

# SERIES-PARALLEL AC NETWORKS

# 16

## OBJECTIVES

- *Develop confidence in the analysis of series-parallel ac networks.*
- *Become proficient in the use of calculators and computer methods to support the analysis of ac series-parallel networks.*
- *Understand the importance of proper grounding in the operation of any electrical system.*

## 16.1 INTRODUCTION

In this chapter, we shall use the fundamental concepts of the previous chapter to develop a technique for solving **series-parallel ac networks**. A brief review of Chapter 7 may be helpful before considering these networks since the approach here is quite similar to that undertaken earlier. The circuits to be discussed have only one source of energy, either potential or current. Networks with two or more sources are considered in Chapters 17 and 18, using methods previously described for dc circuits.

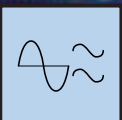
In general, when working with series-parallel ac networks, consider the following approach:

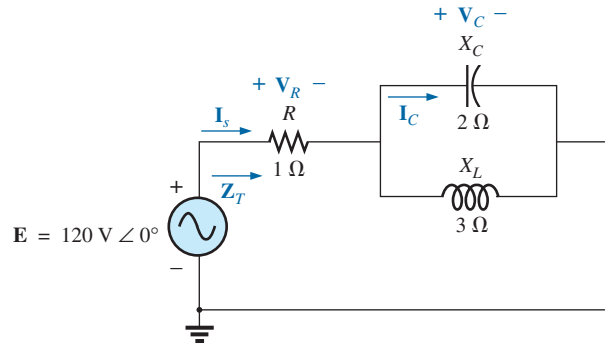
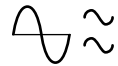
1. *Redraw the network, using block impedances to combine obvious series and parallel elements, which will reduce the network to one that clearly reveals the fundamental structure of the system.*
2. *Study the problem and make a brief mental sketch of the overall approach you plan to use. Doing this may result in time- and energy-saving shortcuts. In some cases, a lengthy, drawn-out analysis may not be necessary. A single application of a fundamental law of circuit analysis may result in the desired solution.*
3. *After the overall approach has been determined, it is usually best to consider each branch involved in your method independently before tying them together in series-parallel combinations. In most cases, work back from the obvious series and parallel combinations to the source to determine the total impedance of the network. The source current can then be determined, and the path back to specific unknowns can be defined. As you progress back to the source, continually define those unknowns that have not been lost in the reduction process. It will save time when you have to work back through the network to find specific quantities.*
4. *When you have arrived at a solution, check to see that it is reasonable by considering the magnitudes of the energy source and the elements in the circuit. If not, either solve the network using another approach, or check over your work very carefully. At this point, a computer solution can be an invaluable asset in the validation process.*

## 16.2 ILLUSTRATIVE EXAMPLES

**EXAMPLE 16.1** For the network in Fig. 16.1:

- a. Calculate  $\mathbf{Z}_T$ .
- b. Determine  $\mathbf{I}_s$ .
- c. Calculate  $\mathbf{V}_R$  and  $\mathbf{V}_C$ .

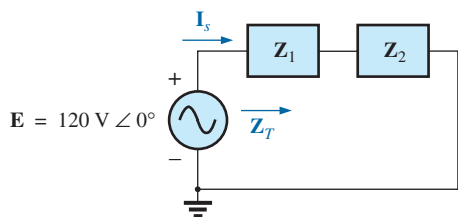




**FIG. 16.1**  
Example 16.1.

- d. Find  $\mathbf{I}_C$ .
- e. Compute the power delivered.
- f. Find  $F_p$  of the network.

**Solutions:**



**FIG. 16.2**

Network in Fig. 16.1 after assigning the block impedances.

- a. As suggested in the introduction, the network has been redrawn with block impedances, as shown in Fig. 16.2. The impedance  $\mathbf{Z}_1$  is simply the resistor  $R$  of  $1 \Omega$ , and  $\mathbf{Z}_2$  is the parallel combination of  $X_C$  and  $X_L$ . The network now clearly reveals that it is fundamentally a series circuit, suggesting a direct path toward the total impedance and the source current. For many such problems, you must work back to the source to find first the total impedance and then the source current. When the unknown quantities are found in terms of these subscripted impedances, the numerical values can then be substituted to find the magnitude and phase angle of the unknown. In other words, try to find the desired solution solely in terms of the subscripted impedances before substituting numbers. This approach will usually enhance the clarity of the chosen path toward a solution while saving time and preventing careless calculation errors. Note also in Fig. 16.2 that all the unknown quantities except  $\mathbf{I}_C$  have been preserved, meaning that we can use Fig. 16.2 to determine these quantities rather than having to return to the more complex network in Fig. 16.1.

The total impedance is defined by

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

with

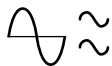
$$\mathbf{Z}_1 = R \angle 0^\circ = 1 \Omega \angle 0^\circ$$

$$\begin{aligned} \mathbf{Z}_2 = \mathbf{Z}_C \parallel \mathbf{Z}_L &= \frac{(X_C \angle -90^\circ)(X_L \angle 90^\circ)}{-jX_C + jX_L} = \frac{(2 \Omega \angle -90^\circ)(3 \Omega \angle 90^\circ)}{-j2 \Omega + j3 \Omega} \\ &= \frac{6 \Omega \angle 0^\circ}{j1} = \frac{6 \Omega \angle 0^\circ}{1 \angle 90^\circ} = 6 \Omega \angle -90^\circ \end{aligned}$$

and

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 1 \Omega - j6 \Omega = \mathbf{6.08 \Omega \angle -80.54^\circ}$$

$$\text{b. } \mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{120 \text{ V} \angle 0^\circ}{6.08 \Omega \angle -80.54^\circ} = \mathbf{19.74 \text{ A} \angle 80.54^\circ}$$



- c. Referring to Fig. 16.2, we find that  $\mathbf{V}_R$  and  $\mathbf{V}_C$  can be found by a direct application of Ohm's law:

$$\mathbf{V}_R = \mathbf{I}_s \mathbf{Z}_1 = (19.74 \text{ A } \angle 80.54^\circ)(1 \Omega \angle 0^\circ) = \mathbf{19.74 \text{ V } \angle 80.54^\circ}$$

$$\begin{aligned} \mathbf{V}_C &= \mathbf{I}_s \mathbf{Z}_2 = (19.74 \text{ A } \angle 80.54^\circ)(6 \Omega \angle -90^\circ) \\ &= \mathbf{118.44 \text{ V } \angle -9.46^\circ} \end{aligned}$$

- d. Now that  $\mathbf{V}_C$  is known, the current  $\mathbf{I}_C$  can also be found using Ohm's law.

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C} = \frac{118.44 \text{ V } \angle -9.46^\circ}{2 \Omega \angle -90^\circ} = \mathbf{59.22 \text{ A } \angle 80.54^\circ}$$

e.  $P_{\text{del}} = I_s^2 R = (19.74 \text{ A})^2(1 \Omega) = \mathbf{389.67 \text{ W}}$

f.  $F_p = \cos \theta = \cos 80.54^\circ = \mathbf{0.164 \text{ leading}}$

The fact that the total impedance has a negative phase angle (revealing that  $\mathbf{I}_s$  leads  $\mathbf{E}$ ) is a clear indication that the network is capacitive in nature and therefore has a leading power factor. The fact that the network is capacitive can be determined from the original network by first realizing that, for the parallel  $L$ - $C$  elements, the smaller impedance predominates and results in an  $R$ - $C$  network.

**EXAMPLE 16.2** For the network in Fig. 16.3:

- If  $\mathbf{I}$  is  $50 \text{ A } \angle 30^\circ$ , calculate  $\mathbf{I}_1$  using the current divider rule.
- Repeat part (a) for  $\mathbf{I}_2$ .
- Verify Kirchhoff's current law at one node.

**Solutions:**

- a. Redrawing the circuit as in Fig. 16.4, we have

$$\mathbf{Z}_1 = R + jX_L = 3 \Omega + j4 \Omega = 5 \Omega \angle 53.13^\circ$$

$$\mathbf{Z}_2 = -jX_C = -j8 \Omega = 8 \Omega \angle -90^\circ$$

Using the current divider rule yields

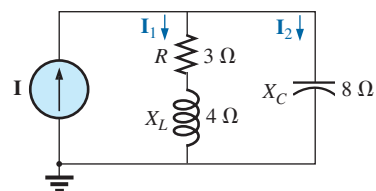
$$\begin{aligned} \mathbf{I}_1 &= \frac{\mathbf{Z}_2 \mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(8 \Omega \angle -90^\circ)(50 \text{ A } \angle 30^\circ)}{(-j8 \Omega) + (3 \Omega + j4 \Omega)} = \frac{400 \angle -60^\circ}{3 - j4} \\ &= \frac{400 \angle -60^\circ}{5 \angle -53.13^\circ} = \mathbf{80 \text{ A } \angle -6.87^\circ} \end{aligned}$$

$$\begin{aligned} \text{b. } \mathbf{I}_2 &= \frac{\mathbf{Z}_1 \mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(5 \Omega \angle 53.13^\circ)(50 \text{ A } \angle 30^\circ)}{5 \Omega \angle -53.13^\circ} = \frac{250 \angle 83.13^\circ}{5 \angle -53.13^\circ} \\ &= \mathbf{50 \text{ A } \angle 136.26^\circ} \end{aligned}$$

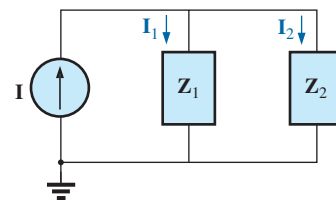
- c.  $\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$   
 $50 \text{ A } \angle 30^\circ = 80 \text{ A } \angle -6.87^\circ + 50 \text{ A } \angle 136.26^\circ$   
 $= (79.43 - j9.57) + (-36.12 + j34.57)$   
 $= 43.31 + j25.0$   
 $50 \text{ A } \angle 30^\circ = 50 \text{ A } \angle 30^\circ \quad (\text{checks})$

**EXAMPLE 16.3** For the network in Fig. 16.5:

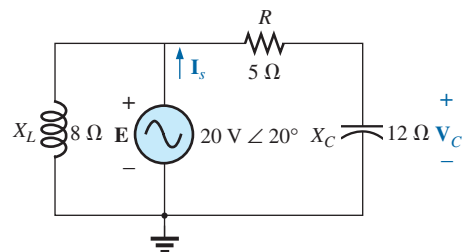
- Calculate the voltage  $\mathbf{V}_C$  using the voltage divider rule.
- Calculate the current  $\mathbf{I}_s$ .



**FIG. 16.3**  
Example 16.2.



**FIG. 16.4**  
Network in Fig. 16.3 after assigning the block impedances.



**FIG. 16.5**  
Example 16.3.

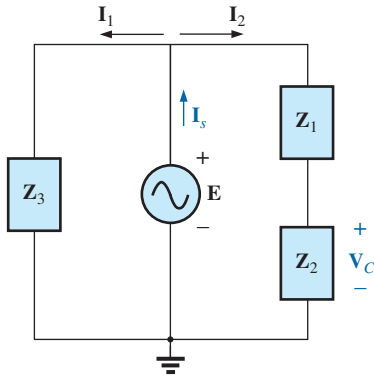
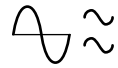


FIG. 16.6

Network in Fig. 16.5 after assigning the block impedances.

**Solutions:**

a. The network is redrawn as shown in Fig. 16.6, with

$$\begin{aligned} Z_1 &= 5 \Omega = 5 \Omega \angle 0^\circ \\ Z_2 &= -j 12 \Omega = 12 \Omega \angle -90^\circ \\ Z_3 &= +j 8 \Omega = 8 \Omega \angle 90^\circ \end{aligned}$$

Since  $V_C$  is desired, we will not combine  $R$  and  $X_C$  into a single block impedance. Note also how Fig. 16.6 clearly reveals that  $E$  is the total voltage across the series combination of  $Z_1$  and  $Z_2$ , permitting the use of the voltage divider rule to calculate  $V_C$ . In addition, note that all the currents necessary to determine  $I_s$  have been preserved in Fig. 16.6, revealing that there is no need to ever return to the network of Fig. 16.5—everything is defined by Fig. 16.6.

$$V_C = \frac{Z_2 E}{Z_1 + Z_2} = \frac{(12 \Omega \angle -90^\circ)(20 \text{ V} \angle 20^\circ)}{5 \Omega - j 12 \Omega} = \frac{240 \text{ V} \angle -70^\circ}{13 \angle -67.38^\circ} = 18.46 \text{ V} \angle -2.62^\circ$$

$$b. I_1 = \frac{E}{Z_3} = \frac{20 \text{ V} \angle 20^\circ}{8 \Omega \angle 90^\circ} = 2.5 \text{ A} \angle -70^\circ$$

$$I_2 = \frac{E}{Z_1 + Z_2} = \frac{20 \text{ V} \angle 20^\circ}{13 \Omega \angle -67.38^\circ} = 1.54 \text{ A} \angle 87.38^\circ$$

and

$$\begin{aligned} I_s &= I_1 + I_2 \\ &= 2.5 \text{ A} \angle -70^\circ + 1.54 \text{ A} \angle 87.38^\circ \\ &= (0.86 - j 2.35) + (0.07 + j 1.54) \\ I_s &= 0.93 - j 0.81 = 1.23 \text{ A} \angle -41.05^\circ \end{aligned}$$

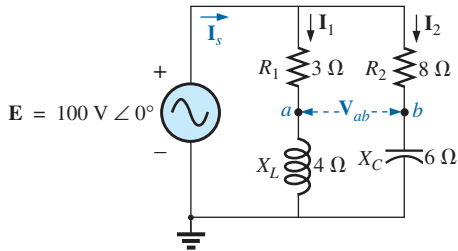


FIG. 16.7

Example 16.4.

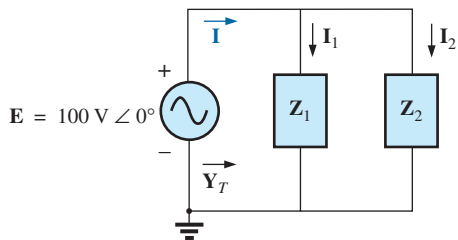


FIG. 16.8

Network in Fig. 16.7 after assigning the block impedances.

**EXAMPLE 16.4** For Fig. 16.7:

- a. Calculate the current  $I_s$ .
- b. Find the voltage  $V_{ab}$ .

**Solutions:**

a. Redrawing the circuit as in Fig. 16.8, we obtain

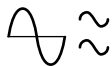
$$\begin{aligned} Z_1 &= R_1 + j X_L = 3 \Omega + j 4 \Omega = 5 \Omega \angle 53.13^\circ \\ Z_2 &= R_2 - j X_C = 8 \Omega - j 6 \Omega = 10 \Omega \angle -36.87^\circ \end{aligned}$$

In this case the voltage  $V_{ab}$  is lost in the redrawn network, but the currents  $I_1$  and  $I_2$  remain defined for future calculations necessary to determine  $V_{ab}$ . Fig. 16.8 clearly reveals that the total impedance can be found using the equation for two parallel impedances:

$$\begin{aligned} Z_T &= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(5 \Omega \angle 53.13^\circ)(10 \Omega \angle -36.87^\circ)}{(3 \Omega + j 4 \Omega) + (8 \Omega - j 6 \Omega)} \\ &= \frac{50 \Omega \angle 16.26^\circ}{11 - j 2} = \frac{50 \Omega \angle 16.26^\circ}{11.18 \angle -10.30^\circ} \\ &= 4.472 \Omega \angle 26.56^\circ \end{aligned}$$

and

$$I_s = \frac{E}{Z_T} = \frac{100 \text{ V} \angle 0^\circ}{4.472 \Omega \angle 26.56^\circ} = 22.36 \text{ A} \angle -26.56^\circ$$



b. By Ohm's law,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{100 \text{ V } \angle 0^\circ}{5 \Omega \angle 53.13^\circ} = \mathbf{20 \text{ A } \angle -53.13^\circ}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{100 \text{ V } \angle 0^\circ}{10 \Omega \angle -36.87^\circ} = \mathbf{10 \text{ A } \angle 36.87^\circ}$$

Returning to Fig. 16.7, we have

$$\mathbf{V}_{R_1} = \mathbf{I}_1 \mathbf{Z}_{R_1} = (20 \text{ A } \angle -53.13^\circ)(3 \Omega \angle 0^\circ) = \mathbf{60 \text{ V } \angle -53.13^\circ}$$

$$\mathbf{V}_{R_2} = \mathbf{I}_2 \mathbf{Z}_{R_2} = (10 \text{ A } \angle +36.87^\circ)(8 \Omega \angle 0^\circ) = \mathbf{80 \text{ V } \angle +36.87^\circ}$$

Instead of using the two steps just shown, we could have determined  $\mathbf{V}_{R_1}$  or  $\mathbf{V}_{R_2}$  in one step using the voltage divider rule:

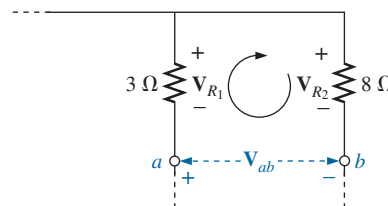
$$\mathbf{V}_{R_1} = \frac{(3 \Omega \angle 0^\circ)(100 \text{ V } \angle 0^\circ)}{3 \Omega \angle 0^\circ + 4 \Omega \angle 90^\circ} = \frac{300 \text{ V } \angle 0^\circ}{5 \angle 53.13^\circ} = \mathbf{60 \text{ V } \angle -53.13^\circ}$$

To find  $\mathbf{V}_{ab}$ , Kirchhoff's voltage law must be applied around the loop (Fig. 16.9) consisting of the  $3 \Omega$  and  $8 \Omega$  resistors. By Kirchhoff's voltage law,

$$\mathbf{V}_{ab} + \mathbf{V}_{R_1} - \mathbf{V}_{R_2} = 0$$

or

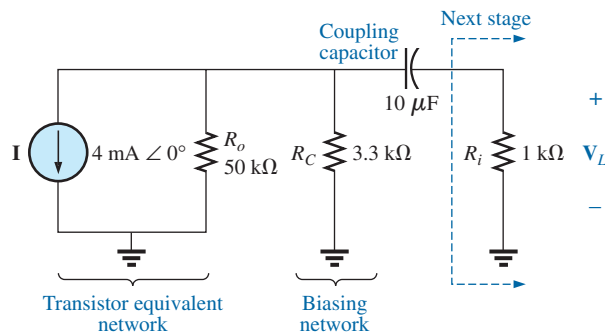
$$\begin{aligned} \mathbf{V}_{ab} &= \mathbf{V}_{R_2} - \mathbf{V}_{R_1} \\ &= 80 \text{ V } \angle 36.87^\circ - 60 \text{ V } \angle -53.13^\circ \\ &= (64 + j48) - (36 - j48) \\ &= 28 + j96 \\ \mathbf{V}_{ab} &= \mathbf{100 \text{ V } \angle 73.74^\circ} \end{aligned}$$



**FIG. 16.9**

Determining the voltage  $\mathbf{V}_{ab}$  for the network in Fig. 16.7.

**EXAMPLE 16.5** The network in Fig. 16.10 is frequently encountered in the analysis of transistor networks. The transistor equivalent circuit includes a current source  $\mathbf{I}$  and an output impedance  $R_o$ . The resistor  $R_C$  is a biasing resistor to establish specific dc conditions, and the resistor  $R_i$  represents the loading of the next stage. The coupling capacitor is designed to be an open circuit for dc and to have as low an impedance as possible for the frequencies of interest to ensure that  $\mathbf{V}_L$  is a maximum value. The frequency range of the example includes the entire audio (hearing) spectrum from 100 Hz to 20 kHz. The purpose of the example is to demonstrate that, for the full audio range, the effect of the capacitor can be ignored. It performs its function as a dc blocking agent but permits the ac to pass through with little disturbance.



**FIG. 16.10**

Basic transistor amplifier.

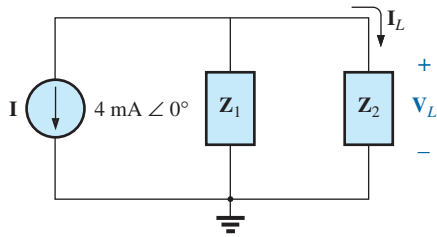
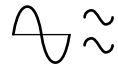


FIG. 16.11

Network in Fig. 16.10 following the assignment of the block impedances.

- Determine  $V_L$  for the network in Fig. 16.10 at a frequency of 100 Hz.
- Repeat part (a) at a frequency of 20 kHz.
- Compare the results of parts (a) and (b).

**Solutions:**

- The network is redrawn with subscripted impedances in Fig. 16.11.

$$\mathbf{Z}_1 = 50 \text{ k}\Omega \angle 0^\circ \parallel 3.3 \text{ k}\Omega \angle 0^\circ = 3.096 \text{ k}\Omega \angle 0^\circ$$

$$\mathbf{Z}_2 = R_i - jX_C$$

$$\text{At } f = 100 \text{ Hz: } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100 \text{ Hz})(10 \mu\text{F})} = 159.16 \Omega$$

$$\text{and } \mathbf{Z}_2 = 1 \text{ k}\Omega - j159.16 \Omega$$

Current divider rule:

$$\begin{aligned} \mathbf{I}_L &= \frac{-\mathbf{Z}_1 \mathbf{I}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{-(3.096 \text{ k}\Omega \angle 0^\circ)(4 \text{ mA} \angle 0^\circ)}{3.096 \text{ k}\Omega + 1 \text{ k}\Omega - j159.16 \Omega} \\ &= \frac{-12.384 \text{ A} \angle 0^\circ}{4096 - j159.16} = \frac{-12.384 \text{ A} \angle 0^\circ}{4099 \angle -2.225^\circ} \\ &= -3.02 \text{ mA} \angle 2.23^\circ = 3.02 \text{ mA} \angle 2.23^\circ + 180^\circ = 3.02 \text{ mA} \angle 182.23^\circ \end{aligned}$$

$$\begin{aligned} \text{and } \mathbf{V}_L &= \mathbf{I}_L \mathbf{Z}_R \\ &= (3.02 \text{ mA} \angle 182.23^\circ)(1 \text{ k}\Omega \angle 0^\circ) \\ &= \mathbf{3.02 \text{ V} \angle 182.23^\circ} \end{aligned}$$

$$\text{b. At } f = 20 \text{ kHz: } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(20 \text{ kHz})(10 \mu\text{F})} = 0.796 \Omega$$

Note the dramatic change in  $X_C$  with frequency. Obviously, the higher the frequency, the better the short-circuit approximation for  $X_C$  for ac conditions.

$$\mathbf{Z}_2 = 1 \text{ k}\Omega - j0.796 \Omega$$

Current divider rule:

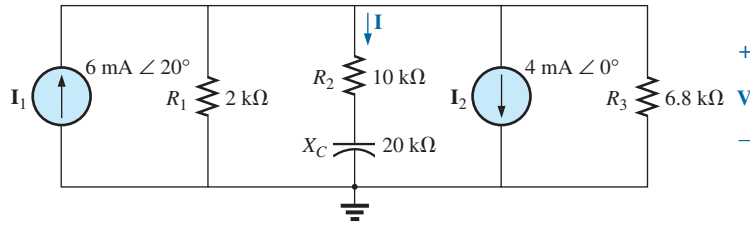
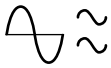
$$\begin{aligned} \mathbf{I}_L &= \frac{-\mathbf{Z}_1 \mathbf{I}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{-(3.096 \text{ k}\Omega \angle 0^\circ)(4 \text{ mA} \angle 0^\circ)}{3.096 \text{ k}\Omega + 1 \text{ k}\Omega - j0.796 \Omega} \\ &= \frac{-12.384 \text{ A} \angle 0^\circ}{4096 - j0.796 \Omega} = \frac{-12.384 \text{ A} \angle 0^\circ}{4096 \angle -0.011^\circ} \\ &= -3.02 \text{ mA} \angle 0.01^\circ = 3.02 \text{ mA} \angle 0.01^\circ + 180^\circ = 3.02 \text{ mA} \angle 180.01^\circ \end{aligned}$$

$$\begin{aligned} \text{and } \mathbf{V}_L &= \mathbf{I}_L \mathbf{Z}_R \\ &= (3.02 \text{ mA} \angle 180.01^\circ)(1 \text{ k}\Omega \angle 0^\circ) \\ &= \mathbf{3.02 \text{ V} \angle 180.01^\circ} \end{aligned}$$

- The results clearly indicate that the capacitor had little effect on the frequencies of interest. In addition, note that most of the supply current reached the load for the typical parameters employed.

**EXAMPLE 16.6** For the network in Fig. 16.12:

- Determine the current  $\mathbf{I}$ .
- Find the voltage  $\mathbf{V}$ .



**FIG. 16.12**  
Example 16.6.

**Solutions:**

- a. The rules for parallel current sources are the same for dc and ac networks. That is, the equivalent current source is their sum or difference (as phasors). Therefore,

$$\begin{aligned} \mathbf{I}_T &= 6 \text{ mA } \angle 20^\circ - 4 \text{ mA } \angle 0^\circ \\ &= 5.638 \text{ mA} + j 2.052 \text{ mA} - 4 \text{ mA} \\ &= 1.638 \text{ mA} + j 2.052 \text{ mA} \\ &= 2.626 \text{ mA } \angle 51.402^\circ \end{aligned}$$

Redrawing the network using block impedances results in the network in Fig. 16.13 where

$$\mathbf{Z}_1 = 2 \text{ k}\Omega \angle 0^\circ \parallel 6.8 \text{ k}\Omega \angle 0^\circ = 1.545 \text{ k}\Omega \angle 0^\circ$$

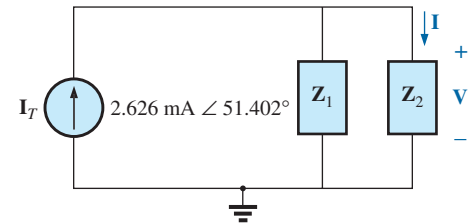
and  $\mathbf{Z}_2 = 10 \text{ k}\Omega - j 20 \text{ k}\Omega = 22.361 \text{ k}\Omega \angle -63.435^\circ$

Note that  $\mathbf{I}$  and  $\mathbf{V}$  are still defined in Fig. 16.13.

Current divider rule:

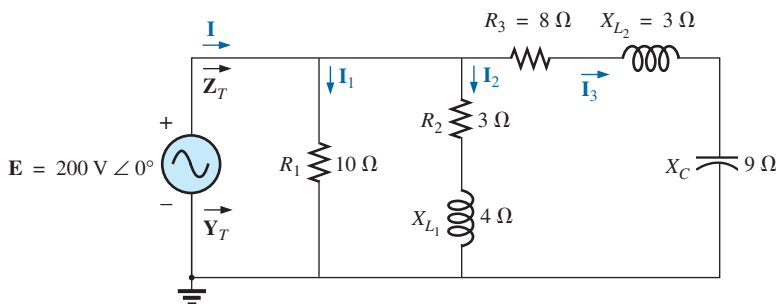
$$\begin{aligned} \mathbf{I} &= \frac{\mathbf{Z}_1 \mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(1.545 \text{ k}\Omega \angle 0^\circ)(2.626 \text{ mA } \angle 51.402^\circ)}{1.545 \text{ k}\Omega + 10 \text{ k}\Omega - j 20 \text{ k}\Omega} \\ &= \frac{4.057 \text{ A } \angle 51.402^\circ}{11.545 \times 10^3 - j 20 \times 10^3} = \frac{4.057 \text{ A } \angle 51.402^\circ}{23.093 \times 10^3 \angle -60.004^\circ} \\ &= \mathbf{0.18 \text{ mA } \angle 111.41^\circ} \end{aligned}$$

- b.  $\mathbf{V} = \mathbf{I} \mathbf{Z}_2$   
 $= (0.176 \text{ mA } \angle 111.406^\circ)(22.36 \text{ k}\Omega \angle -63.435^\circ)$   
 $= \mathbf{3.94 \text{ V } \angle 47.97^\circ}$



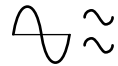
**FIG. 16.13**  
Network in Fig. 16.12 following the assignment of the subscripted impedances.

**EXAMPLE 16.7** For the network in Fig. 16.14:



**FIG. 16.14**  
Example 16.7.





- Compute  $\mathbf{I}$ .
- Find  $\mathbf{I}_1$ ,  $\mathbf{I}_2$ , and  $\mathbf{I}_3$ .
- Verify Kirchhoff's current law by showing that

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

- Find the total impedance of the circuit.

### Solutions:

- Redrawing the circuit as in Fig. 16.15 reveals a strictly parallel network where

$$\mathbf{Z}_1 = R_1 = 10 \Omega \angle 0^\circ$$

$$\mathbf{Z}_2 = R_2 + jX_{L_1} = 3 \Omega + j4 \Omega$$

$$\mathbf{Z}_3 = R_3 + jX_{L_2} - jX_C = 8 \Omega + j3 \Omega - j9 \Omega = 8 \Omega - j6 \Omega$$

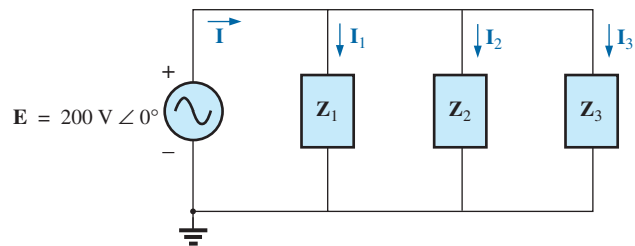


FIG. 16.15

Network in Fig. 16.14 following the assignment of the subscripted impedances.

The total admittance is

$$\begin{aligned} \mathbf{Y}_T &= \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 \\ &= \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{10 \Omega} + \frac{1}{3 \Omega + j4 \Omega} + \frac{1}{8 \Omega - j6 \Omega} \\ &= 0.1 \text{ S} + \frac{1}{5 \Omega \angle 53.13^\circ} + \frac{1}{10 \Omega \angle -36.87^\circ} \\ &= 0.1 \text{ S} + 0.2 \text{ S} \angle -53.13^\circ + 0.1 \text{ S} \angle 36.87^\circ \\ &= 0.1 \text{ S} + 0.12 \text{ S} - j0.16 \text{ S} + 0.08 \text{ S} + j0.06 \text{ S} \\ &= 0.3 \text{ S} - j0.1 \text{ S} = 0.316 \text{ S} \angle -18.435^\circ \end{aligned}$$

**Calculator** The above mathematical exercise presents an excellent opportunity to demonstrate the power of some of today's calculators. Using the TI-89, the above operation is as shown in Fig. 16.16.

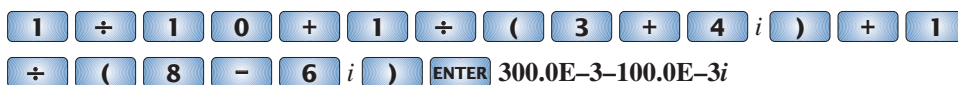
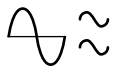


FIG. 16.16

Finding the total admittance for the network in Fig. 16.14 using the TI-89 calculator:

Be sure to use the negative sign for the complex number from the subtraction option and not the sign selection (-). The sign selection is used for negative angles in the polar form.





Converting to polar form requires the sequence shown in Fig. 16.17.



FIG. 16.17

Converting the rectangular form in Fig. 16.16 to polar form.

Converting to polar form:

The current  $\mathbf{I}$ :

$$\begin{aligned}\mathbf{I} &= \mathbf{E}\mathbf{Y}_T = (200 \text{ V } \angle 0^\circ)(0.326 \text{ S } \angle -18.435^\circ) \\ &= \mathbf{63.2 \text{ A } \angle -18.44^\circ}\end{aligned}$$

b. Since the voltage is the same across parallel branches,

$$\begin{aligned}\mathbf{I}_1 &= \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{200 \text{ V } \angle 0^\circ}{10 \Omega \angle 0^\circ} = \mathbf{20 \text{ A } \angle 0^\circ} \\ \mathbf{I}_2 &= \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{200 \text{ V } \angle 0^\circ}{5 \Omega \angle 53.13^\circ} = \mathbf{40 \text{ A } \angle -53.13^\circ} \\ \mathbf{I}_3 &= \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{200 \text{ V } \angle 0^\circ}{10 \Omega \angle -36.87^\circ} = \mathbf{20 \text{ A } \angle +36.87^\circ}\end{aligned}$$

c.  $\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$

$$\begin{aligned}60 - j20 &= 20 \angle 0^\circ + 40 \angle -53.13^\circ + 20 \angle +36.87^\circ \\ &= (20 + j0) + (24 - j32) + (16 + j12) \\ 60 - j20 &= 60 - j20 \quad (\text{checks})\end{aligned}$$

d.  $\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.316 \text{ S } \angle -18.435^\circ}$

$$= \mathbf{3.17 \Omega \angle 18.44^\circ}$$

**EXAMPLE 16.8** For the network in Fig. 16.18:

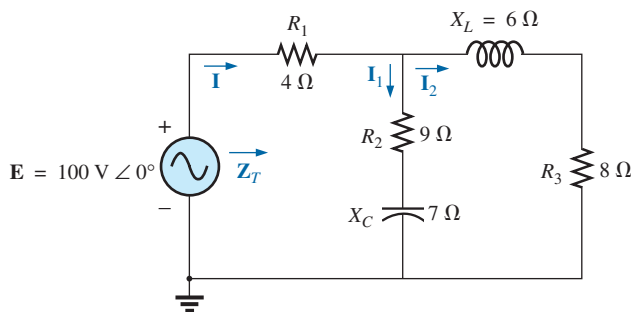
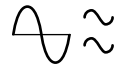


FIG. 16.18

Example 16.8.

- Calculate the total impedance  $\mathbf{Z}_T$ .
- Compute  $\mathbf{I}$ .
- Find the total power factor.
- Calculate  $\mathbf{I}_1$  and  $\mathbf{I}_2$ .
- Find the average power delivered to the circuit.



### Solutions:

a. Redrawing the circuit as in Fig. 16.19, we have

$$\mathbf{Z}_1 = R_1 = 4 \Omega \angle 0^\circ$$

$$\mathbf{Z}_2 = R_2 - jX_C = 9 \Omega - j7 \Omega = 11.40 \Omega \angle -37.87^\circ$$

$$\mathbf{Z}_3 = R_3 + jX_L = 8 \Omega + j6 \Omega = 10 \Omega \angle +36.87^\circ$$

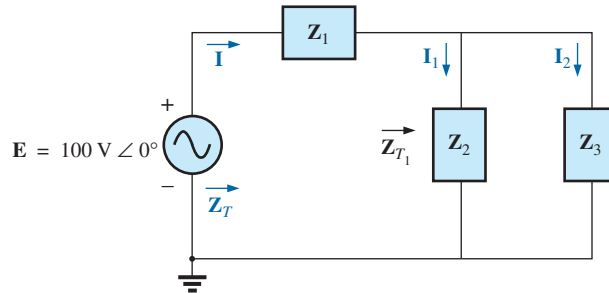


FIG. 16.19

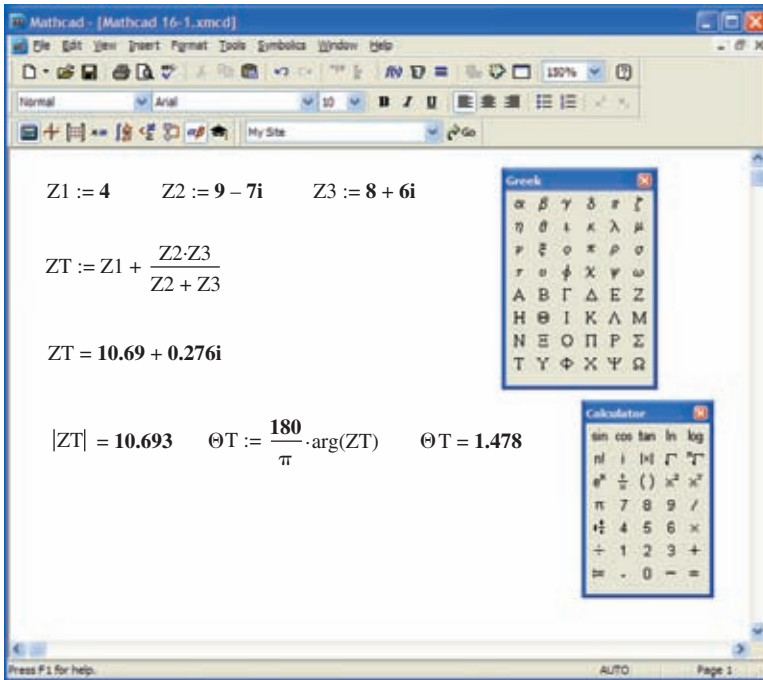
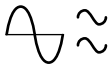
Network in Fig. 16.18 following the assignment of the subscripted impedances.

Notice that all the desired quantities were conserved in the redrawn network. The total impedance:

$$\begin{aligned} \mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_{T_1} \\ &= \mathbf{Z}_1 + \frac{\mathbf{Z}_2 \mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} \\ &= 4 \Omega + \frac{(11.4 \Omega \angle -37.87^\circ)(10 \Omega \angle 36.87^\circ)}{(9 \Omega - j7 \Omega) + (8 \Omega + j6 \Omega)} \\ &= 4 \Omega + \frac{114 \Omega \angle -1.00^\circ}{17.03 \angle -3.37^\circ} = 4 \Omega + 6.69 \Omega \angle 2.37^\circ \\ &= 4 \Omega + 6.68 \Omega + j0.28 \Omega = 10.68 \Omega + j0.28 \Omega \\ \mathbf{Z}_T &= \mathbf{10.68 \Omega \angle 1.5^\circ} \end{aligned}$$

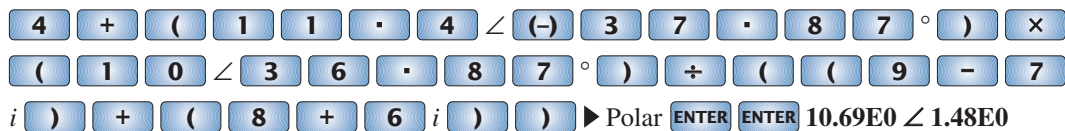
**Mathcad Solution:** The complex algebra just presented in detail provides an excellent opportunity to practice our Mathcad skills with complex numbers. Remember that the  $j$  must follow the numerical value of the imaginary part and **is not** multiplied by the numerical value. Simply type in the numerical value and then  $j$ . Also recall that unless you make a global change in the format, an  $i$  will appear with the imaginary part of the solution. As shown in Fig. 16.20, first define each impedance with **Shift:**. Then enter each impedance in sequence on the same line or succeeding lines. Next, define the equation for the total impedance using the brackets to ensure that the bottom summation is carried out before the division and also to provide the same format to the equation as appearing above. Then enter **ZT**, select the equal sign key, and the rectangular form for the total impedance appears as shown.

The polar form can be obtained by first going to the **Calculator** toolbar to obtain the magnitude operation and inserting **ZT** as shown in Fig. 16.20. Then selecting the equal sign results in the magnitude of  $10.693 \Omega$ . The angle is obtained by first going to the **Greek** toolbar and picking up theta, entering **T**, and defining the variable. The  $\pi$  comes from the **Calculator** toolbar, and the **arg()** from **Insert-f(x)-Function Name-arg**. Finally, enter the variable again and select the equal sign to obtain an angle of  $1.478^\circ$ . The computer solution of  $10.693 \Omega \angle 1.478^\circ$  is an excellent verification of the theoretical solution of  $10.68 \Omega \angle 1.5^\circ$ .


**FIG. 16.20**

Using Mathcad to determine the total impedance for the network in Fig. 16.18.

**Calculator** Another opportunity to demonstrate the versatility of the calculator! For the above operation, however, you must be aware of the priority of the mathematical operations, as demonstrated in the calculator display in Fig. 16.21. In most cases, the operations are performed in the same order they would be if you wrote them longhand.


**FIG. 16.21**

Finding the total impedance for the network in Fig. 16.18 using the TI-89 calculator.

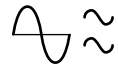
$$\text{b. } \mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100 \text{ V } \angle 0^\circ}{10.684 \Omega \angle 1.5^\circ} = \mathbf{9.36 \text{ A } \angle -1.5^\circ}$$

$$\text{c. } F_p = \cos \theta_T = \frac{R}{Z_T} = \frac{10.68 \Omega}{10.684 \Omega} \cong 1$$

(essentially resistive, which is interesting, considering the complexity of the network)

d. Current divider rule:

$$\begin{aligned} \mathbf{I}_2 &= \frac{\mathbf{Z}_2 \mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(11.40 \Omega \angle -37.87^\circ)(9.36 \text{ A } \angle -1.5^\circ)}{(9 \Omega - j 7 \Omega) + (8 \Omega + j 6 \Omega)} \\ &= \frac{106.7 \text{ A } \angle -39.37^\circ}{17 - j 1} = \frac{106.7 \text{ A } \angle -39.37^\circ}{17.03 \angle -3.37^\circ} \\ \mathbf{I}_2 &= \mathbf{6.27 \text{ A } \angle -36^\circ} \end{aligned}$$



Applying Kirchoff's current law (rather than another application of the current divider rule) yields

$$\mathbf{I}_1 = \mathbf{I} - \mathbf{I}_2$$

$$\begin{aligned} \text{or } \mathbf{I} &= \mathbf{I}_1 + \mathbf{I}_2 \\ &= (9.36 \text{ A } \angle -1.5^\circ) + (6.27 \text{ A } \angle -36^\circ) \\ &= (9.36 \text{ A} - j 0.25 \text{ A}) + (5.07 \text{ A} - j 3.69 \text{ A}) \\ \mathbf{I}_1 &= 4.29 \text{ A} + j 3.44 \text{ A} = \mathbf{5.5 A } \angle \mathbf{38.72^\circ} \end{aligned}$$

$$\begin{aligned} \text{e. } P_T &= EI \cos \theta_T \\ &= (100 \text{ V})(9.36 \text{ A}) \cos 1.5^\circ \\ &= (936)(0.99966) \\ P_T &= \mathbf{935.68 \text{ W}} \end{aligned}$$

### 16.3 LADDER NETWORKS

Ladder networks were discussed in some detail in Chapter 7. This section will simply apply the first method described in Section 7.6 to the general sinusoidal ac ladder network in Fig. 16.22. The current  $\mathbf{I}_6$  is desired.

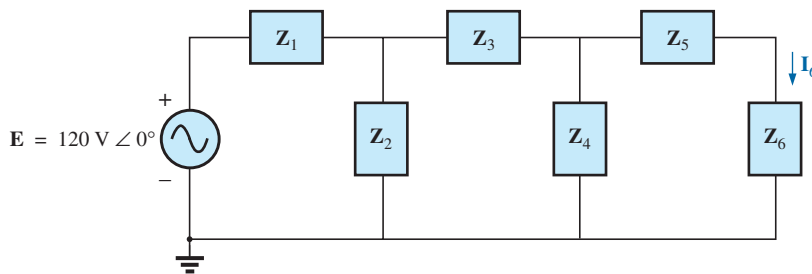


FIG. 16.22

Ladder network.

Impedances  $\mathbf{Z}_T$ ,  $\mathbf{Z}'_T$ , and  $\mathbf{Z}''_T$  and currents  $\mathbf{I}_1$  and  $\mathbf{I}_3$  are defined in Fig. 16.23.

$$\begin{aligned} \mathbf{Z}''_T &= \mathbf{Z}_5 + \mathbf{Z}_6 \\ \text{and } \mathbf{Z}'_T &= \mathbf{Z}_3 + \mathbf{Z}_4 \parallel \mathbf{Z}''_T \\ \text{with } \mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}'_T \\ \text{Then } \mathbf{I} &= \frac{\mathbf{E}}{\mathbf{Z}_T} \end{aligned}$$

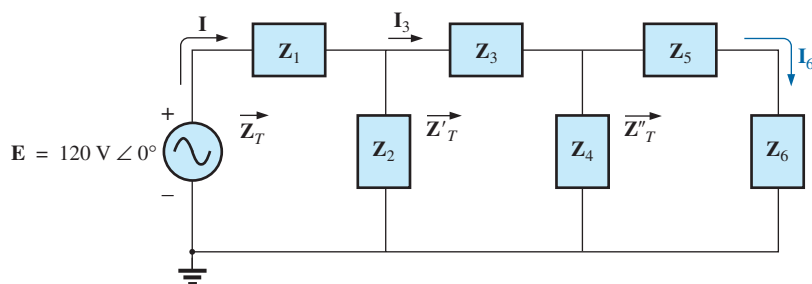
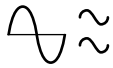


FIG. 16.23

Defining an approach to the analysis of ladder networks.



and

$$\mathbf{I}_3 = \frac{\mathbf{Z}_2 \mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}'_T}$$

with

$$\mathbf{I}_6 = \frac{\mathbf{Z}_4 \mathbf{I}_3}{\mathbf{Z}_4 + \mathbf{Z}''_T}$$

## 16.4 GROUNDING

Although usually treated too lightly in most introductory electrical or electronics texts, the impact of the ground connection and how it can provide a measure of safety to a design are very important topics. *Ground potential is 0 V at every point in a network that has a ground symbol.* Since all points are at the same potential, they can all be connected together, but for purposes of clarity most are left isolated on a large schematic. On a schematic, the voltage levels provided are always with respect to ground. A system can therefore be checked quite rapidly by simply connecting the black lead of the voltmeter to the ground connection and placing the red lead at the various points where the typical operating voltage is provided. A close match normally implies that that portion of the system is operating properly.

There are various types of grounds whose use depends on the application. An *earth ground* is one that is connected directly to the earth by a low-impedance connection. Under typical environmental conditions, *local* ground potentials are fairly uniform and can be defined as equal to zero volts. This local uniformity is due to sufficient conductive agents in the soil such as water and electrolytes to ensure that any difference in voltage on the surface is equalized by a flow of charge between the two points. However, between long distances on the earth's surface there can be significant changes in potential level. Every home has an earth ground, usually established by a long conductive rod driven into the ground and connected to the power panel. The electrical code requires a direct connection from earth ground to the cold-water pipes of a home for safety reasons. A “hot” wire touching a cold-water pipe draws sufficient current because of the low-impedance ground connection to throw the breaker. Otherwise, people in the bathroom could pick up the voltage when they touched the cold-water faucet, thereby risking bodily harm. Because water is a conductive agent, any area of the home with water, such as a bathroom or the kitchen, is of particular concern. Most electrical systems are connected to earth ground primarily for safety reasons. All the power lines in a laboratory, at industrial locations, or in the home are connected to earth ground.

A second type is referred to as a *chassis ground*, which may be *floating* or connected directly to an earth ground. A chassis ground simply stipulates that the chassis has a reference potential for all points of the network. If the chassis is not connected to earth potential (0 V), it is said to be *floating* and can have any other reference voltage for the other voltages to be compared to. For instance, if the chassis is sitting at 120 V, all measured voltages of the network will be referenced to this level. A reading of 32 V between a point in the network and the chassis ground will therefore actually be at 152 V with respect to earth potential. Most high-voltage systems are not left floating, however, because of loss of the safety factor. For instance, if someone should touch the chassis and be standing on a suitable ground, the full 120 V would fall across that individual.

Grounding can be particularly important when working with numerous pieces of measuring equipment in the laboratory. For instance, the supply and oscilloscope in Fig. 16.24(a) are each connected directly to

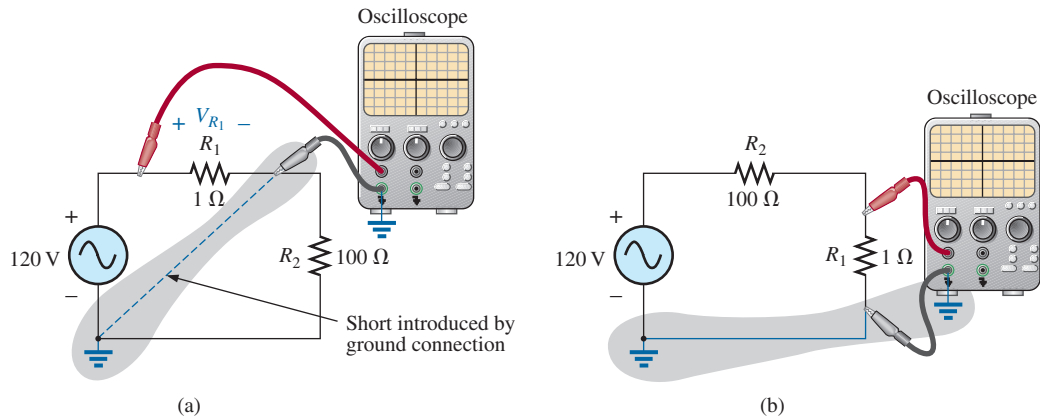
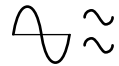


FIG. 16.24

Demonstrating the effect of the oscilloscope ground on the measurement of the voltage across resistor  $R_1$ .

an earth ground through the negative terminal of each. If the oscilloscope is connected as shown in Fig. 16.24(a) to measure the voltage  $V_{R_1}$ , a dangerous situation will develop. The grounds of each piece of equipment are connected together through the earth ground, and they effectively short out the resistor. Since the resistor is the primary current-controlling element in the network, the current will rise to a very high level and possibly damage the instruments or cause dangerous side effects. In this case, the supply or scope should be used in the floating mode, or the resistors interchanged as shown in Fig. 16.24(b). In Fig. 16.24(b), the grounds have a common point and do not affect the structure of the network.

The National Electrical Code requires that the “hot” (or *feeder*) line that carries current to a load be *black*, and the line (called the *neutral*) that carries the current back to the supply be *white*. Three-wire conductors have a ground wire that must be *green* or *bare*, which ensures a common ground but which is not designed to carry current. The components of a three-prong extension cord and wall outlet are shown in Fig. 16.25. Note that on both fixtures, the connection to the hot lead is smaller than the return leg and that the ground connection is partially circular.

The complete wiring diagram for a household outlet is shown in Fig. 16.26. Note that the current through the ground wire is zero and that both

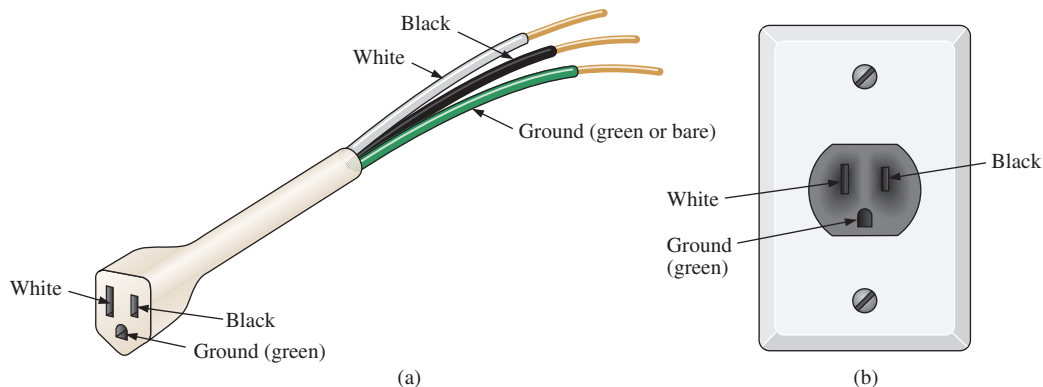
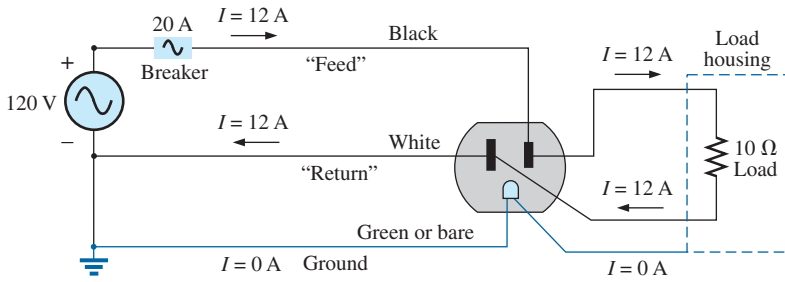
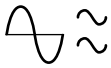


FIG. 16.25

Three-wire conductors: (a) extension cord; (b) home outlet.

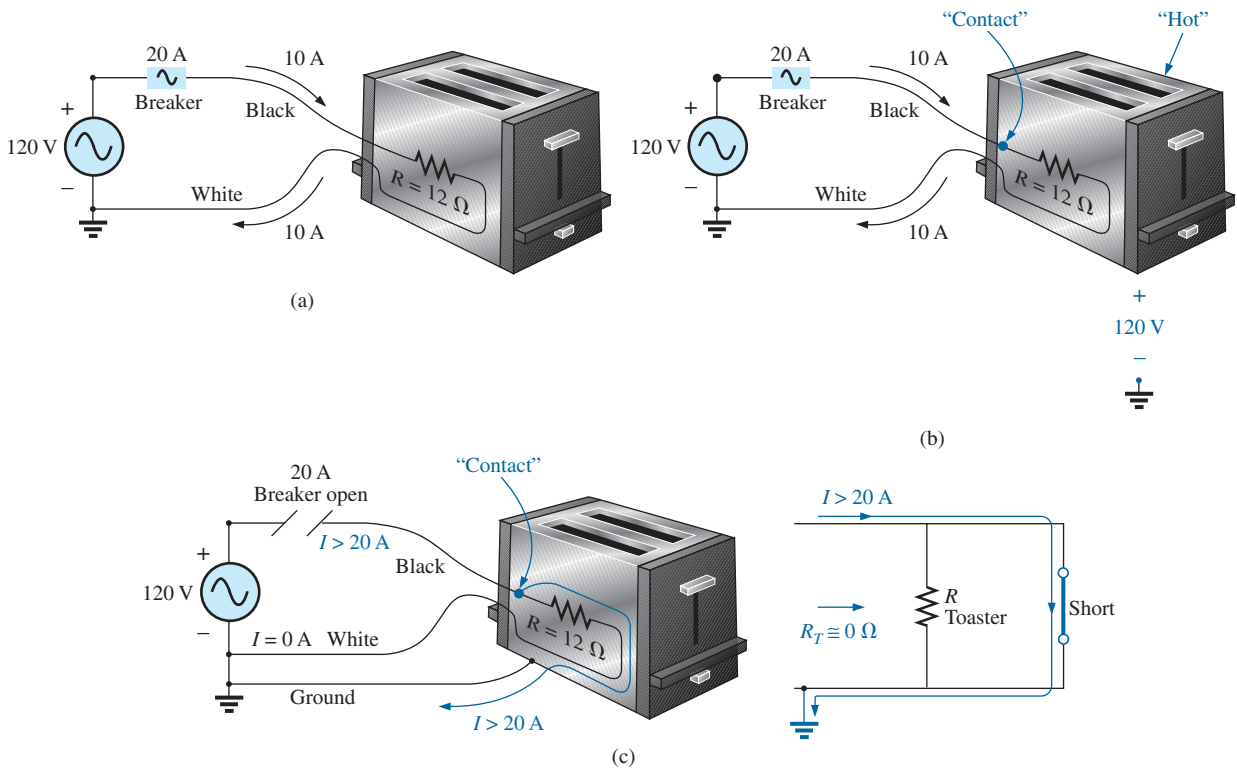


**FIG. 16.26**

Complete wiring diagram for a household outlet with a 10 Ω load.

the return wire and the ground wire are connected to an earth ground. The full current to the loads flows through the feeder and return lines.

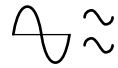
The importance of the ground wire in a three-wire system can be demonstrated by the toaster in Fig. 16.27, rated 1200 W at 120 V. From the power equation  $P = EI$ , the current drawn under normal operating conditions is  $I = P/E = 1200 \text{ W}/120 \text{ V} = 10 \text{ A}$ . If a two-wire line were used as shown in Fig. 16.27(a), the 20 A breaker would be quite comfortable with the 10 A current, and the system would perform normally. However, if abuse to the feeder caused it to become frayed and to touch the metal housing of the toaster, the situation depicted in Fig. 16.27(b) would result. The housing would become “hot,” yet the breaker would not “pop” because the current would still be the rated 10 A. A dangerous condition would exist because anyone touching the toaster would feel the



**FIG. 16.27**

Demonstrating the importance of a properly grounded appliance: (a) ungrounded; (b) ungrounded and undesirable contact; (c) grounded appliance with undesirable contact.





full 120 V to ground. If the ground wire were attached to the chassis as shown in Fig. 16.27(c), a low-resistance path would be created between the short-circuit point and ground, and the current would jump to very high levels. The breaker would “pop,” and the user would be warned that a problem exists.

Although the above discussion does not cover all possible areas of concern with proper grounding or introduce all the nuances associated with the effect of grounds on a system’s performance, you should understand the importance of its impact.

## 16.5 APPLICATIONS

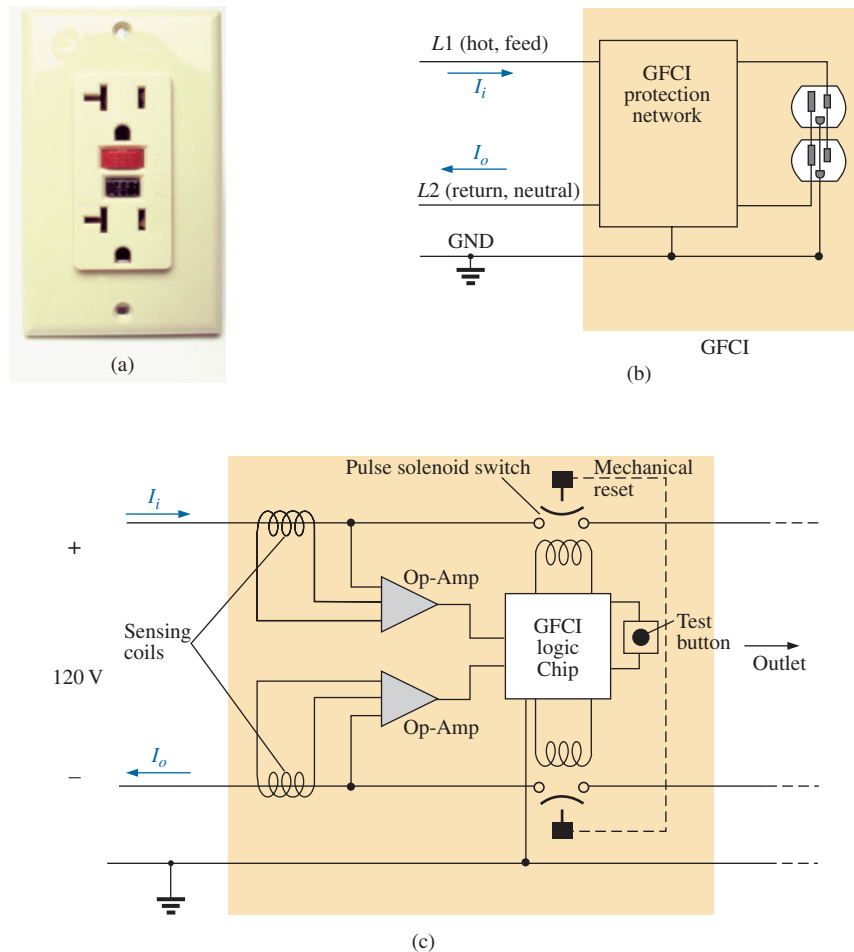
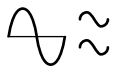
The vast majority of the applications appearing throughout the text have been of the series-parallel variety. The following are series-parallel combinations of elements and systems used to perform important everyday tasks. The ground fault circuit interrupter (GFCI) outlet employs series protective switches and sensing coils and a parallel control system, while the ideal equivalent circuit for the coax cable employs a series-parallel combination of inductors and capacitors.

### GFCI (Ground Fault Circuit Interrupter)

The National Electric Code, the “bible” for all electrical contractors, now requires that ground fault circuit interrupter (GFCI) outlets be used in any area where water and dampness could result in serious injury, such as in bathrooms, pools, marinas, and so on. The outlet looks like any other except that it has a reset button and a test button in the center of the unit as shown in Fig. 16.28(a). The primary difference between it and an ordinary outlet is that it will shut the power off much more quickly than the breaker all the way down in the basement could. You may still feel a shock with a GFCI outlet, but the current cuts off so quickly (in a few milliseconds) that a person in normal health should not receive a serious electrical injury. Whenever in doubt about its use, remember that its cost (relatively inexpensive) is well worth the increased measure of safety.

The basic operation is best described by the network in Fig. 16.28(b). The protection circuit separates the power source from the outlet itself. Note in Fig. 16.28(b) the importance of grounding the protection circuit to the central ground of the establishment (a water pipe, ground bar, and so on, connected to the main panel). In general, the outlet will be grounded to the same connection. Basically, the network shown in Fig. 16.28(b) senses both the current entering ( $I_i$ ) and the current leaving ( $I_o$ ) and provides a direct connection to the outlet when they are equal. If a fault should develop such as that caused by someone touching the hot leg while standing on a wet floor, the return current will be less than the feed current (just a few milliamperes is enough). The protection circuitry senses this difference, establishes an open circuit in the line, and cuts off the power to the outlet.

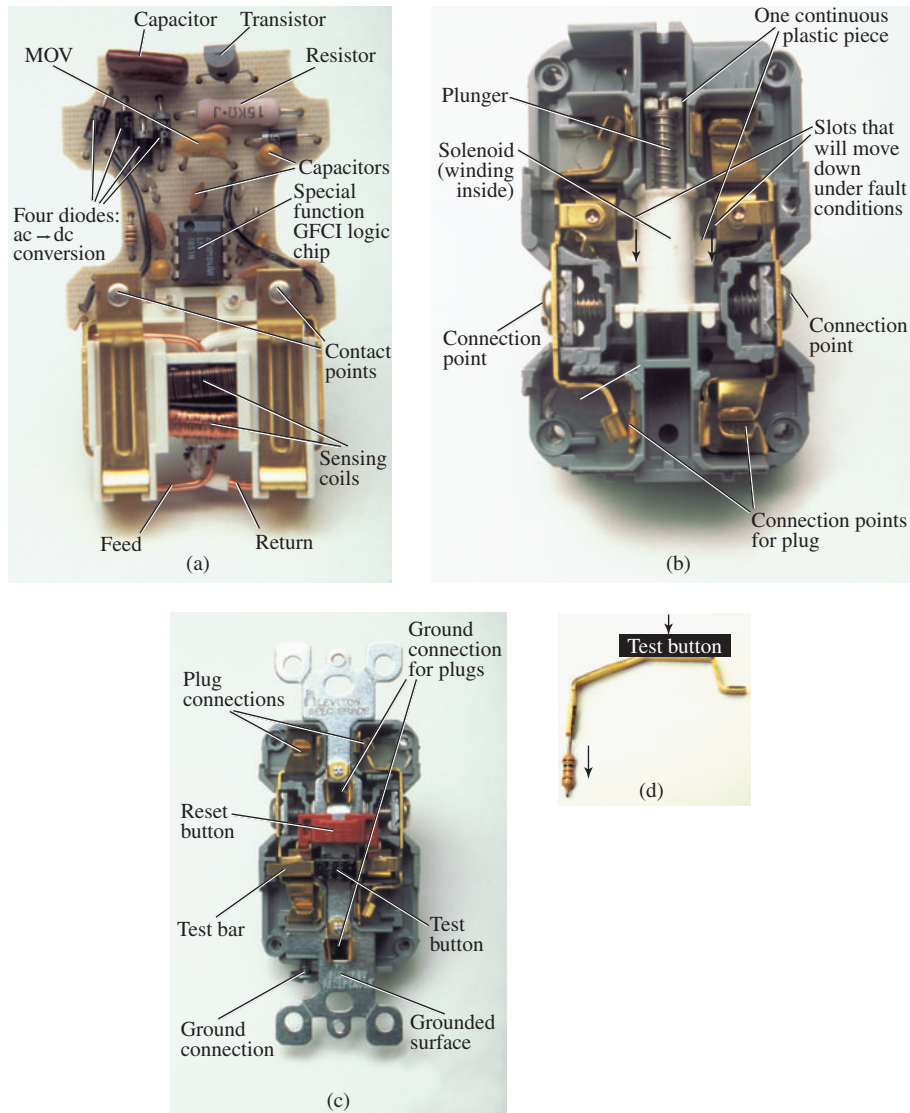
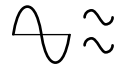
Fig. 16.29(a) shows the feed and return lines passing through the sensing coils. The two sensing coils are separately connected to the printed circuit board. There are two pulse control switches in the line and a return to establish an open circuit under errant conditions. The two contacts in Fig. 16.29(a) are the contacts that provide conduction to the outlet. When a fault develops, another set of similar contacts in the housing slides away, providing the desired open-circuit condition. The separation is created by the solenoid appearing in Fig. 16.29(b). When the solenoid is energized due to a fault condition, it pulls the plunger toward the solenoid, compressing the

**FIG. 16.28**

*GFCI outlet: (a) wall-mounted appearance; (b) basic operation; (c) schematic.*

spring. At the same time, the slots in the lower plastic piece (connected directly to the plunger) shift down, causing a disconnect by moving the structure inserted in the slots. The test button is connected to the brass bar across the unit in Fig. 16.29(c) below the reset button. When pressed, it places a large resistor between the line and ground to “unbalance” the line and cause a fault condition. When the button is released, the resistor is separated from the line, and the unbalance condition is removed. The resistor is actually connected directly to one end of the bar and moves down with pressure on the bar as shown in Fig. 16.29(d). Note in Fig. 16.29(c) how the metal ground connection passes right through the entire unit and how it is connected to the ground terminal of an applied plug. Also note how it is separated from the rest of the network with the plastic housing. Although this unit appears simple on the outside and is relatively small in size, it is beautifully designed and contains a great deal of technology and innovation.

Before leaving the subject, note the logic chip in the center of Fig. 16.29(a) and the various sizes of capacitors and resistors. Note also the four diodes in the upper left region of the circuit board used as a bridge rectifier for the ac-to-dc conversion process. The transistor is the black element with the half-circle appearance. It is part of the driver circuit for



**FIG. 16.29**

*GFCI construction: (a) sensing coils; (b) solenoid control (bottom view); (c) grounding (top view); (d) test bar.*

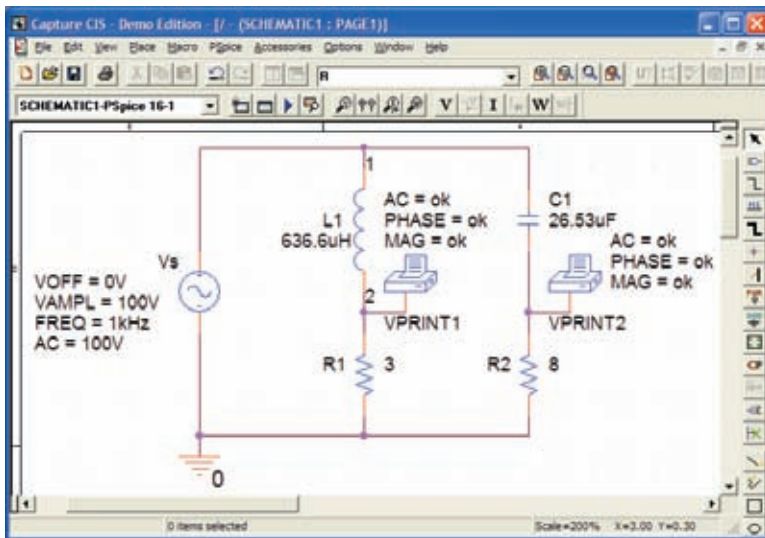
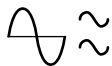
the controlling solenoid. Because of the size of the unit, there wasn't a lot of room to provide the power to quickly open the circuit. The result is the use of a pulse circuit to control the motion of the controlling solenoid. In other words, the solenoid is pulsed for a short period of time to cause the required release. If the design used a system that would hold the circuit open on a continuing basis, the power requirement would be greater and the size of the coil larger. A small coil can handle the required power pulse for a short period of time without any long-term damage.

As mentioned earlier, if unsure, install a GFCI. It provides a measure of safety—at a very reasonable cost—that should not be ignored.

## 16.6 COMPUTER ANALYSIS

### PSpice

**ac Bridge Network** We will use Example 16.4 to demonstrate the power of the **VPRINT** option in the **SPECIAL** library. It permits a di-

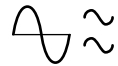
**FIG. 16.30**

Determining the voltage across  $R_1$  and  $R_2$  using the **VPRINT** option of a PSpice analysis.

rect determination of the magnitude and angle of any voltage in an ac network. Similarly, the **IPRINT** option does the same for ac currents. In Example 16.4, the ac voltages across  $R_1$  and  $R_2$  were first determined, and then Kirchhoff's voltage law was applied to determine the voltage between the two known points. Since PSpice is designed primarily to determine the voltage at a point with respect to ground, the network in Fig. 16.7 is entered as shown in Fig. 16.30 to permit a direct calculation of the voltages across  $R_1$  and  $R_2$ .

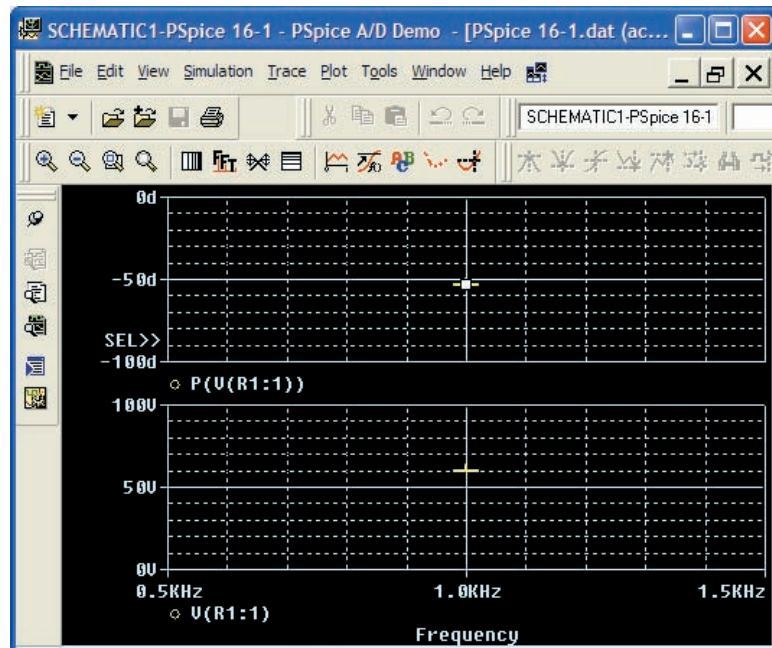
The source and network elements are entered using a procedure that has been demonstrated several times in previous chapters, although for the **AC Sweep** analysis to be performed in this example, the source must carry an **AC** level also. Fortunately, it is the same as **VAMPL** as shown in Fig. 16.30. It is introduced into the source description by double-clicking on the source symbol to obtain the **Property Editor** dialog box. Select the **AC** column and enter 100 V in the box below. Then select **Display** and choose **Name and Value**. Click **OK** followed by **Apply**, and you can exit the dialog box. The result is **AC = 100 V** added to the source description on the diagram and in the system. Using the reactance values in Fig. 16.7, the values for  $L$  and  $C$  were determined using a frequency of 1 kHz. The voltage across  $R_1$  and  $R_2$  can be determined using the **Trace** command in the same manner as described in the previous chapter or by using the **VPRINT** option. Both approaches are discussed in this section because they have application to any ac network.

The **VPRINT** option is under the **SPECIAL** library in the **Place Part** dialog box. Once selected, the printer symbol appears on the screen next to the cursor, and it can be placed near the point of interest. Once the printer symbol is in place, double-click on it to display the **Property Editor** dialog box. Scrolling from left to right, type the word **ok** under **AC**, **MAG**, and **PHASE**. When each is active, select the **Display** key and choose the option **Name and Value** followed by **OK**. When all the entries have been made, choose **Apply** and exit the dialog box. The result appears in Fig. 16.30 for the two applications of the **VPRINT** option. If you prefer, **VPRINT1** and **VPRINT2** can be added to distinguish between the two when you review the output data. To do this, return to the



**Property Editor** dialog box for each by double-clicking on the printer symbol of each and selecting **Value** and then **Display** followed by **Value Only**. You are now ready for the simulation.

The simulation is initiated by selecting the **New Simulation Profile** icon and entering **PSpice 16-1** as the **Name**. Then select **Create** to bring up the **Simulation Settings** dialog box. This time, you want to analyze the network at 1 kHz but are not interested in plots against time. Thus, select the **AC Sweep/Noise** option under **Analysis type** in the **Analysis** section. An **AC Sweep Type** region then appears asking for the **Start Frequency**. Since you are interested in the response at only one frequency, the **Start** and **End Frequency** will be the same: 1 kHz. Since you need only one point of analysis, the **Points/Decade** will be 1. Click **OK**, and select the **Run PSpice** icon. The **SCHEMATIC1** screen appears, and the voltage across  $R_1$  can be determined by selecting **Trace** followed by **Add Trace** and then **V(R1:1)**. The result is the bottom display in Fig. 16.31 with only one plot point at 1 kHz. Since you fixed the frequency of interest at 1 kHz, this is the only frequency with a response. The magnitude of the voltage across  $R_1$  is 60 V to match the longhand solution of Example 16.4. The phase angle associated with the voltage can be determined by the sequence **Plot-Add Plot to Window-Trace-Add Trace-P( )** from the **Functions or Macros** list and then **V(R1:1)** to obtain **P(V(R1:1))** in the **Trace Expression** box. Click **OK**, and the resulting plot shows that the phase angle is near just less than  $-50^\circ$  which is certainly a close match with the  $-53.13^\circ$  obtained in Example 16.4.

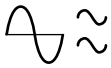


**FIG. 16.31**

The resulting magnitude and phase angle for the voltage  $V_{R_1}$  in Fig. 16.30.

The **VPRINT** option just introduced offers another method for analyzing voltage in a network. When the **SCHEMATIC1** window appears after the simulation, exit the window using the **X**, and select **PSpice** on the top menu bar of the resulting screen. Select **View Output File** from the list that appears. You will see a long list of data about the construction of the network and the results obtained from the simulation. In Fig. 16.32, the por-





```
** Profile: "SCHEMATIC1-PSpice 16-2" [ C:\ICAI1\PSpice\PSpice 16-1-
PSpiceFiles\SCHEMATIC1\PSpice 16-2.sim ]

*** AC ANALYSIS TEMPERATURE = 27.000 DEG C
*****
FREQ VM(N00879) VP(N00879)
1.000E+03 6.000E+01 -5.313E+01

*** AC ANALYSIS TEMPERATURE = 27.000 DEG C
*****
FREQ VM(N00875) VP(N00875)
1.000E+03 8.000E+01 3.687E+01
```

FIG. 16.32

The *VPRINT1* ( $V_{R_1}$ ) and *VPRINT2* ( $V_{R_2}$ ) response for the network in Fig. 16.30.

tion of the output file listing the resulting magnitude and phase angle for the voltages defined by *VPRINT1* and *VPRINT2* is provided. Note that the voltage across  $R_1$  defined by *VPRINT1* is 60 V at an angle of  $53.13^\circ$ . The voltage across  $R_2$  as defined by *VPRINT2* is 80 V at an angle of  $36.87^\circ$ . Both are exact matches of the solutions in Example 16.4. In the future, therefore, if the *VPRINT* option is used, the results will appear in the output file.

Now you can determine the voltage across the two branches from point *a* to point *b*. Return to **SCHEMATIC1**, and select **Trace** followed by **Add Trace** to obtain the list of **Simulation Output Variables**. Then, by applying Kirchhoff's voltage law around the closed loop, you find that the desired voltage is **V(R1:1)-V(R2:1)** which when followed by **OK** results in the plot point in the screen in the bottom of Fig. 16.33. Note that it is exactly 100 V as obtained in the longhand solution. Determine the phase angle through **Plot-Add Plot to Window-Trace-Add Trace**, creating the expression **P(V(R1:1)-V(R2:1))**. Remember that the expression can be generated using the lists of **Output variables** and **Functions**, but it can also be typed in from the keyboard. However, always remember that

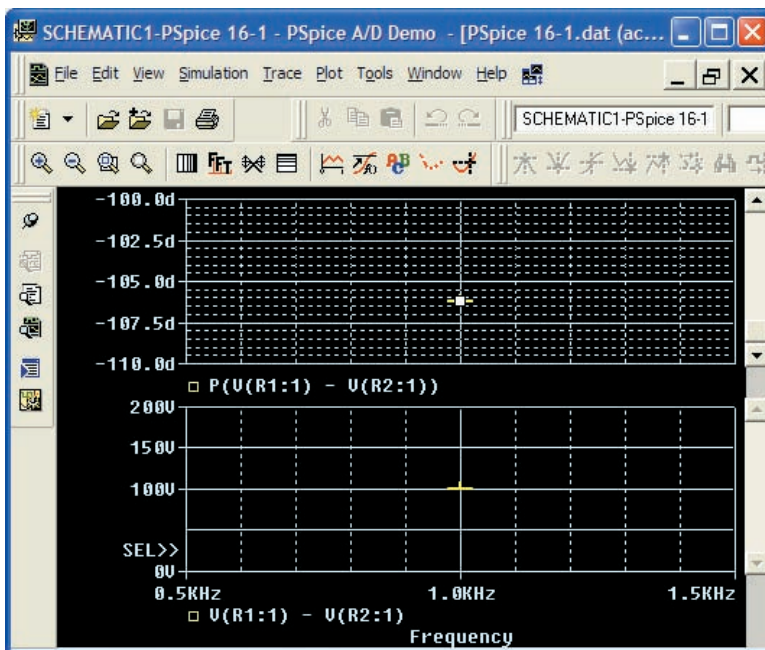
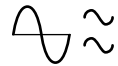


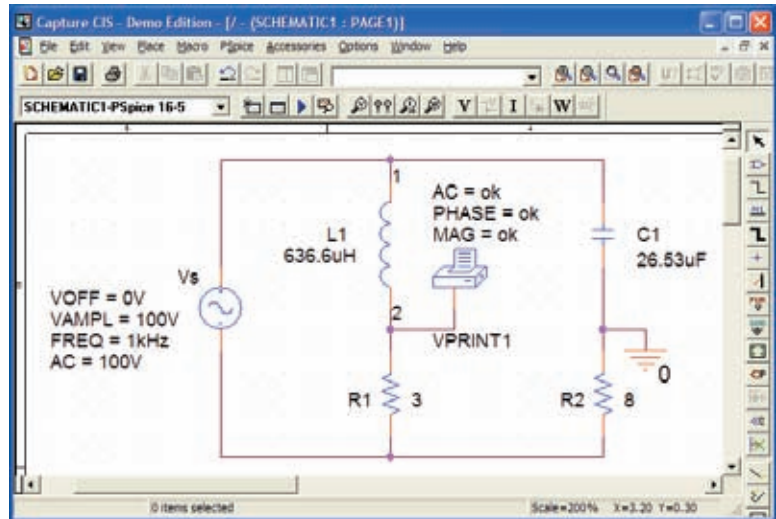
FIG. 16.33

The PSpice response for the voltage between the two points above resistors  $R_1$  and  $R_2$ .



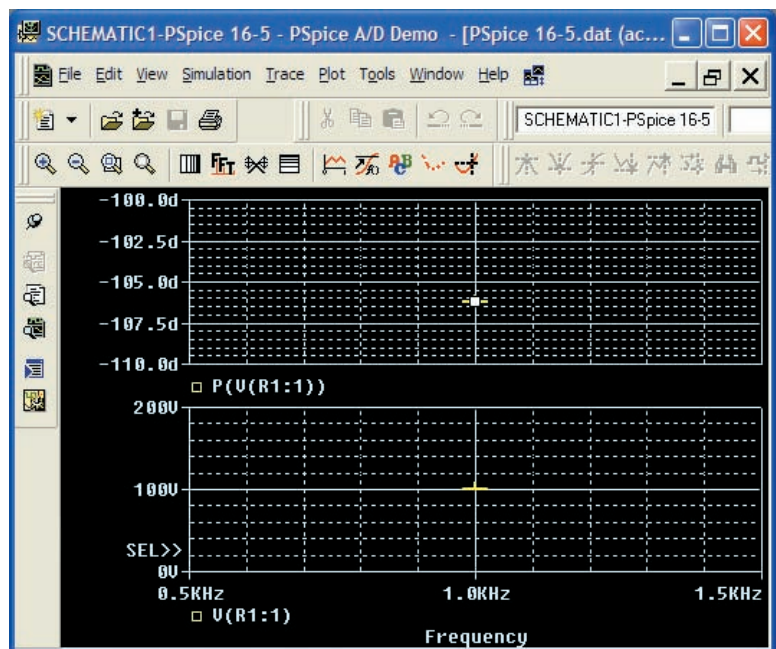
parentheses must be in sets—a left and a right. Click **OK**, and a solution near  $-105^\circ$  appears. A better reading can be obtained by using **Plot-Axis Settings-Y Axis-User Defined** and changing the scale to  $-100^\circ$  to  $-110^\circ$ . The result is the top screen in Fig. 16.33 with an angle closer to  $-106.5^\circ$  or  $73.5^\circ$  which is very close to the theoretical solution of  $73.74^\circ$ .

Finally, the last way to find the desired bridge voltage is to remove the **VPRINT2** option and place the ground at that point as shown in Fig. 16.34. Now the voltage generated from a point above  $R_1$  to ground will be the desired voltage. Repeating a full simulation results in the plot in Fig. 16.35 with the same results as Fig. 16.33. Note, however, that even



**FIG. 16.34**

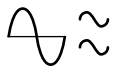
Determining the voltage between the two points above resistors  $R_1$  and  $R_2$  by moving the ground connection in Fig. 16.30 to the position of **VPRINT2**.



**FIG. 16.35**

PSpice response to the simulation of the network in Fig. 16.34.

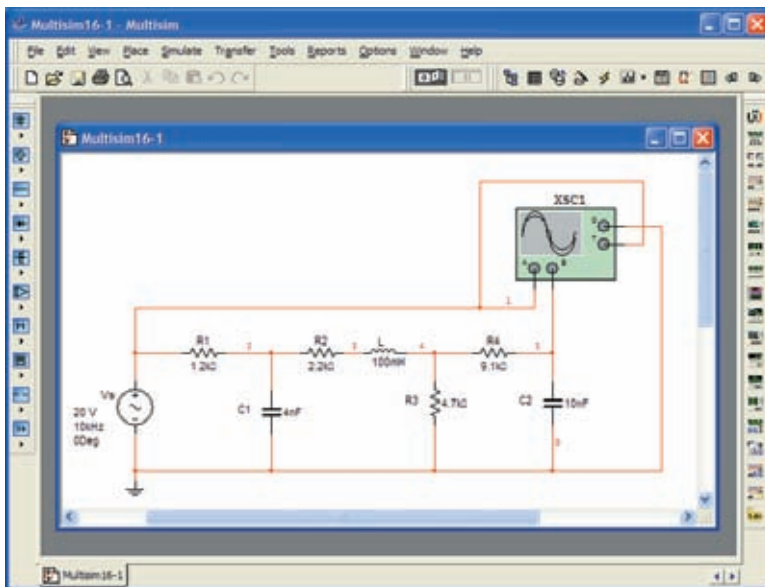




though the two figures look the same, the quantities listed in the bottom left of each plot are different.

## Multisim

Multisim is now used to determine the voltage across the last element of the ladder network in Fig. 16.36. The mathematical content of this chapter suggests that this analysis would be a lengthy exercise in complex algebra, with one mistake (a single sign or an incorrect angle) enough to invalidate the results. However, it takes only a few minutes to “draw” the network on the screen and only a few seconds to generate the results—results you can usually assume are correct if all the parameters were entered correctly. The results are certainly an excellent check against a longhand solution.



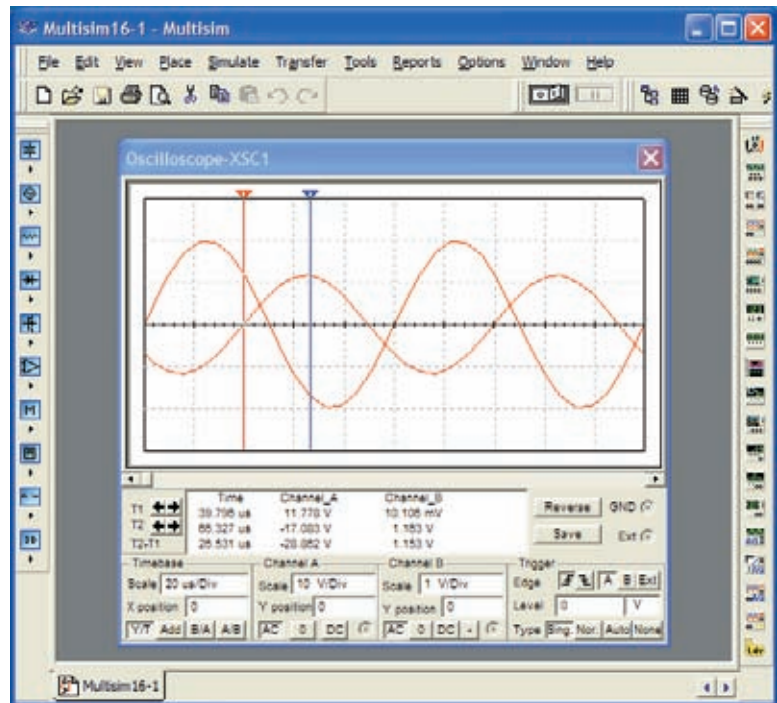
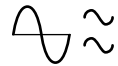
**FIG. 16.36**

Using the Multisim oscilloscope to determine the voltage across the capacitor  $C_2$ .

Our first approach is to use an oscilloscope to measure the amplitude and phase angle of the output voltage as shown in Fig. 16.36. Note that five nodes are defined, with node 5 the desired voltage. The oscilloscope settings include a **Time base** of  $20 \mu\text{s}/\text{div}$ . since the period of the 10 kHz signal is  $100 \mu\text{s}$ . Channel **A** is set on  $10 \text{ V}/\text{div}$ . so that the full 20 V of the applied signal will have a peak value encompassing two divisions. Note that **Channel A** in Fig. 16.36 is connected directly to the source  $V_s$  and to the **Trigger** input for synchronization. Expecting the output voltage to have a smaller amplitude resulted in a vertical sensitivity of  $1 \text{ V}/\text{div}$ . for **Channel B**. The analysis was initiated by placing the **Simulation** switch in the **1** position. It is important to realize that

*when simulation is initiated, it will take time for networks with reactive elements to settle down and for the response to reach its steady-state condition. It is therefore wise to let a system run for a while after simulation before selecting Sing. (Single) on the oscilloscope to obtain a steady waveform for analysis.*

The resulting plots in Fig. 16.37 clearly show that the applied voltage has an amplitude of 20 V and a period of  $100 \mu\text{s}$  (5 div. at  $20 \mu\text{s}/\text{div}$ ). The



**FIG. 16.37**

Using Multisim to display the applied voltage and voltage across the capacitor  $C_2$  for the network in Fig. 16.36.

cursors sit ready for use at the left and right edges of the screen. Clicking on the small red arrow (with number 1) at the top of the oscilloscope screen allows you to drag it to any location on the horizontal axis. As you move the cursor, the magnitude of each waveform appears in the **T1** box below. By comparing positive slopes through the origin, you should see that the applied voltage is leading the output voltage by an angle that is more than  $90^\circ$ . Setting the cursor at the point where the output voltage on channel B passes through the origin with a positive slope, you find that you cannot achieve exactly 0 V; but 0.01 V is certainly very close at  $39.8 \mu\text{s}$  (**T1**).

Knowing that the applied voltage passed through the origin at  $0 \mu\text{s}$  permits the following calculation for the phase angle:

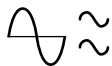
$$\frac{39.8 \mu\text{s}}{100 \mu\text{s}} = \frac{\theta}{360^\circ}$$

$$\theta = 143.28^\circ$$

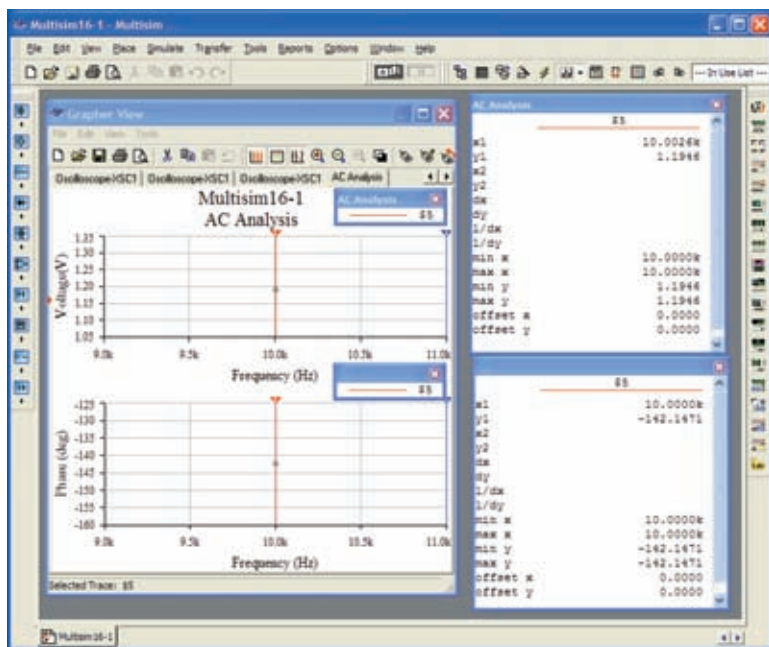
with the result that the output voltage has an angle of  $-143.28^\circ$  associated with it. The second cursor at the right edge of the screen is blue. Selecting it and moving it to the peak value of the output voltage results in 1.16 V at  $66.33 \mu\text{s}$  (**T2**). The result of all the above is

$$\mathbf{V}_{C_2} = 1.16 \text{ V} \angle -143.28^\circ$$

The second approach is to use the **AC Analysis** option under the **Simulate** heading. First, realize that when you use the oscilloscope as you just did, you did not need to pass through the sequence of dialog boxes to choose the desired analysis. All that was necessary was to simulate using either the switch or the **PSpice-Run** sequence—the oscilloscope was there



to measure the output voltage. Remember that the source defined the magnitude of the applied voltage, the frequency, and the phase shift. This time, use the sequence **Simulate-Analyses-AC Analysis** to obtain the **AC Analysis** dialog box in which the **Start** and **Stop frequencies** are 10 kHz and the **Selected variable for analysis** is node 5 only. Selecting **Simulate** results in a magnitude-phase plot with no apparent indicators at 10 kHz. However, this is easily corrected by first selecting one of the plots by clicking on the **Voltage** label at the left of the plot. Then select the **Show/Hide Grid**, **Show/Hide Legend**, and **Show/Hide Cursors** keys to obtain the cursors, legend, and **AC Analysis** dialog box. Hook on the red cursor and move it to 10 kHz. At that location, and that location only, **x1** appears as 10 kHz in the dialog box, and **y1** is  $\cong 1.19$  V as shown in Fig. 16.38. If you then select the **Phase** curve and repeat the procedure, you will find that at 10 kHz (**x1**) the angle is  $-142.15^\circ$  (**y1**) which is very close to the  $-143.28^\circ$  obtained above.


**FIG. 16.38**

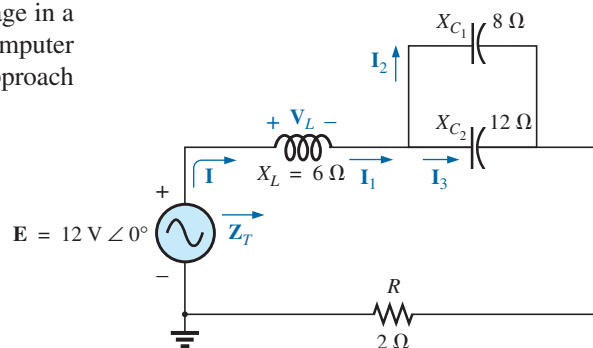
Using the **AC Analysis** option in Multisim to determine the magnitude and phase angle for the voltage  $V_{C_2}$  for the network in Fig. 16.36.

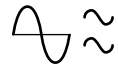
In total, therefore, you have two methods to obtain an ac voltage in a network—one by instrumentation and the other through the computer methods. Both are valid, although, as expected, the computer approach has a higher level of accuracy.

## PROBLEMS

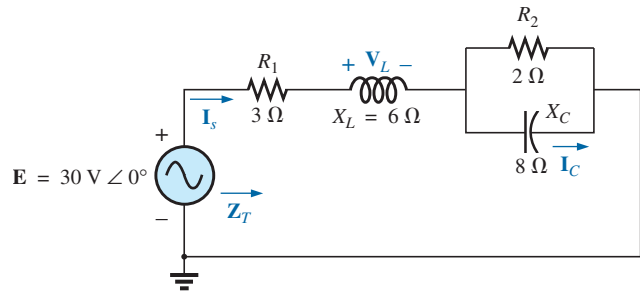
### SECTION 16.2 Illustrative Examples

1. For the series-parallel network in Fig. 16.39:
  - a. Calculate  $Z_T$ .
  - b. Determine  $I$ .
  - c. Determine  $I_1$ .
  - d. Find  $I_2$  and  $I_3$ .
  - e. Find  $V_L$ .


**FIG. 16.39**  
 Problem 1.



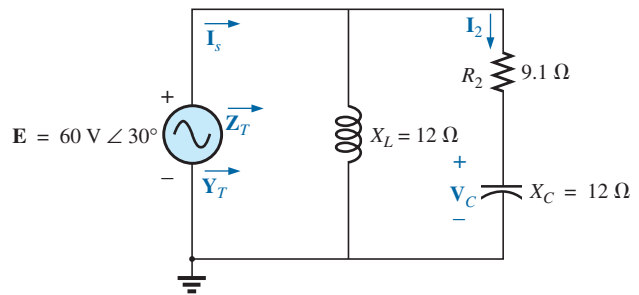
2. For the network in Fig. 16.40:
  - a. Find the total impedance  $Z_T$ .
  - b. Determine the current  $I_s$ .
  - c. Calculate  $I_C$  using the current divider rule.
  - d. Calculate  $V_L$  using the voltage divider rule.



**FIG. 16.40**

Problems 2 and 15.

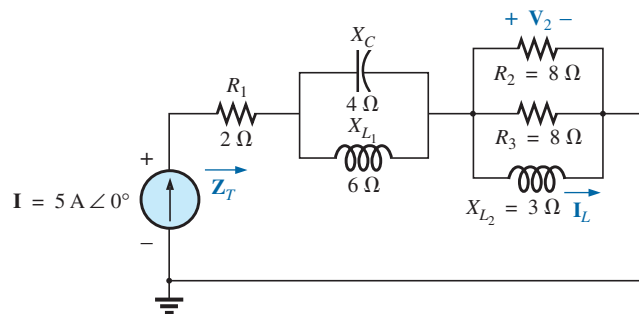
3. For the network in Fig. 16.41:
  - a. Find the total impedance  $Z_T$  and the total admittance  $Y_T$ .
  - b. Find the current  $I_s$ .
  - c. Calculate  $I_2$  using the current divider rule.
  - d. Calculate  $V_C$ .
  - e. Calculate the average power delivered to the network.



**FIG. 16.41**

Problem 3.

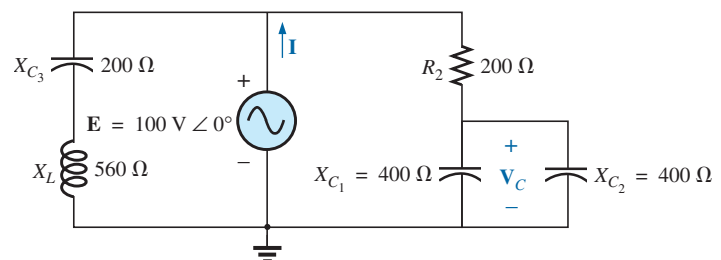
4. For the network in Fig. 16.42:
  - a. Find the total impedance  $Z_T$ .
  - b. Calculate the voltage  $V_2$  and the current  $I_L$ .
  - c. Find the power factor of the network.



**FIG. 16.42**

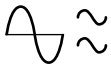
Problem 4.

5. For the network in Fig. 16.43:
  - a. Find the current  $I$ .
  - b. Find the voltage  $V_C$ .
  - c. Find the average power delivered to the network.

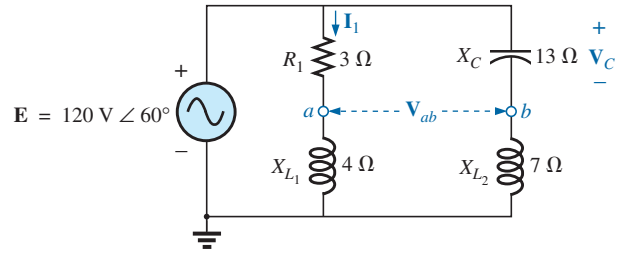


**FIG. 16.43**

Problem 5.

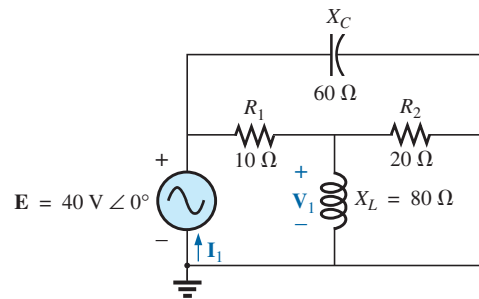


- \*6. For the network in Fig. 16.44:
- Find the current  $I_1$ .
  - Calculate the voltage  $V_C$  using the voltage divider rule.
  - Find the voltage  $V_{ab}$ .



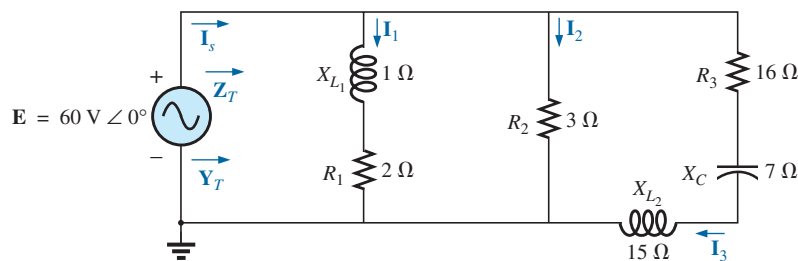
**FIG. 16.44**  
Problem 6.

- \*7. For the network in Fig. 16.45:
- Find the current  $I_1$ .
  - Find the voltage  $V_1$ .
  - Calculate the average power delivered to the network.

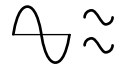


**FIG. 16.45**  
Problems 7 and 16.

8. For the network in Fig. 16.46:
- Find the total impedance  $Z_T$  and the admittance  $Y_T$ .
  - Find the currents  $I_1$ ,  $I_2$ , and  $I_3$ .
  - Verify Kirchhoff's current law by showing that  $I_s = I_1 + I_2 + I_3$ .
  - Find the power factor of the network, and indicate whether it is leading or lagging.



**FIG. 16.46**  
Problem 8.



- \*9. For the network in Fig. 16.47:
- Find the total admittance  $Y_T$ .
  - Find the voltages  $V_1$  and  $V_2$ .
  - Find the current  $I_3$ .

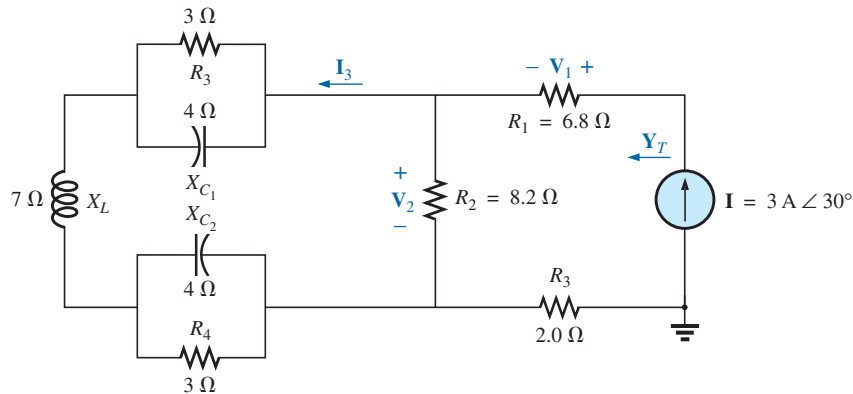


FIG. 16.47  
Problem 9.

- \*10. For the network in Fig. 16.48:
- Find the total impedance  $Z_T$  and the admittance  $Y_T$ .
  - Find the source current  $I_s$  in phasor form.
  - Find the currents  $I_1$  and  $I_2$  in phasor form.
  - Find the voltages  $V_1$  and  $V_{ab}$  in phasor form.
  - Find the average power delivered to the network.
  - Find the power factor of the network, and indicate whether it is leading or lagging.

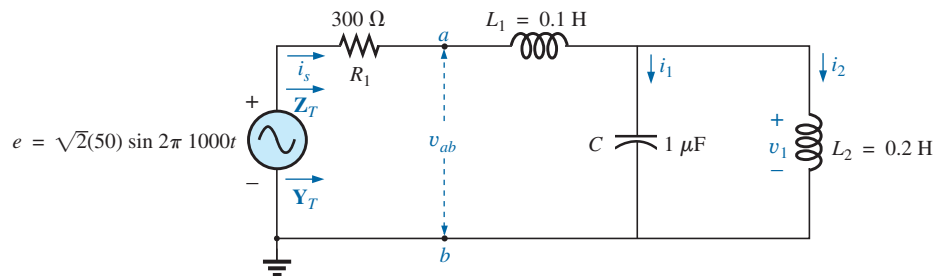


FIG. 16.48  
Problem 10.

- \*11. Find the current  $I$  for the network in Fig. 16.49.

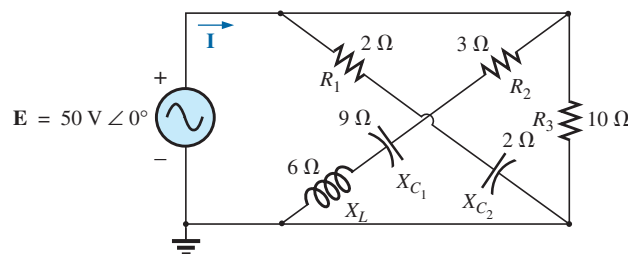
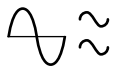


FIG. 16.49  
Problems 11 and 17.



## SECTION 16.3 Ladder Networks

12. Find the current  $I_5$  for the network in Fig. 16.50. Note the effect of one reactive element on the resulting calculations.

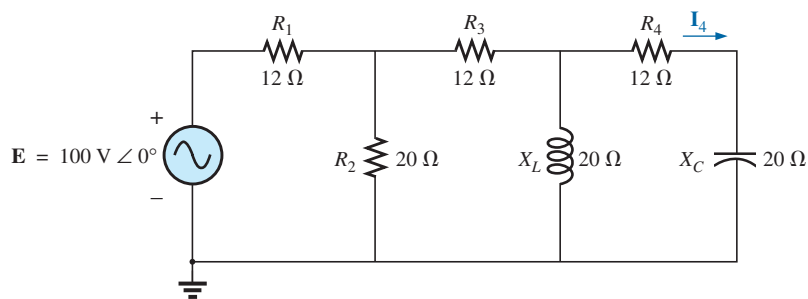


FIG. 16.50  
Problem 12.

13. Find the average power delivered to  $R_4$  in Fig. 16.51.

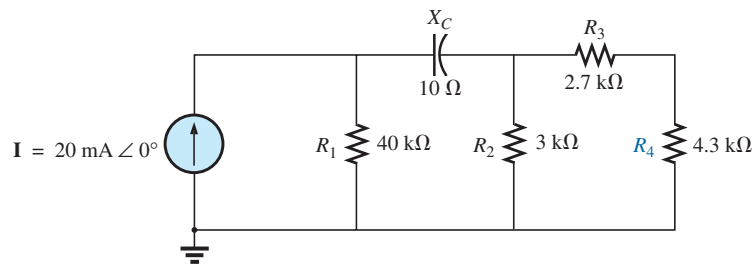


FIG. 16.51  
Problem 13.

14. Find the current  $I_1$  for the network in Fig. 16.52.

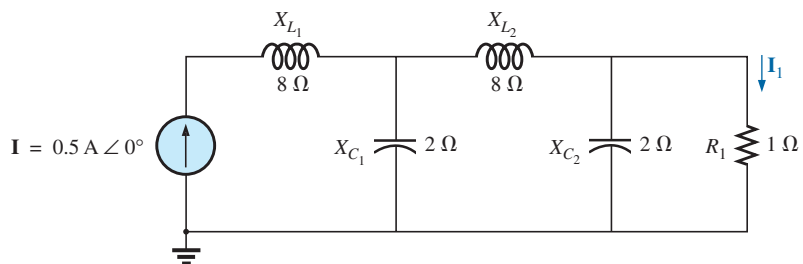
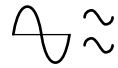


FIG. 16.52  
Problems 14 and 18.





## SECTION 16.6 Computer Analysis

### PSpice or Multisim

For Problems 15 through 18, use a frequency of 1 kHz to determine the inductive and capacitive levels required for the input files. In each case, write the required input file.

- \*15. Repeat Problem 2 using PSpice or Multisim.
- \*16. Repeat Problem 7, parts (a) and (b), using PSpice or Multisim.
- \*17. Repeat Problem 11 using PSpice or Multisim.
- \*18. Repeat Problem 14 using PSpice or Multisim.

## GLOSSARY

**Ladder network** A repetitive combination of series and parallel branches that has the appearance of a ladder.

**Series-parallel ac network** A combination of series and parallel branches in the same network configuration. Each branch may contain any number of elements whose impedance is dependent on the applied frequency.