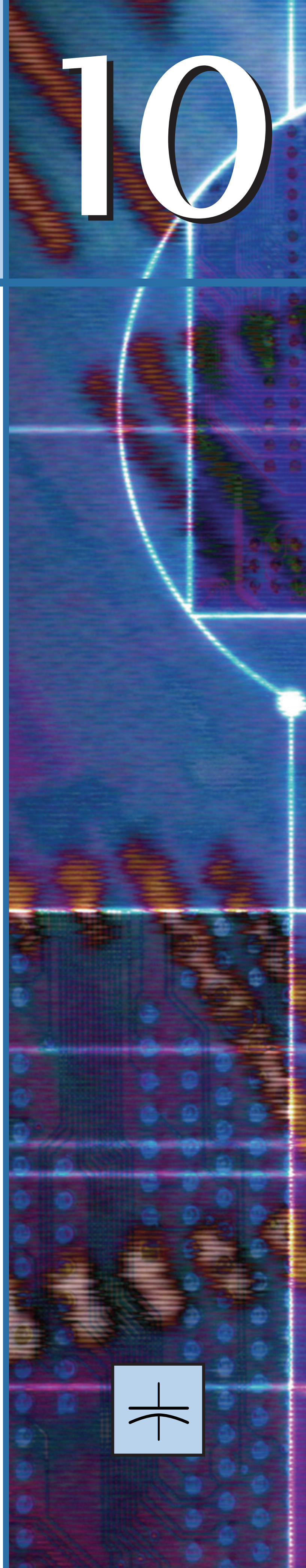
Capacitors

10



Objectives

- *Become familiar with the basic construction of a capacitor and the factors that affect its ability to store charge on its plates.*
- *Be able to determine the transient (time-varying) response of a capacitive network and plot the resulting voltages and currents.*
- *Understand the impact of combining capacitors in series or parallel and how to read the nameplate data.*
- *Develop some familiarity with the use of computer methods to analyze networks with capacitive elements.*

10.1 INTRODUCTION

Thus far, the resistor has been the only network component appearing in our analyses. In this chapter, we introduce the **capacitor,** which has a significant impact on the types of networks that you will be able to design and analyze. Like the resistor, it is a two-terminal device, but its characteristics are totally different from those of a resistor. In fact, *the capacitor displays its true characteristics only when a change in the voltage or current is made in the network.* All the power delivered to a resistor is dissipated in the form of heat. An ideal capacitor, however, stores the energy delivered to it in a form that can be returned to the system.

Although the basic construction of capacitors is actually quite simple, it is a component that opens the door to all types of practical applications, extending from touch pads to sophisticated control systems. A few applications are introduced and discussed in detail later in this chapter.

10.2 THE ELECTRIC FIELD

Recall from Chapter 2 that a force of attraction or repulsion exists between two charged bodies. We now examine this phenomenon in greater detail by considering the electric field that exists in the region around any charged body. This electric field is represented by **electric flux lines,** which are drawn to indicate the strength of the electric field at any point around the charged body. The denser the lines of flux, the stronger the electric field. In Fig. 10.1, for example, the electric field strength is stronger in region *a* than region *b* because the flux lines are denser in region *a* than *b*. That is, the same number of flux lines pass through each region, but the area A_1 is much smaller than area A_2. The symbol for electric flux is the Greek letter ψ (*psi*). The flux per unit area (flux density) is represented by the capital letter D and is determined by

$$D = \frac{\psi}{A} \qquad \text{(flux/unit area)} \qquad \textbf{(10.1)}$$

The larger the charge Q in coulombs, the greater the number of flux lines extending or terminating per unit area, independent of the surrounding medium. Twice the charge produces twice the flux per unit area. The two can therefore be equated:

$$\psi \equiv Q \qquad \text{(coulombs, C)} \qquad \textbf{(10.2)}$$

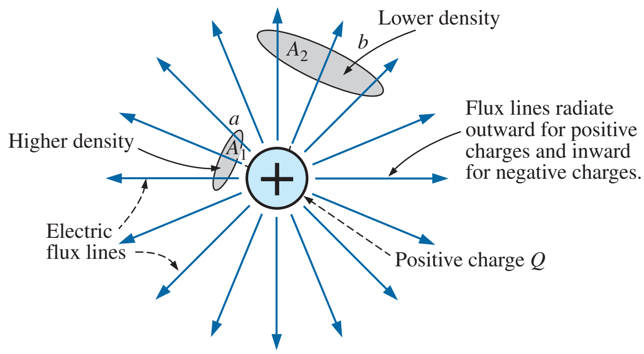


FIG. 10.1
Flux distribution from an isolated positive charge.

By definition, the **electric field strength** (designated by the capital script letter $\mathscr{E}$) at a point is the force acting on a unit positive charge at that point; that is,

$$\boxed{\mathscr{E} = \frac{F}{Q}} \quad \text{(newtons/coulomb, N/C)} \qquad \textbf{(10.3)}$$

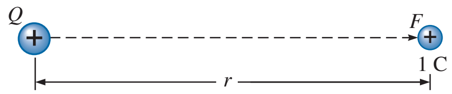


FIG. 10.2
Determining the force on a unit charge r meters from a charge Q of similar polarity.

In Fig. 10.2, the force exerted on a unit (1 coulomb) positive charge by a charge Q, r meters away, can be determined using **Coulomb's law** (Eq. 2.1) as follows:

$$F = k\frac{Q_1Q_2}{r^2} = k\frac{Q(1\text{ C})}{r^2} = \frac{kQ}{r^2} \quad (k = 9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)$$

Substituting the result into Eq. 10.3 for a unit positive charge results in

$$\mathscr{E} = \frac{F}{Q} = \frac{kQ/r^2}{1/\text{C}}$$

and

$$\boxed{\mathscr{E} = \frac{kQ}{r^2}} \quad \text{(N/C)} \qquad \textbf{(10.4)}$$

The result clearly reveals that the electric field strength is directly related to the size of the charge Q. The greater the charge Q, the greater the electric field intensity on a unit charge at any point in the neighborhood. However, the distance is a squared term in the denominator. The result is that the greater the distance from the charge Q, the less the electric field strength, and dramatically so because of the squared term. In Fig. 10.1, the electric field strength at region A_2 is therefore significantly less than at region A_1.

For two charges of similar and opposite polarities, the flux distribution appears as shown in Fig. 10.3. In general,

electric flux lines always extend from a positively charged body to a negatively charged body, always extend or terminate perpendicular to the charged surfaces, and never intersect.

Note in Fig. 10.3(a) that the electric flux lines establish the most direct pattern possible from the positive to negative charge. They are evenly distributed and have the shortest distance on the horizontal between the two charges. This pattern is a direct result of the fact that electric flux

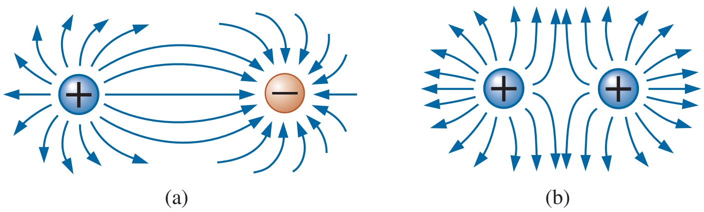


FIG. 10.3
Electric flux distributions: (a) opposite charges; (b) like charges.

lines strive to establish the shortest path from one charged body to another. The result is a natural pressure to be as close as possible. If two bodies of the same polarity are in the same vicinity, as shown in Fig. 10.3(b), the result is the direct opposite. The flux lines tend to establish a buffer action between the two with a repulsive action that grows as the two charges are brought closer to one another.

10.3 CAPACITANCE

Thus far, we have examined only isolated positive and negative spherical charges, but the description can be extended to charged surfaces of any shape and size. In Fig. 10.4, for example, two parallel plates of a material such as aluminum (the most commonly used metal in the construction of capacitors) have been connected through a switch and a resistor to a battery. If the parallel plates are initially uncharged and the switch is left open, no net positive or negative charge exists on either plate. The instant the switch is closed, however, electrons are drawn from the upper plate through the resistor to the positive terminal of the battery. There will be a surge of current at first, limited in magnitude by the resistance present. The level of flow then declines, as will be demonstrated in the sections to follow. This action creates a net positive charge on the top plate. Electrons are being repelled by the negative terminal through the lower conductor to the bottom plate at the same rate they are being drawn to the positive terminal. This transfer of electrons continues until the potential difference across the parallel plates is exactly equal to the battery voltage. The final result is a net positive charge on the top plate and a negative charge on the bottom plate, very similar in many respects to the two isolated charges in Fig. 10.3(a).

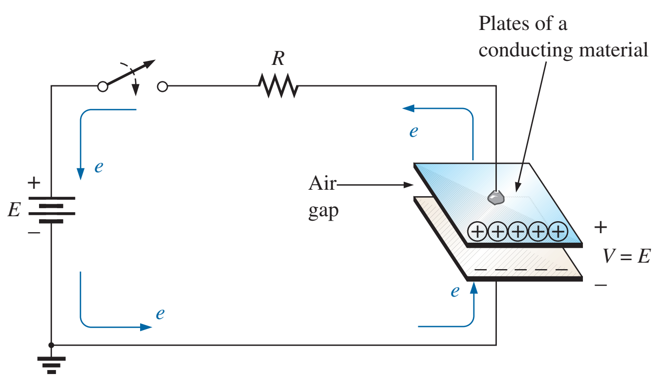


FIG. 10.4
Fundamental charging circuit.

FIG. 10.5
Michael Faraday.
Courtesy of the Smithsonian Institution
Photo No. 51,147

English (London)
(1791–1867)
Chemist and Electrical Experimenter
Honorary Doctorate, Oxford University, 1832

An experimenter with no formal education, he began his research career at the Royal Institute in London as a laboratory assistant. Intrigued by the interaction between electrical and magnetic effects, he discovered *electromagnetic induction,* demonstrating that electrical effects can be generated from a magnetic field (the birth of the generator as we know it today). He also discovered *self-induced currents* and introduced the concept of *lines and fields of magnetic force.* Having received over one hundred academic and scientific honors, he became a Fellow of the Royal Society in 1824 at the young age of 32.

Before continuing, it is important to note that the entire flow of charge is through the battery and resistor—not through the region between the plates. In every sense of the definition, *there is an open circuit between the plates of the capacitor.*

This element, constructed simply of two conducting surfaces separated by the air gap, is called a **capacitor.**

Capacitance is a measure of a capacitor's ability to store charge on its plates—in other words, its storage capacity.

In addition,

the higher the capacitance of a capacitor, the greater the amount of charge stored on the plates for the same applied voltage.

The unit of measure applied to capacitors is the farad (F), named after an English scientist, Michael Faraday, who did extensive research in the field (Fig. 10.5). In particular,

a capacitor has a capacitance of 1 F if 1 C of charge (6.242×10^{18} electrons) is deposited on the plates by a potential difference of 1 V across its plates.

The farad, however, is generally too large a measure of capacitance for most practical applications, so the microfarad (10^{-6}) or picofarad(10^{-12}) are more commonly encountered.

The relationship between the applied voltage, the charge on the plates, and the capacitance level is defined by the following equation:

$$\boxed{C = \frac{Q}{V}} \qquad \begin{aligned} C &= \text{farads (F)} \\ Q &= \text{coulombs (C)} \\ V &= \text{volts (V)} \end{aligned} \tag{10.5}$$

Eq. (10.5) reveals that for the same voltage (V), the greater the charge (Q) on the plates (in the numerator of the equation), the higher the capacitance level (C).

If we write the equation in the following form:

$$\boxed{Q = CV} \qquad \text{(coulombs, C)} \tag{10.6}$$

it becomes obvious through the product relationship that the higher the capacitance (C) or applied voltage (V), the greater the charge on the plates.

EXAMPLE 10.1

a. If 82.4×10^{14} electrons are deposited on the negative plate of a capacitor by an applied voltage of 60 V, find the capacitance of the capacitor.
b. If 40 V are applied across a 470 μF capacitor, find the charge on the plates.

Solutions:

a. First find the number of coulombs of charge as follows:

$$82.4 \times 10^{14}\ \cancel{\text{electrons}}\left(\frac{1\ \text{C}}{6.242 \times 10^{18}\ \cancel{\text{electrons}}}\right) = 1.32\ \text{mC}$$

and then

$$C = \frac{Q}{V} = \frac{1.32 \text{ mC}}{60 \text{ V}} = \mathbf{22\ \mu F} \qquad \text{(a standard value)}$$

b. Applying Eq. (10.6):

$$Q = CV = (470\ \mu\text{F})(40\ \text{V}) = \mathbf{18.8\ mC}$$

A cross-sectional view of the parallel plates in Fig. 10.4 is provided in Fig. 10.6(a). Note the **fringing** that occurs at the edges as the flux lines originating from the points farthest away from the negative plate strive to complete the connection. This fringing, which has the effect of reducing the net capacitance somewhat, can be ignored for most applications. Ideally, and the way we will assume the distribution to be in this text, the electric flux distribution appears as shown in Fig. 10.6(b), where all the flux lines are equally distributed and "fringing" does not occur.

The **electric field strength** between the plates is determined by the voltage across the plates and the distance between the plates as follows:

$$\mathscr{E} = \frac{V}{d} \qquad \begin{aligned} \mathscr{E} &= \text{volts/m (V/m)} \\ V &= \text{volts (V)} \\ d &= \text{meters (m)} \end{aligned} \qquad \textbf{(10.7)}$$

Note that the distance between the plates is measured in meters, not centimeters or inches.

The equation for the electric field strength is determined by two factors only: *the applied voltage and the distance between the plates.* The charge on the plates does not appear in the equation, nor does the size of the capacitor or the plate material.

Many values of capacitance can be obtained for the same set of parallel plates by the addition of certain insulating materials between the plates. In Fig. 10.7, an insulating material has been placed between a set of parallel plates having a potential difference of V volts across them.

Since the material is an insulator, the electrons within the insulator are unable to leave the parent atom and travel to the positive plate. The positive components (protons) and negative components (electrons) of each atom do shift, however [as shown in Fig. 10.7(a)], to form *dipoles.*

When the dipoles align themselves as shown in Fig. 10.7(a), the material is *polarized.* A close examination within this polarized material reveals that the positive and negative components of adjoining dipoles are

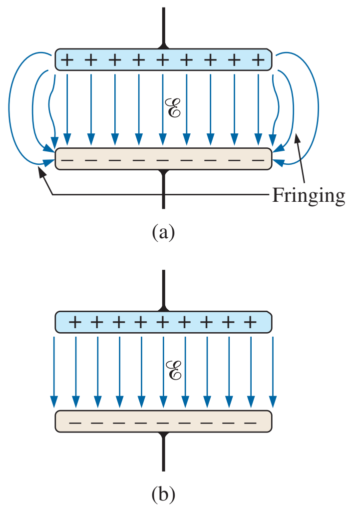


FIG. 10.6
Electric flux distribution between the plates of a capacitor: (a) including fringing; (b) ideal.

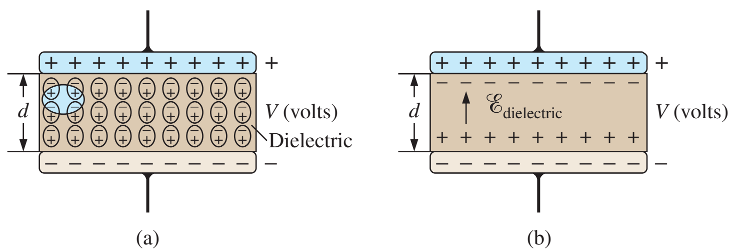


FIG. 10.7
Effect of a dielectric on the field distribution between the plates of a capacitor: (a) alignment of dipoles in the dielectric; (b) electric field components between the plates of a capacitor with a dielectric present.

neutralizing the effects of each other [note the dashed area in Fig. 10.7(a)]. The layer of positive charge on one surface and the negative charge on the other are not neutralized, however, resulting in the establishment of an electric field within the insulator [$\mathscr{E}_{\text{dielectric}}$; Fig. 10.7(b)].

In Fig. 10.8(a), two plates are separated by an air gap and have layers of charge on the plates as established by the applied voltage and the distance between the plates. The electric field strength is $\mathscr{E}_1$ as defined by Eq. (10.7). In Fig. 10.8(b), a slice of mica is introduced which, through an alignment of cells within the dielectric, establishes an electric field $\mathscr{E}_2$ that will oppose electric field $\mathscr{E}_1$. The effect is to try to reduce the electric field strength between the plates. However, Eq. (10.7) states that the electric field strength *must be* the value established by the applied voltage and the distance between the plates. This condition is maintained by placing more charge on the plates, thereby increasing the electric field strength between the plates to a level that cancels out the opposing electric field introduced by the mica sheet. The net result is an increase in charge on the plates and an increase in the capacitance level as established by Eq. (10.5).

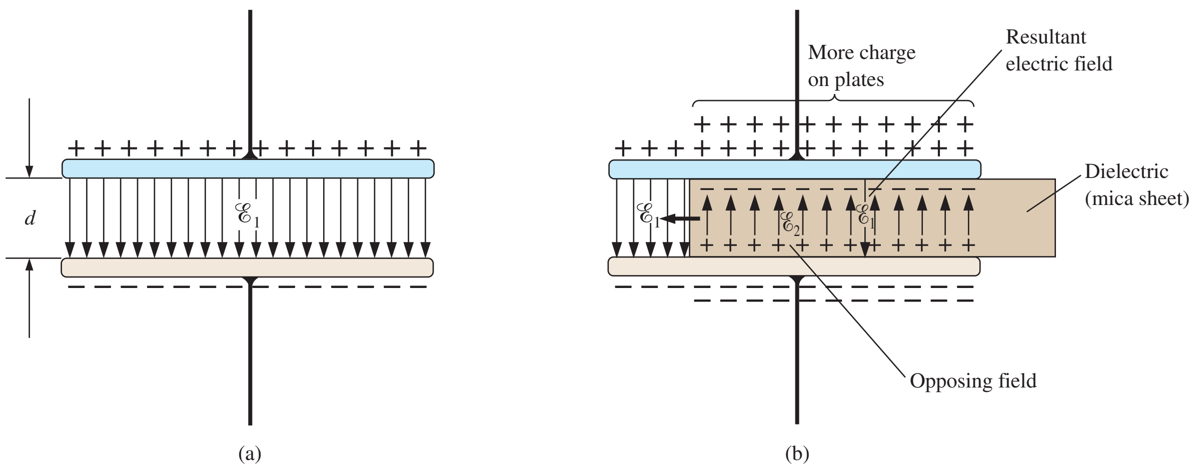


FIG. 10.8
Demonstrating the effect of inserting a dielectric between the plates of a capacitor: (a) air capacitor; (b) dielectric being inserted.

Different materials placed between the plates establish different amounts of additional charge on the plates. All, however, must be insulators and must have the ability to set up an electric field within the structure. A list of common materials appears in Table 10.1 using air as the reference level of 1.* All of these materials are referred to as **dielectrics,** the "di" for *opposing,* and the "electric" from *electric field.* The symbol ϵ_r in Table 10.1 is called the **relative permittivity** (or **dielectric constant**). The term **permittivity** is applied as a measure of how easily a material "permits" the establishment of an electric field in the material. The relative permittivity compares the permittivity of a material to that of air. For instance, Table 10.1 reveals that mica, with a relative permittivity of 5, "permits" the establishment of an opposing electric field in the material five times better than in air. Note the ceramic material at the bottom of the chart with a relative permittivity of 7500—a relative permittivity that makes it a very special dielectric in the manufacture of capacitors.

*Although there is a difference in dielectric characteristics between air and a vacuum, the difference is so small that air is commonly used as the reference level.

TABLE 10.1

Relative permittivity (dielectric constant) ϵ_r of various dielectrics.

Dielectric	ϵ_r (Average Values)
Vacuum	1.0
Air	1.0006
Teflon®	2.0
Paper, paraffined	2.5
Rubber	3.0
Polystyrene	3.0
Oil	4.0
Mica	5.0
Porcelain	6.0
Bakelite®	7.0
Aluminum oxide	7
Glass	7.5
Tantalum oxide	30
Ceramics	20–7500
Barium-strontium titanite (ceramic)	7500.0

Defining ϵ_o as the permittivity of air, the relative permittivity of a material with a permittivity ϵ is defined by Eq. (10.8):

$$\epsilon_r = \frac{\epsilon}{\epsilon_o} \quad \text{(dimensionless)} \tag{10.8}$$

Note that ϵ_r, which (as mentioned previously) is often called the **dielectric constant,** is a dimensionless quantity because it is a ratio of similar quantities. However, permittivity does have the units of farads/meter (F/m) and is 8.85×10^{-12} F/m for air. Although the relative permittivity for the air we breathe is listed as 1.006, a value of 1 is normally used for the relative permittivity of air.

For every dielectric there is a potential that, if applied across the dielectric, will break down the bonds within it and cause current to flow through it. The voltage required per unit length is an indication of its **dielectric strength** and is called the **breakdown voltage.** When breakdown occurs, the capacitor has characteristics very similar to those of a conductor. A typical example of dielectric breakdown is lightning, which occurs when the potential between the clouds and the earth is so high that charge can pass from one to the other through the atmosphere (the dielectric). The average dielectric strengths for various dielectrics are tabulated in volts/mil in Table 10.2 (1 mil = 1/1000 inch).

One of the important parameters of a capacitor is the **maximum working voltage.** It defines the maximum voltage that can be placed across the capacitor on a continuous basis without damaging it or changing its characteristics. For most capacitors, it is the dielectric strength that defines the maximum working voltage.

TABLE 10.2

Dielectric strength of some dielectric materials.

Dielectric	Dielectric Strength (Average Value) in Volts/Mil
Air	75
Barium-strontium titanite (ceramic)	75
Ceramics	75–1000
Porcelain	200
Oil	400
Bakelite®	400
Rubber	700
Paper paraffined	1300
Teflon®	1500
Glass	3000
Mica	5000

10.4 CAPACITORS

Capacitor Construction

We are now aware of the basic components of a capacitor: conductive plates, separation, and dielectric. However, the question remains, How do all these factors interact to determine the capacitance of a capacitor? *Larger plates* permit an increased area for the storage of charge, so the area of the plates should be in the numerator of the defining equation. *The smaller the distance between the plates,* the larger the capacitance so this factor should appear in the numerator of the equation. Finally, since *higher levels of permittivity* result in higher levels of capacitance, the factor ϵ should appear in the numerator of the defining equation.

The result is the following general equation for capacitance:

$$\boxed{C = \epsilon\frac{A}{d}} \qquad \begin{aligned} C &= \text{farads (F)} \\ \epsilon &= \text{permittivity (F/m)} \\ A &= \text{m}^2 \\ d &= \text{m} \end{aligned} \tag{10.9}$$

If we substitute Eq. (10.8) for the permittivity of the material, we obtain the following equation for the capacitance:

$$\boxed{C = \epsilon_o\epsilon_r\frac{A}{d}} \qquad \text{(farads, F)} \tag{10.10}$$

or if we substitute the known value for the permittivity of air, we obtain the following useful equation:

$$\boxed{C = 8.85 \times 10^{-12}\,\epsilon_r\frac{A}{d}} \qquad \text{(farads, F)} \tag{10.11}$$

It is important to note in Eq. (10.11) that the area of the plates (actually the area of only one plate) is in meters squared (m^2); the distance between the plates is measured in meters; and the numerical value of ϵ_r is simply taken from Table 10.1.

You should also be aware that most capacitors are in the μF, nF, or pF range, not the 1 F or greater range. A 1 F capacitor can be as large as a typical flashlight requiring that the housing for the system be quite large. Most capacitors in electronic systems are the size of a thumbnail or smaller.

If we form the ratio of the equation for the capacitance of a capacitor with a specific dielectric to that of the same capacitor with air as the dielectric, the following results:

$$\frac{C = \varepsilon\dfrac{A}{d}}{C_o = \varepsilon_o\dfrac{A}{d}} \Rightarrow \frac{C}{C_o} = \frac{\varepsilon}{\varepsilon_o} = \varepsilon_r$$

and

$$\boxed{C = \epsilon_r C_o} \tag{10.12}$$

The result is that

the capacitance of a capacitor with a dielectric having a relative permittivity of ϵ_r is ϵ_r times the capacitance using air as the dielectric.

The next few examples review the concepts and equations just presented.

EXAMPLE 10.2 In Fig. 10.9, if each air capacitor in the left column is changed to the type appearing in the right column, find the new capacitance level. For each change, the other factors remain the same.

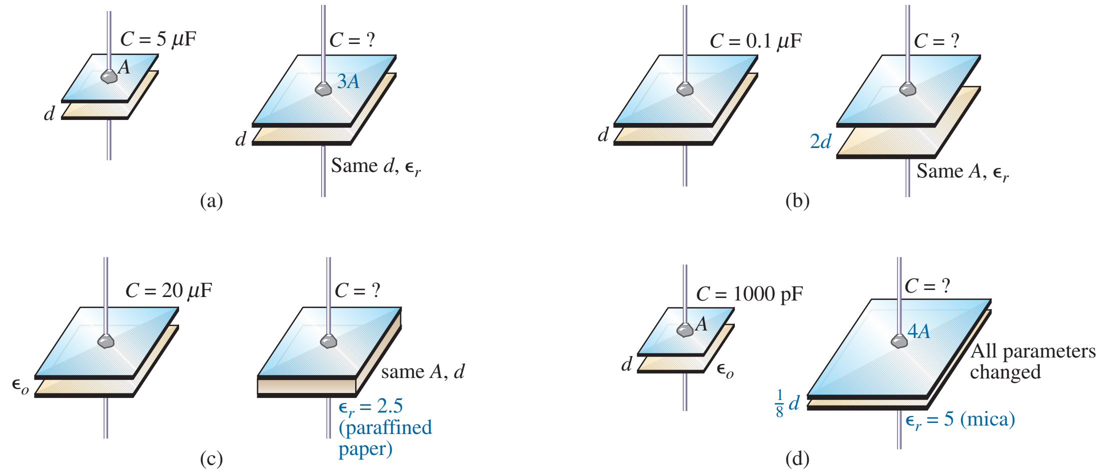


FIG. 10.9
Example 10.2.

Solutions:

a. In Fig. 10.9(a), the area has increased by a factor of three, providing more space for the storage of charge on each plate. Since the area appears in the numerator of the capacitance equation, the capacitance increases by a factor of three. That is,

$$C = 3(C_o) = 3(5\ \mu\text{F}) = \mathbf{15\ \mu F}$$

b. In Fig. 10.9(b), the area stayed the same, but the distance between the plates was increased by a factor of two. Increasing the distance reduces the capacitance level, so the resulting capacitance is one-half of what it was before. That is,

$$C = \frac{1}{2}(0.1\ \mu\text{F}) = \mathbf{0.05\ \mu F}$$

c. In Fig. 10.9(c), the area and the distance between the plates were maintained, but a dielectric of paraffined (waxed) paper was added between the plates. Since the permittivity appears in the numerator of the capacitance equation, the capacitance increases by a factor determined by the relative permittivity. That is,

$$C = \epsilon_r C_o = 2.5(20\ \mu\text{F}) = \mathbf{50\ \mu F}$$

d. In Fig. 10.9(d), a multitude of changes are happening at the same time. However, solving the problem is simply a matter of determining whether the change increases or decreases the capacitance and then placing the multiplying factor in the numerator or denominator of the equation. The increase in area by a factor of four produces a multiplier of four in the numerator, as shown in the equation below. Reducing the distance by a factor of 1/8 will increase the capacitance by its inverse, or a factor of eight. Inserting the mica dielectric increases the capacitance by a factor of five. The result is

$$C = (5)\frac{4}{(1/8)}(C_o) = 160(1000\ \text{pF}) = \mathbf{0.16\ \mu F}$$

In the next example, the dimensions of an air capacitor are provided and the capacitance is to be determined. The example emphasizes the importance of knowing the units of each factor of the equation. Failing to make a conversion to the proper set of units will probably produce a meaningless result, even if the proper equation were used and the mathematics properly executed.

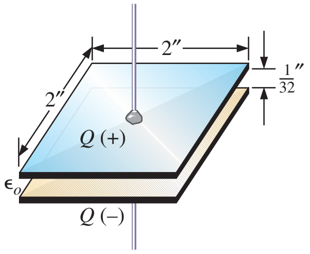


FIG. 10.10
Air capacitor for Example 10.3.

EXAMPLE 10.3 For the capacitor in Fig. 10.10:

a. Find the capacitance.
b. Find the strength of the electric field between the plates if 48 V are applied across the plates.
c. Find the charge on each plate.

Solutions:

a. First, the area and the distance between the plates must be converted to the SI system as required by Eq. (10.11):

$$d = \frac{1}{32}\,\text{in.}\left(\frac{1\text{ m}}{39.37\text{ in.}}\right) = 0.794\text{ mm}$$

and $$A = (2\text{ in.})(2\text{ in.})\left(\frac{1\text{ m}}{39.37\text{ in.}}\right)\left(\frac{1\text{ m}}{39.37\text{ in.}}\right) = 2.581 \times 10^{-3}\text{ m}^2$$

Eq. (10.11):

$$C = 8.85 \times 10^{-12}\,\epsilon_r \frac{A}{d} = 8.85 \times 10^{-12}(1)\frac{(2.581 \times 10^{-3}\text{ m}^2)}{0.794\text{ mm}} = \mathbf{28.8\text{ pF}}$$

b. The electric field between the plates is determined by Eq. (10.7):

$$\mathscr{E} = \frac{V}{d} = \frac{48\text{ V}}{0.794\text{ mm}} = \mathbf{60.5\text{ kV/m}}$$

c. The charge on the plates is determined by Eq. (10.6):

$$Q = CV = (28.8\text{ pF})(48\text{ V}) = \mathbf{1.38\text{ nC}}$$

In the next example, we will insert a ceramic dielectric between the plates of the air capacitor in Fig. 10.10 and see how it affects the capacitance level, electric field, and charge on the plates.

EXAMPLE 10.4

a. Insert a ceramic dielectric with an ϵ_r of 250 between the plates of the capacitor in Fig. 10.10. Then determine the new level of capacitance. Compare your results to the solution in Example 10.3.
b. Find the resulting electric field strength between the plates, and compare your answer to the result in Example 10.3.
c. Determine the charge on each of the plates, and compare your answer to the result in Example 10.3.

Solutions:

a. Using Eq. (10.12), the new capacitance level is

$$C = \epsilon_r C_o = (250)(28.8\text{ pF}) = \mathbf{7200\text{ pF} = 7.2\text{ nF} = 0.0072\ \mu F}$$

which is *significantly higher* than the level in Example 10.3.

b. $\mathscr{E} = \dfrac{V}{d} = \dfrac{48\text{ V}}{0.794\text{ mm}} = \mathbf{60.5\ kV/m}$

Since the applied voltage and the distance between the plates did not change, *the electric field between the plates remains the same.*

c. $Q = CV = (7200\text{ pF})(48\text{ V}) = \mathbf{345.6\ nC = 0.35\ \mu C}$

We now know that the insertion of a dielectric between the plates increases the amount of charge stored on the plates. In Example 10.4, since the relative permittivity increased by a factor of 250, the charge on the plates *increased by the same amount.*

EXAMPLE 10.5 Find the maximum voltage that can be applied across the capacitor in Example 10.4 if the dielectric strength is 80 V/mil.

Solution:

$$d = \frac{1}{32}\,\cancel{\text{in.}}\left(\frac{1000\text{ mils}}{1\ \cancel{\text{in.}}}\right) = 31.25\text{ mils}$$

and

$$V_{max} = 31.25\ \cancel{\text{mils}}\left(\frac{80\text{ V}}{\cancel{\text{mil}}}\right) = \mathbf{2.5\ kV}$$

although the provided working voltage may be only 2 kV to provide a margin of safety.

Types of Capacitors

Capacitors, like resistors, can be listed under two general headings: **fixed** and **variable.** The symbol for the fixed capacitor appears in Fig. 10.11(a). Note that the curved side is normally connected to ground or to the point of lower dc potential. The symbol for variable capacitors appears in Fig. 10.11(b).

FIG. 10.11
Symbols for the capacitor: (a) fixed; (b) variable.

Fixed Capacitors Fixed-type capacitors come in all shapes and sizes. However,

in general, for the same type of construction and dielectric, the larger the required capacitance, the larger the physical size of the capacitor.

In Fig. 10.12(a), the 10,000 μF electrolytic capacitor is significantly larger than the 1 μF capacitor. However, it is certainly not 10,000 times larger. For the polyester-film type of Fig. 10.12(b), the 2.2 μF capacitor is significantly larger than the 0.01 μF capacitor, but again it is not 220 times larger. The 22 μF tantalum capacitor of Fig. 10.12(c) is about 6 times larger than the 1.5 μF capacitor, even though the capacitance level is about 15 times higher. It is particularly interesting to note that due to the difference in dielectric and construction, the 22 μF tantalum capacitor is significantly smaller than the 2.2 μF polyester-film capacitor, and much smaller than 1/5 the size of the 100 μF electrolytic capacitor. The relatively large 10,000 μF electrolytic capacitor is normally used for high-power applications, such as in power supplies and high-output speaker systems. All the others may appear in any commercial electronic system.

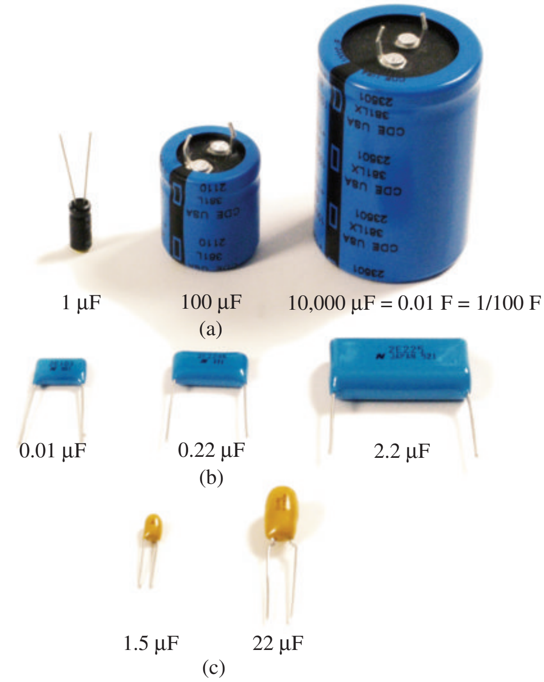


FIG. 10.12
Demonstrating that, in general, for each type of construction, the size of a capacitor increases with the capacitance value: (a) electrolytic; (b) polyester-film; (c) tantalum.

The increase in size is due primarily to the effect of area and thickness of the dielectric on the capacitance level. There are a number of ways to increase the area without making the capacitor too large. One is to lay out the plates and the dielectric in long, narrow strips and then roll them all together, as shown in Fig. 10.13(a). The dielectric (remember that it has the characteristics of an insulator) between the conducting strips ensures the strips never touch. Of course, the dielectric must be the type that can be rolled without breaking up. Depending on how the materials are wrapped, the capacitor can be either a cylindrical or a rectangular, box-type shape.

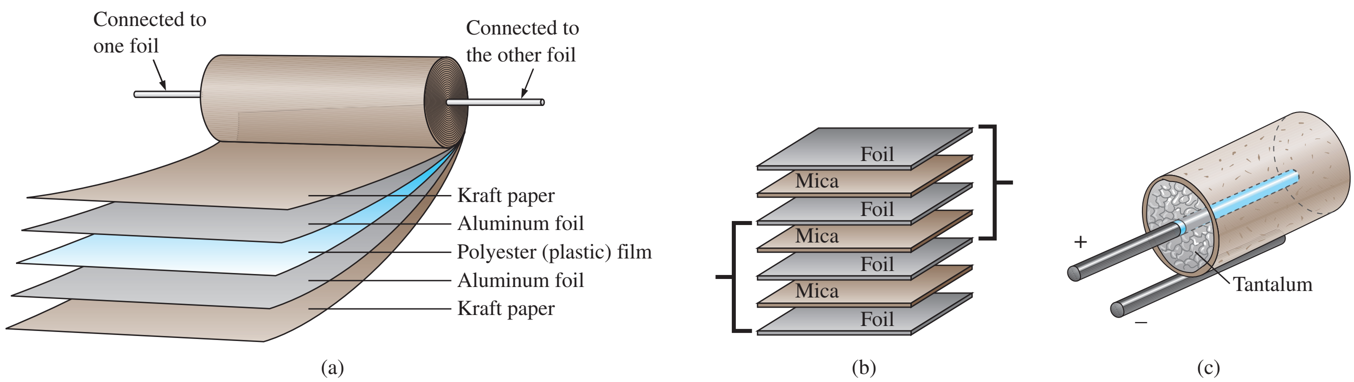


FIG. 10.13
Three ways to increase the area of a capacitor: (a) rolling; (b) stacking; (c) insertion.

A second popular method is to stack the plates and the dielectrics, as shown in Fig. 10.14(b). The area is now a multiple of the number of dielectric layers. This construction is very popular for smaller capacitors.

A third method is to use the dielectric to establish the body shape [a cylinder in Fig. 10.13(c)]. Then simply insert a rod for the positive plate, and coat the surface of the cylinder to form the negative plate, as shown in Fig. 10.13(c). Although the resulting "plates" are not the same in construction or surface area, the effect is to provide a large surface area for storage (the density of electric field lines will be different on the two "plates"), although the resulting distance factor may be larger than desired. Using a dielectric with a high ϵ_r, however, compensates for the increased distance between the plates.

There are other variations of the above to increase the area factor, but the three depicted in Fig. 10.13 are the most popular.

The next controllable factor is the distance between the plates. This factor, however, is very sensitive to how thin the dielectric can be made, with natural concerns because the working voltage (the breakdown voltage) drops as the gap decreases. Some of the thinnest dielectrics are just oxide coatings on one of the conducting surfaces (plates). A very thin polyester material, such as Mylar®, Teflon®, or even paper with a paraffin coating, provides a thin sheet of material than can easily be wrapped for increased areas. Materials such as mica and some ceramic materials can be made only so thin before crumbling or breaking down under stress.

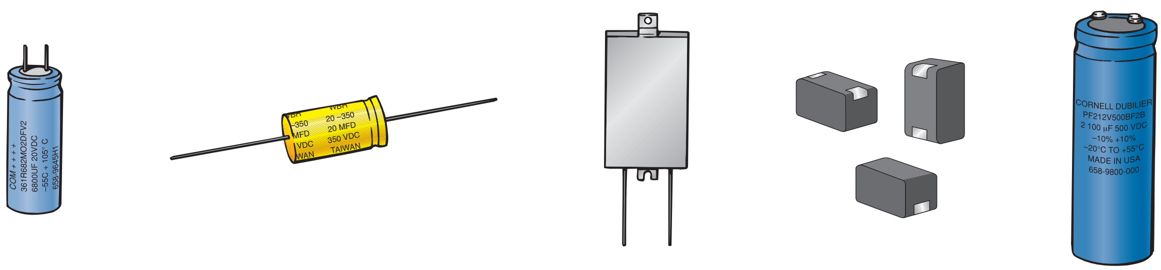
The last factor is the dielectric, for which there is a wide range of possibilities. However, the following factors greatly influence which dielectric is used:

The level of capacitance desired
The resulting size
The possibilities for rolling, stacking and so on
Temperature sensitivity
Working voltage

The range of relative permittivities is enormous, as shown in Table 10.2, but all the factors listed above must be considered in the construction process.

In general, the most common fixed capacitors are the electrolytic, film, polyester, foil, ceramic, mica, dipped, and oil.

The **electrolytic capacitors** in Fig. 10.14 are usually easy to identify by their shape and the fact that they usually have a polarity marking on the body (although special-application electrolytics are available that are not polarized). Few capacitors have a polarity marking, but those that do must be connected with the negative terminal connected to ground or to the point of lower potential. The markings often used to denote the positive terminal or plate include +, □, and Δ. In general, electrolytic



(a) (b) (c) (d) (e)

FIG. 10.14

Various types of electrolytic capacitors: (a) miniature radial leads; (b) axial leads; (c) flatpack; (d) surface-mount; (e) screw-in terminals. (Courtesy of Cornell-Dubilier.)

capacitors offer some of the highest capacitance values available, although their working voltage levels are limited. Typical values range from 0.1 μF to 15,000 μF, with working voltages from 5 V to 450 V. The basic construction uses the rolling process in Fig. 10.13(a) in which a roll of aluminum foil is coated on one side with aluminum oxide—the aluminum being the positive plate, and the oxide the dielectric. A layer of paper or gauze saturated with an electrolyte (a solution or paste that forms the conducting medium between the electrodes of the capacitor) is placed over the aluminum-oxide coating of the positive plate. Another layer of aluminum without the oxide coating is then placed over this layer to assume the role of the negative plate. In most cases, the negative plate is connected directly to the aluminum container, which then serves as the negative terminal for external connections. Because of the size of the roll of aluminum foil, the overall size of the electrolytic capacitor is greater than most.

Film, polyester, foil, polypropylene or **Teflon® capacitors** use a rolling or stacking process to increase the surface area, as shown in Fig. 10.15. The resulting shape can be either round or rectangular, with radial or axial leads. The typical range for such capacitors is 100 pF to 10 μF, with units available up to 100 μF. The name of the unit defines the type of dielectric employed. Working voltages can extend from a few volts to 2000 V, depending on the type of unit.

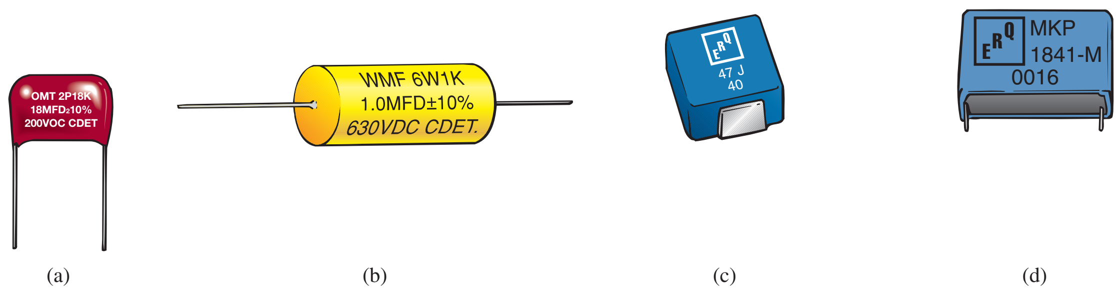


FIG. 10.15
(a) Film/foil polyester radial lead; (b) metalized polyester-film axial lead; (c) surface-mount polyester-film; (d) polypropylene-film, radial lead.

Ceramic capacitors (often called **disc capacitors**) use a ceramic dielectric, as shown in Fig. 10.16(a), to utilize the excellent ϵ_r values and high working voltages associated with a number of ceramic materials.

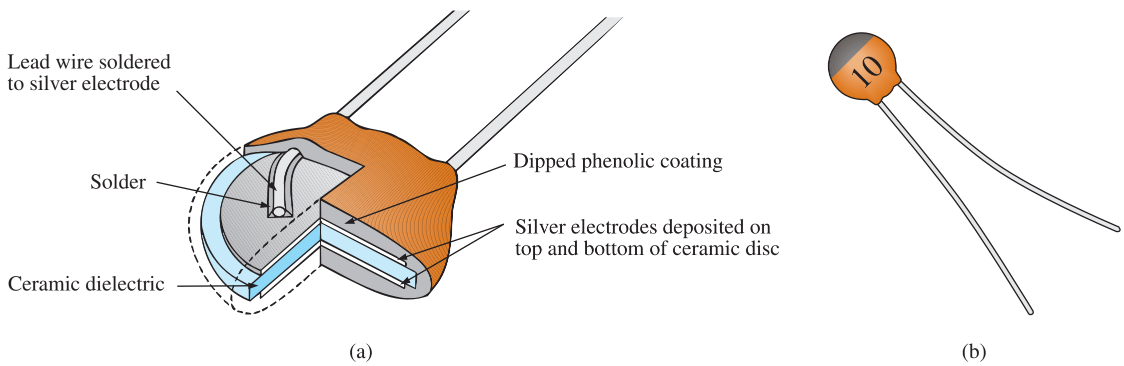
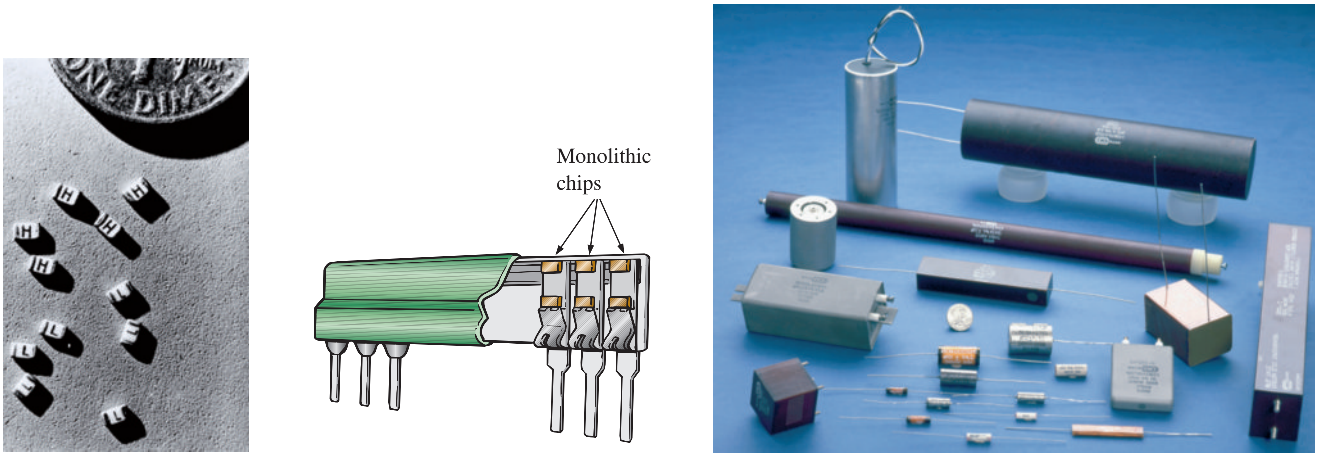


FIG. 10.16
Ceramic (disc) capacitor: (a) construction; (b) appearance.



(a) (b) (c)

FIG. 10.17
Mica capacitors: (a) and (b) surface-mount monolithic chips; (c) high-voltage/temperature mica paper capacitors.
[(a) and (b) courtesy of Vishay Intertechnology, Inc.; (c) courtesy of Custom Electronics, Inc.]

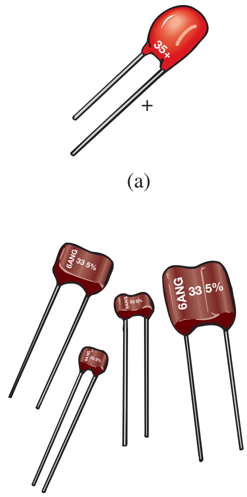
Stacking can also be applied to increase the surface area. An example of the disc variety appears in Fig. 10.16(b). Ceramic capacitors typically range in value from 10 pF to 0.047 μF, with high working voltages that can reach as high as 10 kV.

Mica capacitors use a mica dielectric that can be monolithic (single chip) or stacked. The relatively small size of monolithic mica chip capacitors is demonstrated in Fig. 10.17(a), with their placement shown in Fig. 10.17(b). A variety of high-voltage mica paper capacitors are displayed in Fig. 10.17(c). Mica capacitors typically range in value from 2 pF to several microfarads, with working voltages up to 20 kV.

Dipped capacitors are made by dipping the dielectric (tantalum or mica) into a conductor in a molten state to form a thin, conductive sheet on the dielectric. Due to the presence of an electrolyte in the manufacturing process, dipped tantalum capacitors require a polarity marking to ensure that the positive plate is always at a higher potential than the negative plate, as shown in Fig. 10.18(a). A series of small positive signs is typically applied to the casing near the positive lead. A group of nonpolarized, mica dipped capacitors are shown in Fig. 10.18(b). They typically range in value from 0.1 μF to 680 μF, but with lower working voltages ranging from 6 V to 50 V.

Most **oil capacitors** such as appearing in Fig. 10.19 are used for industrial applications such as welding, high-voltage power supplies, surge protection, and power-factor correction (Chapter 19). They can provide capacitance levels extending from 0.001 μF all the way up to 10,000 μF, with working voltages up to 150 kV. Internally, there are a number of parallel plates sitting in a bath of oil or oil-impregnated material (the dielectric).

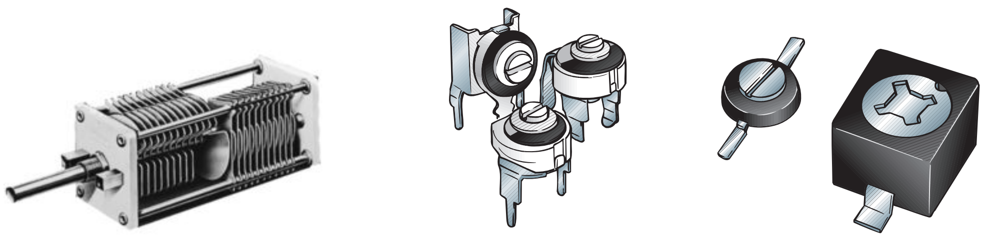
Variable Capacitors All the parameters in Eq. (10.11) can be changed to some degree to create a **variable capacitor.** For example, in Fig. 10.20(a), the capacitance of the variable air capacitor is changed by turning the shaft at the end of the unit. By turning the shaft, you control the amount of common area between the plates: The less common area there is, the less capacitance. In Fig. 10.20(b), we have a much smaller **air trimmer capacitor.** It works under the same principle, but the rotating blades are totally hidden inside the structure. In Fig. 10.20(c), the



(b)

FIG. 10.18
Dipped capacitors: (a) polarized tantalum; (b) nonpolarized mica.

FIG. 10.19
Oil-filled, metallic oval case snubber capacitor (the snubber removes unwanted voltage spikes).
(Courtesy of Cornell-Dubilier.)



(a) (b) (c)

FIG. 10.20

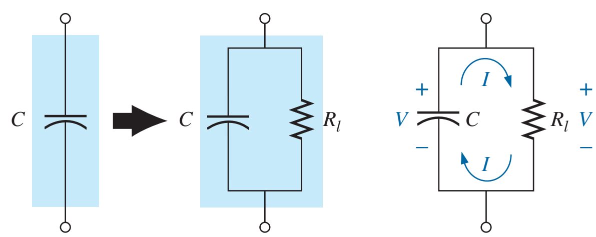
Variable capacitors: (a) air; (b) air trimmer; (c) ceramic dielectric compression trimmer.
[(a) courtesy of James Millen Manufacturing Co.; (c) courtesy of Sprague-Goodman, Inc.]

ceramic trimmer capacitor permits varying the capacitance by changing the common area as above or by applying pressure to the ceramic plate to reduce the distance between the plates.

Leakage Current

Although we would like to think of capacitors as ideal elements, unfortunately, this is not the case. Up to this point, we have assumed that the insulating characteristics of dielectrics prevent any flow of charge between the plates unless the breakdown voltage is exceeded. In reality, however, dielectrics are not perfect insulators, and they do carry a few free electrons in their atomic structure.

When a voltage is applied across a capacitor, a **leakage current** is established between the plates. This current is usually so small that it can be ignored for the application under investigation. The availability of free electrons to support current flow is represented by a large parallel resistor in the equivalent circuit for a capacitor as shown in Fig. 10.21(a). If we apply 10 V across a capacitor with an internal resistance of 1000 MΩ, the current will be 0.01 μA—a level that can be ignored for most applications.



(a) (b)

FIG. 10.21

Leakage current: (a) including the leakage resistance in the equivalent model for a capacitor; (b) internal discharge of a capacitor due to the leakage current.

The real problem associated with leakage currents is not evident until you ask the capacitors to sit in a charged state for long periods of time. As shown in Fig. 10.21(b), the voltage ($V = Q/C$) across a charged capacitor also appears across the parallel leakage resistance and establishes a discharge current through the resistor. In time, the capacitor is totally discharged. Capacitors such as the electrolytic that have high leakage currents (a leakage resistance of 0.5 MΩ is typical) usually have a limited shelf life due to this internal discharge characteristic. Ceramic, tantalum, and mica capacitors typically have unlimited shelf life due to leakage resistances in excess of 1000 MΩ. Thin-film capacitors have lower levels of leakage resistances that result in some concern about shelf life.

Temperature Effects: ppm

Every capacitor is temperature sensitive, with the nameplate capacitance level specified at room temperature. Depending on the type of dielectric, increasing or decreasing temperatures can cause either a drop or a rise in capacitance. If temperature is a concern for a particular application, the manufacturer will provide a temperature plot, such as shown in Fig. 10.22, or a **ppm/°C** (parts per million per degree Celsius) rating for the capacitor. Note in Fig. 10.20 the 0% variation from the nominal (nameplate) value at 25°C (room temperature). At 0°C (freezing), it has dropped 20%, while at 100°C (the boiling point of water), it has dropped 70%—a factor to consider for some applications.

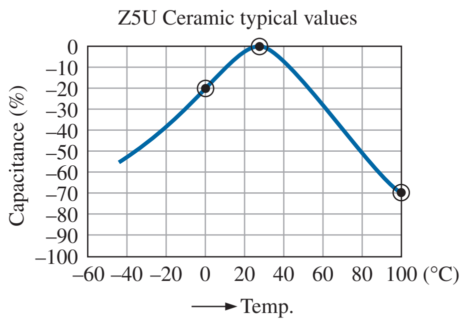


FIG. 10.22
Variation of capacitor value with temperature.

As an example using the ppm level, consider a 100 μF capacitor with a **temperature coefficient** or **ppm** of −150 ppm/°C. It is important to note the negative sign in front of the ppm value, because it reveals that the capacitance will drop with increase in temperature. It takes a moment to fully appreciate a term such as *parts per million.* In equation form, 150 parts per million can be written as

$$-\frac{150}{1{,}000{,}000} \times$$

If we then multiply this term by the capacitor value, we can obtain the change in capacitance for each 1°C change in temperature. That is,

$$-\frac{150}{1{,}000{,}000}(100\ \mu\text{F})/^\circ\text{C} = \mathbf{-0.015\ \mu F/^\circ C} = \mathbf{-15{,}000\ pF/^\circ C}$$

If the temperature should rise 25°C, the capacitance decreases by

$$-\frac{15{,}000\ \text{pF}}{^\circ\cancel{\text{C}}}(25^\circ\cancel{\text{C}}) = \mathbf{-0.38\ \mu F}$$

changing the capacitance level to

$$100\ \mu\text{F} - 0.38\ \mu\text{F} = \mathbf{99.62\ \mu F}$$

Capacitor Labeling

Due to the small size of some capacitors, various marking schemes have been adopted to provide the capacitance level, tolerance, and, if possible, working voltage. In general, however, as pointed out above, *the size of the capacitor is the first indicator of its value.* In fact, most marking schemes do not indicate whether it is in μF or pF. It is assumed that you can make that judgment purely from the size. The smaller units are typically in pF, and the larger units in μF. Unless indicated by an **n** or **N,** most units are not provided in nF. On larger μF units, the value can often be printed on the jacket with the tolerance and working voltage. However, smaller units need to use some form of abbreviation as shown in Fig. 10.23. For very small units such as those in Fig. 10.23(a) with only two numbers, the value is recognized immediately as being in pF with the **K** an indicator of a

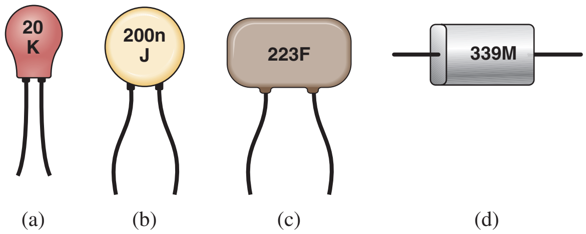


FIG. 10.23
Various marking schemes for small capacitors.

±10% tolerance level. Too often the K is read as a multiplier of 10^3, and the capacitance is read as 20,000 pF or 20 nF rather than the actual 20 pF.

For the unit in Fig. 10.23(b), there was room for a lowercase **n** to represent a multiplier of 10^{-9}, resulting in a value of 200 nF. To avoid unnecessary confusion, the letters used for tolerance do not include **N** or **U** or **P,** so the presence of any of these letters in upper- or lowercase normally refers to the multiplier level. The **J** appearing on the unit in Fig. 10.23(b) represents a ±5% tolerance level. For the capacitor in Fig. 10.23(c), the first two numbers are the numerical value of the capacitor, while the third number is the power of the multiplier (or number of zeros to be added to the first two numbers). The question then remains whether the units are μF or pF. With the 223 representing a number of 22,000, the units are certainly not μF, because the unit is too small for such a large capacitance. It is a 22,000 pF = 22 nF capacitor. The **F** represents a ±1% tolerance level. Multipliers of 0.01 use an 8 for the third digit, while multipliers of 0.1 use a 9. The capacitor in Fig. 10.23(d) is a $33 \times 0.1 = 3.3\ \mu$F capacitor with a tolerance of ±20% as defined by the capital letter **M.** The capacitance is not 3.3 pF because the unit is too large; again, the factor of size is very helpful in making a judgment about the capacitance level. It should also be noted that **MFD** is sometimes used to signify microfarads.

Measurement and Testing of Capacitors

The capacitance of a capacitor can be read directly using a meter such as the Universal LCR Meter in Fig. 10.24. If you set the meter on **C** for *capacitance,* it will automatically choose the most appropriate unit of measurement for the element, that is, F, μF, nF, or pF. Note the polarity markings on the meter for capacitors that have a specified polarity.

The best check is to use a meter such as the one in Fig. 10.24. However, if it is unavailable, an ohmmeter can be used to determine whether the dielectric is still in good working order or whether it has deteriorated due to age or use (especially for paper and electrolytics). As the dielectric breaks down, the insulating qualities of the material decrease to the point where the resistance between the plates drops to a relatively low level. To use an ohmmeter, be sure that the capacitor is fully discharged by placing a lead directly across its terminals. Then hook up the meter (paying attention to the polarities if the unit is polarized) as shown in Fig. 10.25, and note whether the resistance has dropped to a relatively low value (0 to a few kilohms). If so, the capacitor should be discarded. You may find that the reading changes when the meter is first connected. This change is due to the charging of the capacitor by the internal supply of the ohmmeter. In time the capacitor becomes stable, and the correct read-

FIG. 10.24
Digital reading capacitance meter.
(Image provided by B & K Precision Corporation.)

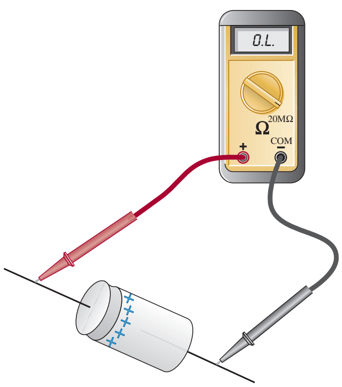


FIG. 10.25
Checking the dielectric of an electrolytic capacitor.

ing can be observed. Typically, it should pin at the highest level on the megohm scales or indicate OL on a digital meter.

The above ohmmeter test is not all-inclusive, since some capacitors exhibit the breakdown characteristics only when a large voltage is applied. The test, however, does help isolate capacitors in which the dielectric has deteriorated.

Standard Capacitor Values

The most common capacitors use the same numerical multipliers encountered for resistors.

The vast majority are available with 5%, 10%, or 20% tolerances. There are capacitors available, however, with tolerances of 1%, 2%, or 3%, if you are willing to pay the price. Typical values include 0.1 μF, 0.15 μF, 0.22 μF, 0.33 μF, 0.47 μF, 0.68 μF; or 1 μF, 1.5 μF, 2.2 μF, 3.3 μF, 4.7 μF, 6.8 μF; and 10 pF, 22 pF, 33 pF, 100 pF; and so on.

10.5 TRANSIENTS IN CAPACITIVE NETWORKS: THE CHARGING PHASE

The placement of charge on the plates of a capacitor does not occur instantaneously. Instead, it occurs over a period of time determined by the components of the network. The charging phase, the phase during which charge is deposited on the plates, can be described by reviewing the response of the simple series circuit in Fig. 10.4. The circuit has been redrawn in Fig. 10.26 with the symbol for a fixed capacitor.

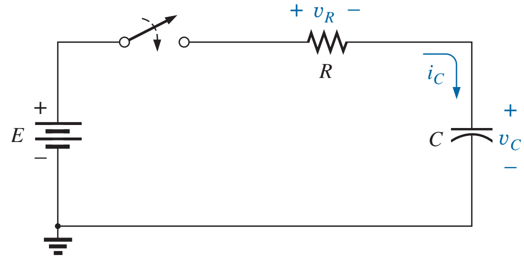


FIG. 10.26
Basic R-C charging network.

Recall that the instant the switch is closed, electrons are drawn from the top plate and deposited on the bottom plate by the battery, resulting in a net positive charge on the top plate and a negative charge on the bottom plate. The transfer of electrons is very rapid at first, slowing down as the potential across the plates approaches the applied voltage of the battery. Eventually, when the voltage across the capacitor equals the applied voltage, the transfer of electrons ceases, and the plates have a net charge determined by $Q = CV_C = CE$. This period of time during which charge is being deposited on the plates is called the **transient period**—a period of time where the voltage or current changes from one steady-state level to another.

Since the voltage across the plates is directly related to the charge on the plates by $V = Q/C$, a plot of the voltage across the capacitor will have the same shape as a plot of the charge on the plates over time. As shown in Fig. 10.27, the voltage across the capacitor is zero volts when the switch is closed ($t = 0$ s). It then builds up very quickly at first since charge is being deposited at a very high rate of speed. As time passes, the charge is deposited at a slower rate, and the change in voltage drops off. The voltage continues to grow, but at a much slower rate. Eventually, as the voltage across the plates approaches the applied voltage, the charging rate is very slow, until finally the voltage across the plates is equal to the applied voltage—the transient phase has passed.

Fortunately, the waveform in Fig. 10.27 from beginning to end can be described using the mathematical function e^{-x}. It is an exponential function that decreases with time, as shown in Fig. 10.28. If we substitute zero for x, we obtain e^{-0} which by definition is 1, as shown in

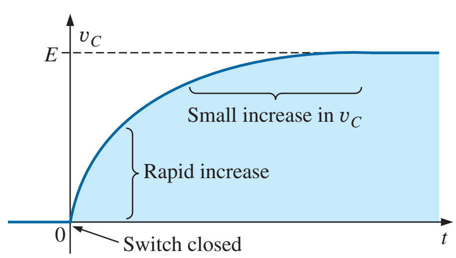


FIG. 10.27
v_C during the charging phase.

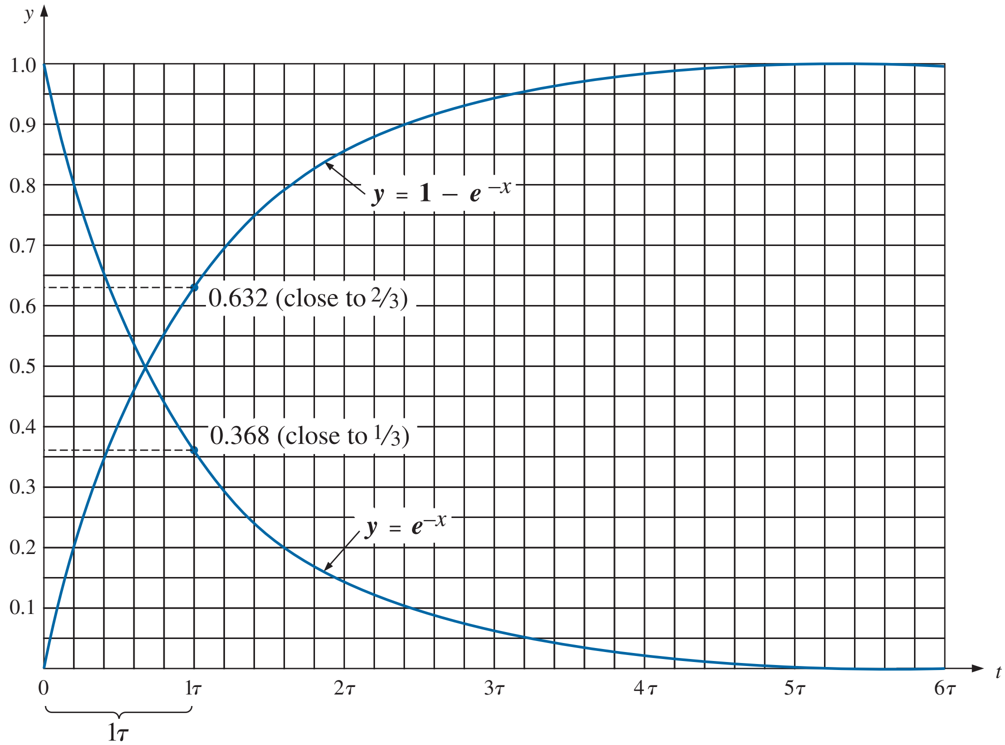


FIG. 10.28
Universal time constant chart.

TABLE 10.3
Selected values of e^{-x}.

$x = 0$	$e^{-x} = e^{-0} = \frac{1}{e^0} = \frac{1}{1} = 1$
$x = 1$	$e^{-1} = \frac{1}{e} = \frac{1}{2.71828\ldots} = 0.3679$
$x = 2$	$e^{-2} = \frac{1}{e^2} = 0.1353$
$x = 5$	$e^{-5} = \frac{1}{e^5} = 0.00674$
$x = 10$	$e^{-10} = \frac{1}{e^{10}} = 0.0000454$
$x = 100$	$e^{-100} = \frac{1}{e^{100}} = 3.72 \times 10^{-44}$

Table 10.3 and on the plot in Fig. 10.28. Table 10.3 reveals that as x increases, the function e^{-x} decreases in magnitude until it is very close to zero after $x = 5$. As noted in Table 10.3, the exponential factor $e^1 = e = 2.71828$.

A plot of $1 - e^{-x}$ is also provided in Fig. 10.28 since it is a component of the voltage v_C in Fig. 10.27. When e^{-x} is 1, $1 - e^{-x}$ is zero as shown in Fig. 10.28 and when e^{-x} decreases in magnitude, $1 - e^{-x}$ approaches 1 as shown in the same figure.

You may wonder how this function can help us if it decreases with time and the curve for the voltage across the capacitor increases with time. We simply place the exponential in the proper mathematical form as shown in Eq. (10.13):

$$\boxed{v_C = E(1 - e^{-t/\tau})}_{\text{charging}} \qquad \text{(volts, V)} \qquad \textbf{(10.13)}$$

First note in Eq. (10.13) that the voltage v_C is written in *lowercase (not capital) italic* to point out that it is a function that will change with time—it is not a constant. The exponent of the exponential function is no longer just x, but now is time (t) divided by a constant τ, the Greek letter *tau*. The quantity τ is defined by

$$\boxed{\tau = RC} \qquad \text{(time, s)} \qquad \textbf{(10.14)}$$

The factor τ, called the **time constant** of the network, has the units of time as shown below using some of the basic equations introduced earlier in this text:

$$\tau = RC = \left(\frac{V}{I}\right)\left(\frac{Q}{V}\right) = \left(\frac{\cancel{V}}{\cancel{Q}/t}\right)\left(\frac{\cancel{Q}}{\cancel{V}}\right) = t\ \text{(seconds)}$$

A plot of Eq. (10.13) results in the curve in Fig. 10.29 whose shape is an exact match with that in Fig. 10.27.

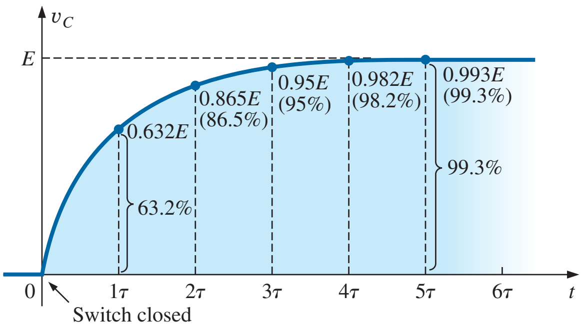


FIG. 10.29
Plotting the equation $v_C = E(1 - e^{-t/\tau})$ versus time (t).

In Eq. (10.13), if we substitute $t = 0$ s, we find that

$$e^{-t/\tau} = e^{-0/\tau} = e^{-0} = \frac{1}{e^0} = \frac{1}{1} = 1$$

and $$v_C = E(1 - e^{-t/\tau}) = E(1 - 1) = \mathbf{0\ V}$$

as appearing in the plot in Fig. 10.29.

It is important to realize at this point that the plot in Fig. 10.29 is not against simply time but against τ, the time constant of the network. If we want to know the voltage across the plates after one time constant, we simply plug $t = 1\tau$ into Eq. (10.13). The result is

$$e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} \cong 0.368$$

and $$v_C = E(1 - e^{-t/\tau}) = E(1 - 0.368) = \mathbf{0.632}\boldsymbol{E}$$

as shown in Fig. 10.29.

At $t = 2\tau$:

$$e^{-t/\tau} = e^{-2\tau/\tau} = e^{-2} \cong 0.135$$

and $$v_C = E(1 - e^{-t/\tau}) = E(1 - 0.135) = \mathbf{0.865}\boldsymbol{E}$$

as shown in Fig. 10.29.

As the number of time constants increases, the voltage across the capacitor does indeed approach the applied voltage.

At $t = 5\tau$:

$$e^{-t/\tau} = e^{-5\tau/\tau} = e^{-5} \cong 0.007$$

and $$v_C = E(1 - e^{-t/\tau}) = E(1 - 0.007) = \mathbf{0.993}\boldsymbol{E} \cong E$$

In fact, we can conclude from the results just obtained that

the voltage across a capacitor in a dc network is essentially equal to the applied voltage after five time constants of the charging phase have passed.

Or, in more general terms,

the transient or charging phase of a capacitor has essentially ended after five time constants.

It is indeed fortunate that the same exponential function can be used to plot the current of the capacitor versus time. When the switch is first closed, the flow of charge or current jumps very quickly to a value limited by the applied voltage and the circuit resistance, as shown in Fig. 10.30. The rate of deposit, and hence the current, then decreases quite rapidly, until eventually charge is not being deposited on the plates and the current drops to zero amperes.

The equation for the current is:

$$i_C = \frac{E}{R}e^{-t/\tau} \quad \text{charging} \qquad \text{(amperes, A)} \qquad \textbf{(10.15)}$$

In Fig. 10.26, the current (conventional flow) has the direction shown since electrons flow in the opposite direction.

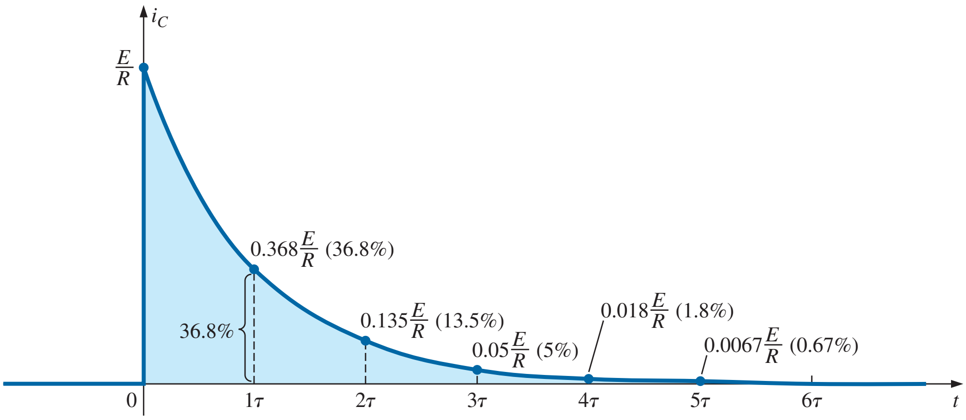


FIG. 10.30

Plotting the equation $i_C = \frac{E}{R}e^{-t/\tau}$ *versus time (t).*

At $t = 0$ s:

$$e^{-t/\tau} = e^{-0} = 1$$

and
$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{E}{R}(1) = \boldsymbol{\frac{E}{R}}$$

At $t = 1\tau$:

$$e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} \cong 0.368$$

and
$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{E}{R}(0.368) = \mathbf{0.368}\,\boldsymbol{\frac{E}{R}}$$

In general, Figure 10.30 clearly reveals that

the current of a capacitive dc network is essentially zero amperes after five time constants of the charging phase have passed.

It is also important to recognize that

during the charging phase, the major change in voltage and current occurs during the first time constant.

The voltage across the capacitor reaches about 63.2% (about 2/3) of its final value, whereas the current drops to 36.8% (about 1/3) of its peak value. During the next time constant, the voltage increases only about 23.3%, whereas the current drops to 13.5%. The first time constant is therefore a very dramatic time for the changing parameters. Between the fourth and fifth time constants, the voltage increases only about 1.2%, whereas the current drops to less than 1% of its peak value.

Returning to Figs. 10.29 and 10.30, note that when the voltage across the capacitor reaches the applied voltage *E,* the current drops to zero amperes, as reviewed in Fig. 10.31. These conditions match those of an open circuit, permitting the following conclusion:

A capacitor can be replaced by an open-circuit equivalent once the charging phase in a dc network has passed.

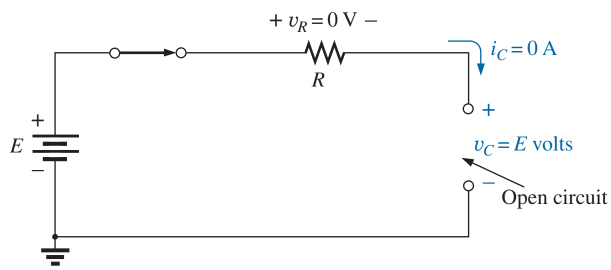


FIG. 10.31
Demonstrating that a capacitor has the characteristics of an open circuit after the charging phase has passed.

This conclusion will be particularly useful when analyzing dc networks that have been on for a long period of time or have passed the transient phase that normally occurs when a system is first turned on.

A similar conclusion can be reached if we consider the instant the switch is closed in the circuit in Fig. 10.26. Referring to Figs. 10.29 and 10.30 again, we find that the current is a peak value at $t = 0$ s, whereas the voltage across the capacitor is 0 V, as shown in the equivalent circuit in Fig. 10.32. The result is that

a capacitor has the characteristics of a short-circuit equivalent at the instant the switch is closed in an uncharged series R-C circuit.

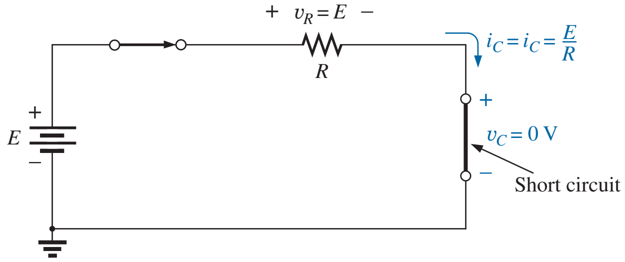


FIG. 10.32
Revealing the short-circuit equivalent for the capacitor that occurs when the switch is first closed.

In Eq. (10.13), the time constant τ will always have some value because some resistance is always present in a capacitive network. In some cases, the value of τ may be very small, but five times that value of τ, no matter how small, must therefore always exist; it cannot be zero. The result is the following very important conclusion:

The voltage across a capacitor cannot change instantaneously.

In fact, we can take this statement a step further by saying that the capacitance of a network is a measure of how much it will oppose a change in voltage in a network. The larger the capacitance, the larger the time constant, and the longer it will take the voltage across the capacitor to reach the applied value. This can prove very helpful when lightning arresters and surge suppressors are designed to protect equipment from unexpected high surges in voltage.

Since the resistor and the capacitor in Fig. 10.26 are in series, the current through the resistor is the same as that associated with the capacitor. The voltage across the resistor can be determined by using Ohm's law in the following manner:

$$\upsilon_R = i_R R = i_C R$$

so that
$$\upsilon_R = \left(\frac{E}{R}e^{-t/\tau}\right)R$$

and
$$\boxed{\upsilon_R = Ee^{-t/\tau}}_{\text{charging}} \qquad \text{(volts, V)} \qquad \textbf{(10.16)}$$

A plot of the voltage as shown in Fig. 10.33 has the same shape as that for the current because they are related by the constant R. Note, however, that the voltage across the resistor starts at a level of E volts because the voltage across the capacitor is zero volts and Kirchhoff's voltage law must always be satisfied. When the capacitor has reached the applied voltage, the voltage across the resistor must drop to zero volts for the same reason. Always remember that

Kirchhoff's voltage law is applicable at any instant of time for any type of voltage in any type of network.

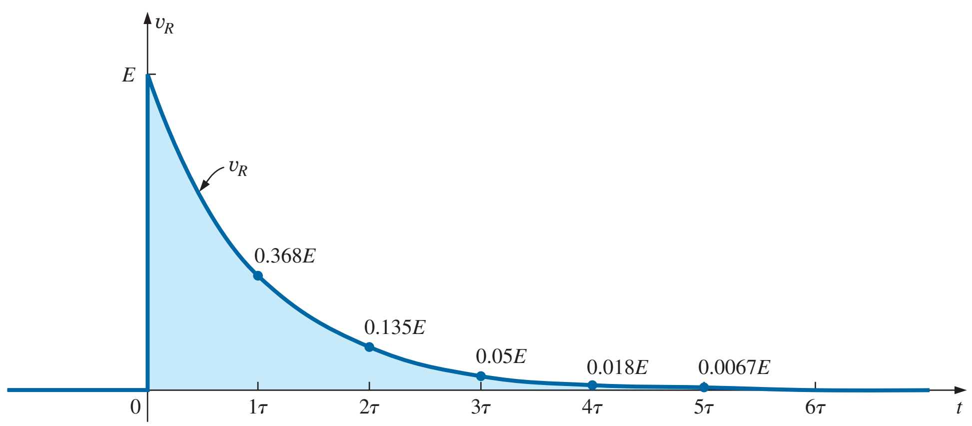


FIG. 10.33
Plotting the equation $\upsilon_R = Ee^{-t/\tau}$ versus time (t).

Using the Calculator to Solve Exponential Functions

Before looking at an example, we will first discuss the use of the TI-89 calculator with exponential functions. The process is actually quite simple for a number such as $e^{-1.2}$. Just select the 2nd function (diamond) key, followed by the function e^x. Then insert the (−) sign from the numerical keyboard (not the mathematical functions), and insert the number 1.2 followed by ENTER to obtain the result of 0.301, as shown in Fig. 10.34. The use of the computer software program Mathcad is demonstrated in a later example.

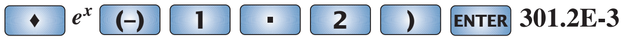


FIG. 10.34
Calculator key strokes to determine $e^{-1.2}$.

EXAMPLE 10.6 For the circuit in Fig. 10.35:

a. Find the mathematical expression for the transient behavior of υ_C, i_C, and υ_R if the switch is closed at $t = 0$ s.
b. Plot the waveform of υ_C versus the time constant of the network.
c. Plot the waveform of υ_C versus time.
d. Plot the waveforms of i_C and υ_R versus the time constant of the network.
e. What is the value of υ_C at $t = 20$ ms?
f. On a practical basis, how much time must pass before we can assume that the charging phase has passed?
g. When the charging phase has passed, how much charge is sitting on the plates?
h. If the capacitor has a leakage resistance of 10,000 MΩ, what is the initial leakage current? Once the capacitor is separated from the circuit, how long will it take to totally discharge, assuming a linear (unchanging) discharge rate?

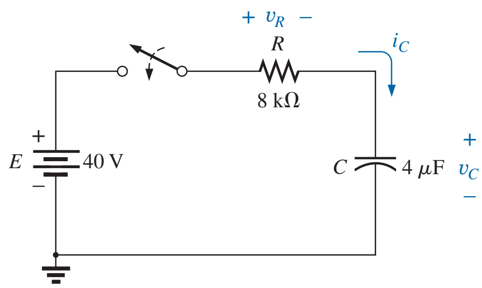


FIG. 10.35
Transient network for Example 10.6.

Solutions:

a. The time constant of the network is

$$\tau = RC = (8\text{ k}\Omega)(4\ \mu\text{F}) = 32\text{ ms}$$

resulting in the following mathematical equations:

$$\upsilon_C = E(1 - e^{-t/\tau}) = \mathbf{40\ V(1 - e^{-t/32ms})}$$

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{40\text{ V}}{8\text{ k}\Omega}e^{-t/32\text{ms}} = \mathbf{5\ mAe^{-t/32ms}}$$

$$\upsilon_R = Ee^{-t/\tau} = \mathbf{40\ Ve^{-t/32ms}}$$

b. The resulting plot appears in Fig. 10.36.
c. The horizontal scale will now be against time rather than time constants, as shown in Fig. 10.37. The plot points in Fig. 10.37 were taken from Fig. 10.36.

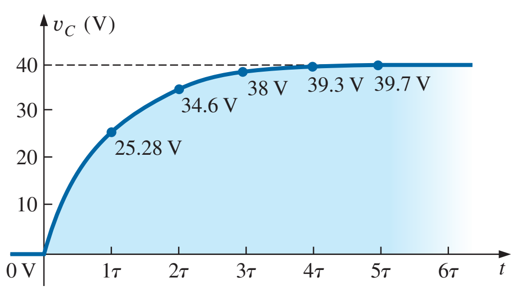


FIG. 10.36
υ_C versus time for the charging network in Fig. 10.35.

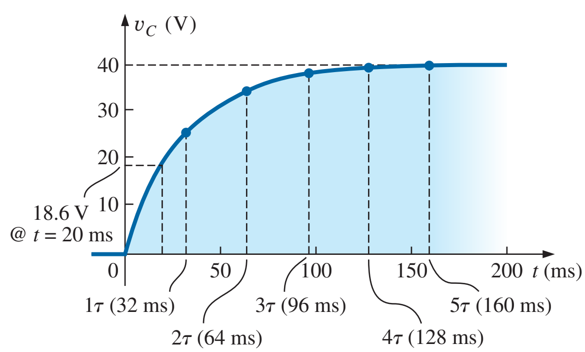


FIG. 10.37
Plotting the waveform in Fig. 10.36 versus time (t).

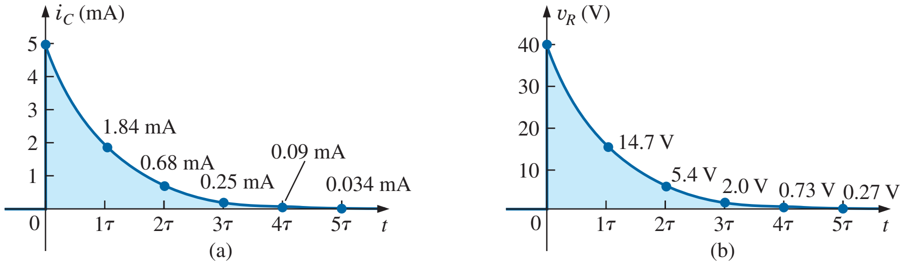


FIG. 10.38
i_C and v_R for the charging network in Fig. 10.36.

d. Both plots appear in Fig. 10.38.

e. Substituting the time $t = 20$ ms results in the following for the exponential part of the equation:

$$e^{-t/\tau} = e^{-20\text{ms}/32\text{ms}} = e^{-0.625} = 0.535 \text{ (using a calculator)}$$

so that $v_C = 40\text{ V}(1 - e^{-t/32\text{ms}}) = 40\text{ V}(1 - 0.535)$
$= (40\text{ V})(0.465) = \mathbf{18.6\ V}$ (as verified by Fig. 10.37)

f. Assuming a full charge in five time constants results in

$$5\tau = 5(32\text{ ms}) = \mathbf{160\ ms} = \mathbf{0.16\ s}$$

g. Using Eq. (10.6):

$$Q = CV = (4\ \mu\text{F})(40\text{ V}) = \mathbf{160\ \mu C}$$

h. Using Ohm's law:

$$I_{\text{leakage}} = \frac{40\text{ V}}{10{,}000\text{ M}\Omega} = 4\text{ nA}$$

Finally, the basic equation $I = Q/t$ results in

$$t = \frac{Q}{I} = \frac{160\ \mu\text{C}}{4\text{ nA}} = (40{,}000\ \cancel{\text{s}})\left(\frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}}\right)\left(\frac{1\text{ h}}{60\ \cancel{\text{min}}}\right) = \mathbf{11.11\ h}$$

10.6 TRANSIENTS IN CAPACITIVE NETWORKS: THE DISCHARGING PHASE

We now investigate how to discharge a capacitor while exerting some control on how long the discharge time will be. You can, of course, place a lead directly across a capacitor to discharge it very quickly—and possibly cause a visible spark. For larger capacitors such those in TV sets, this procedure should not be attempted because of the high voltages involved—unless, of course, you are trained in the maneuver.

In Fig. 10.39(a), a second contact for the switch was added to the circuit in Fig. 10.26 to permit a controlled discharge of the capacitor. With the switch in position 1, we have the charging network described in the last section. Following the full charging phase, if we move the switch to position 1, the capacitor can be discharged through the resulting circuit in Fig. 10.39(b). In Fig. 10.39(b), the voltage across the capacitor appears directly across the resistor to establish a discharge current. Initially, the current jumps to a relatively high value; then it begins to drop. It drops with time because charge is leaving the plates of the capacitor, which in turn reduces the voltage across the capacitor and thereby the voltage across the resistor and the resulting current.

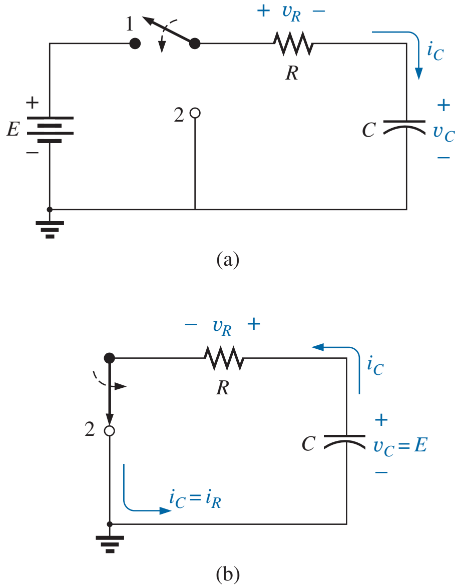


FIG. 10.39
(a) Charging network; (b) discharging configuration.

Before looking at the wave shapes for each quantity of interest, note that current i_C has now reversed direction as shown in Fig. 10.39(b). As shown in parts (a) and (b) in Fig. 10.39, the voltage across the capacitor does not reverse polarity, but the current reverses direction. We will show the reversals on the resulting plots by sketching the waveforms in the negative regions of the graph. In all the waveforms, note that all the mathematical expressions use the same e^{-x} factor appearing during the charging phase.

For the voltage across the capacitor that is decreasing with time, the mathematical expression is:

$$\boxed{\upsilon_C = Ee^{-t/\tau}}_{\text{discharging}} \tag{10.17}$$

For this circuit, the time constant τ is defined by the same equation as used for the charging phase. That is,

$$\boxed{\tau = RC}_{\text{discharging}} \tag{10.18}$$

Since the current decreases with time, it will have a similar format:

$$\boxed{i_C = \frac{E}{R}e^{-t/\tau}}_{\text{discharging}} \tag{10.19}$$

For the configuration in Fig. 10.39(b), since $\upsilon_R = \upsilon_C$ (in parallel), the equation for the voltage υ_R has the same format:

$$\boxed{\upsilon_R = Ee^{-t/\tau}}_{\text{discharging}} \tag{10.20}$$

The complete discharge will occur, for all practical purposes, in five time constants. If the switch is moved between terminals 1 and 2 every five time constants, the wave shapes in Fig. 10.40 will result for υ_C, i_C,

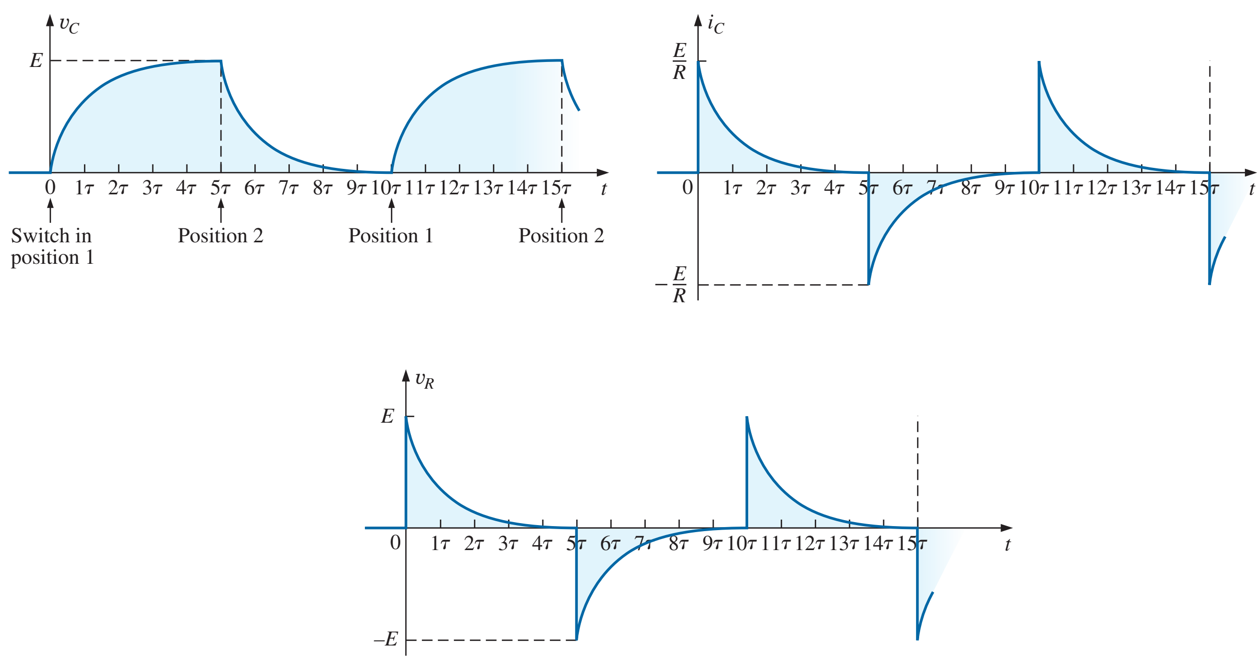


FIG. 10.40
υ_C, i_C, and υ_R for 5τ switching between contacts in Fig. 10.39(a).

and υ_R. For each curve, the current directions and voltage polarities are as defined by the configurations in Fig. 10.39. Note, as pointed out above, that the current reverses direction during the discharge phase.

The discharge rate does not have to equal the charging rate if a different switching arrangement is used. In fact, Example 10.8 will demonstrate how to change the discharge rate.

EXAMPLE 10.7 Using the values in Example 10.6, plot the waveforms for υ_C and i_C resulting from switching between contacts 1 and 2 in Fig. 10.39 every five time constants.

Solution: The time constant is the same for the charging and discharging phases. That is,

$$\tau = RC = (8\ \text{k}\Omega)(4\ \mu\text{F}) = 32\ \text{ms}$$

For the discharge phase, the equations are

$$\upsilon_C = Ee^{-t/\tau} = \mathbf{40\ V}e^{-t/32\text{ms}}$$

$$i_C = -\frac{E}{R}e^{-t/\tau} = \frac{40\ \text{V}}{8\ \text{k}\Omega}e^{-t/32\text{ms}} = -\mathbf{5\ mA}e^{-t/32\text{ms}}$$

$$\upsilon_R = \upsilon_C = \mathbf{40\ V}e^{-t/32\text{ms}}$$

A continuous plot for the charging and discharging phases appears in Fig. 10.41.

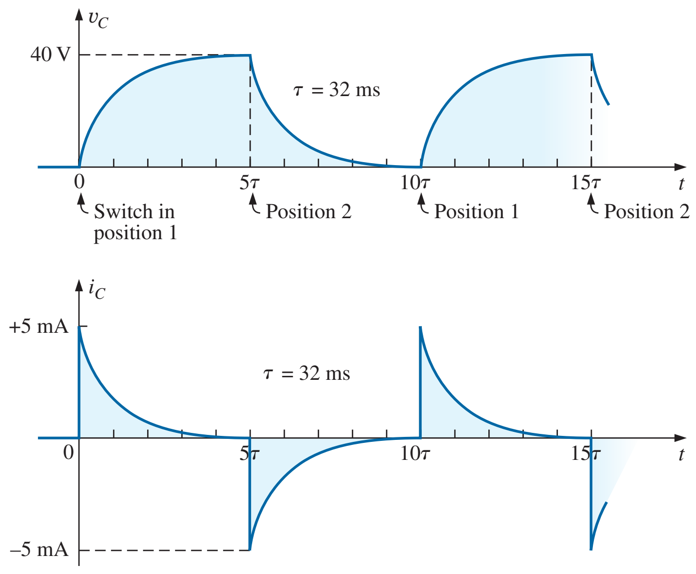


FIG. 10.41
υ_C and i_C for the network in Fig. 10.39(a) with the values in Example 10.6.

The Effect of τ on the Response

In Example 10.7, if the value of τ were changed by changing the resistance, or the capacitor, or both, *the resulting waveforms would appear the same because they were plotted against the time constant of the network.* If they were plotted against time, there could be a dramatic change in the appearance of the resulting plots. In fact, on an oscilloscope, an instru-

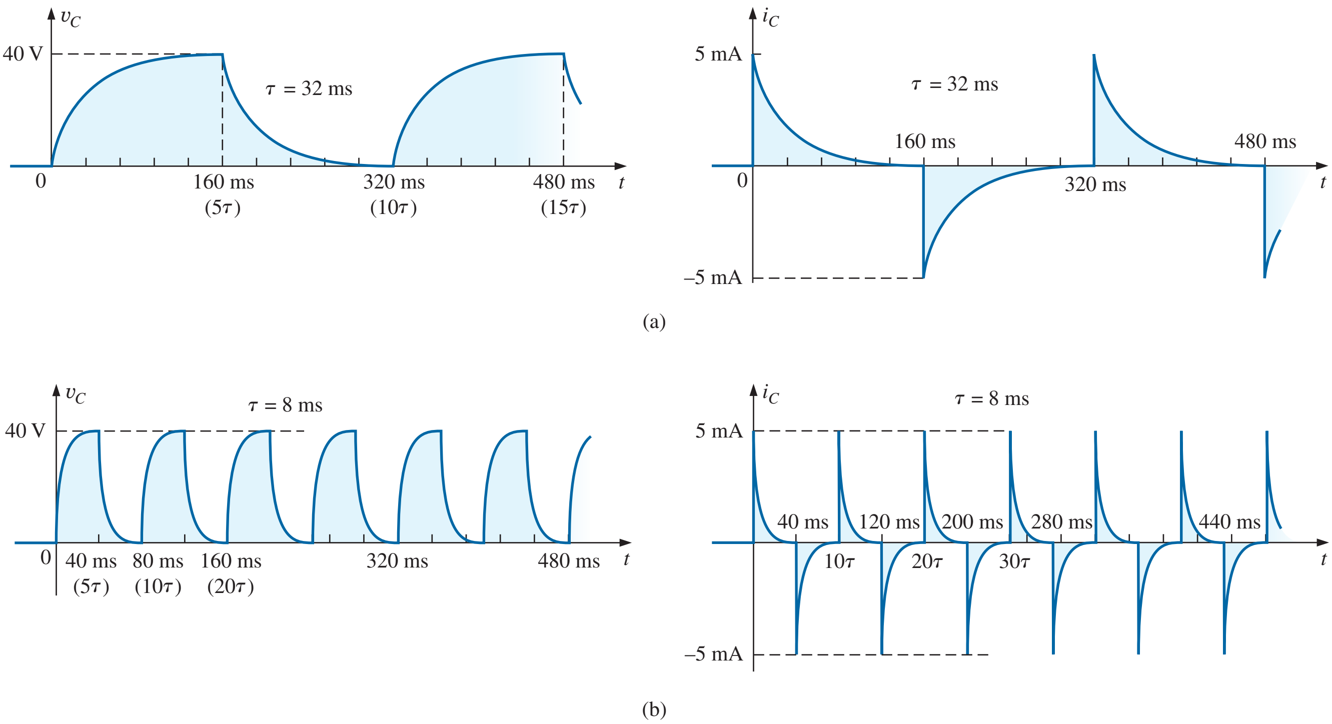


FIG. 10.42
Plotting v_C and i_C versus time in ms: (a) $\tau = 32$ ms; (b) $\tau = 8$ ms.

ment designed to display such waveforms, the plots are against time, and the change will be immediately apparent. In Fig. 10.42(a), the waveforms in Fig. 10.41 for v_C and i_C were plotted against time. In Fig. 10.42(b), the capacitance was decreased to 1 μF which reduces the time constant to 8 ms. Note the dramatic effect on the appearance of the waveform.

For a fixed-resistance network, the effect of increasing the capacitance is clearly demonstrated in Fig. 10.43. The larger the capacitance, and hence the time constant, the longer it takes the capacitor to charge up—there is more charge to be stored. The same effect can be created by holding the capacitance constant and increasing the resistance, but now the longer time is due to the lower currents that are a result of the higher resistance.

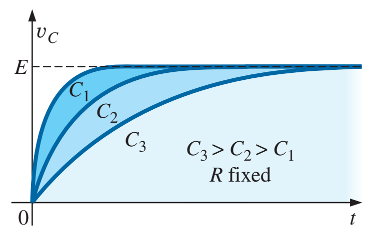


FIG. 10.43
Effect of increasing values of C (with R constant) on the charging curve for v_C.

EXAMPLE 10.8 For the circuit in Fig. 10.44:

a. Find the mathematical expressions for the transient behavior of the voltage v_C and the current i_C if the capacitor was initially uncharged and the switch is thrown into position 1 at $t = 0$ s.

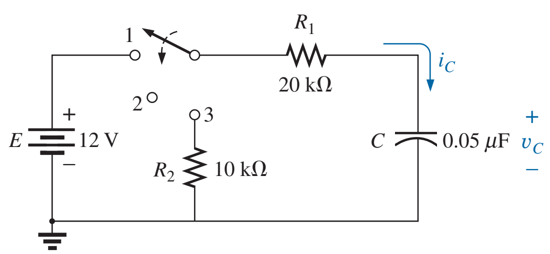


FIG. 10.44
Network to be analyzed in Example 10.8.

b. Find the mathematical expressions for the voltage υ_C and the current i_C if the switch is moved to position 2 at $t = 10$ ms. (Assume that the leakage resistance of the capacitor is infinite ohms; that is, there is no leakage current.)
c. Find the mathematical expressions for the voltage υ_C and the current i_C if the switch is thrown into position 3 at $t = 20$ ms.
d. Plot the waveforms obtained in parts (a)–(c) on the same time axis using the defined polarities in Fig. 10.44.

Solutions:

a. *Charging phase:*

$$\tau = R_1C = (20\text{ k}\Omega)(0.05\ \mu\text{F}) = 1\text{ ms}$$
$$\upsilon_C = E(1 - e^{-t/\tau}) = \mathbf{12\ V(1 - e^{-t/1ms})}$$
$$i_C = \frac{E}{R_1}e^{-t/\tau} = \frac{12\text{ V}}{20\text{ k}\Omega}e^{-t/1\text{ms}} = \mathbf{0.6\ mA}e^{-t/1\text{ms}}$$

b. *Storage phase:* At 10 ms, a period of time equal to 10τ has passed, permitting the assumption that the capacitor is fully charged. Since $R_{leakage} = \infty\ \Omega$, the capacitor will hold its charge indefinitely. The result is that both υ_C and i_C will remain at a fixed value of

$$\upsilon_C = \mathbf{12\ V}$$
$$i_C = \mathbf{0\ A}$$

c. *Discharge phase* (using 20 ms as the new $t = 0$ s for the equations): The new time constant is

$$\tau' = RC = (R_1 + R_2)C = (20\text{ k}\Omega + 10\text{ k}\Omega)(0.05\ \mu\text{F}) = 1.5\text{ ms}$$
$$\upsilon_C = Ee^{-t/\tau'} = \mathbf{12\ V}e^{-t/1.5\text{ms}}$$
$$i_C = -\frac{E}{R}e^{-t/\tau'} = -\frac{E}{R_1 + R_2}e^{-t/\tau'}$$
$$= -\frac{12\text{ V}}{20\text{ k}\Omega + 10\text{ k}\Omega}e^{-t/1.5\text{ms}} = \mathbf{-0.4\ mA}e^{-t/1.5\text{ms}}$$

d. See Fig. 10.45.

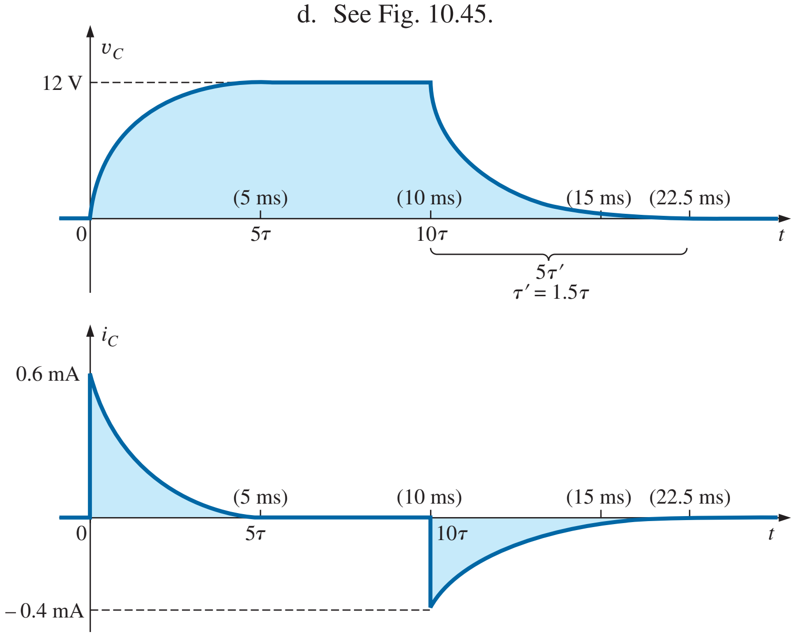


FIG. 10.45
υ_C and i_C for the network in Fig. 10.44.

EXAMPLE 10.9 For the network in Fig. 10.46:

a. Find the mathematical expression for the transient behavior of the voltage across the capacitor if the switch is thrown into position 1 at $t = 0$ s.
b. Find the mathematical expression for the transient behavior of the voltage across the capacitor if the switch is moved to position 2 at $t = 1\tau$.
c. Plot the resulting waveform for the voltage v_C as determined by parts (a) and (b).
d. Repeat parts (a)–(c) for the current i_C.

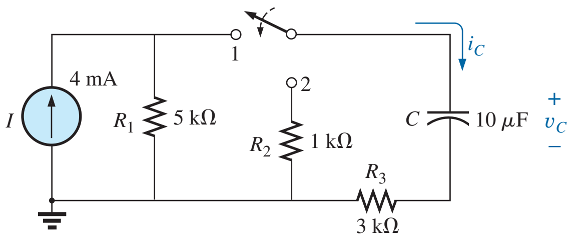


FIG. 10.46
Network to be analyzed in Example 10.9.

Solutions:

a. Converting the current source to a voltage source results in the configuration in Fig. 10.47 for the charging phase.

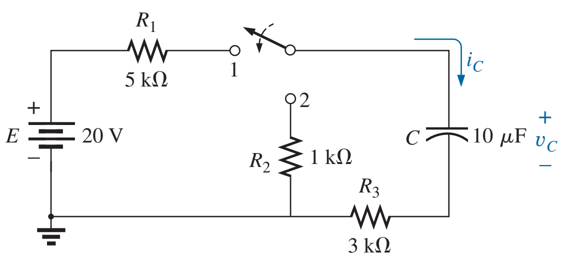


FIG. 10.47
The charging phase for the network in Fig. 10.46.

For the source conversion: $E = IR = (4\text{ mA})(5\text{ k}\Omega) = 20\text{ V}$

and $R_s = R_p = 5\text{ k}\Omega$

$$\tau = RC = (R_1 + R_3)C = (5\text{ k}\Omega + 3\text{ k}\Omega)(10\ \mu\text{F}) = 80\text{ ms}$$

$$v_C = E(1 - e^{-t/\tau}) = \mathbf{20\ V\ (1 - e^{-t/80ms})}$$

b. With the switch in position 2, the network appears as shown in Fig. 10.48. The voltage at 1τ can be found by using the fact that the voltage is 63.2% of its final value of 20 V, so that 0.632(20 V) = 12.64 V. Or you can substitute into the derived equation as follows:

$$e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} = 0.368$$

and
$$v_C = 20\text{ V}(1 - e^{-t/80\text{ms}}) = 20\text{ V}(1 - 0.368)$$
$$= (20\text{ V})(0.632) = 12.64\text{ V}$$

Using this voltage as the starting point and substituting into the discharge equation results in

$$\tau' = RC = (R_2 + R_3)C = (1\text{ k}\Omega + 3\text{ k}\Omega)(10\ \mu\text{F}) = 40\text{ ms}$$

$$v_C = Ee^{-t/\tau'} = \mathbf{12.64\ Ve^{-t/40ms}}$$

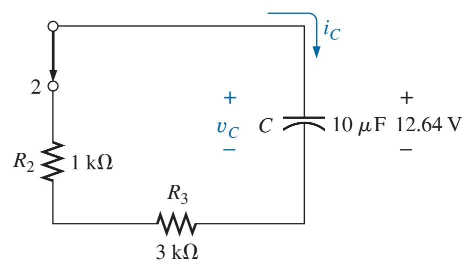


FIG. 10.48
Network in Fig. 10.47 when the switch is moved to position 2 at $t = 1\tau_1$.

c. See Fig. 10.49.

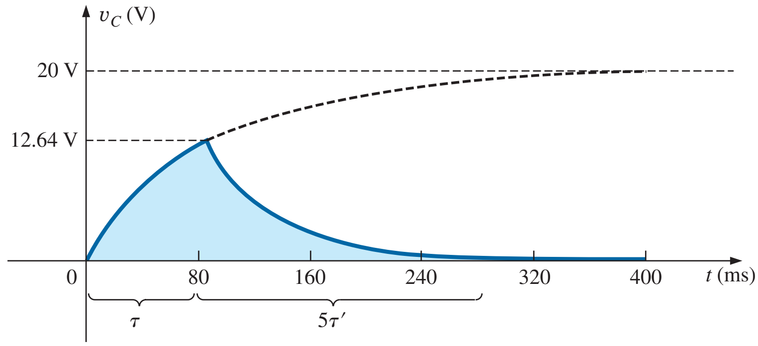


FIG. 10.49
v_C for the network in Fig. 10.47.

d. The charging equation for the current is

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{E}{R_1 + R_3}e^{-t/\tau} = \frac{20\text{ V}}{8\text{ k}\Omega}e^{-t/80\text{ms}} = \mathbf{2.5\text{ mA}e^{-t/80\text{ms}}}$$

which, at $t = 80$ ms, results in

$$i_C = 2.5\text{ mA}e^{-80\text{ms}/80\text{ms}} = 2.5\text{ mA}e^{-1} = (2.5\text{ mA})(0.368) = 0.92\text{ mA}$$

When the switch is moved to position 2, the 12.64 V across the capacitor appears across the resistor to establish a current of 12.64 V/4 kΩ = 3.16 mA. Substituting into the discharge equation with $V_i = 12.64$ V and $\tau' = 40$ ms yields

$$i_C = -\frac{V_i}{R_2 + R_3}e^{-t/\tau'} = -\frac{12.64\text{ V}}{1\text{ k}\Omega + 3\text{ k}\Omega}e^{-t/40\text{ms}}$$
$$= -\frac{12.64\text{ V}}{4\text{ k}\Omega}e^{-t/40\text{ms}} = \mathbf{-3.16\text{ mA}e^{-t/40\text{ms}}}$$

The equation has a minus sign because the direction of the discharge current is opposite to that defined for the current in Fig. 10.48. The resulting plot appears in Fig. 10.50.

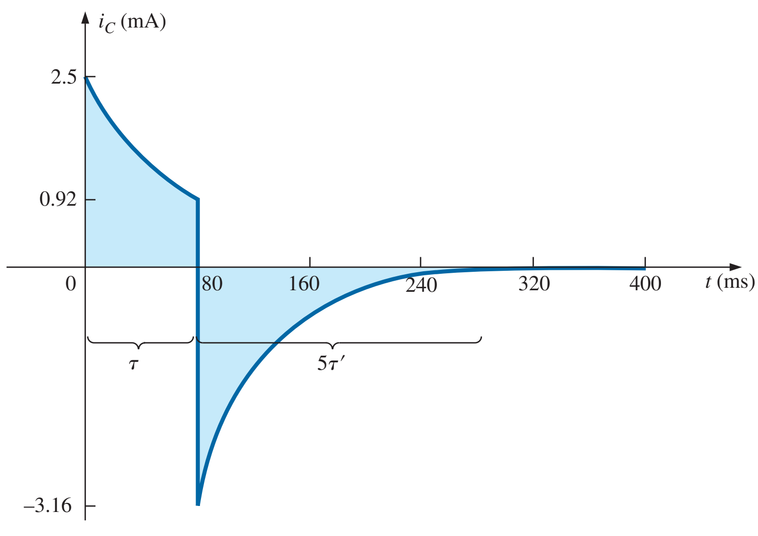


FIG. 10.50
i_c for the network in Fig. 10.47.

10.7 INITIAL CONDITIONS

In all the examples in the previous sections, the capacitor was uncharged before the switch was thrown. We now examine the effect of a charge, and therefore a voltage ($V = Q/C$), on the plates at the instant the switching action takes place. The voltage across the capacitor at this instant is called the **initial value,** as shown for the general waveform in Fig. 10.51.

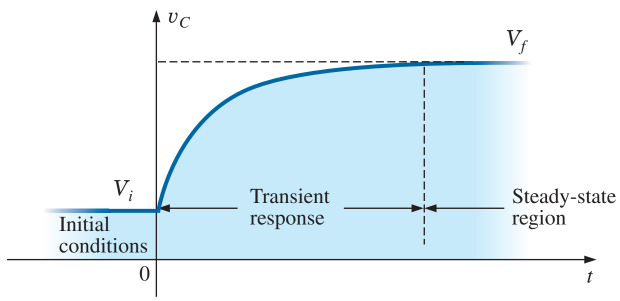


FIG. 10.51
Defining the regions associated with a transient response.

Once the switch is thrown, the transient phase commences until a leveling off occurs after five time constants. This region of relatively fixed value that follows the transient response is called the **steady-state region,** and the resulting value is called the **steady-state** or **final value.** The steady-state value is found by substituting the open-circuit equivalent for the capacitor and finding the voltage across the plates. Using the transient equation developed in the previous section, an equation for the voltage v_C can be written for the entire time interval in Fig. 10.51. That is, for the transient period, the voltage rises from V_i (previously 0 V) to a final value of V_f. Therefore,

$$v_C = E(1 - e^{-t/\tau}) = (V_f - V_i)(1 - e^{-t/\tau})$$

Adding the starting value of V_i to the equation results in

$$v_C = V_i + (V_f - V_i)(1 - e^{-t/\tau})$$

However, by multiplying through and rearranging terms:

$$\begin{aligned} v_C &= V_i + V_f - V_f e^{-t/\tau} - V_i + V_i e^{-t/\tau} \\ &= V_f - V_f e^{-t/\tau} + V_i e^{-t/\tau} \end{aligned}$$

we find

$$\boxed{v_C = V_f + (V_i - V_f)e^{-t/\tau}} \qquad \textbf{(10.21)}$$

Now that the equation has been developed, it is important to recognize that

Eq. (10.21) is a universal equation for the transient response of a capacitor.

That is, it can be used whether or not the capacitor has an initial value. If the initial value is 0 V as it was in all the previous examples, simply set V_i equal to zero in the equation, and the desired equation results. The final value is the voltage across the capacitor when the open-circuit equivalent is substituted.

EXAMPLE 10.10 The capacitor in Fig. 10.52 has an initial voltage of 4 V.

a. Find the mathematical expression for the voltage across the capacitor once the switch is closed.
b. Find the mathematical expression for the current during the transient period.
c. Sketch the waveform for each from initial value to final value.

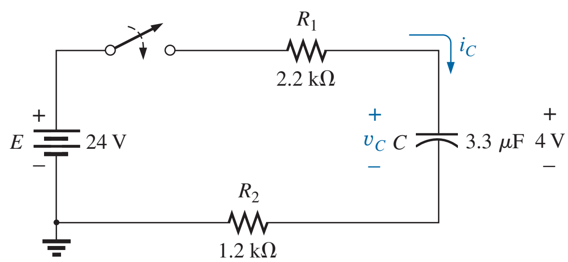


FIG. 10.52
Example 10.10.

Solutions:

a. Substituting the open-circuit equivalent for the capacitor results in a final or steady-state voltage v_C of 24 V.

   The time constant is determined by

$$\tau = (R_1 + R_2)C$$
$$= (2.2\text{ k}\Omega + 1.2\text{ k}\Omega)(3.3\ \mu\text{F}) = 11.22\text{ ms}$$

   with $\quad 5\tau = 56.1\text{ ms}$

   Applying Eq. (10.21):

$$v_C = V_f + (V_i - V_f)e^{-t/\tau} = 24\text{ V} + (4\text{ V} - 24\text{ V})e^{-t/11.22\text{ms}}$$

   and $v_C = \mathbf{24\ V - 20\ V}\boldsymbol{e^{-t/11.22\text{ms}}}$

b. Since the voltage across the capacitor is constant at 4 V prior to the closing of the switch, the current (whose level is sensitive only to changes in voltage across the capacitor) must have an initial value of 0 mA. At the instant the switch is closed, the voltage across the capacitor cannot change instantaneously, so the voltage across the resistive elements at this instant is the applied voltage less the initial voltage across the capacitor. The resulting peak current is

$$I_m = \frac{E - V_C}{R_1 + R_2} = \frac{24\text{ V} - 4\text{ V}}{2.2\text{ k}\Omega + 1.2\text{ k}\Omega} = \frac{20\text{ V}}{3.4\text{ k}\Omega} = 5.88\text{ mA}$$

   The current then decays (with the same time constant as the voltage v_C) to zero because the capacitor is approaching its open-circuit equivalence.

   The equation for i_C is therefore:

$$i_C = \mathbf{5.88\ mA}\boldsymbol{e^{-t/11.22\text{ms}}}$$

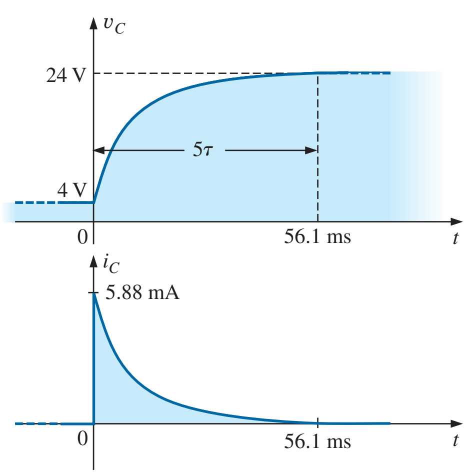


FIG. 10.53
v_C and i_C for the network in Fig. 10.52.

c. See Fig. 10.53. The initial and final values of the voltage were drawn first, and then the transient response was included between these levels. For the current, the waveform begins and ends at zero, with the peak value having a sign sensitive to the defined direction of i_C in Fig. 10.52.

Let us now test the validity of the equation for v_C by substituting $t = 0$ s to reflect the instant the switch is closed.

$$e^{-t/\tau} = e^{-0} = 1$$

and $$v_C = 24\text{ V} - 20\text{ V}e^{-t/\tau} = 24\text{ V} - 20\text{ V} = 4\text{ V}$$

When $t > 5\tau$,

$$e^{-t/\tau} \cong 0$$

and $$v_C = 24\text{ V} - 20\text{ V}e^{-t/\tau} = 24\text{ V} - 0\text{ V} = 24\text{ V}$$

Eq. (10.21) can also be applied to the discharge phase by applying the correct levels of V_i and V_f.

For the discharge pattern in Fig. 10.54, $V_f = 0$ V, and Eq. (10.21) becomes

$$v_C = V_f + (V_i - V_f)e^{-t/\tau} = 0\text{ V} + (V_i - 0\text{ V})e^{-t/\tau}$$

and $$\boxed{v_C = V_i e^{-t/\tau}}_{\text{discharging}} \qquad \textbf{(10.22)}$$

Substituting $V_i = E$ volts results in Eq. (10.17).

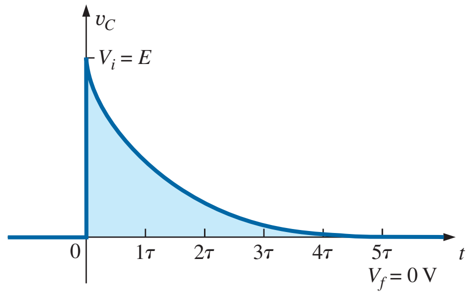


FIG. 10.54
Defining the parameters in Eq. (10.21) for the discharge phase.

10.8 INSTANTANEOUS VALUES

Occasionally, you may need to determine the voltage or current at a particular instant of time that is not an integral multiple of τ, as in the previous sections. For example, if

$$v_C = 20\text{ V}(1 - e^{(-t/2\text{ms})}$$

the voltage v_C may be required at $t = 5$ ms, which does not correspond to a particular value of τ. Fig. 10.28 reveals that $(1 - e^{-t/\tau})$ is approximately 0.93 at $t = 5\text{ ms} = 2.5\tau$, resulting in $v_C = 20(0.93) = 18.6$ V. Additional accuracy can be obtained by substituting $v = 5$ ms into the equation and solving for v_C using a calculator or table to determine $e^{-2.5}$. Thus,

$$\begin{aligned} v_C &= 20\text{ V}(1 - e^{-5\text{ms}/2\text{ms}}) = (20\text{ V})(1 - e^{-2.5}) = (20\text{ V})(1 - 0.082) \\ &= (20\text{ V})(0.918) = \mathbf{18.36\text{ V}} \end{aligned}$$

The results are close, but accuracy beyond the tenths place is suspect using Fig. 10.29. The above procedure can also be applied to any other equation introduced in this chapter for currents or other voltages.

Occasionally, you may need to determine the time required to reach a particular voltage or current. The procedure is complicated somewhat by the use of natural logs ($\log_e$, or ln), but today's calculators are equipped to handle the operation with ease.

For example, solving for t in the equation

$$v_C = V_f + (V_i - V_f)\, e^{-t/\tau}$$

results in

$$\boxed{t = \tau(\log_e)\frac{(V_i - V_f)}{(v_C - V_f)}} \qquad \textbf{(10.23)}$$

For example, suppose that

$$v_C = 20\text{ V}(1 - e^{-t/2\text{ms}})$$

and the time t to reach 10 V is desired. Since $V_i = 0$ V, and $V_f = 20$ V, we have

$$t = \tau(\log_e)\frac{(V_i - V_f)}{(v_C - V_f)} = (2\text{ ms})(\log_e)\frac{(0\text{ V} - 20\text{ V})}{(10\text{ V} - 20\text{ V})}$$

$$= (2\text{ ms})\left[\log_e\left(\frac{-20\text{ V}}{-10\text{ V}}\right)\right] = (2\text{ ms})(\log_e 2) = (2\text{ ms})(0.693)$$

$$= \mathbf{1.386\text{ ms}}$$

The TI-89 calculator key strokes appear in Fig. 10.55.

2 EE (–) 3 × 2ND LN 2) ENTER **1.39E-3**

FIG. 10.55

Key strokes to determine (2 ms)($\log_e 2$) using the TI-89 calculator.

For the discharge equation,

$$v_C = Ee^{-t/\tau} = V_i\,(e^{-t/\tau}) \quad \text{with } V_f = 0\text{ V}$$

Using Eq. (10.23):

$$t = \tau(\log_e)\frac{(V_i - V_f)}{(v_C - V_f)} = \tau(\log_e)\frac{(V_i - 0\text{ V})}{(v_C - 0\text{ V})}$$

and

$$t = \tau \log_e \frac{V_i}{v_C} \qquad \textbf{(10.24)}$$

For the current equation,

$$i_C = \frac{E}{R}e^{-t/\tau} \qquad I_i = \frac{E}{R} \qquad I_f = 0\text{ A}$$

and

$$t = \log_e \frac{I_i}{i_C} \qquad \textbf{(10.25)}$$

Using Mathcad to Perform Transient Analysis

Mathcad will now be applied to Eq. (10.13), the equation for the voltage across a capacitor as it changes to the supply voltage of the series circuit. The value of t must be defined before the expression is written, or the value can simply be inserted in the equation. The former approach is often the better choice, because changing the defined value of t results in an immediate change in the result. In other words, the value can be used for further calculations. In Fig. 10.56, the value of t was defined as 5 ms. The equation was then entered using the e function from the **Calculator** palette obtained from **View-Toolbars-Caculator.** Be sure to insert a multiplication operator between the initial 20 and the main left bracket. Also, be careful that the control bracket is in the correct place before placing the right bracket to enclose the equation. It takes some practice to ensure

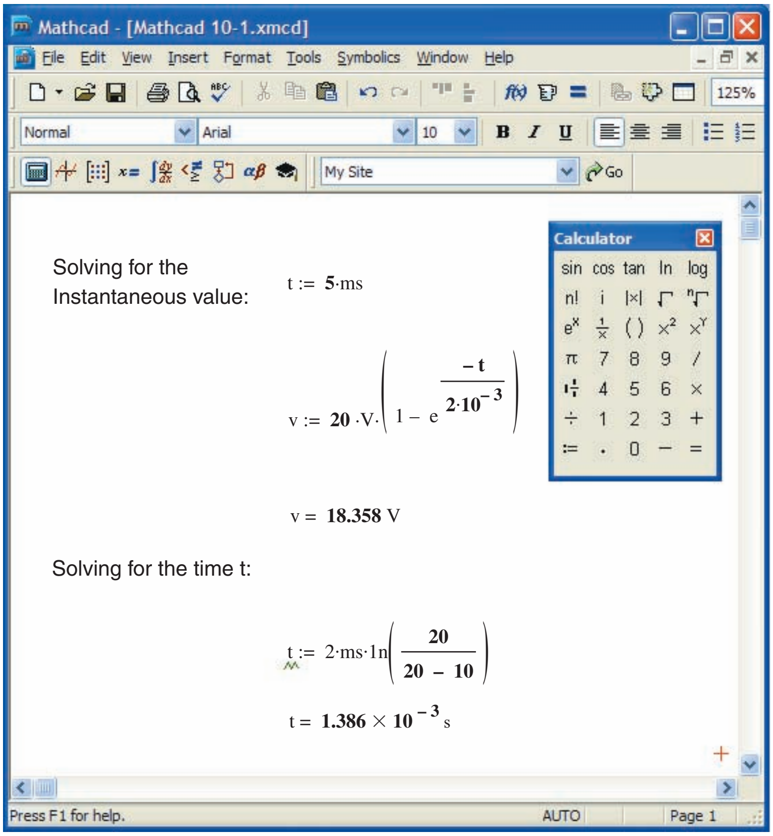


FIG. 10.56

Applying Mathcad to the transient R-C equations.

that the insertion bracket is in the correct place before entering a parameter, but in time you will find that it is a fairly direct procedure. The **−3** is placed using the shift operator over the number **6** on the standard keyboard. The result is displayed by entering v again, followed by an equals sign. The result for $t = 1$ ms can now be obtained by changing the defined value for t. The result of 7.869 V appears immediately.

For the example above, the equation for t can be entered directly as shown in the bottom of Fig. 10.56. The **ln** from the **Calculator** is for a base e calculation, whereas **log** is for a base 10 calculation. The result appears the instant the equals sign is placed after the t on the bottom line.

The text you see on the screen to define each operation is obtained from **Insert-Text Region.** Then simply type in the text material. Change to boldface by clicking on the text material and swiping the text to establish a black background. Then select **B** from the toolbar, and the boldface appears.

10.9 THÉVENIN EQUIVALENT: $\tau = R_{Th}C$

You may encounter instances in which the network does not have the simple series form in Fig. 10.26. You then need to find the Thévenin equivalent circuit for the network external to the capacitive element. E_{Th} will be the source voltage E in Eqs. (10.13) through (10.25), and R_{Th} will be the resistance R. The time constant is then $\tau = R_{Th}C$.

EXAMPLE 10.11 For the network in Fig. 10.57:

a. Find the mathematical expression for the transient behavior of the voltage v_C and the current i_C following the closing of the switch (position 1 at $t = 0$ s).
b. Find the mathematical expression for the voltage v_C and the current i_C as a function of time if the switch is thrown into position 2 at $t = 9$ ms.
c. Draw the resultant waveforms of parts (a) and (b) on the same time axis.

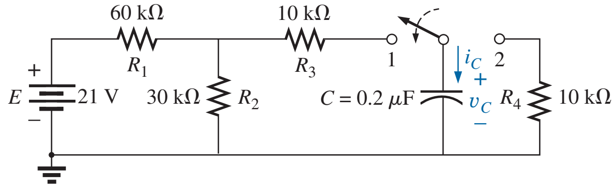


FIG. 10.57
Example 10.11.

Solutions:

a. Applying Thévenin's theorem to the 0.2 μF capacitor, we obtain Fig. 10.58.

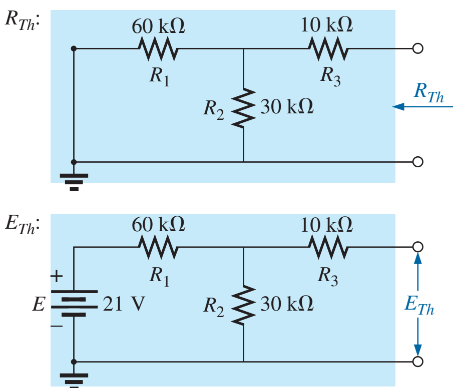


FIG. 10.58
Applying Thévenin's theorem to the network in Fig. 10.57.

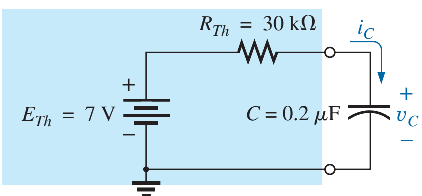


FIG. 10.59
Substituting the Thévenin equivalent for the network in Fig. 10.57.

$$R_{Th} = R_1 \parallel R_2 + R_3 = \frac{(60\text{ k}\Omega)(30\text{ k}\Omega)}{90\text{ k}\Omega} + 10\text{ k}\Omega$$
$$= 20\text{ k}\Omega + 10\text{ k}\Omega = 30\text{ k}\Omega$$
$$E_{Th} = \frac{R_2 E}{R_2 + R_1} = \frac{(30\text{ k}\Omega)(21\text{ V})}{30\text{ k}\Omega + 60\text{ k}\Omega} = \frac{1}{3}(21\text{ V}) = 7\text{ V}$$

The resultant Thévenin equivalent circuit with the capacitor replaced is shown in Fig. 10.59.

Using Eq. (10.21) with $V_f = E_{Th}$ and $V_i = 0$ V, we find that

$$v_C = V_f + (V_i - V_f)e^{-t/\tau}$$

becomes $$v_C = E_{Th} + (0\text{ V} - E_{Th})e^{-t/\tau}$$

or $$v_C = E_{Th}(1 - e^{-t/\tau})$$

with $$\tau = RC = (30\text{ k}\Omega)(0.2\ \mu\text{F}) = 6\text{ ms}$$

Therefore, $$v_C = \mathbf{7\ V(1 - e^{-t/6ms})}$$

For the current i_C:

$$i_C = \frac{E_{Th}}{R}e^{-t/RC} = \frac{7\text{ V}}{30\text{ k}\Omega}e^{-t/6\text{ms}}$$
$$= \mathbf{0.23\ mAe^{-t/6ms}}$$

b. At $t = 9$ ms,

$$v_C = E_{Th}(1 - e^{-t/\tau}) = 7\text{ V}(1 - e^{-(9\text{ms}/6\text{ms})})$$
$$= (7\text{ V})(1 - e^{-1.5}) = (7\text{ V})(1 - 0.223)$$
$$= (7\text{ V})(0.777) = 5.44\text{ V}$$

and $$i_C = \frac{E_{Th}}{R}e^{-t/\tau} = 0.23\text{ mA}e^{-1.5}$$
$$= (0.23 \times 10^{-3})(0.233) = 0.052 \times 10^{-3} = 0.05\text{ mA}$$

Using Eq. (10.21) with $V_f = 0$ V and $V_i = 5.44$ V, we find that

$$\upsilon_C = V_f + (V_i - V_f)e^{-t/\tau'}$$

becomes $\upsilon_C = 0\text{ V} + (5.44\text{ V} - 0\text{ V})e^{-t/\tau'}$
$= 5.44\text{ V}e^{-t/\tau'}$

with $\tau' = R_4C = (10\text{ k}\Omega)(0.2\ \mu\text{F}) = 2\text{ ms}$

and $\upsilon_C = \mathbf{5.44\ Ve^{-t/2ms}}$

By Eq. (10.19):

$$I_i = \frac{5.44\text{ V}}{10\text{ k}\Omega} = 0.54\text{ mA}$$

and $i_C = I_ie^{-t/\tau} = \mathbf{-0.54\ mAe^{-t/2ms}}$

c. See Fig. 10.60.

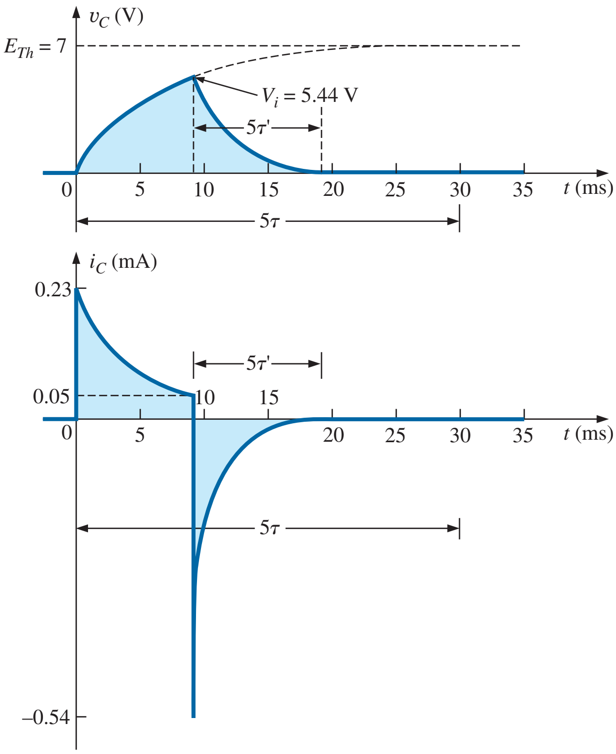


FIG. 10.60
The resulting waveforms for the network in Fig. 10.57.

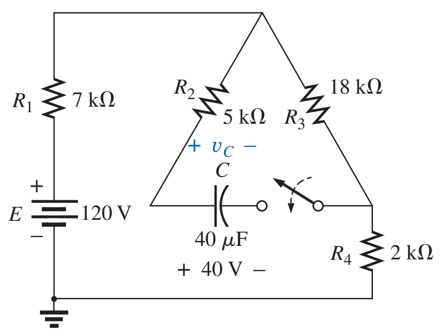


FIG. 10.61
Example 10.12.

EXAMPLE 10.12 The capacitor in Fig. 10.61 is initially charged to 40 V. Find the mathematical expression for υ_C after the closing of the switch. Plot the waveform for υ_C.

Solution: The network is redrawn in Fig. 10.62.

E_{Th}:

$$E_{Th} = \frac{R_3E}{R_3 + R_1 + R_4} = \frac{(18\text{ k}\Omega)(120\text{ V})}{18\text{ k}\Omega + 7\text{ k}\Omega + 2\text{ k}\Omega} = 80\text{ V}$$

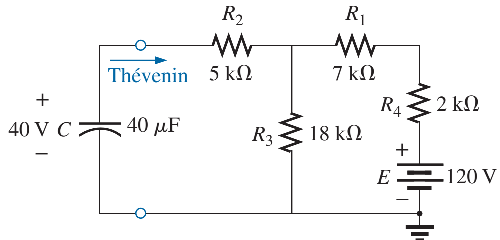


FIG. 10.62
Network in Fig. 10.61 redrawn.

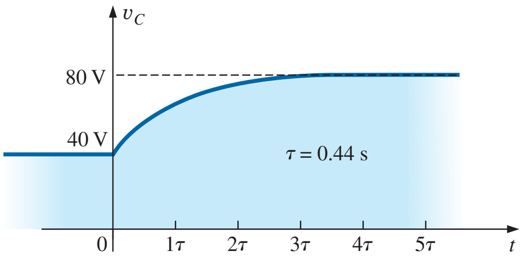


FIG. 10.63
v_C for the network in Fig. 10.61.

R_{Th}:

$$R_{Th} = 5\text{ k}\Omega + (18\text{ k}\Omega) \parallel (7\text{ k}\Omega + 2\text{ k}\Omega)$$
$$= 5\text{ k}\Omega + 6\text{ k}\Omega = 11\text{ k}\Omega$$

Therefore, $V_i = 40\text{ V}$ and $V_f = 80\text{ V}$

and $\tau = R_{Th}C = (11\text{ k}\Omega)(40\ \mu\text{F}) = 0.44\text{ s}$

Eq. (10.21): $v_C = V_f + (V_i - V_f)e^{-t/\tau}$

$$= 80\text{ V} + (40\text{ V} - 80\text{ V})e^{-t/0.44\text{s}}$$

and $\mathbf{v_C = 80\ V - 40\ Ve^{-t/0.44s}}$

The waveform appears as in Fig. 10.63.

EXAMPLE 10.13 For the network in Fig. 10.64, find the mathematical expression for the voltage v_C after the closing of the switch (at $t = 0$).

Solution:

$$R_{Th} = R_1 + R_2 = 6\ \Omega + 10\ \Omega = 16\ \Omega$$
$$E_{Th} = V_1 + V_2 = IR_1 + 0$$
$$= (20 \times 10^{-3}\text{ A})(6\ \Omega) = 120 \times 10^{-3}\text{ V} = 0.12\text{ V}$$

and $\tau = R_{Th}C = (16\ \Omega)(500 \times 10^{-6}\text{ F}) = 8\text{ ms}$

so that $\mathbf{v_C = 0.12\ V(1 - e^{-t/8ms})}$

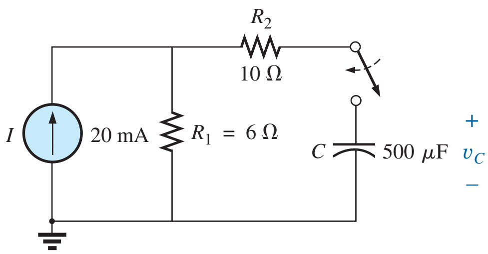


FIG. 10.64
Example 10.13.

10.10 THE CURRENT i_C

There is a very special relationship between the current of a capacitor and the voltage across it. For the resistor, it is defined by Ohm's law: $i_R = v_R/R$. The current through and the voltage across the resistor are related by a constant R—a very simple direct linear relationship. For the capacitor, it is the more complex relationship defined by

$$i_C = C\frac{dv_C}{dt} \tag{10.26}$$

The factor C reveals that the higher the capacitance, the greater the resulting current. Intuitively, this relationship makes sense, because higher capacitance levels result in increased levels of stored charge, providing a source for increased current levels. The second term, dv_C/dt, is sensitive to the *rate of change* of v_C with time. The function dv_C/dt is called the **derivative** (calculus) of the voltage v_C with respect to time t. The faster the voltage v_C changes with time, the larger the factor dv_C/dt will be and the larger the resulting current i_C will be. That is why the current jumps to its maximum of E/R in a charging circuit the instant the switch is closed. At that instant, if you look at the charging curve for v_C, the voltage is *changing* at its greatest rate. As it approaches its final value, the rate of change decreases, and, as confirmed by Eq. (10.26), the level of current decreases.

Take special note of the following:

The capacitive current is directly related to the rate of change of the voltage across the capacitor, not the levels of voltage involved.

For example, the current of a capacitor will be *greater* when the voltage changes from 1 V to 10 V in 1 ms than when it changes from 10 V to 100 V in 1 s; in fact, it will be 100 times more.

If the voltage fails to change over time, then

$$\frac{dv_C}{dt} = 0$$

and
$$i_C = C\frac{dv_C}{dt} = C(0) = 0 \text{ A}$$

In an effort to develop a clearer understanding of Eq. (10.26), let us calculate the **average current** associated with a capacitor for various voltages impressed across the capacitor. The average current is defined by the equation

$$i_{C_{av}} = C\frac{\Delta v_C}{\Delta t} \quad \textbf{(10.27)}$$

where Δ indicates a finite (measurable) change in voltage or time. The instantaneous current can be derived from Eq. (10.27) by letting Δt become vanishingly small; that is,

$$i_{C_{inst}} = \lim_{\Delta t \to 0} C\frac{\Delta v_C}{\Delta t} = C\frac{dv_C}{dt}$$

In the following example, the change in voltage Δv_C will be considered for each slope of the voltage waveform. If the voltage increases with time, the average current is the change in voltage divided by the change in time, with a positive sign. If the voltage decreases with time, the average current is again the change in voltage divided by the change in time, but with a negative sign.

EXAMPLE 10.14 Find the waveform for the average current if the voltage across a 2 μF capacitor is as shown in Fig. 10.65.

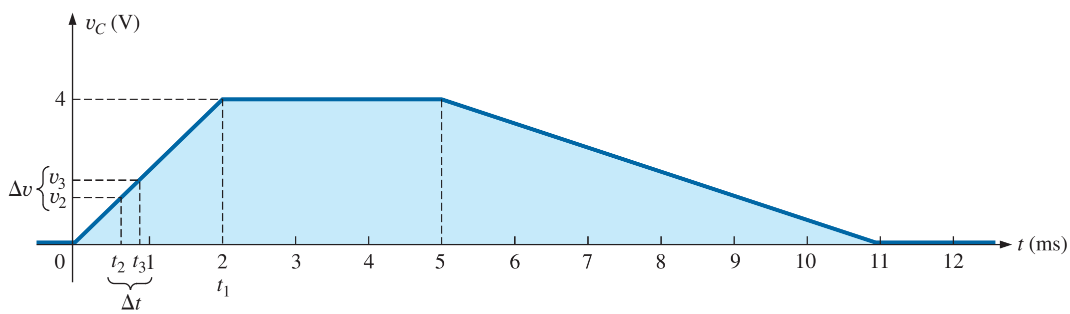


FIG. 10.65
v_C for Example 10.14.

Solutions:

a. From 0 ms to 2 ms, the voltage increases linearly from 0 V to 4 V; the change in voltage $\Delta v = 4 \text{ V} - 0 = 4 \text{ V}$ (with a positive sign since the voltage increases with time). The change in time $\Delta t = 2 \text{ ms} - 0 = 2 \text{ ms}$, and

$$i_{C_{av}} = C\frac{\Delta v_C}{\Delta t} = (2 \times 10^{-6} \text{ F})\left(\frac{4 \text{ V}}{2 \times 10^{-3} \text{ s}}\right)$$
$$= 4 \times 10^{-3} \text{ A} = \mathbf{4 \text{ mA}}$$

b. From 2 ms to 5 ms, the voltage remains constant at 4 V; the change in voltage $\Delta v = 0$. The change in time $\Delta t = 3$ ms, and

$$i_{C_{av}} = C\frac{\Delta v_C}{\Delta t} = C\frac{0}{\Delta t} = 0\ \mathbf{mA}$$

c. From 5 ms to 11 ms, the voltage decreases from 4 V to 0 V. The change in voltage Δv is, therefore, 4 V − 0 = 4 V (with a negative sign since the voltage is decreasing with time). The change in time $\Delta t = 11\text{ ms} - 5\text{ ms} = 6\text{ ms}$, and

$$i_{C_{av}} = C\frac{\Delta v_C}{\Delta t} = -(2 \times 10^{-6}\text{ F})\left(\frac{4\text{ V}}{6 \times 10^{-3}\text{ s}}\right)$$
$$= -1.33 \times 10^{-3}\text{ A} = \mathbf{-1.33\ mA}$$

d. From 11 ms on, the voltage remains constant at 0 and $\Delta v = 0$, so $i_{C_{av}} = 0$ mA. The waveform for the average current for the impressed voltage is as shown in Fig. 10.66.

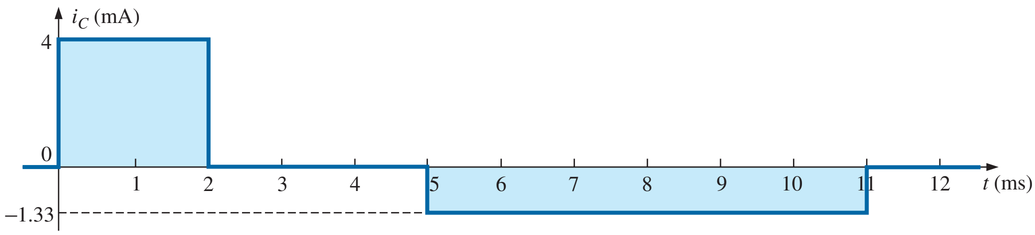


FIG. 10.66
The resulting current i_C for the applied voltage in Fig. 10.65.

Note in Example 10.14 that, in general, the steeper the slope, the greater the current, and when the voltage fails to change, the current is zero. In addition, the average value is the same as the instantaneous value at any point along the slope over which the average value was found. For example, if the interval Δt is reduced from $0 \rightarrow t_1$ to $t_2 - t_3$, as noted in Fig. 10.65, $\Delta v/\Delta t$ is still the same. In fact, no matter how small the interval Δt, the slope will be the same, and therefore the current $i_{C_{av}}$ will be the same. If we consider the limit as $\Delta t \rightarrow 0$, the slope will still remain the same, and therefore $i_{C_{av}} = i_{C_{inst}}$ at any instant of time between 0 and t_1. The same can be said about any portion of the voltage waveform that has a constant slope.

An important point to be gained from this discussion is that it is not the magnitude of the voltage across a capacitor that determines the current but rather how quickly the voltage *changes* across the capacitor. An applied steady dc voltage of 10,000 V would (ideally) not create any flow of charge (current), but a change in voltage of 1 V in a very brief period of time could create a significant current.

The method described above is only for waveforms with straight-line (linear) segments. For nonlinear (curved) waveforms, a method of calculus (differentiation) must be used.

10.11 CAPACITORS IN SERIES AND IN PARALLEL

Capacitors, like resistors, can be placed in series and in parallel. Increasing levels of capacitance can be obtained by placing capacitors in parallel, while decreasing levels can be obtained by placing capacitors in series.

For capacitors in series, the charge is the same on each capacitor (Fig. 10.67):

$$Q_T = Q_1 = Q_2 = Q_3 \tag{10.28}$$

Applying Kirchhoff's voltage law around the closed loop gives

$$E = V_1 + V_2 + V_3$$

However, $$V = \frac{Q}{C}$$

so that $$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

Using Eq. (10.28) and dividing both sides by Q yields

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \tag{10.29}$$

which is similar to the manner in which we found the total resistance of a parallel resistive circuit. The total capacitance of two capacitors in series is

$$C_T = \frac{C_1 C_2}{C_1 + C_2} \tag{10.30}$$

The voltage across each capacitor in Fig. 10.67 can be found by first recognizing that

$$Q_T = Q_1$$

or $$C_T E = C_1 V_1$$

Solving for V_1: $$V_1 = \frac{C_T E}{C_1}$$

and substituting for C_T:

$$V_1 = \left(\frac{1/C_1}{1/C_1 + 1/C_2 + 1/C_3}\right)E \tag{10.31}$$

A similar equation results for each capacitor of the network.

For capacitors in parallel, as shown in Fig. 10.68, the voltage is the same across each capacitor, and the total charge is the sum of that on each capacitor:

$$Q_T = Q_1 + Q_2 + Q_3 \tag{10.32}$$

However, $$Q = CV$$

Therefore, $$C_T E = C_1 V_1 = C_2 V_2 = C_3 V_3$$

but $$E = V_1 = V_2 = V_3$$

Thus, $$C_T = C_1 + C_2 + C_3 \tag{10.33}$$

which is similar to the manner in which the total resistance of a series circuit is found.

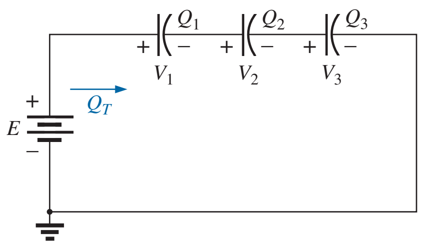


FIG. 10.67
Series capacitors.

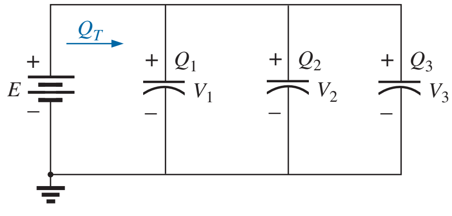


FIG. 10.68
Parallel capacitors.

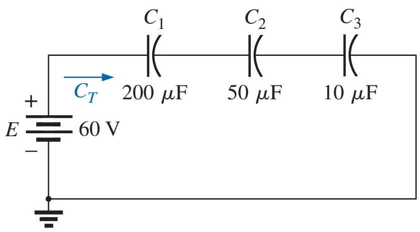


FIG. 10.69
Example 10.15.

EXAMPLE 10.15 For the circuit in Fig. 10.69:

a. Find the total capacitance.
b. Determine the charge on each plate.
c. Find the voltage across each capacitor.

Solutions:

a. $$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{200 \times 10^{-6}\,\text{F}} + \frac{1}{50 \times 10^{-6}\,\text{F}} + \frac{1}{10 \times 10^{-6}\,\text{F}}$$
$$= 0.005 \times 10^6 + 0.02 \times 10^6 + 0.1 \times 10^6$$
$$= 0.125 \times 10^6$$

and $$C_T = \frac{1}{0.125 \times 10^6} = \mathbf{8\,\mu F}$$

b. $$Q_T = Q_1 = Q_2 = Q_3$$
$$= C_T E = (8 \times 10^{-6}\,\text{F})\,(60\,\text{V}) = \mathbf{480\,\mu C}$$

c. $$V_1 = \frac{Q_1}{C_1} = \frac{480 \times 10^{-6}\,\text{C}}{200 \times 10^{-6}\,\text{F}} = \mathbf{2.4\,V}$$
$$V_2 = \frac{Q_2}{C_2} = \frac{480 \times 10^{-6}\,\text{C}}{50 \times 10^{-6}\,\text{F}} = \mathbf{9.6\,V}$$
$$V_3 = \frac{Q_3}{C_3} = \frac{480 \times 10^{-6}\,\text{C}}{10 \times 10^{-6}\,\text{F}} = \mathbf{48.0\,V}$$

and $E = V_1 + V_2 + V_3 = 2.4\,\text{V} + 9.6\,\text{V} + 48\,\text{V} = \mathbf{60\,V}$ (checks)

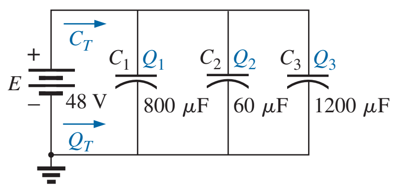


FIG. 10.70
Example 10.16.

EXAMPLE 10.16 For the network in Fig. 10.70:

a. Find the total capacitance.
b. Determine the charge on each plate.
c. Find the total charge.

Solutions:

a. $C_T = C_1 + C_2 + C_3 = 800\,\mu\text{F} + 60\,\mu\text{F} + 1200\,\mu\text{F} = \mathbf{2060\,\mu F}$
b. $Q_1 = C_1 E = (800 \times 10^{-6}\,\text{F})(48\,\text{V}) = \mathbf{38.4\,mC}$
$Q_2 = C_2 E = (60 \times 10^{-6}\,\text{F})(48\,\text{V}) = \mathbf{2.88\,mC}$
$Q_3 = C_3 E = (1200 \times 10^{-6}\,\text{F})(48\,\text{V}) = \mathbf{57.6\,mC}$
c. $Q_T = Q_1 + Q_2 + Q_3 = 38.4\,\text{mC} + 2.88\,\text{mC} + 57.6\,\text{mC} = \mathbf{98.88\,mC}$

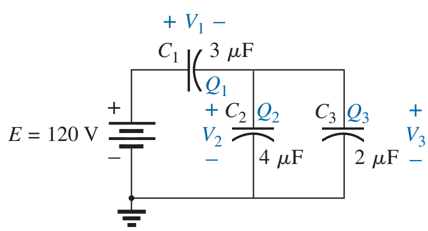


FIG. 10.71
Example 10.17.

EXAMPLE 10.17 Find the voltage across and the charge on each capacitor for the network in Fig. 10.71.

Solution:

$$C'_T = C_2 + C_3 = 4\,\mu\text{F} + 2\,\mu\text{F} = 6\,\mu\text{F}$$
$$C_T = \frac{C_1 C'_T}{C_1 + C'_T} = \frac{(3\,\mu\text{F})(6\,\mu\text{F})}{3\,\mu\text{F} + 6\,\mu\text{F}} = 2\,\mu\text{F}$$
$$Q_T = C_T E = (2 \times 10^{-6}\,\text{F})(120\,\text{V}) = \mathbf{240\,\mu C}$$

An equivalent circuit (Fig. 10.72) has

$$Q_T = Q_1 = Q'_T$$

and, therefore, $$Q_1 = \mathbf{240\ \mu C}$$

and $$V_1 = \frac{Q_1}{C_1} = \frac{240 \times 10^{-6}\ \text{C}}{3 \times 10^{-6}\ \text{F}} = \mathbf{80\ V}$$

$$Q'_T = 240\ \mu\text{C}$$

Therefore, $$V'_T = \frac{Q'_T}{C'_T} = \frac{240 \times 10^{-6}\ \text{C}}{6 \times 10^{-6}\ \text{F}} = \mathbf{40\ V}$$

and $$Q_2 = C_2 V'_T = (4 \times 10^{-6}\ \text{F})(40\ \text{V}) = \mathbf{160\ \mu C}$$

$$Q_3 = C_3 V'_T = (2 \times 10^{-6}\ \text{F})(40\ \text{V}) = \mathbf{80\ \mu C}$$

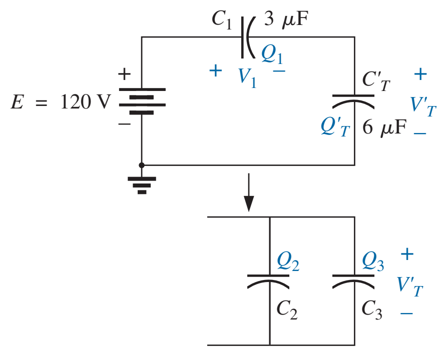


FIG. 10.72
Reduced equivalent for the network in Fig. 10.71.

EXAMPLE 10.18 Find the voltage across and the charge on capacitor C_1 in Fig. 10.73 after it has charged up to its final value.

Solution: As previously discussed, the capacitor is effectively an open circuit for dc after charging up to its final value (Fig. 10.74).

Therefore,

$$V_C = \frac{(8\ \Omega)(24\ \text{V})}{4\ \Omega + 8\ \Omega} = \mathbf{16\ V}$$

$$Q_1 = C_1 V_C = (20 \times 10^{-6}\ \text{F})(16\ \text{V}) = \mathbf{320\ \mu C}$$

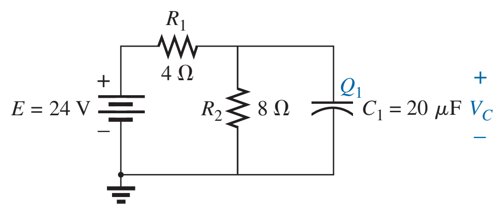


FIG. 10.73
Example 10.18.

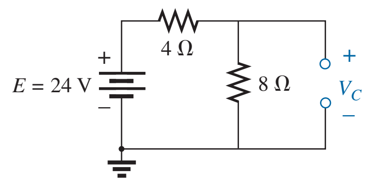


FIG. 10.74
Determining the final (steady-state) value for v_C.

EXAMPLE 10.19 Find the voltage across and the charge on each capacitor of the network in Fig. 10.75(a) after each has charged up to its final value.

Solution: See Fig. 10.75(b).

$$V_{C_2} = \frac{(7\ \Omega)(72\ \text{V})}{7\ \Omega + 2\ \Omega} = \mathbf{56\ V}$$

$$V_{C_1} = \frac{(2\ \Omega)(72\ \text{V})}{2\ \Omega + 7\ \Omega} = \mathbf{16\ V}$$

$$Q_1 = C_1 V_{C_1} = (2 \times 10^{-6}\ \text{F})(16\ \text{V}) = \mathbf{32\ \mu C}$$

$$Q_2 = C_2 V_{C_2} = (3 \times 10^{-6}\ \text{F})(56\ \text{V}) = \mathbf{168\ \mu C}$$

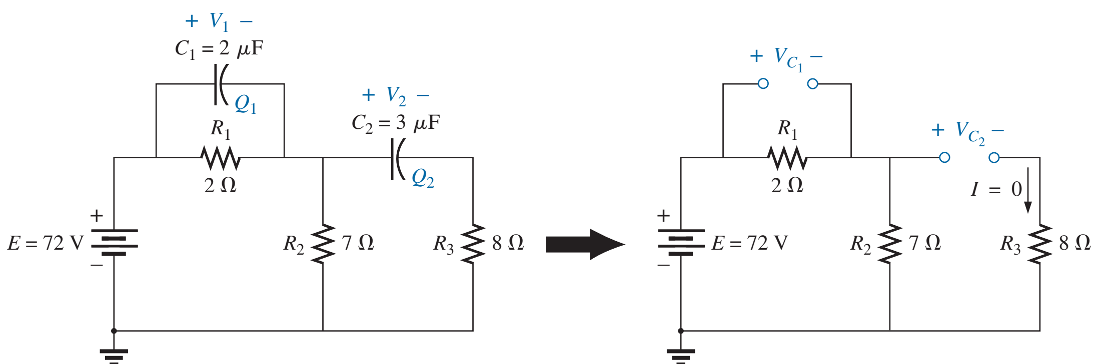


FIG. 10.75
Example 10.19.

10.12 ENERGY STORED BY A CAPACITOR

An ideal capacitor does not dissipate any of the energy supplied to it. It stores the energy in the form of an electric field between the conducting surfaces. A plot of the voltage, current, and power to a capacitor during the charging phase is shown in Fig. 10.76. The power curve can be obtained by finding the product of the voltage and current at selected instants of time and connecting the points obtained. *The energy stored is represented by the shaded area under the power curve.* Using calculus, we can determine the area under the curve:

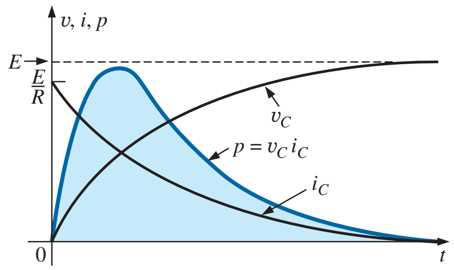


FIG. 10.76
Plotting the power to a capacitive element during the transient phase.

$$W_C = \frac{1}{2}CE^2$$

In general,

$$W_C = \frac{1}{2}CV^2 \quad \text{(J)} \qquad \textbf{(10.34)}$$

where V is the steady-state voltage across the capacitor. In terms of Q and C,

$$W_C = \frac{1}{2}C\left(\frac{Q}{C}\right)^2$$

or

$$W_C = \frac{Q^2}{2C} \quad \text{(J)} \qquad \textbf{(10.35)}$$

EXAMPLE 10.20 For the network in Fig. 10.75(a), determine the energy stored by each capacitor.

Solution: For C_1:

$$W_C = \frac{1}{2}CV^2$$
$$= \frac{1}{2}(2 \times 10^{-6}\,\text{F})(16\,\text{V})^2 = (1 \times 10^{-6})(256) = \mathbf{256\,\mu J}$$

For C_2:

$$W_C = \frac{1}{2}CV^2$$
$$= \frac{1}{2}(3 \times 10^{-6}\,\text{F})(56\,\text{V})^2 = (1.5 \times 10^{-6})(3136) = \mathbf{4704\,\mu J}$$

Due to the squared term, the energy stored increases rapidly with increasing voltages.

10.13 STRAY CAPACITANCES

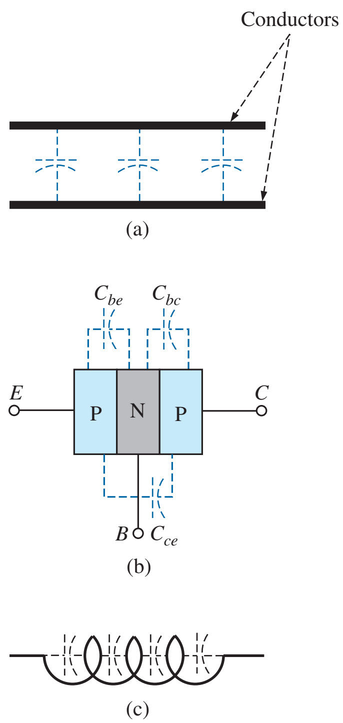


FIG. 10.77
Examples of stray capacitance.

In addition to the capacitors discussed so far in this chapter, there are **stray capacitances** that exist not through design but simply because two conducting surfaces are relatively close to each other. Two conducting wires in the same network have a capacitive effect between them, as shown in Fig. 10.77(a). In electronic circuits, capacitance levels exist between conducting surfaces of the transistor, as shown in Fig. 10.77(b). In Chapter 11, we will discuss another element called the *inductor,* which has capacitive effects between the windings [Fig. 10.77(c)]. Stray capacitances can often lead to serious errors in system design if they are not considered carefully.

10.14 APPLICATIONS

This section includes both a description of the operation of one of the less expensive, throwaway cameras that have become so popular today and a discussion of the use of capacitors in the line conditioners (surge protectors) that are used in many homes and throughout the business world. Additional examples of the use of capacitors appear in Chapter 11.

Flash Lamp

The basic circuitry for the flash lamp of the popular, inexpensive, throwaway camera in Fig. 10.78(a) is provided in Fig. 10.78(b). The physical circuitry is in Fig. 10.78(c) on the next page. The labels added to Fig. 10.78(c) identify broad areas of the design and some individual components. The major components of the electronic circuitry include a large 160 μF, 330 V, polarized electrolytic capacitor to store the necessary charge for the flash lamp, a flash lamp to generate the required light, a dc battery

FIG. 10.78(a)
Flash camera: general appearance.

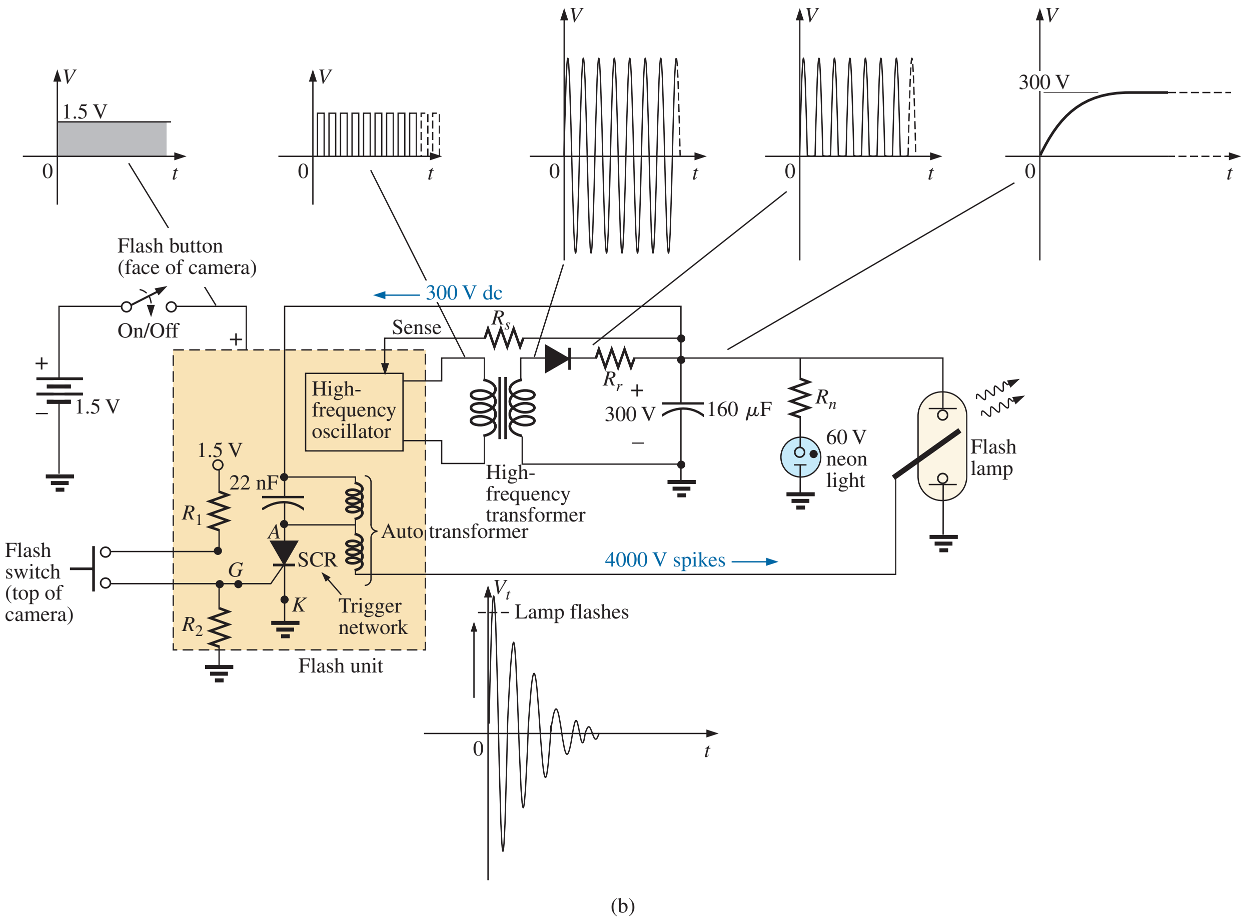


FIG. 10.78(b)
Flash camera: basic circuitry.

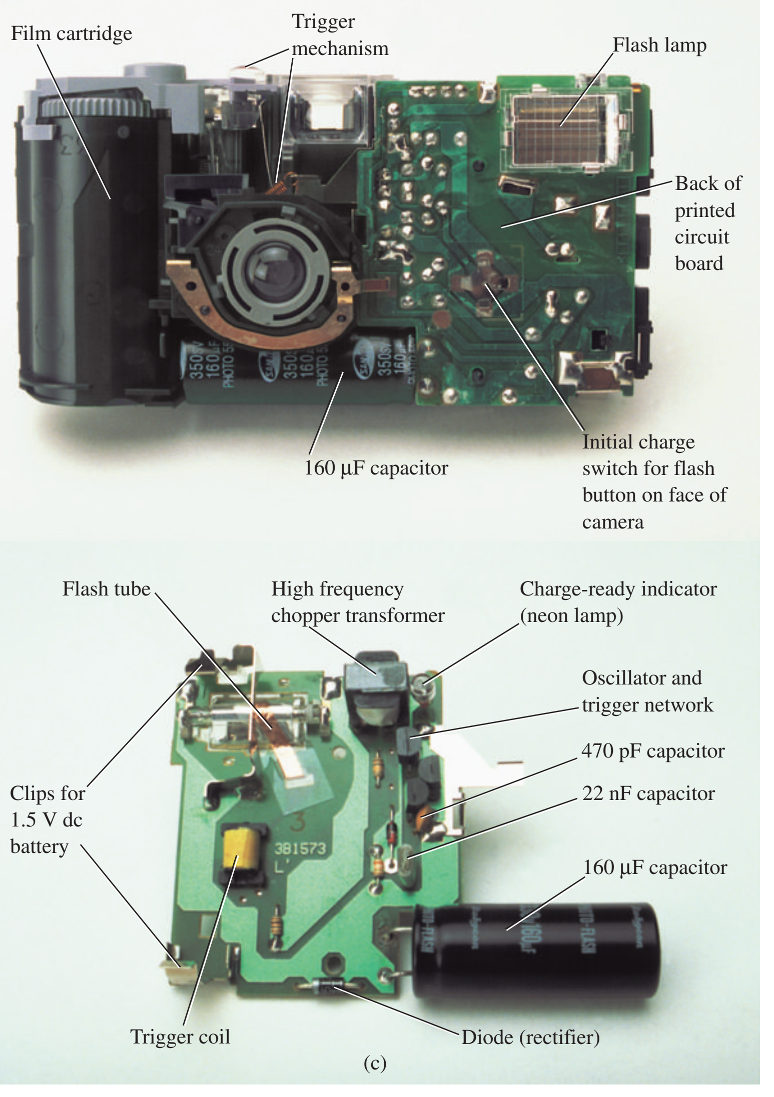


FIG. 10.78(c)
Flash camera: internal construction.

of 1.5 V, a chopper network to generate a dc voltage in excess of 300 V, and a trigger network to establish a few thousand volts for a very short period of time to fire the flash lamp. There are both a 22 nF capacitor in the trigger network as shown in Fig. 10.78(b) and (c) and a third capacitor of 470 pF in the high-frequency oscillator of the chopper network. In particular, note that the size of each capacitor is directly related to its capacitance level. It should certainly be of some interest that a single source of energy of only 1.5 V dc can be converted to one of a few thousand volts (albeit for a very short period of time) to fire the flash lamp. In fact, that single, small battery has sufficient power for the entire run of film through the camera. Always keep in mind that energy is related to power and time by $W = Pt = (VI)t$. That is, a high level of voltage can be generated for a defined energy level as long as the factors I and t are sufficiently small.

When you first use the camera, you are directed to press the flash button on the face of the camera and wait for the flash-ready light to come on. As soon as the flash button is depressed, the full 1.5 V of the dc battery are applied to an electronic network (a variety of networks can perform the same function) that generates an oscillating waveform of very high frequency (with a high repetitive rate) as shown in Fig. 10.78(c). The high-frequency transformer then significantly increases the magnitude of the generated voltage and passes it on to a half-wave rectification system (introduced in earlier chapters), resulting in a dc voltage of about 300 V across the 160 μF capacitor to charge the capacitor (as determined by $Q = CV$). Once the 300 V level is reached, the lead marked "sense" in Fig. 10.78(b) feeds the information back to the oscillator and turns it off until the output dc voltage drops to a low threshold level. When the capacitor is fully charged, the neon light in parallel with the capacitor turns on (labeled "flash-ready lamp" on the camera) to let you know that the camera is ready to use. The entire network from the 1.5 V dc level to the final 300 V level is called a *dc-dc converter.* The terminology *chopper network* comes from the fact that the applied dc voltage of 1.5 V was chopped up into one that changes level at a very high frequency so that the transformer can perform its function.

Even though the camera may use a 60 V neon light, the neon light and series resistor R_n must have a full 300 V across the branch before the neon light turns on. Neon lights are simply bulbs with a neon gas that support conduction when the voltage across the terminals reaches a sufficiently high level. There is no filament, or hot wire as in a light bulb, but simply conduction through the gaseous medium. For new cameras, the first charging sequence may take 12 s to 15 s. Succeeding charging cycles may only take some 7 s or 8 s because the capacitor still has some residual charge on its plates. If the flash unit is not used, the neon light begins to drain the 300 V dc supply with a drain current in microamperes. As the terminal voltage drops, the neon light eventually turns off. For the unit in Fig. 10.78, it takes about 15 min before the light turns off. Once off, the neon light no longer drains the capacitor, and the terminal voltage of the capacitor remains fairly constant. Eventually, however, the capacitor discharges due to its own leakage current, and the terminal voltage drops to very low levels. The discharge process is very rapid when the flash unit is used, causing the terminal voltage to drop very quickly ($V = Q/C$) and, through the feedback-sense connection signal, causing the oscillator to start up again and recharge the capacitor. You may have noticed when using a camera of this type that once the camera has its initial charge, you do not need to press the charge button between pictures—it is done automatically. However, if the camera sits for a long period of time, you must depress the charge button, but the charge time is only 3 s or 4 s due to the residual charge on the plates of the capacitor.

The 300 V across the capacitor are insufficient to fire the flash lamp. Additional circuitry, called the *trigger network,* must be incorporated to generate the few thousand volts necessary to fire the flash lamp. The resulting high voltage is one reason that there is a CAUTION note on each camera regarding the high internal voltages generated and the possibility of electrical shock if the camera is opened.

The thousands of volts required to fire the flash lamp require a discussion that introduces elements and concepts beyond the current level of the text. This description is simply a first exposure to some of the interesting possibilities available from the right mix of elements. When the flash switch at the bottom left of Fig. 10.78(b) is closed, it establishes a connection between the resistors R_1 and R_2. Through a voltage divider action, a dc voltage appears at the gate (G) terminal of the SCR (silicon-controlled rectifier—a device whose state is controlled by the voltage at the gate terminal). This dc voltage turns the SCR "on" and establishes a very low resistance path (like a short circuit) between its anode (*A*) and cathode (*K*) terminals. At this point the trigger capacitor, which is connected directly to the 300 V sitting across the capacitor, rapidly charges to 300 V because it now has a direct, low-resistance path to ground through the SCR. Once it reaches 300 V, the charging current in this part of the network drops to 0 A, and the SCR opens up again since it is a device that needs a steady current in the anode circuit to stay on. The capacitor then sits across the parallel coil (with no connection to ground through the SCR) with its full 300 V and begins to quickly discharge through the coil because the only resistance in the circuit affecting the time constant is the resistance of the parallel coil. As a result, a rapidly changing current through the coil generates a high voltage across the coil for reasons to be introduced in Chapter 11.

When the capacitor decays to zero volts, the current through the coil will be zero amperes, but a strong magnetic field has been established around the coil. This strong magnetic field then quickly collapses, establishing a current in the parallel network that recharges the capacitor again. This continual exchange between the two storage elements continues for a period of time, depending on the resistance in the circuit. The more the resistance, the shorter the "ringing" of the voltage at the output. This action of the energy "flying back" to the other element is the basis for the "flyback" effect that is frequently used to generate high dc voltages such as needed in TVs. In Fig. 10.78(b), you will find that the trigger coil is connected directly to a second coil to form an autotransformer (a transformer with one end connected). Through transformer action, the high voltage generated across the trigger coil increases further, resulting in the 4000 V necessary to fire the flash lamp. Note in Fig. 10.78(c) that the 4000 V are applied to a grid that actually lies on the surface of the glass tube of the flash lamp (not internally connected or in contact with the gases). When the trigger voltage is applied, it excites the gases in the lamp, causing a very high current to develop in the bulb for a very short period of time and producing the desired bright light. The current in the lamp is supported by the charge on the 160 μF capacitor which is dissipated very quickly. The capacitor voltage drops very quickly, the photo lamp shuts down, and the charging process begins again. If the entire process didn't occur as quickly as it does, the lamp would burn out after a single use.

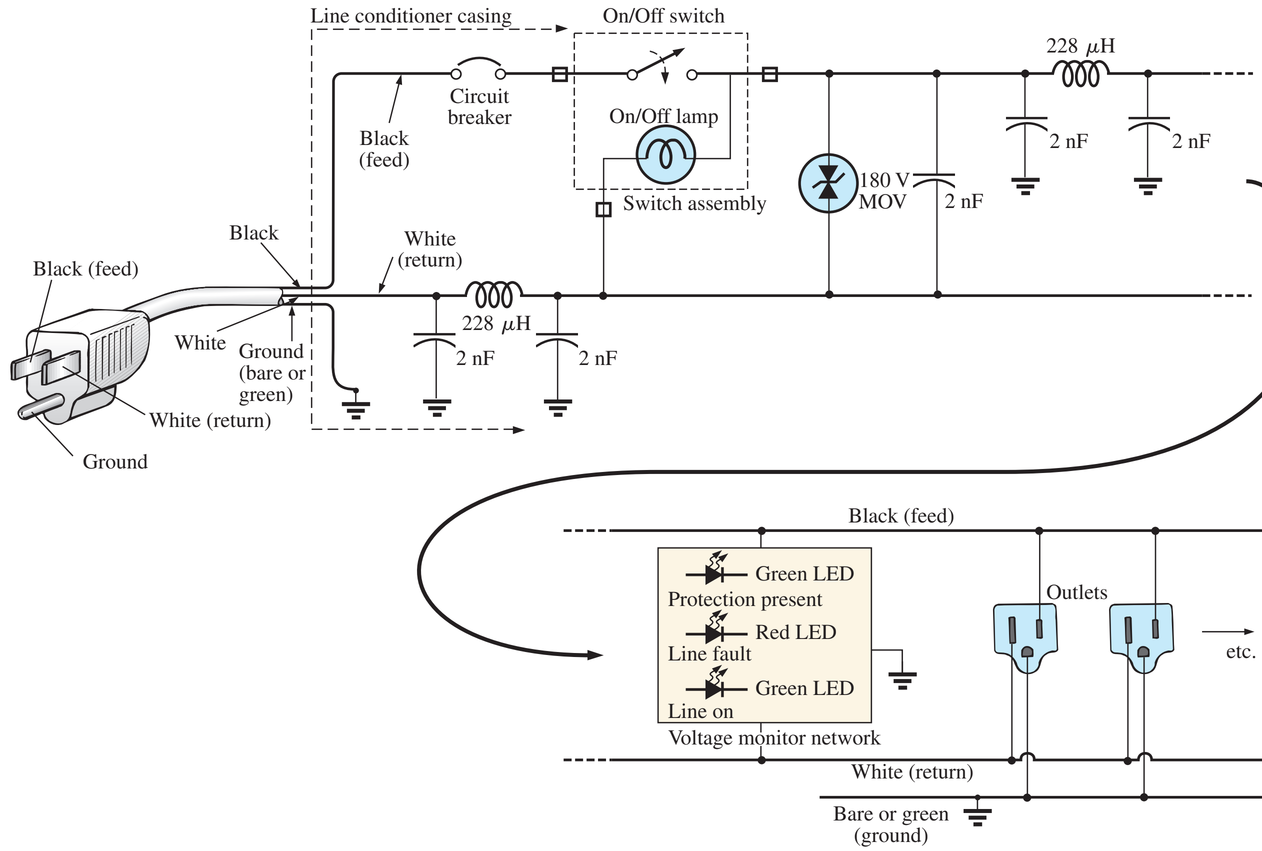
Surge Protector (Line Conditioner)

In recent years we have all become familiar with the surge protector as a safety measure for our computers, TVs, DVD players, and other sensitive instrumentation. In addition to protecting equipment from unexpected surges in voltage and current, most quality units also filter out (remove) electromagnetic interference (EMI) and radio-frequency interference (RFI). EMI encompasses any unwanted disturbances down the power line established by any combination of electromagnetic effects such as those generated by motors on the line, power equipment in the area emitting signals picked up by the power line acting as an antenna, and so on. RFI includes all signals in the air in the audio range and beyond which may also be picked up by power lines inside or outside the house.

(a)

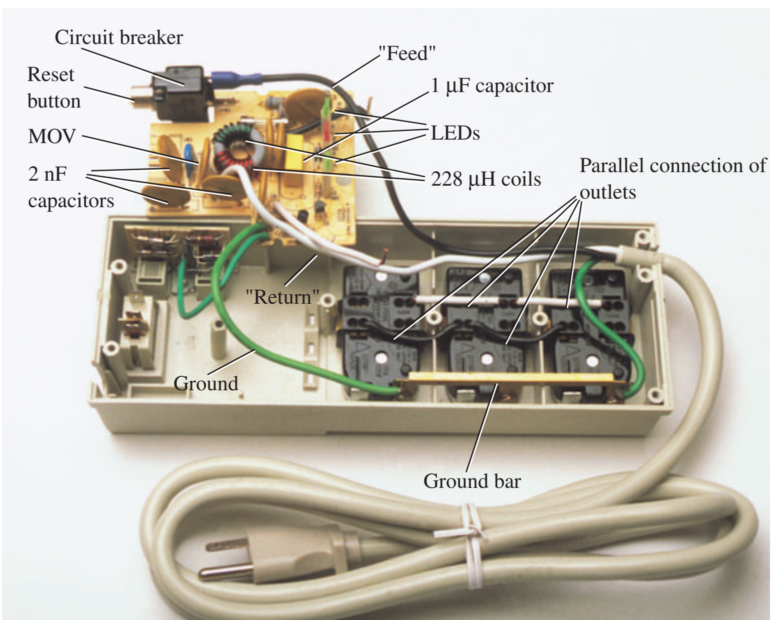
FIG. 10.79(a)
Surge protector: general appearance.

The unit in Fig. 10.79 has all the design features expected in a good line conditioner. Figure 10.79(a) reveals that it can handle the power drawn by six outlets and that it is set up for FAX/MODEM protection. Also note that it has both LED (light-emitting diode) displays which



(b)

FIG. 10.79(b)
Electrical schematic.



(c)

FIG. 10.79(c)
Internal construction.

reveal whether there is fault on the line or whether the line is OK and an external circuit breaker to reset the system. In addition, when the surge protector is on, a red light is visible at the power switch.

The schematic in Fig. 10.79(b) does not include all the details of the design, but it does include the major components that appear in most good line conditioners. First note in the photograph in Fig. 10.79(c) that the outlets are all connected in parallel, with a ground bar used to establish a ground connection for each outlet. The circuit board had to be flipped over to show the components, so it will take some adjustment to relate the position of the elements on the board to the casing. The *feed line* or *hot lead wire* (black in the actual unit) is connected directly from the line to the circuit breaker. The other end of the circuit breaker is connected to the other side of the circuit board. All the large discs that you see are 2 nF capacitors [not all have been included in Fig 10.79(b) for clarity]. There are quite a few capacitors to handle all the possibilities. For instance, there are capacitors from line to return (black wire to white wire), from line to ground (black to green), and from return to ground (white to ground). Each has two functions. The first and most obvious function is to prevent any spikes in voltage that may come down the line because of external effects such as lightning from reaching the equipment plugged into the unit. Recall from this chapter that the voltage across capacitors cannot change instantaneously and, in fact, acts to squelch any rapid change in voltage across its terminals. The capacitor, therefore, prevents the line to neutral voltage from changing too quickly, and any spike that tries to come down the line has to find another point in the feed circuit to fall across. In this way, the appliances plugged into the surge protector are well protected.

The second function requires some knowledge of the reaction of capacitors to different frequencies and is discussed in more detail in later chapters. For the moment, let it suffice to say that the capacitor has a different impedance to different frequencies, thereby preventing undesired frequencies, such as those associated with EMI and RFI disturbances, from affecting the operation of units connected to the line conditioner. The rectangular-shaped capacitor of 1 μF near the center of the board is connected directly across the line to take the brunt of a strong voltage spike down the line. Its larger size is clear evidence that it is designed to absorb a fairly high energy level that may be established by a large voltage—significant current over a period of time that may exceed a few milliseconds.

The large, toroidal-shaped structure in the center of the circuit board in Fig. 10.79(c) has two coils (Chapter 11) of 228 μH that appear in the line and neutral in Fig. 10.79(b). Their purpose, like that of the capacitors, is twofold: to block spikes in current from coming down the line and to block unwanted EMI and RFI frequencies from getting to the connected systems. In the next chapter you will find that coils act as "chokes" to quick changes in current; that is, the current through a coil cannot change instantaneously. For increasing frequencies, such as those associated with EMI and RFI disturbances, the reactance of a coil increases and absorbs the undesired signal rather than let it pass down the line. Using a choke in both the line and the neutral makes the conditioner network balanced to ground. In total, capacitors in a line conditioner have the effect of *bypassing* the disturbances, whereas inductors *block* the disturbance.

The smaller disc (blue) between two capacitors and near the circuit breaker is an MOV (metal-oxide varistor) which is the heart of most line conditioners. It is an electronic device whose terminal characteristics change with the voltage applied across its terminals. For the normal range of voltages down the line, its terminal resistance is sufficiently large to be considered an open circuit, and its presence can be ignored. However, if the voltage is too large, its terminal characteristics change from a very large resistance to a very small resistance that can essentially be considered a short circuit. This variation in resistance with applied voltage is the reason for the name *varistor*. For MOVs in North America where the line voltage is 120 V, the MOVs are 180 V or more. The reason for the 60 V difference is that the 120 V rating is an effective value related to dc voltage levels, whereas the waveform for the voltage at any 120 V outlet has a peak value of about 170 V. A great deal more will be said about this topic in Chapter 13.

Taking a look at the symbol for an MOV in Fig. 10.79(b), note that it has an arrow in each direction, revealing that the MOV is bidirectional and blocks voltages with either polarity. In general, therefore, for normal operating conditions, the presence of the MOV can be ignored; but, if a large spike should appear down the line, exceeding the MOV rating, it acts as a short across the line to protect the connected circuitry. It is a significant improvement to simply putting a fuse in the line because it is voltage sensitive, can react much quicker than a fuse, and displays its low-resistance characteristics for only a short period of time. When the spike has passed, it returns to its normal open-circuit characteristic. If you're wondering where the spike goes if the load is protected by a short circuit, remember that all sources of disturbance, such as lightning, generators, inductive motors (such as in air conditioners, dishwashers, power saws, and so on), have their own "source resistance," and there is always some resistance down the line to absorb the disturbance.

Most line conditioners, as part of their advertising, mention their energy absorption level. The rating of the unit in Fig. 10.79 is 1200 J which is actually higher than most. Remembering that $W = Pt = EIt$ from the earlier discussion of cameras, we now realize that if a 5000 V spike occurred, we would be left with the product $It = W/E = 1200\text{ J}/5000\text{ V} = 240\text{ mAs}$. Assuming a linear relationship between all quantities, the rated energy level reveals that a current of 100 A could be sustained for $t = 240\text{ mAs}/100\text{ A} = 2.4\text{ ms}$, a current of 1000 A for 240 μs, and a current of 10,000 A for 24 μs. Obviously, the higher the power product of E and I, the less the time element.

The technical specifications of the unit in Fig. 10.79 include an instantaneous response time in the order of ps, with a phone line protection of 5 ns. The unit is rated to dissipate surges up to 6000 V and current spikes up to 96,000 A. It has a very high noise suppression ratio (80 dB; see Chapter 21) at frequencies from 50 kHz to 1000 MHz, and (a credit to the company) it has a lifetime warranty.

10.15 COMPUTER ANALYSIS

PSpice

Transient *RC* Response We now use PSpice to investigate the transient response for the voltage across the capacitor in Fig. 10.80. In all the examples in the text involving a transient response, a switch appeared in series with the source as shown in Fig. 10.81(a). When applying PSpice, we establish this instantaneous change in voltage level by applying a pulse waveform as shown in Fig. 10.81(b) with a pulse width *(PW)* longer than the period (5τ) of interest for the network.

To obtain a pulse source, start with the sequence **Place part** key-**Libraries-SOURCE-VPULSE-OK.** Once in place, set the label and all the parameters by double-clicking on each to obtain the **Display**

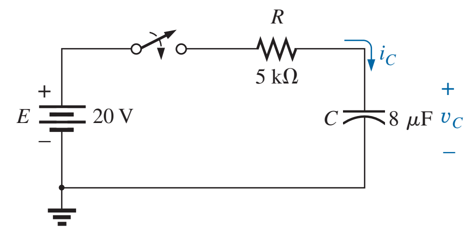


FIG. 10.80
Circuit to be analyzed using PSpice.

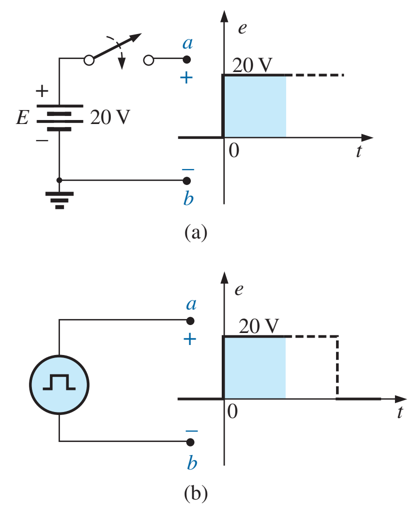


FIG. 10.81
Establishing a switching dc voltage level: (a) series dc voltage-switch combination; (b) PSpice pulse option.

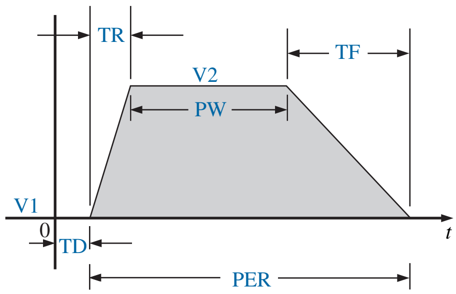


FIG. 10.82
*The defining parameters of PSpice **VPULSE.***

Properties dialog box. As you scroll down the list of attributes, you will see the following parameters defined by Fig. 10.82:

V1 is the initial value.
V2 is the pulse level.
TD is the delay time.
TR is the rise time.
TF is the fall time.
PW is the pulse width at the V_2 level.
PER is the period of the waveform.

All the parameters have been set as shown on the schematic in Fig. 10.83 for the network in Fig. 10.80. Since a rise and fall time of 0 s is unrealistic from a practical standpoint, 0.1 ms was chosen for each in this example. Further, since $\tau = RC = (5\text{ k}\Omega) \times (8\ \mu\text{F}) = 20$ ms and $5\tau = 200$ ms, a pulse width of 500 ms was selected. The period was simply chosen as twice the pulse width.

Now for the simulation process. First select the **New Simulation Profile** key to obtain the **New Simulation** dialog box in which **PSpice 10-1** is inserted for the **Name** and **Create** is chosen to leave the dialog box. The **Simulation Settings-PSpice 10-1** dialog box results, and under **Analysis,** choose the **Time Domain (Transient)** option under **Analysis type.** Set the **Run to time** at 200 ms so that only the first five time constants will be plotted. Set the **Start saving data after** option at 0 s to ensure that the data are collected immediately. The **Maximum step size** is 1 ms to provide sufficient data points for a good plot. Click **OK,** and you are ready to select the **Run PSpice** key. The result is a graph without a plot (since it has not been defined yet) and an x-axis that extends from 0 s to 200 ms as defined above. To obtain a plot of the voltage across the capacitor versus time, apply the following sequence: **Add Trace** key-**Add Traces** dialog box-**V1(C)-OK.** The plot in Fig. 10.84 results. The color and thickness of the plot and the axis can be changed by placing the cursor on the plot line and right-clicking. Select **Properties** from the list that appears. A **Trace Properties** dialog box appears in which you can change the color and thickness of the line. Since the plot is against a black background, a better printout occurred when yellow was selected and the line was made thicker as shown in Fig. 10.84. For comparison, plot the

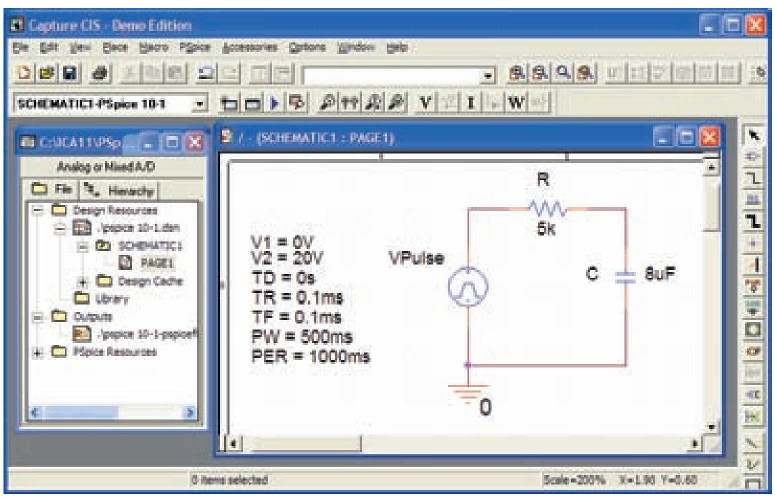


FIG. 10.83
Using PSpice to investigate the transient response of the series R-C circuit in Fig. 10.80.

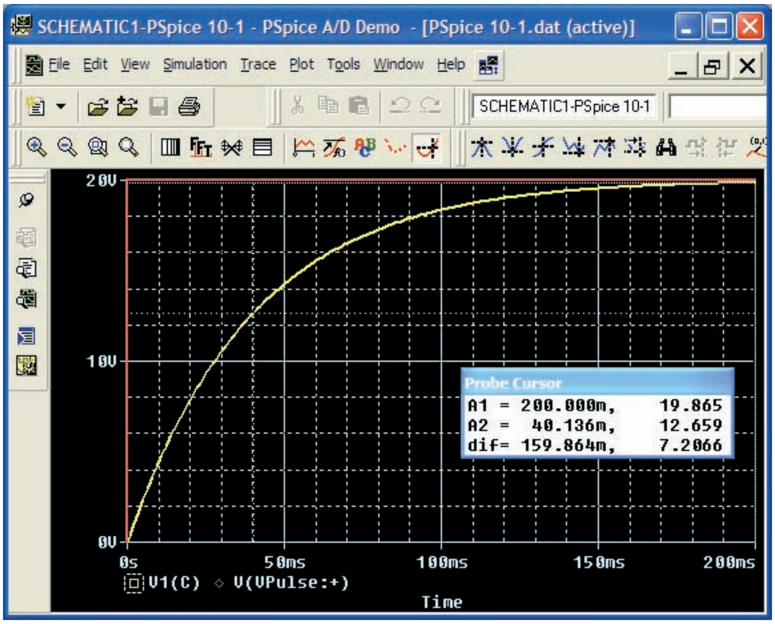


FIG. 10.84

Transient response for the voltage across the capacitor in Fig. 10.80 when ***VPulse*** *is applied.*

applied pulse signal also. This is accomplished by going back to **Trace** and selecting **Add Trace** followed by **V(Vpulse:+)** and **OK.** Now both waveforms appear on the same screen as shown in Fig. 10.84. In this case, the plot has a reddish tint so it can be distinguished from the axis and the other plot. Note that it follows the left axis to the top and travels across the screen at 20 V.

If you want the magnitude of either plot at any instant, simply select the **Toggle cursor** key. Then click on **V1(C)** at the bottom left of the screen. A box appears around **V1(C)** that reveals the spacing between the dots of the cursor on the screen. This is important when more than one cursor is used. By moving the cursor to 200 ms, you find that the magnitude **(A1)** is 19.865 V (in the **Probe Cursor** dialog box), clearly showing how close it is to the final value of 20 V. A second cursor can be placed on the screen with a right click and then a click on the same **V1(C)** on the bottom of the screen. The box around **V1(C)** cannot show two boxes, but the spacing and the width of the lines of the box have definitely changed. There is no box around the **Pulse** symbol since it was not selected—although it could have been selected by either cursor. If you now move the second cursor to one time constant of 40 ms, you find that the voltage is 12.659 V as shown in the **Probe Cursor** dialog box. This confirms that the voltage should be 63.2% of its final value of 20 V in one time constant ($0.632 \times 20\text{ V} = 12.4\text{ V}$). Two separate plots could have been obtained by going to **Plot-Add Plot to Window** and then using the trace sequence again.

Average Capacitive Current As an exercise in using the pulse source and to verify our analysis of the average current for a purely capacitive network, the description to follow verifies the results of Example 10.14. For the pulse waveform in Fig. 10.65, the parameters of the pulse supply appear in Fig. 10.85. Note that the rise time is now 2 ms, starting at 0 s, and the fall time is 6 ms. The period was set at 15 ms to permit monitoring the current after the pulse had passed.

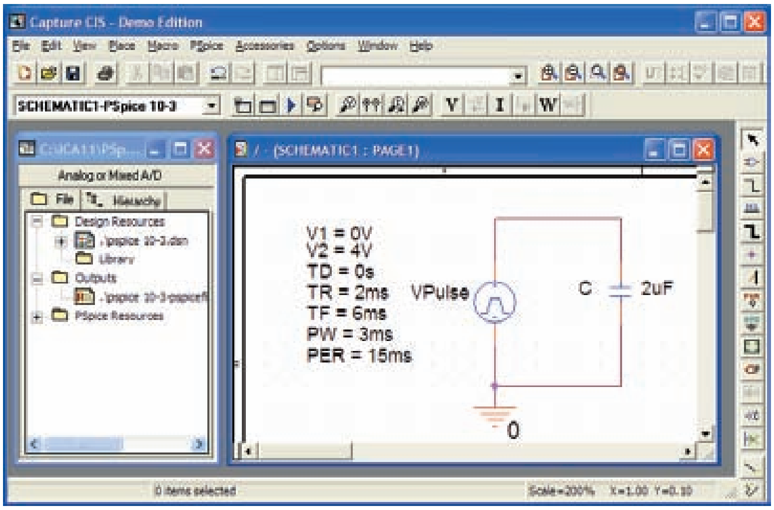


FIG. 10.85
Using PSpice to verify the results in Example 10.14.

Initiate simulation by first selecting the **New Simulation Profile** key to obtain the **New Simulation** dialog box in which **AverageIC** is entered as the **Name.** Choose **Create** to obtain the **Simulation Settings-AverageIC** dialog box. Select **Analysis** and choose **Time Domain(Transient)** under the **Analysis type** options. Set the **Run to time** to 15 ms to encompass the period of interest, and set the **Start saving data after** at 0 s to ensure data points starting at $t = 0$ s. Select the **Maximum step size** from 15 ms/1000 = 15 μs to ensure 1000 data points for the plot. Click **OK,** and select the **Run PSpice** key. A window appears with a horizontal scale that extends from 0 to 15 ms as defined above. Then select the **Add Trace** key, and choose **I(C)** to appear in the **Trace Expression** below. Click **OK,** and the plot of **I(C)** appears in the bottom of Fig. 10.86. This time it would be

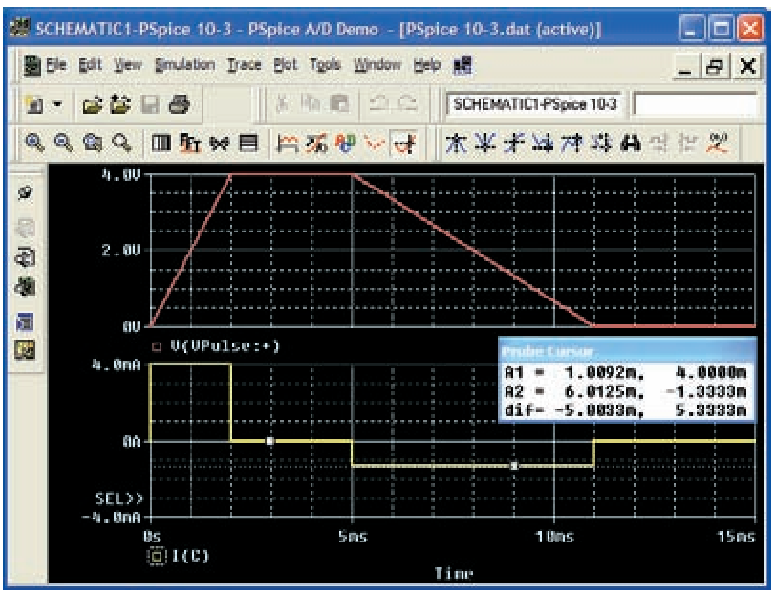


FIG. 10.86
The applied pulse and resulting current for the 2 μF capacitor in Fig. 10.85.

nice to see the pulse waveform in the same window but as a separate plot. Therefore, continue with **Plot-Add Plot to Window-Trace-Add Trace-V(Vpulse:+)-OK,** and both plots appear as shown in Fig. 10.86.

Now use the cursors to measure the resulting average current levels. First, select the **I(C)** plot to move the **SEL>>** notation to the lower plot. The **SEL>>** defines which plot for multiplot screens is active. Then select the **Toggle cursor** key, and left-click on the **I(C)** plot to establish the crosshairs of the cursor. Set the value at 1 ms, and the magnitude **A1** is displayed as 4 mA. Right-click on the same plot, and a second cursor results that can be placed at 6 ms to get a response of −1.33 mA (**A2**) as expected from Example 10.14. The plot for **I(C)** was set in the yellow color with a wider line by right-clicking on the curve and choosing **Properties.**

PROBLEMS

SECTION 10.2 The Electric Field

1. **a.** Find the electric field strength at a point 2 m from a charge of 4 μC.
   b. Find the electric field strength at a point 1 mm from the same charge as part (a) and compare results.
2. The electric field strength is 72 newtons/coulomb (N/C) at a point r meters from a charge of 2 μC. Find the distance r.

SECTIONS 10.3 AND 10.4 Capacitance and Capacitors

3. Find the capacitance of a parallel plate capacitor if 1200 μC of charge are deposited on its plates when 10 V are applied across the plates.
4. How much charge is deposited on the plates of a 0.15 μF capacitor if 45 V are applied across the capacitor?
5. Find the electric field strength between the plates of a parallel plate capacitor if 100 mV are applied across the plates and the plates are 2 mm apart.
6. Repeat Problem 5 if the plates are separated by 10 mils.
7. A 4 μF parallel plate capacitor has 160 μC of charge on its plates. If the plates are 5 mm apart, find the electric field strength between the plates.
8. Find the capacitance of a parallel plate capacitor if the area of each plate is 0.1 m^2 and the distance between the plates is 2 mm. The dielectric is air.
9. Repeat Problem 8 if the dielectric is paraffin-coated paper.
10. Find the distance in mils between the plates of a 2 μF capacitor if the area of each plate is 0.15 m^2 and the dielectric is transformer oil.
11. The capacitance of a capacitor with a dielectric of air is 1200 pF. When a dielectric is inserted between the plates, the capacitance increases to 6 nF. Of what material is the dielectric made?
12. The plates of a parallel plate air capacitor are 0.2 mm apart and have an area of 0.08 m^2, and 200 V are applied across the plates.
    a. Determine the capacitance.
    b. Find the electric field intensity between the plates.
    c. Find the charge on each plate if the dielectric is air.
13. A sheet of Bakelite 0.2 mm thick having an area of 0.08 m^2 is inserted between the plates of Problem 12.
    a. Find the electric field strength between the plates.
    b. Determine the charge on each plate.
    c. Determine the capacitance.
14. A parallel plate air capacitor has a capacitance of 5 μF. Find the new capacitance if:
    a. The distance between the plates is doubled (everything else remains the same).
    b. The area of the plates is doubled (everything else remains the same as for the 5 μF level).
    c. A dielectric with a relative permittivity of 20 is inserted between the plates (everything else remains the same as for the 5 μF level).
    d. A dielectric is inserted with a relative permittivity of 4, and the area is reduced to 1/3 and the distance to 1/4 of their original dimensions.

*15. Find the maximum voltage that can be applied across a parallel plate capacitor of 6 nF if the area of one plate is 0.02 m^2 and the dielectric is mica. Assume a linear relationship between the dielectric strength and the thickness of the dielectric.

*16. Find the distance in micrometers between the plates of a parallel plate mica capacitor if the maximum voltage that can be applied across the capacitor is 1200 V. Assume a linear relationship between the breakdown strength and the thickness of the dielectric.

17. A 22 μF capacitor has −200 ppm/°C at room temperature of 20°C. What is the capacitance if the temperature increases to 100°C, the boiling point of water?
18. What is the capacitance of a small teardrop capacitor labeled 40 J? What is the range of expected values as established by the tolerance?
19. A large, flat, mica capacitor is labeled 220M. What are the capacitance and the expected range of values guaranteed by the manufacturer?
20. A small flat disc ceramic capacitor is labeled 333K. What are the capacitance level and the expected range of values?

SECTION 10.5 Transients in Capacitive Networks: The Charging Phase

21. For the circuit in Fig. 10.87, composed of standard values:
- **a.** Determine the time constant of the circuit.
- **b.** Write the mathematical equation for the voltage v_C following the closing of the switch.
- **c.** Determine the voltage v_C after one, three, and five time constants.
- **d.** Write the equations for the current i_C and the voltage v_R.
- **e.** Sketch the waveforms for v_C and i_C.

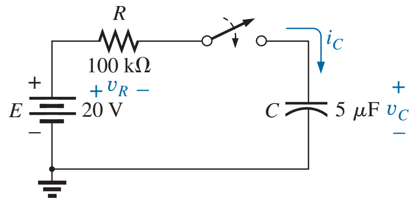


FIG. 10.87
Problems 21 and 22.

22. Repeat Problem 21 for $R = 1\ \text{M}\Omega$, and compare the results.

23. For the circuit in Fig. 10.88, composed of standard values:
- **a.** Determine the time constant of the circuit.
- **b.** Write the mathematical equation for the voltage v_C following the closing of the switch.
- **c.** Determine v_C after one, three, and five time constants.
- **d.** Write the equations for the current i_C and the voltage v_R.
- **e.** Sketch the waveforms for v_C and i_C.

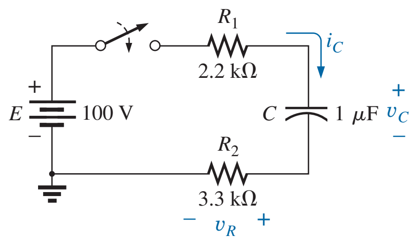


FIG. 10.88
Problem 23.

24. For the circuit in Fig. 10.89, composed of standard values:
- **a.** Determine the time constant of the circuit.
- **b.** Write the mathematical equation for the voltage v_C following the closing of the switch.

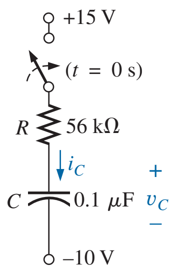


FIG. 10.89
Problem 24.

- **c.** Write the mathematical expression for the current i_C following the closing of the switch.
- **d.** Sketch the waveforms of v_C and i_C.

25. Given the voltage $v_C = 60\ \text{mV}(1 - e^{-t/5\text{ms}})$:
- **a.** What is the time constant?
- **b.** What is the voltage at $t = 2$ ms?
- **c.** What is the voltage at $t = 100$ ms?

26. The voltage across a 10 μF capacitor in a series *R-C* circuit is $v_C = 12\ \text{V}(1 - e^{-t/40\text{ms}})$.
- **a.** On a practical basis, how much time must pass before the charging phase has passed?
- **b.** What is the resistance of the circuit?
- **c.** What is the voltage at $t = 20$ ms?
- **d.** What is the voltage at 10 time constants?
- **e.** Under steady-state conditions, how much charge is on the plates?
- **f.** If the leakage resistance is 1000 MΩ, how long will it take (in hours) for the capacitor to discharge if we assume that the discharge rate is constant throughout the discharge period?

SECTION 10.6 Transients in Capacitive Networks: The Discharging Phase

27. For the *R-C* circuit in Fig. 10.90, composed of standard values:
- **a.** Determine the time constant of the circuit when the switch is thrown into position 1.
- **b.** Find the mathematical expression for the voltage across the capacitor and the current after the switch is thrown into position 1.
- **c.** Determine the magnitude of the voltage v_C and the current i_C the instant the switch is thrown into position 2 at $t = 1$ s.
- **d.** Determine the mathematical expression for the voltage v_C and the current i_C for the discharge phase.
- **e.** Plot the waveforms of v_C and i_C for a period of time extending from 0 to 2 s from when the switch was thrown into position 1.

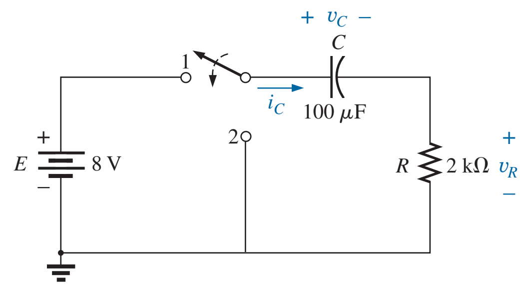


FIG. 10.90
Problem 27.

28. For the network in Fig. 10.91, composed of standard values:
- **a.** Write the mathematical expressions for the voltages v_C, and v_{R_1} and the current i_C after the switch is thrown into position 1.
- **b.** Find the values of v_C, v_{R_1}, and i_C when the switch is moved to position 2 at $t = 100$ ms.

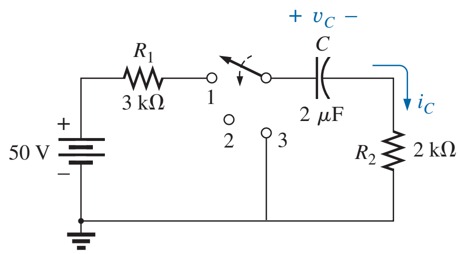


FIG. 10.91
Problems 28 and 29.

c. Write the mathematical expressions for the voltages v_C and v_{R_2} and the current i_C if the switch is moved to position 3 at $t = 200$ ms.

d. Plot the waveforms of v_C, v_{R_2}, and i_C for the time period extending from 0 to 300 ms.

29. Repeat Problem 28 for a capacitance of 20 μF assuming that the leakage resistance of the capacitor is ∞ Ω.

30. For the network in Fig. 10.92, composed of standard values:

a. Find the mathematical expressions for the voltage v_C and the current i_C when the switch is thrown into position 1.

b. Find the mathematical expressions for the voltage v_C and the current i_C if the switch is thrown into position 2 at a time equal to five time constants of the charging circuit.

c. Plot the waveforms of v_C and i_C for a period of time extending from 0 to 30 μs.

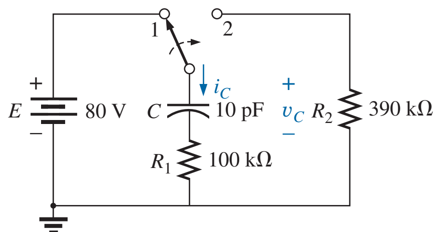


FIG. 10.92
Problem 30.

31. The 1000 μF capacitor in Fig. 10.93 is charged to 6 V. To discharge the capacitor before further use, a wire with a resistance of 2 mΩ is placed across the capacitor.

a. How long will it take to discharge the capacitor?

b. What is the peak value of the current?

c. Based on the answer to part (b), is a spark expected when contact is made with both ends of the capacitor?

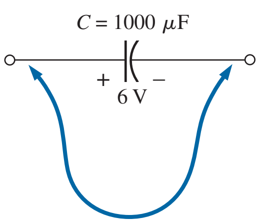


FIG. 10.93
Problem 31.

SECTION 10.7 Initial Conditions

32. The capacitor in Fig. 10.94 is initially charged to 10 V with the polarity shown.

a. Write the expression for the voltage v_C after the switch is closed.

b. Write the expression for the current i_C after the switch is closed.

c. Plot the results of parts (a) and (b).

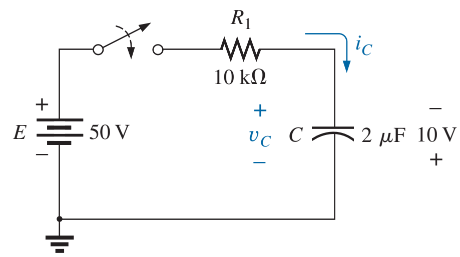


FIG. 10.94
Problem 32.

33. The capacitor in Fig. 10.95 is initially charged to 40 V before the switch is closed. Write the expressions for the voltages v_C and v_R and the current i_C following the closing of the switch. Plot the resulting waveforms.

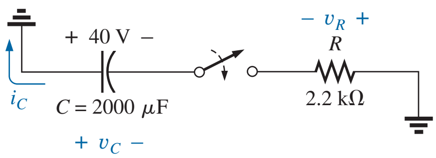


FIG. 10.95
Problem 33.

***34.** The capacitor in Fig. 10.96 is initially charged to 20 V with the polarity shown. Write the expressions for the voltage v_C and the current i_C following the closing of the switch. Plot the resulting waveforms.

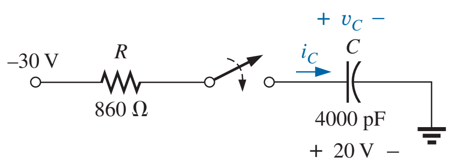


FIG. 10.96
Problem 34.

***35.** The capacitor in Fig. 10.97 is initially charged to 12 V with the polarity shown.

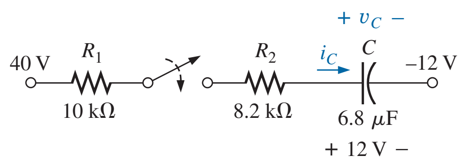


FIG. 10.97
Problem 35.

a. Find the mathematical expressions for the voltage v_C and the current i_C when the switch is closed.

b. Sketch the waveforms of v_C and i_C.

SECTION 10.8 Instantaneous Values

36. Given the expression $v_C = 12\text{ V}(1 - e^{-t/20\mu s})$:

a. Determine v_C at $t = 10\ \mu s$.

b. Determine v_C at $t = 10\tau$.

c. Find the time t for v_C to reach 6 V.

d. Find the time t for v_C to reach 11.98 V.

37. For the network in Fig. 10.98, V_L must be 8 V before the system is activated. If the switch is closed at $t = 0$ s, how long will it take for the system to be activated?

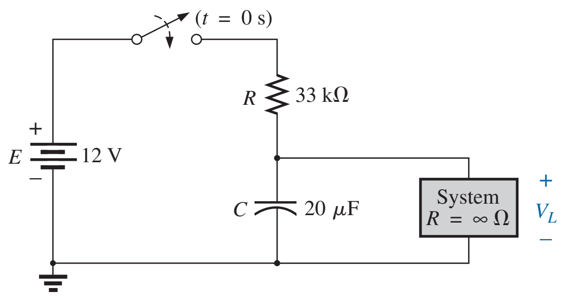


FIG. 10.98
Problem 37.

***38.** Design the network in Fig. 10.99 such that the system turns on 10 s after the switch is closed.

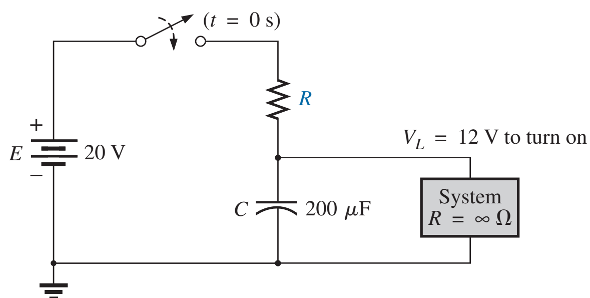


FIG. 10.99
Problem 38.

39. For the circuit in Fig. 10.100:

a. Find the time required for v_C to reach 60 V following the closing of the switch.

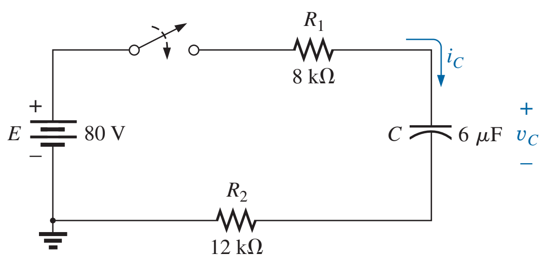


FIG. 10.100
Problem 39.

b. Calculate the current i_C at the instant $v_C = 60$ V.

c. Determine the power delivered by the source at the instant $t = 2\tau$.

***40.** For the network in Fig. 10.101:

a. Calculate v_C, i_C, and v_{R_1} at 0.5 s and 1 s after the switch makes contact with position 1.

b. The network sits in position 1 10 min before the switch is moved to position 2. How long after making contact with position 2 will it take for the current i_C to drop to 8 μA? How much *longer* will it take for v_C to drop to 10 V?

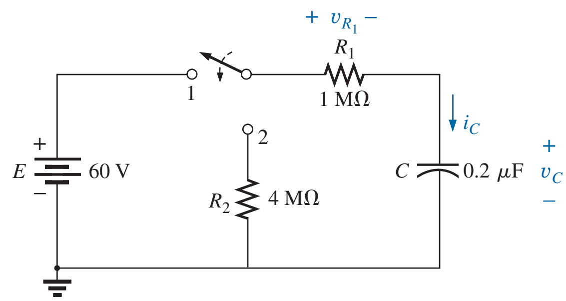


FIG. 10.101
Problem 40.

41. For the system in Fig. 10.102, using a DMM with a 10 MΩ internal resistance in the voltmeter mode:

a. Determine the voltmeter reading one time constant after the switch is closed.

b. Find the current i_C two time constants after the switch is closed.

c. Calculate the time that must pass after the closing of the switch for the voltage v_C to be 50 V.

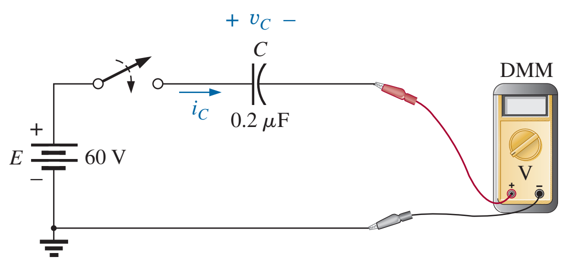


FIG. 10.102
Problem 41.

SECTION 10.9 Thévenin Equivalent: $\tau = R_{Th}C$

42. For the circuit in Fig. 10.103:

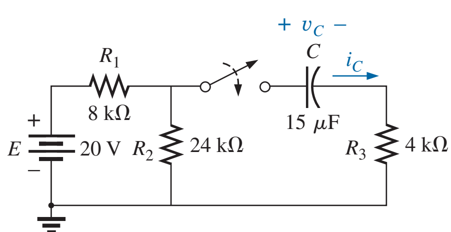


FIG. 10.103
Problem 42.

   a. Find the mathematical expressions for the transient behavior of the voltage v_C and the current i_C following the closing of the switch.
   b. Sketch the waveforms of v_C and i_C.

43. The capacitor in Fig. 10.104 is initially charged to 2 V with the polarity shown.
   a. Write the mathematical expressions for the voltage v_C and the current i_C when the switch is closed.
   b. Sketch the waveforms of v_C and i_C.

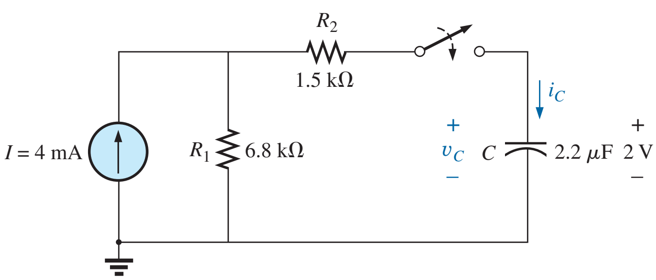


FIG. 10.104
Problem 43.

44. The capacitor in Fig. 10.105 is initially charged to 4 V with the polarity shown.
   a. Write the mathematical expressions for the voltage v_C and the current i_C when the switch is closed.
   b. Sketch the waveforms of v_C and i_C.

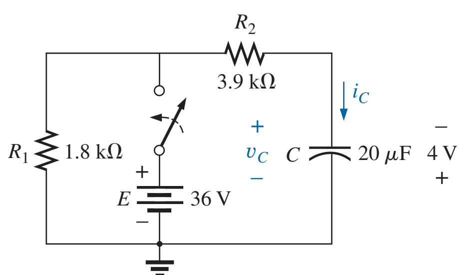


FIG. 10.105
Problem 44.

45. For the circuit in Fig. 10.106:
   a. Find the mathematical expressions for the transient behavior of the voltage v_C and the current i_C following the closing of the switch.
   b. Sketch the waveforms of v_C and i_C.

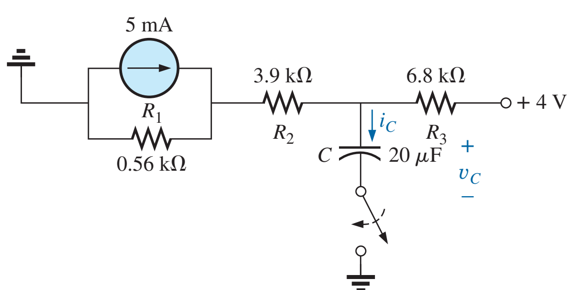


FIG. 10.106
Problem 45.

***46.** The capacitor in Fig. 10.107 is initially charged to 3 V with the polarity shown.
   a. Write the mathematical expressions for the voltage v_C and the current i_C when the switch is closed.
   b. Sketch the waveforms of v_C and i_C.

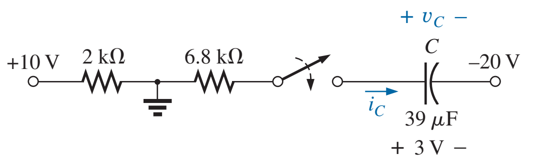


FIG. 10.107
Problem 46.

47. For the system in Fig. 10.108, using a DMM with a 10 MΩ internal resistance in the voltmeter mode:
   a. Determine the voltmeter reading four time constants after the switch is closed.
   b. Find the time that must pass before i_C drops to 3 μA.
   c. Find the time that must pass after the closing of the switch for the voltage across the meter to reach 10 V.

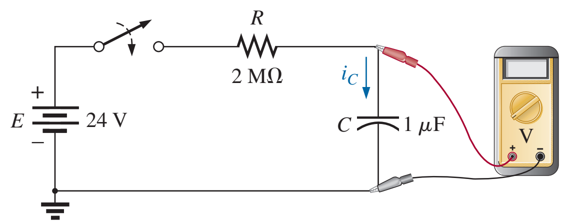


FIG. 10.108
Problem 47.

SECTION 10.10 The Current i_C

48. Find the waveform for the average current if the voltage across the 2 μF capacitor is as shown in Fig. 10.109.

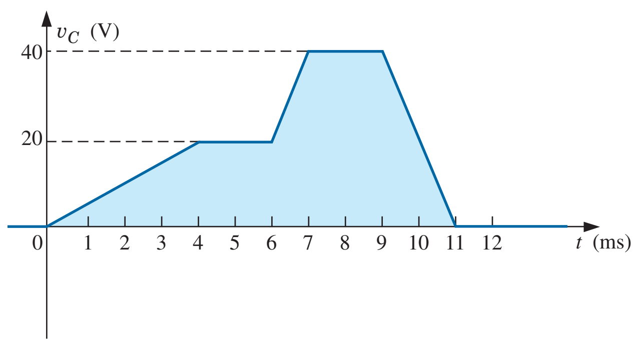


FIG. 10.109
Problem 48.

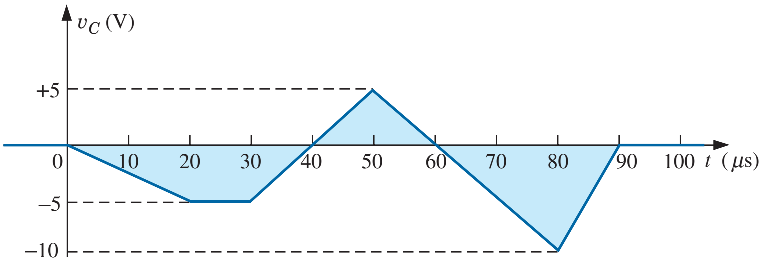


FIG. 10.110
Problem 49.

49. Find the waveform for the average current if the voltage across the 4.7 μF capacitor is as shown in Fig. 10.110.

50. Given the waveform in Fig. 10.111 for the current of a 20 μF capacitor, sketch the waveform of the voltage v_C across the capacitor if $v_C = 0$ V at $t = 0$ s.

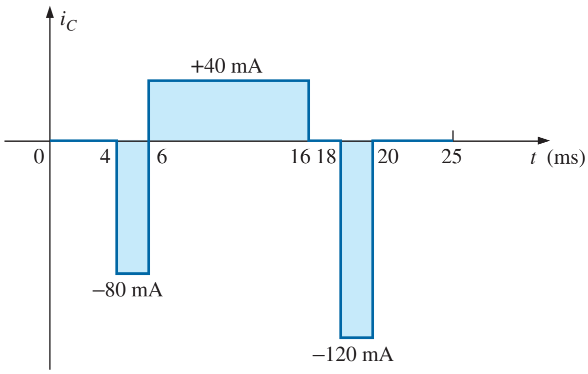


FIG. 10.111
Problem 50.

SECTION 10.11 Capacitors in Series and in Parallel

51. Find the total capacitance C_T for the circuit in Fig. 10.112.

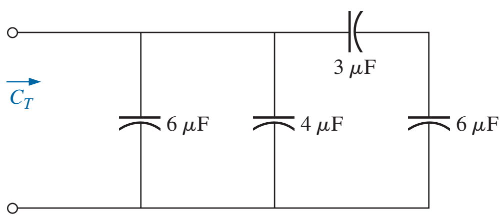


FIG. 10.112
Problem 51.

52. Find the total capacitance C_T for the circuit in Fig. 10.113.

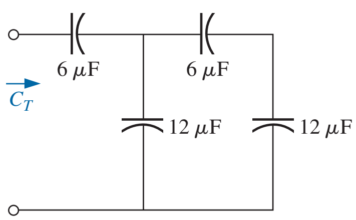


FIG. 10.113
Problem 52.

53. Find the voltage across and the charge on each capacitor for the circuit in Fig. 10.114.

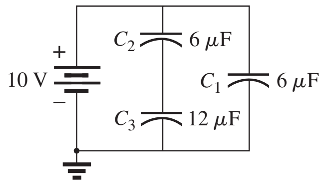


FIG. 10.114
Problem 53.

54. Find the voltage across and the charge on each capacitor for the circuit in Fig. 10.115.

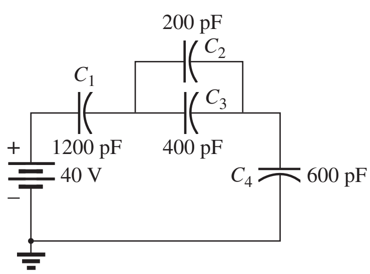


FIG. 10.115
Problem 54.

55. For the configuration in Fig. 10.116, determine the voltage across each capcitor and the charge on each capacitor.

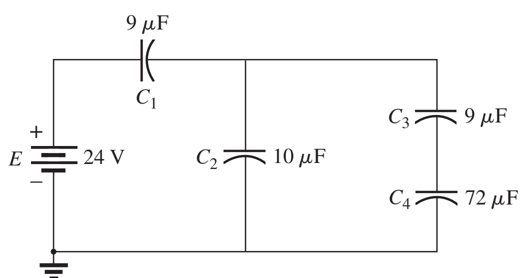


FIG. 10.116
Problem 55.

56. For the configuration in Fig. 10.117, determine the voltage across each capacitor and the charge on each capacitor.

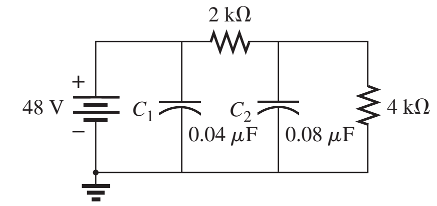


FIG. 10.117
Problem 56.

SECTION 10.12 Energy Stored by a Capacitor

57. Find the energy stored by a 120 pF capacitor with 12 V across its plates.

58. If the energy stored by a 6 μF capacitor is 1200 J, find the charge Q on each plate of the capacitor.

59. For the network in Fig. 10.118:
- **a.** Determine the energy stored by each capacitor under steady-state conditions.
- **b.** Repeat part (a) if the capacitors are in series.

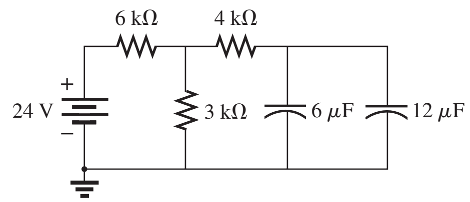


FIG. 10.118
Problem 59.

***60.** An electronic flashgun has a 1000 μF capacitor that is charged to 100 V.
- **a.** How much energy is stored by the capacitor?
- **b.** What is the charge on the capacitor?
- **c.** When the photographer takes a picture, the flash fires for 1/2000 s. What is the average current through the flashtube?
- **d.** Find the power delivered to the flashtube.
- **e.** After a picture is taken, the capacitor has to be recharged by a power supply that delivers a maximum current of 10 mA. How long will it take to charge the capacitor?

SECTION 10.15 Computer Analysis

61. Using PSpice or Multisim, verify the results in Example 10.6.

62. Using the initial condition operator, verify the results in Example 10.8 for the charging phase using PSpice or Multisim.

63. Using PSpice or Multisim, verify the results for v_C during the charging phase in Example 10.11.

64. Using PSpice or Multisim, verify the results in Problem 48.

GLOSSARY

Average current The current defined by a linear (straight line) change in voltage across a capacitor for a specific period of time.

Breakdown voltage Another term for *dielectric strength,* listed below.

Capacitance A measure of a capacitor's ability to store charge; measured in farads (F).

Capacitor A fundamental electrical element having two conducting surfaces separated by an insulating material and having the capacity to store charge on its plates.

Coulomb's law An equation relating the force between two like or unlike charges.

Derivative The instantaneous change in a quantity at a particular instant in time.

Dielectric The insulating material between the plates of a capacitor that can have a pronounced effect on the charge stored on the plates of a capacitor.

Dielectric constant Another term for *relative permittivity,* listed below.

Dielectric strength An indication of the voltage required for unit length to establish conduction in a dielectric.

Electric field strength The force acting on a unit positive charge in the region of interest.

Electric flux lines Lines drawn to indicate the strength and direction of an electric field in a particular region.

Fringing An effect established by flux lines that do not pass directly from one conducting surface to another.

Initial value The steady-state voltage across a capacitor before a transient period begins.

Leakage current The current that results in the total discharge of a capacitor if the capacitor is disconnected from the charging network for a sufficient length of time.

Maximum working voltage That voltage level at which a capacitor can perform its function without concern about breakdown or change in characteristics.

Permittivity A measure of how well a dielectric *permits* the establishment of flux lines within the dielectric.

Relative permittivity The permittivity of a material compared to that of air.

Steady-state region A period of time defined by the fact that the voltage across a capacitor has reached a level that, for all practical purposes, remains constant.

Stray capacitance Capacitances that exist not through design but simply because two conducting surfaces are relatively close to each other.

Temperature coefficient An indication of how much the capacitance value of a capacitor will change with change in temperature.

Time constant A period of time defined by the parameters of the network that defines how long the transient behavior of the voltage or current of a capacitor will last.

Transient period That period of time where the voltage across a capacitor or the current of a capacitor will change in value at a rate determined by the time constant of the network.