_1, the quantity $(1/2)\rho(v_1^2 - v_2^2)$ is a negative quantity and when we subtract it from p_1, p_2 must be less than p_1. *Thus, not only does the fluid speed up in the constricted tube, but the pressure in the constricted tube also decreases.*

Example 13.19

When the velocity increases, the pressure decreases. In example 13.16, associated with figure 13.9, the velocity v_1 in area A_1 was 0.322 m/s and the velocity v_2 in area A_2 was found to be 2.01 m/s. If the pressure in the left pipe is 2.94 $\times$ 10^3 Pa, what is the pressure p_2 in the constricted pipe?

Solution

The pressure p_2, found from equation 13.66, is

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$= 2.94 \times 10^3 \text{ Pa} + (1/2)(1 \times 10^3 \text{ kg/m}^3)[(0.322 \text{ m/s})^2 - (2.01 \text{ m/s})^2]$$

$$= 2.94 \times 10^3 \text{ N/m}^2 - 1.97 \times 10^3 \text{ N/m}^2 = 9.7 \times 10^2 \text{ Pa}$$

Thus, the pressure of the water in the constricted portion of the tube has decreased to 9.7×10^2 Pa. Note that in example 13.18 of section 13.7 the pressure in the constricted area of the pipe was greater than in the larger area of the pipe. This is because in that example the pipe was not all at the same level (i.e., $h_1 \neq h_2$). An additional pressure arose on the right side because of the differences in the heights of the two pipes.

To go to this Interactive Example click on this sentence.

The effect of the decrease in pressure with the increase in speed of the fluid in a horizontal pipe is called the ***Venturi effect****,* and a simple device called a **Venturi meter,** based on this Venturi effect, is used to measure the velocity of fluids in pipes. A Venturi meter is shown schematically in figure 13.11(b). The device is basically the same as the pipe in 13.11(a) except for the two vertical pipes connected to the main pipe as shown. These open vertical pipes allow some of the water in the pipe to flow upward into the vertical pipes. The height that the water rises in the vertical pipes is a function of the pressure in the horizontal pipe. As just seen, the pressure in pipe 1 is greater than in pipe 2 and thus the height of the vertical column of water in pipe 1 will be greater than the height in pipe 2. By actually measuring the height of the fluid in the vertical columns the pressure in the horizontal pipe can be determined by the hydrostatic equation 13.7. Thus, the pressure in pipe 1 is

$$p_1 = \rho g h_{01}$$

and the pressure in pipe 2 is

$$p_2 = \rho g h_{02}$$

where h_{01} and h_{02} are the heights shown in figure 13.11(b). We can now write Bernoulli's equation 13.65 as

$$\rho g h_{01} + \frac{1}{2}\rho v_1^2 = \rho g h_{02} + \frac{1}{2}\rho v_2^2$$

Replacing v_2 by its value from the continuity equation 13.65, we get

$$\rho g h_{01} + \tfrac{1}{2}\rho v_1^2 = \rho g h_{02} + \tfrac{1}{2}\rho\left[\left(\frac{A_1}{A_2}\right)v_1\right]^2$$

$$\rho g h_{01} - \rho g h_{02} = +\tfrac{1}{2}\rho\frac{A_1^2}{A_2^2}v_1^2 - \tfrac{1}{2}\rho v_1^2$$

$$\rho g\left(h_{01} - h_{02}\right) = +\tfrac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} - 1\right)v_1^2$$

Solving for v_1^2, we have

$$v_1^2 = \frac{\rho g\left(h_{01} - h_{02}\right)}{\tfrac{1}{2}\rho\left[\left(A_1^2 / A_2^2\right) - 1\right]}$$

Solving for v_1, we get

$$v_1 = \sqrt{\frac{2g\left(h_{01} - h_{02}\right)}{\left(A_1^2 / A_2^2\right) - 1}} \tag{13.67}$$

Equation 13.67 now gives us a simple means of determining the velocity of fluid flow in a pipe. The main pipe containing the fluid is opened and the Venturi meter is connected between the opened pipes. When the fluid starts to move, the heights h_{01} and h_{02} are measured. Since the cross-sectional areas are easily determined by measuring the diameters of the pipes, the velocity of the fluid flow is easily calculated from equation 13.67.

Example 13.20

A Venturi meter. A Venturi meter reads heights of h_{01} = 30.0 cm and h_{02} = 10.0 cm. Find the velocity of flow v_1 in the pipe. The area $A_1 = 7.85 \times 10^{-3}$ m² and area $A_2 = 1.26 \times 10^{-3}$ m².

Solution

The velocity of flow v_1 in the main pipe, found from equation 13.67, is

$$v_1 = \sqrt{\frac{2g\left(h_{01} - h_{02}\right)}{\left(A_1^2 / A_2^2\right) - 1}}$$

$$v_1 = \sqrt{\frac{2(9.80\ \text{m/s}^2)(0.300\ \text{m} - 0.100\ \text{m})}{\frac{\left(7.85\times10^{-3}\ \text{m}^2\right)^2}{\left(1.26\times10^{-3}\ \text{m}^2\right)^2} - 1}}$$

$$= 0.322\ \text{m/s}$$

To go to this Interactive Example click on this sentence.

The Flow of a Liquid Through an Orifice

Let us consider the large tank of water shown in figure 13.12. Let the top of the fluid be location 1 and the orifice be location 2. Bernoulli's theorem applied to the tank, taken from equation 13.63, is

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

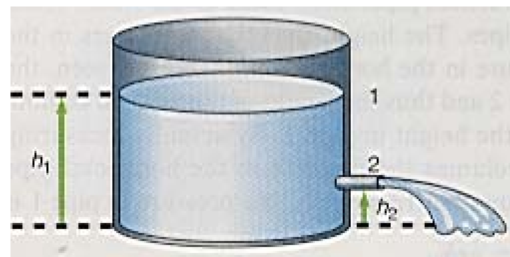

Figure 13.12 Flow from an orifice.

But the pressure at the top of the tank and the outside pressure at the orifice are both p_0, the normal atmospheric pressure. Also, because of the very large volume of fluid, the small loss through the orifice causes an insignificant vertical motion of the top of the fluid. Thus, $v_1 \approx 0$. Bernoulli's equation becomes

$$p_0 + \rho g h_1 + = p_0 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

The pressure term p_0 on both sides of the equation cancels out. Also h_2 is very small compared to h_1 and it can be neglected, leaving

$$\rho g h_1 = \frac{1}{2}\rho v_2^2$$

Solving for the velocity of efflux, we get

$$v_2 = \sqrt{2gh_1} \tag{13.68}$$

Notice that the velocity of efflux is equal to the velocity that an object would acquire when dropped from the height h_1.

Example 13.21

The velocity of efflux. A large water tank, 10.0 m high, springs a leak at the bottom of the tank. Find the velocity of the escaping water.

Solution

The velocity of efflux, found from equation 13.68, is

$$v_2 = \sqrt{2gh_1} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(10.0\ \text{m})}$$

$$= 14.0\ \text{m/s}$$

To go to this Interactive Example click on this sentence.

The Curving Baseball

When a nonspinning ball is thrown through the air it follows the straight line path shown in figure 13.13(a). The air moves over the top and bottom of the ball with a speed v_A. If the ball is now released with a downward spin, as shown in figure 13.13(b), then the spinning ball drags some air around with it. At the top of the ball, there is a velocity of the air v_A to the left, and a velocity of the dragged air on the spinning baseball v_S to the right. Thus, the relative velocity of the air with respect to the ball is $v_A - v_S$ at the top of the ball. At the bottom of the ball the dragged air caused by the spin of the baseball v_S is in the same direction as the velocity of the air v_A moving past the ball. Thus, the relative velocity of the air with respect to the bottom of the ball is $v_A + v_S$. Hence, the velocity of the air at the top of the ball, $v_A - v_S$, is less than the velocity of the air at the bottom of the ball, $v_A + v_S$. By the Venturi principle, the pressure of the fluid is smaller where the velocity is greater. Thus, the pressure on the bottom of the ball is less than the pressure on the top, that is,

$$p_{top} < p_{bottom}$$

But the pressure is related to the force by $p = F/A$. Hence, the force acting on the top of the ball is greater than the force acting on the bottom of the ball, that is,

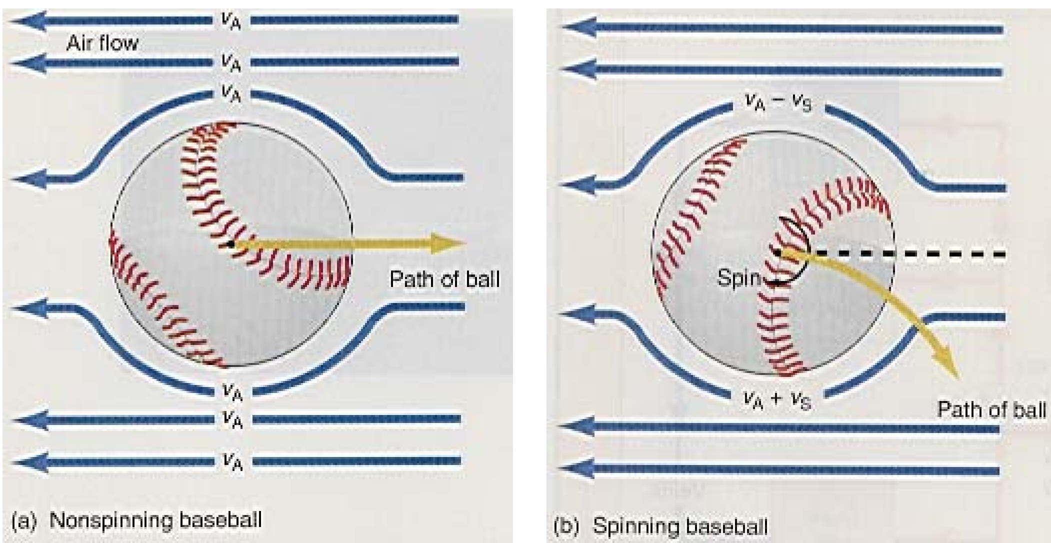

Figure 13.13 The curving baseball.

$$F_{top} < F_{bottom}$$

Therefore, the ball curves downward, or sinks, as it approaches the batter. By spinning the ball to the right (i.e., clockwise) as viewed from above, the ball curves toward the right. By spinning the ball to the left (i.e., counterclockwise) as viewed from above the ball, the ball curves toward the left. Spins about various axes through the ball can cause the ball to curve to the left and downward, to the left and upward, and so on.

Lift on an Airplane Wing

Another example of the Venturi effect can be seen with an aircraft wing, as shown in figure 13.14. The air flowing over the top of the wing has a greater distance to travel than the air flowing under the bottom of the wing. In order for the flow to be streamlined and for the air at the leading edge of the wing to arrive at the trailing edge at the same time, whether it goes above or below the wing, the velocity of the air over the top of the wing must be

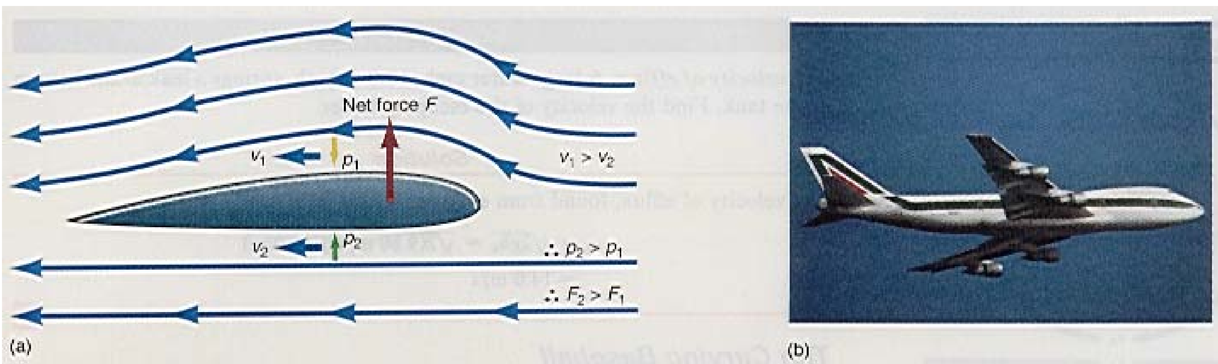

Figure 13.14 An airfoil.

greater than the velocity of the air at the bottom of the wing. But by the Venturi principle, if the velocity is greater at the top of the wing, the pressure must be less there than at the bottom of the wing. Thus, p_2 is greater than p_1 and therefore $F_2 < F_1$. That is, there is a net positive force $F_2 - F_1$ acting upward on the wing, producing lift on the airplane wing.

Have you ever wondered . . . ?
An Essay on the Application of Physics
The Flow of Blood in the Human Body

Human blood consists of a plasma, the fluid, and red and white corpuscles that are immersed in the plasma. Because blood is a fluid, the laws of physics can be applied to the flow of blood throughout the body. A schematic diagram of the circulatory system, which transports blood and oxygen around the body, is shown in figure 1. It consists of (1) the heart, which is the pump that is responsible for supplying the pressure to move the blood; (2) the lungs, which are the source of oxygen for all the cells of the body; (3) the arteries, which are connecting blood vessels that pass the blood from the heart to various parts of the body; (4) the capillaries, which are extremely small blood vessels that bring the oxygenated blood down to the layer of human cells; and (5) the veins, which are blood vessels that return deoxygenated blood to the heart to complete the circulatory system.

The heart is the pump that circulates the blood throughout the body and a diagram of it is shown in figure 2. Blood, containing carbon dioxide, returns to the heart by the veins and enters the right auricle. It is then pumped from the right ventricle to the pulmonary artery to the lungs where it dumps the waste carbon dioxide and picks up a new supply of oxygen. It then returns to the left auricle of the heart. The left ventricle then pumps this oxygen rich blood to the aorta, the main artery of the body, for distribution to the rest of the body.

For a person at rest, the heart pumps approximately 5.00 liters of blood per minute (8.33×10^{-5} m^3/s) at a rate of about 70 beats per minute. For a person engaged in very strenuous exercise the heart can pump up to 25.0 liters of blood per minute (41.7×10^{-5} m^3/s) at a rate of about 180 beats per minute. We can determine the speed of the blood as it enters the aorta by a generalization of equation 13.36, as

$$\frac{\Delta V}{\Delta t} = A_A v_A \tag{13H.1}$$

where $\Delta V/\Delta t$ is the rate at which the blood is flowing from the heart into the aorta, A_A is the cross-sectional area of the aorta, and v_A is the speed of the blood in the aorta. The diameter of the aorta is about 2.00 cm giving an area of

$$A = \pi r^2$$
$$= \pi(0.01 \text{ m})^2 = 3.14 \times 10^{-4} \text{ m}^2$$

The speed of the blood in the aorta is therefore

$$v_A = \frac{\Delta V/\Delta t}{A_A} \tag{13H.2}$$
$$= \frac{8.33 \times 10^{-5} \text{ m}^3/\text{s}}{3.14 \times 10^{-4} \text{ m}^2}$$
$$= 0.265 \text{ m/s} = 26.5 \text{ cm/s}$$

We can determine the speed of the blood in the capillaries by the continuity equation 13.47, as

$$A_A v_A = A_c v_C \tag{13H.3}$$

where A_A is the cross-sectional area of the aorta, which was just determined as 3.14×10^{-4} m²; v_A is the speed of the blood in the aorta, which was just found to be 26.5 cm/s;

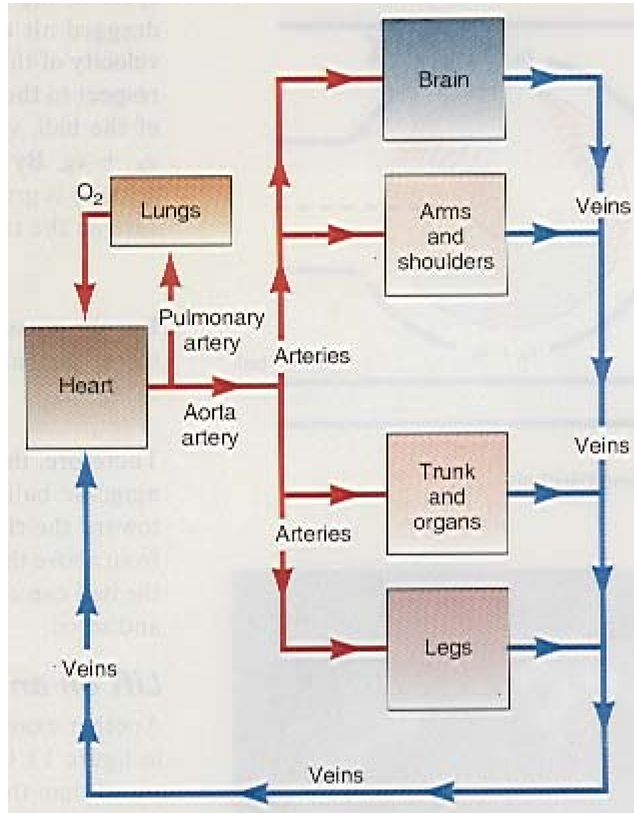

Figure 1 The circulatory system.

and A_C is the cross-sectional area of a capillary tube, which is quite small. However, because there are literally billions of these capillaries the effective cross-sectional area of all these capillaries combined is approximately 2500×10^{-4} m². The speed of the blood in the capillary becomes

$$v_C = \frac{A_A}{A_C} v_A$$
$$= \left(\frac{3.14 \times 10^{-4} \text{ m}^2}{2500 \times 10^{-4} \text{ m}^2}\right)(26.5 \text{ cm/s})$$
$$= 0.0333 \text{ cm/s}$$

Thus, the blood moves relatively slowly at the level of the capillaries.

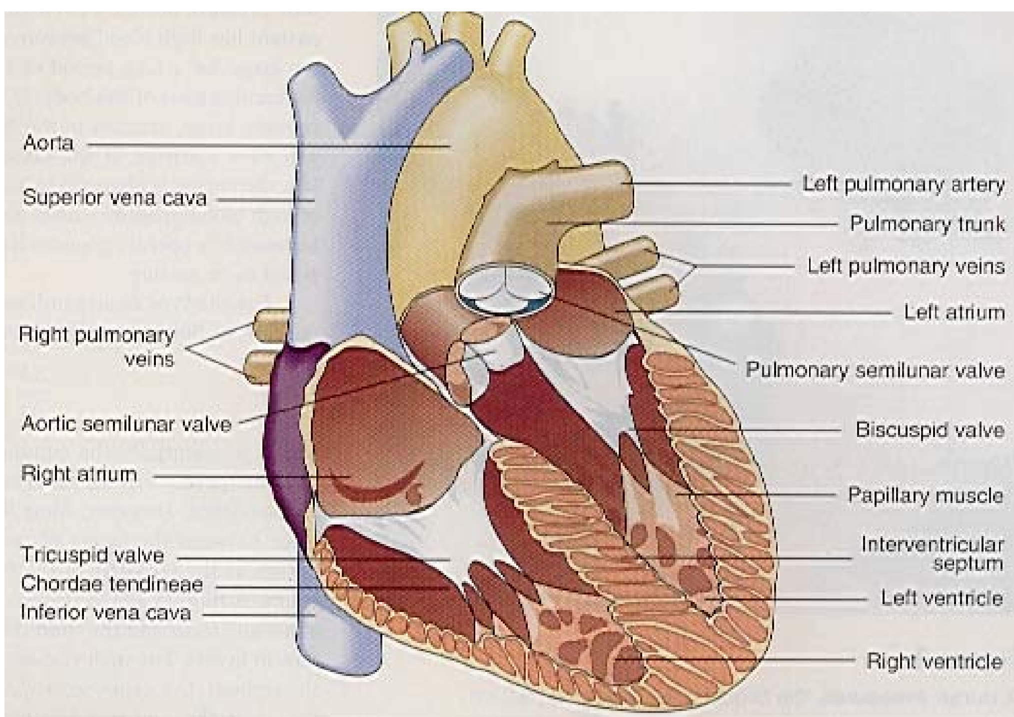

Figure 2 The human heart.

Finally, we should note that the body controls the flow of blood through the arteries by muscles that surround the arteries. When the muscles contract, the diameter of the artery is reduced. From the equation of continuity, Av = constant. By decreasing the diameter of the artery, the cross-sectional area of the artery decreases and hence the speed of blood must increase through the artery. Alternatively, when the muscles are relaxed, the diameter of the artery increases to its former size, the cross-sectional area increases, and the speed of the blood decreases. With advancing age the arterial muscles lose some of this ability to contract, a situation called hardening of the arteries, and the control of blood flow is somewhat diminished.

A good indication of how well the heart is functioning is obtained by measuring the pressure that the heart exerts when pumping blood, and when at rest. The device used to measure blood pressure is called a sphygmomanometer. (The word is derived from the Greek word *sphygmos,* meaning pulse, and the word *manometer,* which is a pressure measuring device. Hence, a sphygmomanometer is a device for measuring pulse pressure, or blood pressure.) The device consists of an air bag, called a cuff, that is wrapped around the upper arm of the patient at the level of the heart. A hand pump is used to inflate the cuff, and the pressure exerted by the cuff on the arm is measured by the mercury manometer. The pressure exerted by the cuff is increased until the pressure is great enough to collapse the brachial artery in the arm, cutting off the blood supply to the rest of the arm. A stethoscope is placed over the brachial artery and the pressure in the cuff is slowly decreased. When the pressure in the cuff becomes low enough, the pressure exerted by the heart is large enough to force the artery open and some blood squirts through. This blood flowing through the narrow restriction becomes turbulent and makes a noise as it enters the open portion of the artery. The physician hears this noise through the stethoscope, and simultaneously observes the pressure indicated on the manometer, expressed in terms of mm of Hg. At this point the pressure exerted by the heart, called the systolic pressure, is equal to the pressure exerted by the cuff. A normal systolic pressure is around 120 mm of Hg.

Figure 3 A nurse measures the blood pressure of a patient.

As the pressure in the cuff is decreased the turbulent flow noise is still heard in the stethoscope until the lowest pressure exerted by the heart, the diastolic pressure, is equal to the pressure exerted by the cuff. At this point the artery is completely open and the blood is no longer in turbulent flow and the characteristic noise disappears. The pressure is read from the mercury manometer at this point. This pressure is the pressure that the heart exerts when it is at rest. The normal diastolic pressure is around 80 mm of Hg. The combined systolic and diastolic pressures are usually indicated in the form 120/80. If the systolic pressure becomes too high, above about 150 mm of Hg, the patient has high blood pressure. If the systolic pressure becomes too large for a long period of time, damage can be done to the different organs of the body. If the systolic pressure becomes extremely large, arteries in the brain can rupture and the person will have a stroke. If the diastolic pressure exceeds 90 mm of Hg, the person is also said to have high blood pressure. This type of high blood pressure causes eventual damage to the heart itself, because it is operating under high pressures even while it is supposed to be resting.

For the type of streamlined flow considered in this chapter the flow of fluid per unit time was shown to be

$$\frac{\Delta V}{\Delta t} = Av \tag{13.36}$$

which is essentially the equation of continuity. In this type of flow the speed v was the same throughout the cross-sectional area A considered. However, some fluids have a significant frictional force between the layers of the fluid, and this frictional effect, known as the *viscosity* of the fluid, must then be taken into account. A fluid in which frictional effects are significant is called a *viscous fluid* and the fluid flow is referred to as *laminar flow,* flow in layers. For such viscous fluids the speed v is not the same throughout the cross-sectional area A. The maximum speed occurs at the center of the pipe or tube, whereas the speed is essentially zero at the walls of the pipe. Experimental work by J. L. Poiseuille (1799-1869), a French scientist, and subsequently confirmed by theory, showed that the flow rate for viscous fluids is given by

$$\frac{\Delta V}{\Delta t} = \frac{(\Delta p)\pi R^4}{8\eta L} \tag{13H.4}$$

where Δp is the pressure difference between both ends of the pipe, R is the radius of the pipe, L is the length of the pipe, and η is the coefficient of viscosity of the fluid. Equation 13H.4 is called *Poiseuille's equation*. Note that the flow rate is inversely proportional to the coefficient of viscosity of the fluid. Thus, a very viscous fluid (high value of η) flows very slowly compared to a fluid of low viscosity. That is, everything else being equal, molasses flows at a slower rate than water. Human blood is a viscous fluid, the greater the number of red corpuscles in the blood the greater the viscosity. The viscosity of human blood varies from about 1.50×10^{-3} (N/m²)s for plasma, to about 4.00×10^{-3} (N/m²)s for whole blood. Also note that the flow rate depends on the fourth power of the radius of the pipe. If the radius is doubled, the flow rate is multiplied by a factor of 16. This relation is important in the selection of the size of hypodermic needles.

Example 13H.1

A blood transfusion. A person is receiving a blood transfusion. The bottle containing the blood is elevated 75.0 cm above the arm of the person. The needle is 4.00 cm long and has a diameter of 0.500 mm. Find the rate at which the blood flows through the needle.

Solution

The rate of flow of blood is found from equation 13H.4, where η, the viscosity of blood, is 4.00×10^{-3} Ns/m². Let us assume that the total pressure differential is obtained by the effects of gravity from the hydrostatic equation, equation 13.7. The density of blood is about 1050 kg/m³. Thus,

$$\begin{aligned}\Delta p &= \rho g h \\ &= (1050 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.750 \text{ m}) \\ &= 7.72 \times 10^3 \text{ Pa}\end{aligned}$$

The blood flow rate now obtained is

$$\begin{aligned}\frac{\Delta V}{\Delta t} &= \frac{(\Delta p)\pi R^4}{8\eta L} \qquad (13H.4) \\ &= \frac{(7.72 \times 10^3 \text{ N/m}^2)(\pi)(0.250 \times 10^{-3} \text{ m})^4}{8(4.00 \times 10^{-3} \text{ Ns/m}^2)(0.0400 \text{ m})} \\ &= 7.40 \times 10^{-8} \text{ m}^3/\text{s}\end{aligned}$$

The Language of Physics

Fluids
A fluid is any substance that can flow. Hence, liquids and gases are both considered to be fluids (p.).

Fluid statics or hydrostatics
The study of fluids at rest (p.).

Fluid dynamics or hydrodynamics
The study of fluids in motion (p.).

Density
The amount of mass in a unit volume of a substance (p.).

Pressure
The magnitude of the normal force acting per unit surface area (p.).

The hydrostatic equation
An equation that gives the pressure of a fluid at a particular depth (p.).

Barometer
An instrument that measures atmospheric pressure (p.).

Gauge pressure
The pressure indicated on a pressure measuring gauge. It is equal to the absolute pressure minus normal atmospheric pressure (p.).

Pascal's principle
If the pressure at any point in an enclosed fluid at rest is changed, the pressure changes by an equal amount at all points in the fluid (p.).

Archimedes' principle
A body immersed in a fluid is buoyed up by a force that is equal to the weight of the fluid displaced. A body floats when the weight of the body is equal to the weight of the fluid displaced (p.).

Law of conservation of mass
In any ordinary mechanical or chemical process, mass is neither created nor destroyed (p.).

The equation of continuity
An equation based on the law of conservation of mass, that indicates that when the cross-sectional area of a pipe gets smaller, the velocity of the fluid must become greater. Conversely, when the cross-sectional area increases, the velocity of the fluid must decrease (p.).

Bernoulli's theorem
The sum of the pressure, the potential energy per unit volume, and the kinetic energy per unit volume at any one location of the fluid is equal to the sum of the pressure, the potential energy per unit volume, and the kinetic energy per unit volume at any other location in the fluid, for a nonviscous, incompressible fluid in streamlined flow (p.).

Venturi effect
The effect of the decrease in pressure with the increase in speed of the fluid in a horizontal pipe (p.).

Venturi meter
A device that uses the Venturi effect to measure the velocity of fluids in pipes (p.).

Summary of Important Equations

Density $\rho = \frac{m}{V}$ (13.1)

Mass $m = \rho V$ (13.2)

Pressure $p = \frac{F}{A}$ (13.3)

Hydrostatic equation
$$p = \rho g h \quad (13.7)$$

Force $F = pA$ (13.12)

Absolute and gauge pressure
$$p_{abs} = p_{gauge} + p_0 \quad (13.15)$$

Hydraulic lift $F = \frac{A}{a} f$ (13.18)

$$y_1 = \frac{A}{a} y_2 \quad (13.23)$$

Archimedes' principle
Buoyant force = Weight of water displaced (13.30)

Mass flow rate $\frac{\Delta m}{\Delta t} = \rho A v$ (13.38)

Equation of continuity
$$A_1 v_1 = A_2 v_2 \quad (13.47)$$

$$Av = \text{constant} \quad (13.49)$$

Work done in moving a fluid
$$W = p\Delta V \quad (13.50)$$

Bernoulli's theorem
$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \quad (13.63)$$

and
$$p + \rho g h + \frac{1}{2}\rho v^2 = \text{constant} \quad (13.64)$$

Questions for Chapter 13

1. Discuss the differences between solids, liquids, and gases.

*2. Hieron II, King of Syracuse in ancient Greece, asked his relative Archimedes to determine if the gold crown made for him by the local goldsmith, was solid gold or a mixture of gold and silver. How did Archimedes, or how could you, determine whether or not the crown was pure gold?

3. When you fly in an airplane you find that your ears keep "popping" when the plane is ascending or descending. Explain why.

4. Using a barometer and the direction of the wind, describe how you could make a reasonable weather forecast.

*5. A pilot uses an aneroid barometer as an altimeter that is calibrated to a standard atmosphere. What happens to the aircraft if the temperature of the atmosphere does not coincide with the standard atmosphere?

*6. Does a sphygmomanometer measure gauge pressure or absolute pressure?

7. How would you define a mechanical advantage for the hydraulic lift?

8. In example 13.13, could the iron block sink to a depth of 39.4 cm in a pool of water 100 cm deep and then float at that point? Why or why not?

9. How does eating foods very high in cholesterol have an effect on the arteries and hence the flow of blood in the body?

*10. Why is an intravenous bottle placed at a height h above the arm of a patient?

Problems for Chapter 13

13.2 Density

1. A cylinder 3.00 cm in diameter and 3.00 cm high has a mass of 15.0 g. What is its density?

2. Find the mass of a cube of iron 10.0 cm on a side.

3. A gold ingot is 50.0 cm by 20.0 cm by 10.0 cm. Find (a) its mass and (b) its weight.

4. Find the mass of the air in a room 6.00 m by 8.00 m by 3.00 m.

5. Assume that the earth is a sphere. Compute the average density of the earth.

6. Find the weight of 1.00 liter of air.

7. A crown, supposedly made of gold, has a mass of 8.00 kg. When it is placed in a full container of water, 691 cm^3 of water overflows. Is the crown made of pure gold or is it mixed with some other materials?

8. A solid brass cylinder 10.0 cm in diameter and 25.0 cm long is soldered to a solid iron cylinder 10.0 cm in diameter and 50.0 cm long. Find the weight of the combined cylinder.

9. An annular cylinder of 2.50-cm inside radius and 4.55-cm outside radius is 10.5 cm high. If the cylinder has a mass of 5.35 kg, find its density.

13.3 Pressure

10. As mentioned in the text, a non-SI unit of pressure is the torr, named after Torricelli, which is equal to the pressure exerted by a column of mercury 1 mm high. Express a pressure of 2.53×10^5 Pa in torrs.

*11. From the knowledge of normal atmospheric pressure at the surface of the earth, compute the approximate mass of the atmosphere.

12. A barometer reads a height of 72.0 cm of Hg. Express this atmospheric pressure in terms of (a) in. of Hg, (b) mb, (c) lb/in.^{2}, and (d) Pa.

13. (a) A "high" pressure area of 1030 mb moves into an area. What is this pressure expressed in N/m^2 and lb/in.^{2}? (b) A "low" pressure area of 980 mb moves into an area. What is this pressure expressed in N/m^2 and lb/in.^{2}?

14. Normal systolic blood pressure is approximately 120 mm of Hg and normal diastolic pressure is 80 mm of Hg. Express these pressures in terms of Pa and lb/in.^{2}.

15. The point of a 10-penny nail has a diameter of 1.00 mm. If the nail is driven into a piece of wood with a force of 150 N, find the pressure that the tip of the nail exerts on the wood.

16. The gauge pressure in the tires of your car is 2.42×10^5 N/m^2. What is the absolute pressure of the air in the tires?

17. What is the water pressure and the absolute pressure in a swimming pool at depths of (a) 1.00 m, (b) 2.00 m, (c) 3.00 m, and (d) 4.00 m?

18. Find the force exerted by normal atmospheric pressure on the top of a table 1.00 m high, 1.00 m long, 0.75 m wide, and 0.10 m thick. What is the force on the underside of the table top exerted by normal atmospheric pressure?

19. A portion of the roof of a home is 12.2 m long and 6.50 m high, and makes an angle of 40.0^0 with the horizontal. What force is exerted on the top of this roof by normal atmospheric pressure?

20. If normal atmospheric pressure can support a column of Hg 76.0 cm high, how high a column will it support of (a) water, (b) benzene, (c) alcohol, and (d) glycerine?

21. What is the minimum pressure of water entering a building if the pressure at the second floor faucet, 4.60 m above the ground, is to be 3.45×10^4 N/m^2 ?

22. The water main pressure entering a house is 31.0 N/cm^2. What is the pressure at the second floor faucet, 6.00 m above the ground? What is the maximum height of any faucet such that water will still flow from it?

23. A barometer reads 76.0 cm of Hg at the base of a tall building. The barometer is carried to the roof of the building and now reads 75.6 cm of Hg. If the average density of the air is 1.28 kg/m^3, what is the height of the building?

24. The hatch of a submarine is 100 cm by 50.0 cm. What force is exerted on this hatch by the water when the submarine is 50.0 m below the surface?

13.4 Pascal's Principle

25. In the hydraulic lift of figure 13.5, the diameter d_1 = 10.0 cm and d_2 = 50.0 cm. If a force of 10.0 N is applied at the small piston, (a) what force will appear at the large piston? (b) If the large piston is to move through a height of 2.00 m, what must the total displacement of the small piston be?

26. In a hydraulic lift, the large piston exerts a force of 25.0 N when a force of 3.50 N is applied to the smaller piston. If the smaller piston has a radius of 12.5 cm, and the lift is 65.0% efficient, what must be the radius of the larger piston?

27. The theoretical mechanical advantage (TMA) of a hydraulic lift is equal to the ratio of the force that you get out of the lift to the force that you must put into the lift. Show that the theoretical mechanical advantage of the hydraulic lift is given by

$$\text{TMA} = \frac{F_{out}}{F_{in}} = \frac{A_{out}}{A_{in}} = \frac{y_{in}}{y_{out}}$$

where A_{out} is the area of the output piston, A_{in} is the area of the input piston, y_{in} is the distance that the input piston moves, and y_{out} is the distance that the output piston moves.

13.5 Archimedes' Principle

28. Find the weight of a cubic block of iron 5.00 cm on a side. This block is now hung from a spring scale such that the block is totally submerged in water. What would the scale indicate for the weight (called the apparent weight) of the block?

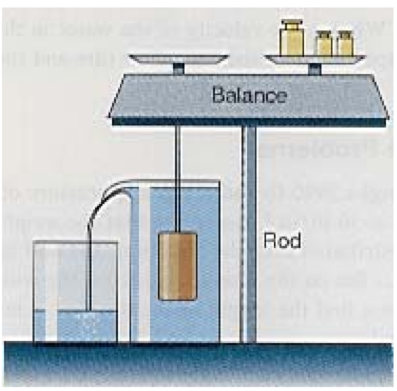

Diagram for problem 28.

29. A copper cylinder 5.00 cm high and 3.00 cm in diameter is hung from a spring scale such that the cylinder is totally submerged in ethyl alcohol. Find the apparent weight of the block.

30. Find the buoyant force on a brass block 10.5 cm long by 12.3 cm wide by 15.0 cm high when placed in (a) water, (b) glycerine, and (c) mercury.

31. If the iron block in example 13.13 were placed in a pool of mercury instead of the water would it float or sink? If it floats, to what depth does it sink before it floats?

*32. A block of wood sinks 8.00 cm in pure water. How far will it sink in salt water?

33. A weather balloon contains 33.5 m^3 of helium at the surface of the earth. Find the largest load this balloon is capable of lifting. The density of helium is 0.1785 kg/m^3.

13.6 The Equation of Continuity

34. A 2.50-cm pipe is connected to a 0.900-cm pipe. If the velocity of the fluid in the 2.50-cm pipe is 1.50 m/s, what is the velocity in the 0.900-cm pipe? How much water flows per second from the 0.900-cm pipe?

35. A duct for a home air-conditioning unit is 35.0 cm in diameter. If the duct is to remove the air in a room 9.00 m by 6.00 m by 3.00 m high every 15.0 min, what must the velocity of the air in the duct be?

13.7 Bernoulli's Theorem

36. Water enters the house from a main at a pressure of 1.5×10^5 Pa at a speed of 40.0 cm/s in a pipe 4.00 cm in diameter. What will be the pressure in a 2.00-cm pipe located on the second floor 6.00 m high when no water is flowing from the upstairs pipe? When the water starts flowing, at what velocity will it emerge from the upstairs pipe?

37. A can of water 30.0 cm high sits on a table 80.0 cm high. If the can develops a leak 5.00 cm from the bottom, how far away from the table will the water hit the floor?

38. Water rises to a height h_{01} = 35.0 cm, and h_{02} = 10.0 cm, in a Venturi meter, figure 13.11(b). The diameter of the first pipe is 4.00 cm, whereas the diameter of the second pipe is 2.00 cm. What is the velocity of the water in the first and second pipe? What is the mass flow rate and the volume flow rate?

Additional Problems

39. A car weighs 12,500 N and the gauge pressure of the air in each tire is 2.00×10^5 N/m^2. Assuming that the weight of the car is evenly distributed over the four tires, (a) find the area of each tire that is flat on the ground and (b) if the width of the tire is 15.0 cm, find the length of the tire that is in contact with the ground.

40. A certain portion of a rectangular, concrete flood wall is 12.0 m high and 30.0 m long. During severe flooding of the river, the water level rises to a height of 10.0 m. Find (a) the water pressure at the base of the flood wall, (b) the average water pressure exerted on the flood wall, and (c) the average force exerted on the flood wall by the water.

41. The Vehicle Assembly Building at the Kennedy Space Center is 160 m high. Assuming the density of air to be a constant, find the difference in atmospheric pressure between the ground floor and the ceiling of the building.

42. If the height of a water tower is 20.0 m, what is the pressure of the water as it comes out of a pipe at the ground?

*43. A 20.0-g block of wood floats in water to a depth of 5.00 cm. A 10.0-g block is now placed on top of the first block, but it does not touch the water. How far does the combination sink?

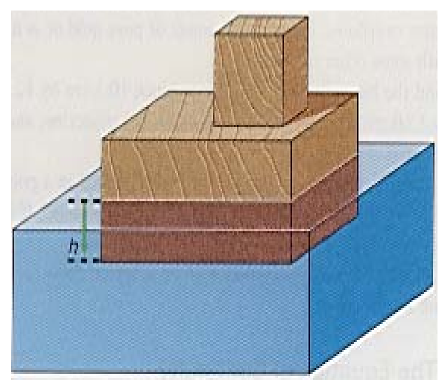

Diagram for problem 43.

*44. An iron ball, 4.00 cm in diameter, is dropped into a tank of water. Assuming that the only forces acting on the ball are gravity and the buoyant force, determine the acceleration of the ball. Discuss the assumption made in this problem.

*45. If 80% of a floating cylinder is beneath the water, what is the density of the cylinder?

*46. From knowing that the density of an ice cube is 920 kg/m^3 can you determine what percentage of the ice cube will be submerged when in a glass of water?

*47. Find the equation for the length of the side of a cube of material that will give the same buoyant force as (a) a sphere of radius r and (b) a cylinder of radius r and height h, if both objects are completely submerged.

*48. Find the radius of a solid cylinder that will experience the same buoyant force as an annular cylinder of radii r_2 = 4.00 cm and r_1 = 3.00 cm. Both cylinders have the same height h.

*49. A cone of maximum radius r_0 and height h_0, is placed in a fluid, as shown in the diagram. The volume of a right circular cone is given by

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

(a) Find the equation for the weight of the cone. (b) If the cone sinks so that a height h_1 remains out of the fluid, find the equation for the volume of the cone that is immersed in the fluid. (c) Find the equation for the buoyant force acting on the cone. (d) Show that the height h_1 that remains out of the fluid is given by

$$h_1 = \sqrt[3]{1-\left(\rho_c/\rho_f\right)}\ ho$$

where ρ_c is the density of the cone and ρ_f is the density of the fluid. (e) If we approximate an iceberg by a cone, find the percentage height of the iceberg that sticks out of the salt water, and the percentage volume of the iceberg that is below the water.

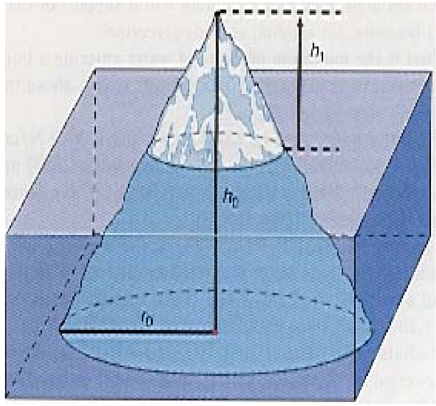

Diagram for problem 49.

*50. A can 30.0 cm high is filled to the top with water. Where should a hole be made in the side of the can such that the escaping water reaches the maximum distance x in the horizontal direction? *(Hint:* calculate the distance x for values of h from 0 to 30.0 cm in steps of 5.00 cm.)

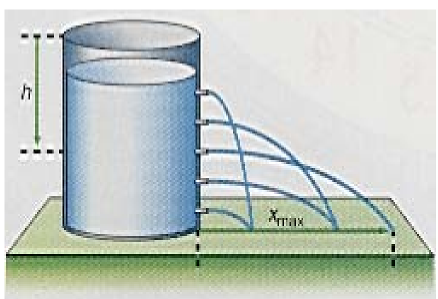

Diagram for problem 50.

51. In the flow of fluid from an orifice in figure 13.12, it was assumed that the vertical motion of the water at the top of the tank was very small, and hence v_1 was set equal to zero. Show that if this assumption does not hold, the velocity of the fluid from the orifice v_2 can be given by

$$v_2 = \sqrt{\frac{2gh}{1-\left(d_2^4/d_1^4\right)}}$$

where d_1 is the diameter of the tank and d_2 is the diameter of the orifice.

*52. A wind blows over the roof of a house at 136 km/hr. What is the difference in pressure acting on the roof because of this velocity? *(Hint:* the air inside the attic is still, that is, v = 0 inside the house.)

*53. If air moves over the top of an airplane wing at 150 m/s and 120 m/s across the bottom of the wing, find the difference in pressure between the top of the wing and the bottom of the wing. If the area of the wing is 15.0 m², find the force acting upward on the wing.

Interactive Tutorials

54. *Buoyant force.* Find the buoyant force BF and apparent weight AW of a solid sphere of radius r = 0.500 m and density ρ = 7.86 × 10³ kg/m³, when immersed in a fluid whose density is ρ_f = 1.00 × 10³ kg/m³.

55. *Archimedes' principle.* A solid block of wood of length L = 15.0 cm, width W = 20.0 cm, and height h_0 = 10.0 cm, is placed into a pool of water. The density of the block is 680 kg/m³. (a) Will the block sink or float? (b) If it floats, how deep will the block be submerged when it floats? (c) What percentage of the original volume is submerged?

56. *The equation of continuity and flow rate.* Water flows in a pipe of diameter d_1 = 4.00 cm at a velocity of 35.0 cm/s, as shown in figure 13.9. The diameter of the tapered part of the pipe is d_2 = 2.55 cm. Find (a) the velocity of the fluid in the tapered part of the pipe, (b) the mass flow rate, and (c) the volume flow rate of the fluid.

57. *Bernoulli's theorem.* Water flows in an elevated, tapered pipe, as shown in figure 13.10. The first part of the pipe is at a height h_1 = 3.58 m above the ground and the water is at a pressure p_1 = 5000 N/m², the diameter d_1 = 25.0 cm, and the velocity of the water is v_1 = 0.553 m/s. If the diameter of the tapered part of the pipe is d_2 = 10.0 cm and the height of the pipe above the ground is h_2 = 1.25 m, find (a) the velocity v_2 of the fluid in the tapered part of the pipe and (b) the pressure p_2 of the water in the tapered part of the pipe.

To go to these Interactive Tutorials click on this sentence.

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