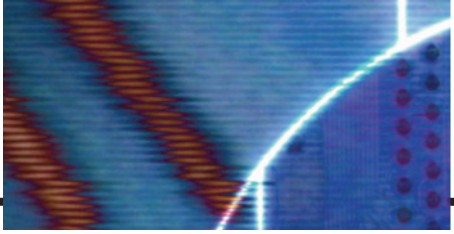
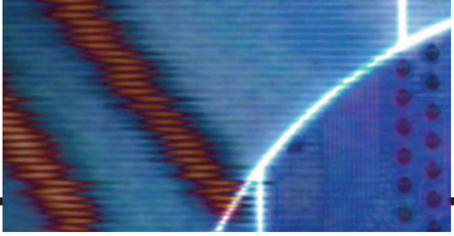
Appendixes





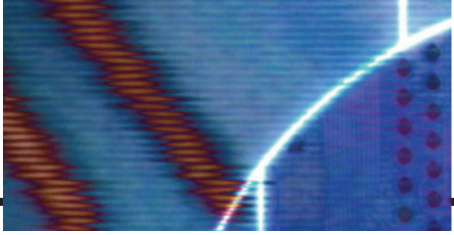
Appendix A

CONVERSION FACTORS

To Convert from	To	Multiply by
Btus	Calorie-grams	251.996
	Ergs	1.054×10^{10}
	Foot-pounds	777.649
	Hp-hours	0.000393
	Joules	1054,35
	Kilowatthours	0.000293
	Wattseconds	1054.35
Centimeters	Angstrom units	1×10^{8}
	Feet	0.0328
	Inches	0.3937
	Meters	0.01
	Miles (statute)	6.214×10^{-6}
	Millimeters	10
Circular mils	Square centimeters	5.067×10^{-6}
	Square inches	7.854×10^{-7}
Cubic inches	Cubic centimeters	16.387
	Gallons (U.S. liquid)	0.00433
Cubic meters	Cubic feet	35.315
Days	Hours	24
	Minutes	1440
	Seconds	86,400
Dynes	Gallons (U.S. liquid)	264.172
	Newtons	0.00001
	Pounds	2.248×10^{-6}
Electronvolts	Ergs	1.60209×10^{-12}
Ergs	Dyne-centimeters	1.0
	Electronvolts	6.242×10^{11}
	Foot-pounds	7.376×10^{-8}
	Joules	1×10^{-7}
	Kilowatthours	2.777×10^{-14}
Feet	Centimeters	30.48
	Meters	0.3048
Foot-candles	Lumens/square foot	1.0
	Lumens/square meter	10.764
Foot-pounds	Dyne-centimeters	1.3558×10^{7}
	Ergs	1.3558×10^{7}
	Horsepower-hours	5.050×10^{-7}
	Joules	1.3558
	Newton-meters	1.3558
Gallons (U.S. liquid)	Cubic inches	231
	Liters	3.785
	Ounces	128
	Pints	8

To Convert from	To	Multiply by
Gauss	Maxwells/square centimeter	1.0
	Lines/square centimeter	1.0
	Lines/square inch	6.4516
Gilberts	Ampere-turns	0.7958
Grams	Dynes	980.665
	Ounces	0.0353
	Pounds	0.0022
Horsepower	Btus/hour	2547.16
	Ergs/second	7.46×10^9
	Foot-pounds/second	550.221
	Joules/second	746
	Watts	746
Hours	Seconds	3600
Inches	Angstrom units	2.54×10^8
	Centimeters	2.54
	Feet	0.0833
	Meters	0.0254
Joules	Btus	0.000948
	Ergs	1×10^7
	Foot-pounds	0.7376
	Horsepower-hours	3.725×10^{-7}
	Kilowatthours	2.777×10^{-7}
	Wattseconds	1.0
Kilograms	Dynes	980,665
	Ounces	35.2
	Pounds	2.2
Lines	Maxwells	1.0
Lines/square centimeter	Gauss	1.0
Lines/square inch	Gauss	0.1550
	Webers/square inch	1×10^{-8}
Liters	Cubic centimeters	1000.028
	Cubic inches	61.025
	Gallons (U.S. liquid)	0.2642
	Ounces (U.S. liquid)	33.815
	Quarts (U.S. liquid)	1.0567
Lumens	Candle power (spher.)	0.0796
Lumens/square centimeter	Lamberts	1.0
Lumens/square foot	Foot-candles	1.0
Maxwells	Lines	1.0
	Webers	1×10^{-8}
Meters	Angstrom units	1×10^{10}
	Centimeters	100
	Feet	3.2808
	Inches	39.370
	Miles (statute)	0.000621

To Convert from	To	Multiply by
Miles (statute)	Feet	5280
	Kilometers	1.609
	Meters	1609.344
Miles/hour	Kilometers/hour	1.609344
Newton–meters	Dyne–centimeters	1×10^7
	Kilogram-meters	0.10197
Oersteds	Ampere-turns/inch	2.0212
	Ampere-turns/meter	79.577
	Gilberts/centimeter	1.0
Quarts (U.S. liquid)	Cubic centimeters	946.353
	Cubic inches	57.75
	Gallons (U.S. liquid)	0.25
	Liters	0.9463
	Pints (U.S. liquid)	2
	Ounces (U.S. liquid)	32
Radians	Degrees	57.2958
Slugs	Kilograms	14.5939
	Pounds	32.1740
Watts	Btus/hour	3.4144
	Ergs/second	1×10^7
	Horsepower	0.00134
	Joules/second	1.0
Webers	Lines	1×10^8
	Maxwells	1×10^8
Years	Days	365
	Hours	8760
	Minutes	525,600
	Seconds	3.1536×10^7



Appendix B

TI-86 CALCULATOR

This appendix provides information for those students who have a TI-86 calculator. The coverage parallels that given for the TI-89 in the text material.

INITIAL SETTINGS

For the TI-86 calculator, pressing the 2nd function (yellow) key followed by the MODE key provides a list of options for the initial settings of the calculator. For each item in the MODE list, use the scroll keys to make a selection and then select the ENTER key.

ORDER OF OPERATIONS

The order of operations is the same as for the TI-89 calculator.

DETERMINANTS

The determinant operator is obtained by the sequence: **2nd FUNCTION-MATRX-MATH.** The **det** operator appears at the far left of the listing at the bottom of the screen. Select it by pressing the **F1** key. Once selected, enter all the parameters of the matrix within a set of brackets. Enter the first row of the determinant within a second set of brackets with a comma between each entry. Enter the second row in the same manner. After adding the closing bracket, select the **ENTER** key to provide the solution. Always be aware that the number of brackets forming a left enclosure must equal the number forming a right enclosure.

Following are the determinants for the current I_1 and the calculator input required:

$$I_1 = \frac{\begin{vmatrix} 2 & 4 \\ 6 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & 4 \\ 4 & 5 \end{vmatrix}} = \frac{10 - 24}{30 - 16} = \frac{-14}{14} = \mathbf{-1\ A}$$

det[[2,4][6,5]]/det[[6,4][4,5]] ENTER	–1

Following is a third-order determinant and the calculator input required:

$$I_3 = I_{10\Omega} = \frac{\begin{vmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{vmatrix}} = \mathbf{1.22\ A}$$

det[[11,−3,15][−3, 10,0][−8,−5,0]]/det[[11,−3,−8][−3,10,−5][−8,−5,23]] ENTER	1.22

EXPONENTIAL FUNCTION

The exponential function e^x is obtained through the sequence 2nd Function e^x.

For the equation

$$v = 20\text{ V}(1 - e^{-2.5})$$

the calculator sequence would be the following:

20 (1 − 2nd e^x (−) 2.5) ENTER	18.3

LOGARITHMS

The natural logarithm ($\log_e$) is obtained using the LN key. That is, $\log_e 2$ is obtained using the following sequence:

LN 2 ENTER	693.15E−3

The common logarithm ($\log_{10}$) is obtained using the LOG key. That is, $\log_{10} 2$ is obtained using the following sequence:

LOG 2 ENTER	301.03E−3

The antilogarithm of a natural logarithm permits working backwards on the above operation. That is, it provides the number we have to take the natural logarithm of to obtain the given number. For instance, the natural logarithm of what number will result in 693.15E-3? This is expressed in equation form as follows:

LN (?) = 693.15E−3

The result is obtained using the exponential function in the following manner:

$$e^{693.15\text{E}-3} = 2.00$$

The calculator sequence is:

2nd e^x 693.15EE(−)3 ENTER	2.00

The common logarithm,

LOG (?) = 301.03E−3

is processed with

2nd 10^x 301.03EE(−)3 ENTER	2.00

TRIGONOMETRIC FUNCTIONS

Degrees vs. Radians

The choice is made by selecting one or the other from the **MODE** listing. Don’t forget the **ENTER** operation before leaving the screen!

Sin, Cos, and Tan

Select the appropriate key followed by the angle in degrees as in the following:

SIN 30 ENTER	0.50

Sin^{-1}, Cos^{-1}, and Tan^{-1}

The 2nd function must be used to obtain the desired function as shown below:

2nd SIN^{-1} 0.5 ENTER	30

COMPLEX NUMBERS

To convert complex numbers on the TI-86 calculator, you must first call up the 2nd function **CPLX** from the keyboard, which results in a menu at the bottom of the display including conj, real, imag, abs, and angle. If you choose the key **MORE, ▶ Rec** and **▶ Pol** appear as options (for the conversion process). To convert from one form to another, enter the current form in brackets with a comma between components for the rectangular form and an angle symbol for the polar form. Follow this form with the operation to be performed, and select **ENTER.** The result appears on the screen in the desired format.

To convert $3 - j4$ to polar form:

(3, −4) ▶ Pol ENTER	(5.00E0 ∠−53.13E0)

To convert $0.006^{20.6}$ to rectangular form:

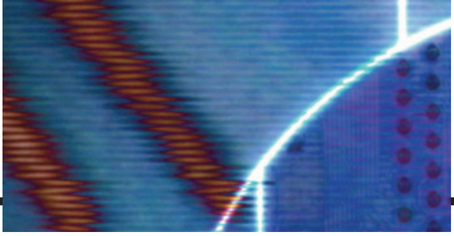
(0.006 ∠20.6) ▶ Rec ENTER	(5.62E−3, 2.11E−3)

To solve this equation:

$$\frac{(2\angle 20^\circ)^2(3 + j\,4)}{8 - j\,6} = ?$$

use this sequence:

$((2\ \angle 20)^2 * (3,4))/(8,-6)$ ▶ Pol ENTER	(2.00E0 ∠130.00E0)



Appendix C

PSPICE, MULTISIM, AND MATHCAD

PSPICE 10.0

The PSpice software package used throughout this text is derived from programs developed at the University of California at Berkeley during the early 1970s. SPICE is an acronym for Simulation Program with Integrated Circuit Emphasis. Although a number of companies have customized SPICE for their particular use, Cadence Design Systems offers both a commercial and a demo version of OrCAD. The commercial or professional versions that engineering companies use can be quite expensive, so Cadence offers a free distribution of a demo version to provide an introduction to the power of this simulation package. This text uses the OrCAD 10.0 Demo version. An application for a free demo version is available online through the web site www.orcad.com. Select **OrCAD Capture** and then **OrCAD Demo CD Request**. Fill out the provided form to receive a copy of the demo version.

You can also contact EMA Mid Atlantic at 1-877-EMA-EDA1 or visit the EMA web site at www.ema-eda.com

Minimum system requirements:

Pentium 4 (32-bit) equivalent or faster
Edition, or Windows 2000 (SP4)
256 MB RAM (512 MB recommended)
300 MB swap space (or more)
CD-ROM drive
32,768 color Windows display

MULTISIM 8

Multisim is a product of Electronics Workbench, a subsidiary of National Instruments. Its web site is www.electronicsworkbench.com and phone number in the U.S. and Canada is 1 800-263-5552. In Europe, the web site is www.ewbeurope.com and phone number is 31-35-694-4444.

Copies can be purchased at the above web sites and through Prentice Hall at www.prenhall.com.

Minimum system requirements:

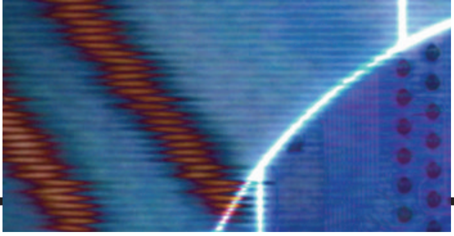
Windows 2000/XP (Windows XP Professional recommended)
Pentium III (Pentium 4 recommended)
128 MB RAM (256 MB recommended)
150 MB hard disk space (500 MB recommended)
CD-ROM drive
800×600 screen resolution (1024×768 recommended)

MATHCAD 12

Mathcad is a product of MathSoft Engineering and Education, Inc. located at 101 Main Street, Cambridge, MA 02142-1521. The web site is www.mathsoft.com. The academic version of Mathcad is available online for purchase at www.edu.com and www.journeyed.com. Professors can purchase the current Mathcad 12 Academic Edition by calling the customer service department at 800-628-4223 or 617-444-8000 or email sales-info@mathsoft.com.

Minimum system requirements for Mathcad 12:

PC with 300 MHz or higher processor clock speed (400 MHz or higher recommended)
Windows 2000, XP, or higher
128 MB RAM (256 MB or higher recommended)
Video card and monitor support for 16-bit or higher color depth and 800×600 or higher screen resolution
100 MB disk space
CD-ROM or DVD drive



Appendix D

DETERMINANTS

Determinants are used to find the mathematical solutions for the variables in two or more simultaneous equations. Once the procedure is properly understood, solutions can be obtained with a minimum of time and effort and usually with fewer errors than when using other methods.

Consider the following equations, where x and y are the unknown variables and a_1, a_2, b_1, b_2, c_1, and c_2 are constants:

$$\begin{array}{ccccc} \text{Col. 1} & & \text{Col. 2} & & \text{Col. 3} \\ \hline a_1x & + & b_1y & = & c_1 \\ a_2x & + & b_2y & = & c_2 \end{array} \qquad \begin{array}{r} \\ \textbf{(D.1a)} \\ \textbf{(D.1b)} \end{array}$$

It is certainly possible to solve for one variable in Eq. (D.1a) and substitute into Eq. (D.1b). That is, solving for x in Eq. (D.1a),

$$x = \frac{c_1 - b_1y}{a_1}$$

and substituting the result in Eq. (D.1b),

$$a_2\left(\frac{c_1 - b_1y}{a_1}\right) + b_2y = c_2$$

It is now possible to solve for y, since it is the only variable remaining, and then substitute into either equation for x. This is acceptable for two equations, but it becomes a very tedious and lengthy process for three or more simultaneous equations.

Using determinants to solve for x and y requires that the following formats be established for each variable:

$$\begin{array}{cc} \begin{array}{cc} \text{Col.} & \text{Col.} \\ 1 & 2 \end{array} & \begin{array}{cc} \text{Col.} & \text{Col.} \\ 1 & 2 \end{array} \\ \hline x = \dfrac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} & y = \dfrac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \end{array} \qquad \textbf{(D.2)}$$

First note that only constants appear within the vertical brackets and that the denominator of each is the same. In fact, the denominator is simply the coefficients of x and y in the same arrangement as in Eqs. (D.1a) and (D.1b). When solving for x, replace the coefficients of x in the numerator by the constants to the right of the equal sign in Eqs. (D.1a) and (D.1b), and repeat the coefficients of the y variable. When solving for y, replace the y coefficients in the numerator by the constants to the right of the equal sign, and repeat the coefficients of x.

Each configuration in the numerator and denominator of Eq. (D.2) is referred to as a *determinant (D)*, which can be evaluated numerically in the following manner:

$$\text{Determinant} = D = \begin{array}{c} \text{Col. Col.} \\ \begin{vmatrix} \underline{1} & \underline{2} \\ a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \end{array} = a_1b_2 - a_2b_1 \qquad \textbf{(D.3)}$$

The expanded value is obtained by first multiplying the top left element by the bottom right and then subtracting the product of the lower left and upper right elements. This particular determinant is referred to as a *second-order* determinant, since it contains two rows and two columns.

It is important to remember when using determinants that the columns of the equations, as indicated in Eqs. (D.1a) and (D.1b), must be placed in the same order within the determinant configuration. That is, since a_1 and a_2 are in column 1 of Eqs. (D.1a) and (D.1b), they must be in column 1 of the determinant. (The same is true for b_1 and b_2.)

Expanding the entire expression for x and y, we have the following:

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1} \qquad \textbf{(D.4a)}$$

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1} \qquad \textbf{(D.4b)}$$

EXAMPLE D.1 Evaluate the following determinants:

a. $\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = (2)(4) - (3)(2) = 8 - 6 = \mathbf{2}$

b. $\begin{vmatrix} 4 & -1 \\ 6 & 2 \end{vmatrix} = (4)(2) - (6)(-1) = 8 + 6 = \mathbf{14}$

c. $\begin{vmatrix} 0 & -2 \\ -2 & 4 \end{vmatrix} = (0)(4) - (-2)(-2) = 0 - 4 = \mathbf{-4}$

d. $\begin{vmatrix} 0 & 0 \\ 3 & 10 \end{vmatrix} = (0)(10) - (3)(0) = \mathbf{0}$

EXAMPLE D.2 Solve for x and y:

$$\begin{aligned} 2x + y &= 3 \\ 3x + 4y &= 2 \end{aligned}$$

Solution:

$$x = \frac{\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix}} = \frac{(3)(4) - (2)(1)}{(2)(4) - (3)(1)} = \frac{12 - 2}{8 - 3} = \frac{10}{5} = \mathbf{2}$$

$$y = \frac{\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}}{5} = \frac{(2)(2) - (3)(3)}{5} = \frac{4 - 9}{5} = \frac{-5}{5} = \mathbf{-1}$$

Check:

$$\begin{aligned} 2x + y &= (2)(2) + (-1) \\ &= 4 - 1 = 3 \qquad \text{(checks)} \\ 3x + 4y &= (3)(2) + (4)(-1) \\ &= 6 - 4 = 2 \qquad \text{(checks)} \end{aligned}$$

EXAMPLE D.3 Solve for x and y:

$$\begin{aligned} -x + 2y &= 3 \\ \underline{3x - 2y} &\underline{= -2} \end{aligned}$$

Solution: In this example, note the effect of the minus sign and the use of parentheses to ensure that the proper sign is obtained for each product:

$$x = \frac{\begin{vmatrix} 3 & 2 \\ -2 & -2 \end{vmatrix}}{\begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix}} = \frac{(3)(-2) - (-2)(2)}{(-1)(-2) - (3)(2)}$$

$$= \frac{-6 + 4}{2 - 6} = \frac{-2}{-4} = \mathbf{\frac{1}{2}}$$

$$y = \frac{\begin{vmatrix} -1 & 3 \\ 3 & -2 \end{vmatrix}}{-4} = \frac{(-1)(-2) - (3)(3)}{-4}$$

$$= \frac{2 - 9}{-4} = \frac{-7}{-4} = \mathbf{\frac{7}{4}}$$

EXAMPLE D.4 Solve for x and y:

$$\begin{aligned} x &= \quad 3 - 4y \\ \underline{20y} &\underline{= -1 + 3x} \end{aligned}$$

Solution: In this case, the equations must first be placed in the format of Eqs. (D.1a) and (D.1b):

$$\begin{aligned} x + \quad 4y &= 3 \\ \underline{-3x + 20y} &\underline{= -1} \end{aligned}$$

$$x = \frac{\begin{vmatrix} 3 & 4 \\ -1 & 20 \end{vmatrix}}{\begin{vmatrix} 1 & 4 \\ -3 & 20 \end{vmatrix}} = \frac{(3)(20) - (-1)(4)}{(1)(20) - (-3)(4)}$$

$$= \frac{60 + 4}{20 + 12} = \frac{64}{32} = \mathbf{2}$$

$$y = \frac{\begin{vmatrix} 1 & 3 \\ -3 & -1 \end{vmatrix}}{32} = \frac{(1)(-1) - (-3)(3)}{32}$$

$$= \frac{-1 + 9}{32} = \frac{8}{32} = \mathbf{\frac{1}{4}}$$

The use of determinants is not limited to the solution of two simultaneous equations; determinants can be applied to any number of simultaneous linear equations. First we examine a shorthand method that is applicable to third-order determinants only, since most of the problems in the text are limited to this level of difficulty. We then investigate the general procedure for solving any number of simultaneous equations.

Consider the three following simultaneous equations:

$$\begin{array}{ccccccc}
\text{Col. 1} & & \text{Col. 2} & & \text{Col. 3} & & \text{Col. 4} \\
\hline
a_1x & + & b_1y & + & c_1z & = & d_1 \\
a_2x & + & b_2y & + & c_2z & = & d_2 \\
a_3x & + & b_3y & + & c_3z & = & d_3
\end{array}$$

in which x, y, and z are the variables, and $a_{1,2,3}$, $b_{1,2,3}$, $c_{1,2,3}$, and $d_{1,2,3}$ are constants.

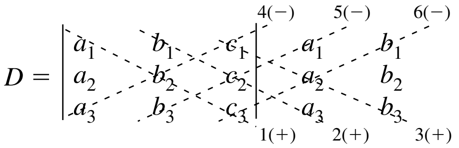
The determinant configuration for x, y, and z can be found in a manner similar to that for two simultaneous equations. That is, to solve for x, find the determinant in the numerator by replacing column 1 with the elements to the right of the equal sign. The denominator is the determinant of the coefficients of the variables (the same applies to y and z). Again, the denominator is the same for each variable.

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{D}, \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{D}, \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{D}$$

where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

A shorthand method for evaluating the third-order determinant consists of repeating the first two columns of the determinant to the right of the determinant and then summing the products along specific diagonals as shown below:



The products of the diagonals 1, 2, and 3 are positive and have the following magnitudes:

$$+a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3$$

The products of the diagonals 4, 5, and 6 are negative and have the following magnitudes:

$$-a_3b_2c_1 - b_3c_2a_1 - c_3a_2b_1$$

The total solution is the sum of the diagonals 1, 2, and 3 minus the sum of the diagonals 4, 5, and 6:

$$\boxed{+\,(a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3) - (a_3b_2c_1 + b_3c_2c_1 + c_3a_2b_1)} \qquad \textbf{(D.5)}$$

Warning: This method of expansion is good only for third-order determinants! It cannot be applied to fourth- and higher-order systems.

EXAMPLE D.5 Evaluate the following determinant:

$$\begin{vmatrix} 1 & 2 & 3 \\ -2 & 1 & 0 \\ 0 & 4 & 2 \end{vmatrix} \longrightarrow \begin{array}{ccc|cc} & & (-) & (-) & (-) \\ 1 & 2 & 3 & 1 & 2 \\ -2 & 1 & 0 & -2 & 1 \\ 0 & 4 & 2 & 0 & 4 \\ & & (+) & (+) & (+) \end{array}$$

Solution:

$$\begin{aligned} &[(1)(1)(2) + (2)(0)(0) + (3)(-2)(4)] \\ &\qquad - [(0)(1)(3) + (4)(0)(1) + (2)(-2)(2)] \\ &= (2 + 0 - 24) - (0 + 0 - 8) = (-22) - (-8) \\ &= -22 + 8 = \mathbf{-14} \end{aligned}$$

EXAMPLE D.6 Solve for *x*, *y*, and *z*:

$$\begin{aligned} 1x + 0y - 2z &= -1 \\ 0x + 3y + 1z &= +2 \\ 1x + 2y + 3z &= 0 \end{aligned}$$

Solution:

$$x = \frac{\left|\begin{array}{ccc|cc} -1 & 0 & -2 & -1 & 0 \\ 2 & 3 & 1 & 2 & 3 \\ 0 & 2 & 3 & 0 & 2 \end{array}\right.}{\left|\begin{array}{ccc|cc} 1 & 0 & -2 & 1 & 0 \\ 0 & 3 & 1 & 0 & 3 \\ 1 & 2 & 3 & 1 & 2 \end{array}\right.}$$

$$= \frac{[(-1)(3)(3) + (0)(1)(0) + (-2)(2)(2)] - [(0)(3)(-2) + (2)(1)(-1) + (3)(2)(0)]}{[(1)(3)(3) + (0)(1)(1) + (-2)(0)(2)] - [(1)(3)(-2) + (2)(1)(1) + (3)(0)(0)]}$$

$$= \frac{(-9 + 0 - 8) - (0 - 2 + 0)}{(9 + 0 + 0) - (-6 + 2 + 0)}$$

$$= \frac{-17 + 2}{9 + 4} = -\mathbf{\frac{15}{13}}$$

$$y = \frac{\left|\begin{array}{ccc|cc} 1 & -1 & -2 & 1 & -1 \\ 0 & 2 & 1 & 0 & 2 \\ 1 & 0 & 3 & 1 & 0 \end{array}\right.}{13}$$

$$= \frac{[(1)(2)(3) + (-1)(1)(1) + (-2)(0)(0)] - [(1)(2)(-2) + (0)(1)(1) + (3)(0)(-1)]}{13}$$

$$= \frac{(6 - 1 + 0) - (-4 + 0 + 0)}{13}$$

$$= \frac{5 + 4}{13} = \mathbf{\frac{9}{13}}$$

$$z = \frac{\left|\begin{array}{ccc}1 & 0 & -1\\ 0 & 3 & 2\\ 1 & 2 & 0\end{array}\right|\begin{array}{cc}1 & 0\\ 0 & 3\\ 1 & 2\end{array}}{13}$$

$$= \frac{[(1)(3)(0) + (0)(2)(1) + (-1)(0)(2)] - [(1)(3)(-1) + (2)(2)(1) + (0)(0)(0)]}{13}$$

$$= \frac{(0 + 0 + 0) - (-3 + 4 + 0)}{13}$$

$$= \frac{0 - 1}{13} = \mathbf{-\frac{1}{13}}$$

or from $0x + 3y + 1z = +2$,

$$z = 2 - 3y = 2 - 3\left(\frac{9}{13}\right) = \frac{26}{13} - \frac{27}{13} = \mathbf{-\frac{1}{13}}$$

Check:

$$\left.\begin{array}{l}1x + 0y - 2z = -1\\[2ex] 0x + 3y + 1z = +2\\[2ex] 1x + 2y + 3z = 0\end{array}\right\}\begin{array}{l}-\dfrac{15}{13} + 0 + \dfrac{2}{13} = -1\\[2ex] 0 + \dfrac{27}{13} + \dfrac{-1}{13} = +2\\[2ex] -\dfrac{15}{13} + \dfrac{18}{13} + \dfrac{-3}{13} = 0\end{array}\left.\right\}\begin{array}{l}-\dfrac{13}{13} = -1 \checkmark\\[2ex] \dfrac{26}{13} = +2 \checkmark\\[2ex] -\dfrac{18}{13} + \dfrac{18}{13} = 0 \checkmark\end{array}$$

The general approach to third-order or higher determinants requires that the determinant be expanded in the following form. There is more than one expansion that will generate the correct result, but this form is typically used when the material is first introduced.

$$D = \begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix} = \underset{\substack{\uparrow\\ \text{Multiplying}\\ \text{factor}}}{a_1}\underbrace{\left(+\underbrace{\begin{vmatrix}b_2 & c_2\\ b_3 & c_3\end{vmatrix}}_{\text{Minor}}\right)}_{\text{Cofactor}} + \underset{\substack{\uparrow\\ \text{Multiplying}\\ \text{factor}}}{b_1}\underbrace{\left(-\underbrace{\begin{vmatrix}a_2 & c_2\\ a_3 & c_3\end{vmatrix}}_{\text{Minor}}\right)}_{\text{Cofactor}} + \underset{\substack{\uparrow\\ \text{Multiplying}\\ \text{factor}}}{c_1}\underbrace{\left(+\underbrace{\begin{vmatrix}a_2 & b_2\\ a_3 & b_3\end{vmatrix}}_{\text{Minor}}\right)}_{\text{Cofactor}}$$

This expansion was obtained by multiplying the elements of the first row of D by their corresponding cofactors. It is not a requirement that the first row be used as the multiplying factors. In fact, any *row* or *column* (not diagonals) may be used to expand a third-order determinant.

The sign of each cofactor is dictated by the position of the multiplying factors (a_1, b_1, and c_1 in this case) as in the following standard format:

$$\left|\begin{array}{ccccc}+ & \rightarrow & - & & +\\ \downarrow & & & & \\ - & & + & & -\\ & & & & \\ + & & - & & +\end{array}\right|$$

Note that the proper sign for each element can be obtained by assigning the upper left element a positive sign and then changing signs as you move horizontally or vertically to the neighboring position.

For the determinant *D,* the elements would have the following signs:

$$\begin{vmatrix} a_1^{(+)} & b_1^{(-)} & c_1^{(+)} \\ a_2^{(-)} & b_2^{(+)} & c_2^{(-)} \\ a_3^{(+)} & b_3^{(-)} & c_3^{(+)} \end{vmatrix}$$

The minors associated with each multiplying factor are obtained by covering up the row and column in which the multiplying factor is located and writing a second-order determinant to include the remaining elements in the same relative positions that they have in the third-order determinant.

Consider the cofactors associated with a_1 and b_1 in the expansion of *D.* The sign is positive for a_1 and negative for b_1 as determined by the standard format. Following the procedure outlined above, we can find the minors of a_1 and b_1 as follows:

$$a_{1(\text{minor})} = \begin{vmatrix} \cancel{a_1} & \cancel{b_1} & \cancel{c_1} \\ \cancel{a_2} & b_2 & c_2 \\ \cancel{a_3} & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$$

$$b_{1(\text{minor})} = \begin{vmatrix} \cancel{a_1} & \cancel{b_1} & \cancel{c_1} \\ a_2 & \cancel{b_2} & c_2 \\ a_3 & \cancel{b_3} & c_3 \end{vmatrix} = \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}$$

It was pointed out that any row or column may be used to expand the third-order determinant, and the same result will still be obtained. Using the first column of *D,* we obtain the expansion

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\left(+\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}\right) + a_2\left(-\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix}\right) + a_3\left(+\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}\right)$$

The proper choice of row or column can often effectively reduce the amount of work required to expand the third-order determinant. For example, in the following determinants, the first column and third row, respectively, would reduce the number of cofactors in the expansion:

$$D = \begin{vmatrix} 2 & 3 & -2 \\ 0 & 4 & 5 \\ 0 & 6 & 7 \end{vmatrix} = 2\left(+\begin{vmatrix} 4 & 5 \\ 6 & 7 \end{vmatrix}\right) + 0 + 0 = 2(28 - 30)$$

$$= \mathbf{-4}$$

$$D = \begin{vmatrix} 1 & 4 & 7 \\ 2 & 6 & 8 \\ 2 & 0 & 3 \end{vmatrix} = 2\left(+\begin{vmatrix} 4 & 7 \\ 6 & 8 \end{vmatrix}\right) + 0 + 3\left(+\begin{vmatrix} 1 & 4 \\ 2 & 6 \end{vmatrix}\right)$$

$$= 2(32 - 42) + 3(6 - 8) = 2(-10) + 3(-2)$$

$$= \mathbf{-26}$$

EXAMPLE D.7 Expand the following third-order determinants:

a. $D = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 1\left(+\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}\right) + 3\left(-\begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix}\right) + 2\left(+\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix}\right)$

$= 1[6 - 1] + 3[-(6 - 3)] + 2[2 - 6]$

$= 5 + 3(-3) + 2(-4)$

$= 5 - 9 - 8$

$= \mathbf{-12}$

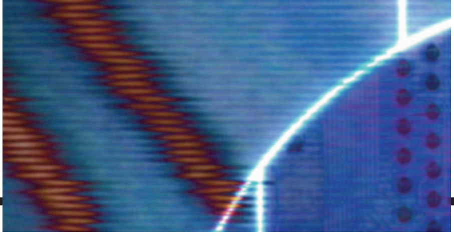
b. $D = \begin{vmatrix} 0 & 4 & 6 \\ 2 & 0 & 5 \\ 8 & 4 & 0 \end{vmatrix} = 0 + 2\left(-\begin{vmatrix} 4 & 6 \\ 4 & 0 \end{vmatrix}\right) + 8\left(+\begin{vmatrix} 4 & 6 \\ 0 & 5 \end{vmatrix}\right)$

$= 0 + 2[-(0 - 24)] + 8[(20 - 0)]$

$= 0 + 2(24) + 8(20)$

$= 48 + 160$

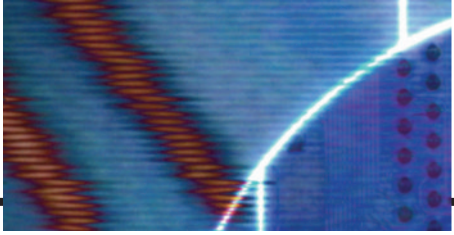
$= \mathbf{208}$



Appendix E

GREEK ALPHABET

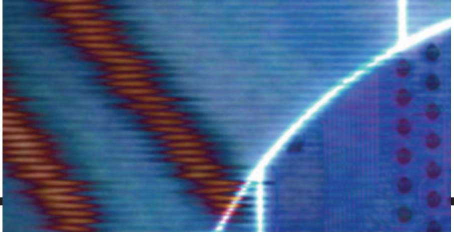
Letter	Capital	Lowercase	Some Applications
Alpha	Α	α	Area, angles, coefficients
Beta	Β	β	Angles, coefficients, flux density
Gamma	Γ	γ	Specific gravity, conductivity
Delta	Δ	δ	Density, variation
Epsilon	Ε	ε	Base of natural logarithms
Zeta	Ζ	ζ	Coefficients, coordinates, impedance
Eta	Η	η	Efficiency, hysteresis coefficient
Theta	Θ	θ	Phase angle, temperature
Iota	Ι	ι	
Kappa	Κ	κ	Dielectric constant, susceptibility
Lambda	Λ	λ	Wavelength
Mu	Μ	μ	Amplification factor, micro, permeability
Nu	Ν	ν	Reluctivity
Xi	Ξ	ξ	
Omicron	Ο	o	
Pi	Π	π	3.1416
Rho	Ρ	ρ	Resistivity
Sigma	Σ	σ	Summation
Tau	Τ	τ	Time constant
Upsilon	Υ	υ	
Phi	Φ	ϕ	Angles, magnetic flux
Chi	Χ	χ	
Psi	Ψ	ψ	Dielectric flux, phase difference
Omega	Ω	ω	Ohms, angular velocity



Appendix F

MAGNETIC PARAMETER CONVERSIONS

	SI (MKS)	CGS	English
Φ	webers (Wb) 1 Wb	maxwells $= 10^8$ maxwells	lines $= 10^8$ lines
B	T $1\text{T} = 1\ \text{Wb/m}^2$	gauss (maxwells/cm^2) $= 10^4$ gauss	lines/in.^2 $= 6.452 \times 10^4$ lines/in.^2
A	$1\ \text{m}^2$	$= 10^4\ \text{cm}^2$	$= 1550\ \text{in.}^2$
μ_o	$4\pi \times 10^{-7}$ Wb/Am	= 1 gauss/oersted	= 3.20 lines/Am
$\mathscr{F}$	NI (ampere-turns, At) 1 At	$0.4\pi NI$ (gilberts) = 1.257 gilberts	NI (At) 1 gilbert = 0.7958 At
H	NI/l (At/m) 1At/m	$0.4\pi NI/l$ (oersteds) $= 1.26 \times 10^{-2}$ oersted	NI/l(At/in.) $= 2.54 \times 10^{-2}$ At/in.
H_g	$7.97 \times 10^5 B_g$ (At/m)	B_g (oersteds)	$0.313B_g$ (At/in.)



Appendix G

MAXIMUM POWER TRANSFER CONDITIONS

Derivation of maximum power transfer conditions for the situation where the resistive component of the load is adjustable, but the load reactance is set in magnitude.*

For the circuit in Fig. G.1, the power delivered to the load is determined by

$$P = \frac{V_{R_L}^2}{R_L}$$

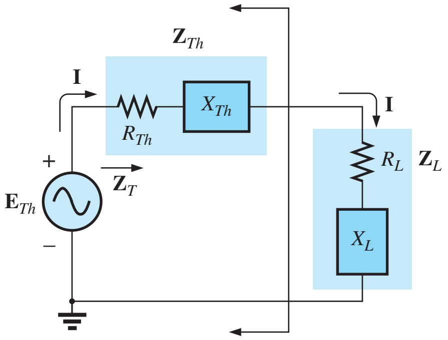


FIG. G.1

Applying the voltage divider rule:

$$\mathbf{V}_{R_L} = \frac{R_L \mathbf{E}_{Th}}{R_L + R_{Th} + X_{Th}\angle 90^\circ + X_L\angle 90^\circ}$$

The magnitude of $\mathbf{V}_{R_L}$ is determined by

$$V_{R_L} = \frac{R_L E_{Th}}{\sqrt{(R_L + R_{Th})^2 + (X_{Th} + X_L)^2}}$$

and
$$V_{R_L}^2 = \frac{R_L^2 E_{Th}^2}{(R_L + R_{Th})^2 + (X_{Th} + X_L)^2}$$

with
$$P = \frac{V_{R_L}^2}{R_L} = \frac{R_L E_{Th}^2}{(R_L + R_{Th})^2 + (X_{Th} + X_L)^2}$$

Using differentiation (calculus), maximum power will be transferred when $dP/dR_L = 0$. The result of the preceding operation is that

$$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_L)^2} \qquad \text{[Eq. (18.21)]}$$

The magnitude of the total impedance of the circuit is

$$Z_T = \sqrt{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

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*With sincerest thanks for the input of Professor Harry J. Franz of the Beaver Campus of Pennsylvania State University.

Substituting this equation for R_L and applying a few algebraic maneuvers will result in

$$Z_T = 2R_L(R_L + R_{Th})$$

and the power to the load R_L will be

$$\begin{aligned} P = I^2R_L &= \frac{E_{Th}^2}{Z_T^2}R_L = \frac{E_{Th}^2R_L}{2R_L(R_L + R_{Th})} \\ &= \frac{E_{Th}^2}{4\left(\dfrac{R_L + R_{Th}}{2}\right)} \\ &= \frac{E_{Th}^2}{4R_{\text{av}}} \end{aligned}$$

with $$R_{\text{av}} = \frac{R_L + R_{Th}}{2}$$