

## OBJECTIVES

- *Become familiar with the operation of a three-phase generator and the magnitude and phase relationship between the three phase voltages.*
- *Be able to calculate the voltages and currents for a three-phase Y-connected generator and Y-connected load.*
- *Understand the significance of the phase sequence on the generated voltages of a three-phase Y-connected or  $\Delta$ -connected generator.*
- *Be able to calculate the voltages and currents for a three-phase  $\Delta$ -connected generator and  $\Delta$ -connected load.*
- *Understand how to calculate the real, reactive, and apparent power to all the elements of a Y- or  $\Delta$ -connected load and be able to measure the power to the load.*

## 23.1 INTRODUCTION

An ac generator designed to develop a single sinusoidal voltage for each rotation of the shaft (rotor) is referred to as a **single-phase ac generator**. If the number of coils on the rotor is increased in a specified manner, the result is a **polyphase ac generator**, which develops more than one ac phase voltage per rotation of the rotor. In this chapter, the three-phase system is discussed in detail since it is the most frequently used for power transmission.

In general, three-phase systems are preferred over single-phase systems for the transmission of power for many reasons, including the following:

1. Thinner conductors can be used to transmit the same kVA at the same voltage, which reduces the amount of copper required (typically about 25% less) and in turn reduces construction and maintenance costs.
2. The lighter lines are easier to install, and the supporting structures can be less massive and farther apart.
3. Three-phase equipment and motors have preferred running and starting characteristics compared to single-phase systems because of a more even flow of power to the transducer than can be delivered with a single-phase supply.
4. In general, most larger motors are three phase because they are essentially self-starting and do not require a special design or additional starting circuitry.

The frequency generated is determined by the number of poles on the *rotor* (the rotating part of the generator) and the speed with which the shaft is turned. In the United States, the line frequency is 60 Hz, whereas in Europe the chosen standard is 50 Hz. Both frequencies were chosen primarily because they can be generated by a relatively efficient and stable mechanical design that is sensitive to the size of the generating systems and the demand that must be met during peak periods. On aircraft and ships, the demand levels permit the use of a 400 Hz line frequency.



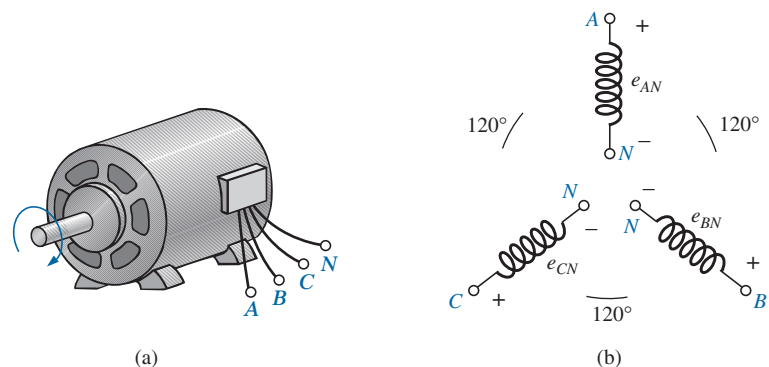


The three-phase system is used by almost all commercial electric generators. This does not mean that single-phase and two-phase generating systems are obsolete. Most small emergency generators, such as the gasoline type, are one-phase generating systems. The two-phase system is commonly used in servomechanisms, which are self-correcting control systems capable of detecting and adjusting their own operation. Servomechanisms are used in ships and aircraft to keep them on course automatically, or, in simpler devices such as a thermostatic circuit, to regulate heat output. In many cases, however, where single-phase and two-phase inputs are required, they are supplied by one and two phases of a three-phase generating system rather than generated independently.

The number of **phase voltages** that can be produced by a polyphase generator is not limited to three. Any number of phases can be obtained by spacing the windings for each phase at the proper angular position around the stator. Some electrical systems operate more efficiently if more than three phases are used. One such system involves the process of rectification, which is used to convert an alternating output to one having an average, or dc, value. The greater the number of phases, the smoother the dc output of the system.

## 23.2 THREE-PHASE GENERATOR

The three-phase generator in Fig. 23.1(a) has three induction coils placed  $120^\circ$  apart on the stator, as shown symbolically by Fig. 23.1(b). Since the three coils have an equal number of turns, and each coil rotates with the same angular velocity, the voltage induced across each coil has the same peak value, shape, and frequency. As the shaft of the generator is turned by some external means, the induced voltages  $e_{AN}$ ,  $e_{BN}$ , and  $e_{CN}$  are generated simultaneously, as shown in Fig. 23.2. Note the  $120^\circ$  phase shift between waveforms and the similarities in appearance of the three sinusoidal functions.



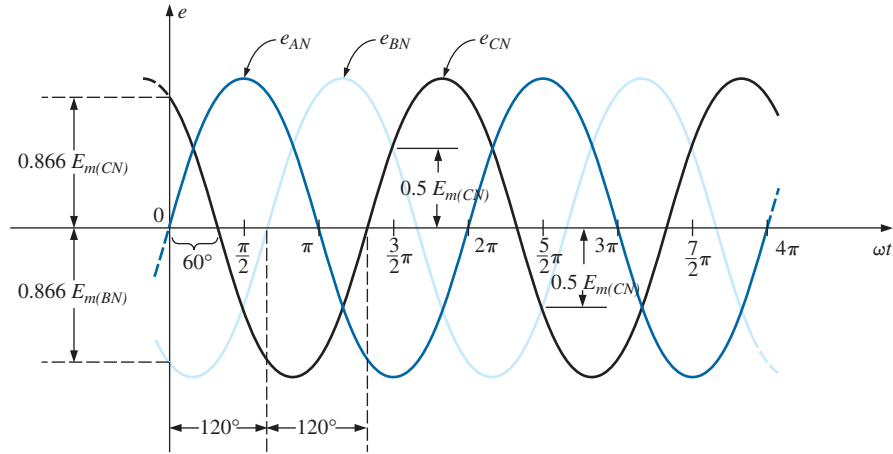
**FIG. 23.1**

(a) Three-phase generator; (b) induced voltages of a three-phase generator.

In particular, note that

**at any instant of time, the algebraic sum of the three phase voltages of a three-phase generator is zero.**

This is shown at  $\omega t = 0$  in Fig. 23.2, where it is also evident that *when one induced voltage is zero, the other two are 86.6% of their positive or negative maximums. In addition, when any two are equal in magnitude*



**FIG. 23.2**

Phase voltages of a three-phase generator.

and sign (at  $0.5E_m$ ), the remaining induced voltage has the opposite polarity and a peak value.

The sinusoidal expression for each of the induced voltages in Fig. 23.2 is

$$\begin{aligned} e_{AN} &= E_{m(AN)} \sin \omega t \\ e_{BN} &= E_{m(BN)} \sin(\omega t - 120^\circ) \\ e_{CN} &= E_{m(CN)} \sin(\omega t - 240^\circ) = E_{m(CN)} \sin(\omega t + 120^\circ) \end{aligned} \quad (23.1)$$

The phasor diagram of the induced voltages is shown in Fig. 23.3, where the effective value of each is determined by

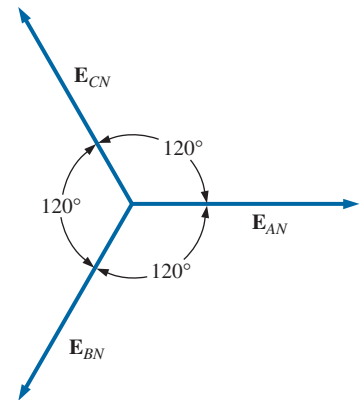
$$\begin{aligned} E_{AN} &= 0.707E_{m(AN)} \\ E_{BN} &= 0.707E_{m(BN)} \\ E_{CN} &= 0.707E_{m(CN)} \end{aligned}$$

and

$$\begin{aligned} \mathbf{E}_{AN} &= E_{AN} \angle 0^\circ \\ \mathbf{E}_{BN} &= E_{BN} \angle -120^\circ \\ \mathbf{E}_{CN} &= E_{CN} \angle +120^\circ \end{aligned}$$

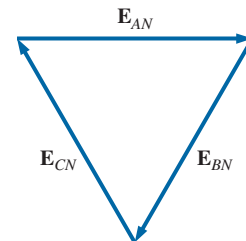
By rearranging the phasors as shown in Fig. 23.4 and applying a law of vectors which states that the vector sum of any number of vectors drawn such that the “head” of one is connected to the “tail” of the next, and that the head of the last vector is connected to the tail of the first is zero, we can conclude that the phasor sum of the phase voltages in a three-phase system is zero. That is,

$$\mathbf{E}_{AN} + \mathbf{E}_{BN} + \mathbf{E}_{CN} = 0 \quad (23.2)$$



**FIG. 23.3**

Phasor diagram for the phase voltages of a three-phase generator.

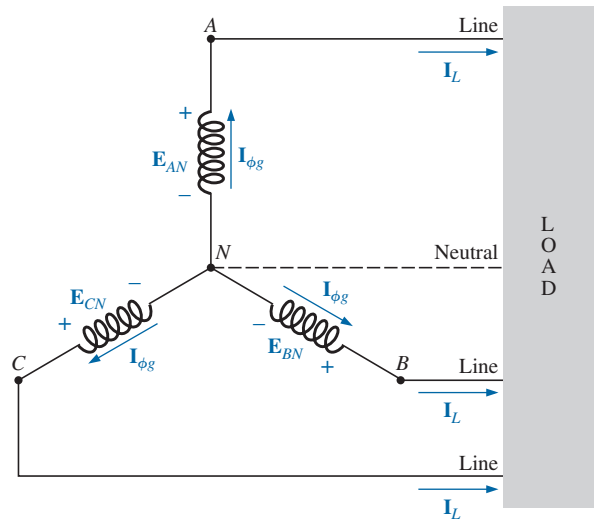


**FIG. 23.4**

Demonstrating that the vector sum of the phase voltages of a three-phase generator is zero.

### 23.3 Y-CONNECTED GENERATOR

If the three terminals denoted  $N$  in Fig. 23.1(b) are connected together, the generator is referred to as a **Y-connected three-phase generator**



**FIG. 23.5**  
Y-connected generator.

(Fig. 23.5). As indicated in Fig. 23.5, the Y is inverted for ease of notation and for clarity. The point at which all the terminals are connected is called the *neutral point*. If a conductor is not attached from this point to the load, the system is called a *Y-connected, three-phase, three-wire generator*. If the neutral is connected, the system is a *Y-connected, three-phase, four-wire generator*. The function of the neutral will be discussed in detail when we consider the load circuit.

The three conductors connected from A, B, and C to the load are called *lines*. For the Y-connected system, it should be obvious from Fig. 23.5 that the **line current** equals the **phase current** for each phase; that is,

$$I_L = I_{\phi g} \tag{23.3}$$

where  $\phi$  is used to denote a phase quantity and  $g$  is a generator parameter.

The voltage from one line to another is called a **line voltage**. On the phasor diagram (Fig. 23.6) it is the phasor drawn from the end of one phase to another in the counterclockwise direction.

Applying Kirchhoff's voltage law around the indicated loop in Fig. 23.6, we obtain

$$E_{AB} - E_{AN} + E_{BN} = 0$$

or

$$E_{AB} = E_{AN} - E_{BN} = E_{AN} + E_{NB}$$

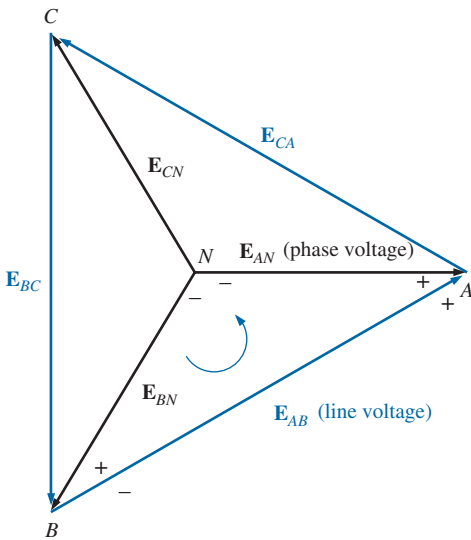
The phasor diagram is redrawn to find  $E_{AB}$  as shown in Fig. 23.7. Since each phase voltage, when reversed ( $E_{NB}$ ), bisects the other two,  $\alpha = 60^\circ$ . The angle  $\beta$  is  $30^\circ$  since a line drawn from opposite ends of a rhombus divides in half both the angle of origin and the opposite angle. Lines drawn between opposite corners of a rhombus also bisect each other at right angles.

The length  $x$  is

$$x = E_{AN} \cos 30^\circ = \frac{\sqrt{3}}{2} E_{AN}$$

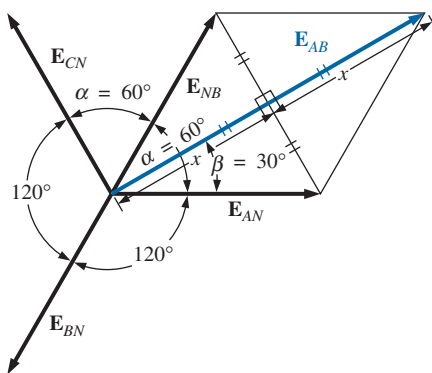
and

$$E_{AB} = 2x = (2) \frac{\sqrt{3}}{2} E_{AN} = \sqrt{3} E_{AN}$$



**FIG. 23.6**

Line and phase voltages of the Y-connected three-phase generator.



**FIG. 23.7**

Determining a line voltage for a three-phase generator.



Noting from the phasor diagram that  $\theta$  of  $\mathbf{E}_{AB} = \beta = 30^\circ$ , the result is

$$\mathbf{E}_{AB} = E_{AB} \angle 30^\circ = \sqrt{3}E_{AN} \angle 30^\circ$$

and 
$$\mathbf{E}_{CA} = \sqrt{3}E_{CN} \angle 150^\circ$$

$$\mathbf{E}_{BC} = \sqrt{3}E_{BN} \angle 270^\circ$$

In words, the magnitude of the line voltage of a Y-connected generator is  $\sqrt{3}$  times the phase voltage:

$$E_L = \sqrt{3}E_\phi \quad (23.4)$$

with the phase angle between any line voltage and the nearest phase voltage at  $30^\circ$ .

In sinusoidal notation,

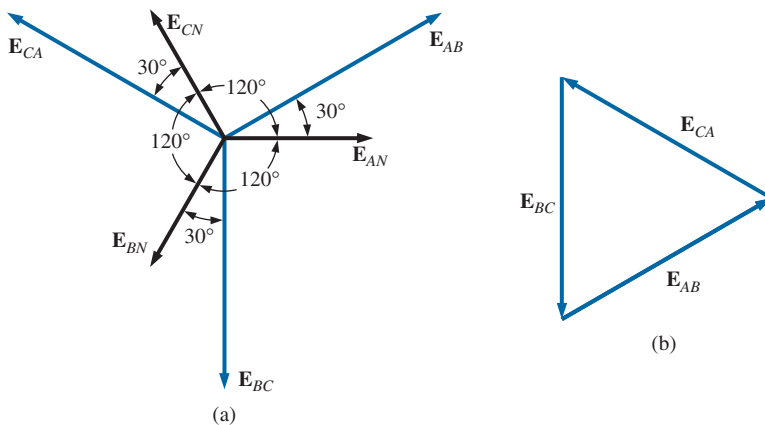
$$e_{AB} = \sqrt{2}E_{AB} \sin(\omega t + 30^\circ)$$

$$e_{CA} = \sqrt{2}E_{CA} \sin(\omega t + 150^\circ)$$

and 
$$e_{BC} = \sqrt{2}E_{BC} \sin(\omega t + 270^\circ)$$

The phasor diagram of the line and phase voltages is shown in Fig. 23.8. If the phasors representing the line voltages in Fig. 23.8(a) are rearranged slightly, they will form a closed loop [Fig. 23.8(b)]. Therefore, we can conclude that the sum of the line voltages is also zero; that is,

$$\mathbf{E}_{AB} + \mathbf{E}_{CA} + \mathbf{E}_{BC} = 0 \quad (23.5)$$

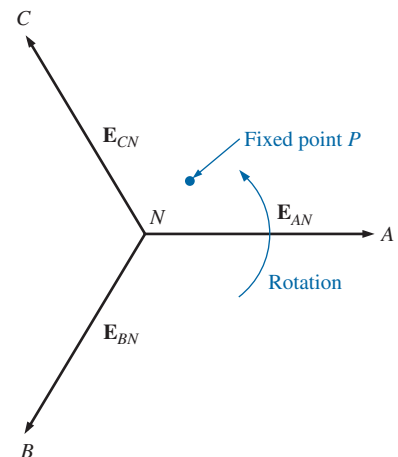


**FIG. 23.8**

(a) Phasor diagram of the line and phase voltages of a three-phase generator;  
 (b) demonstrating that the vector sum of the line voltages of a three-phase system is zero.

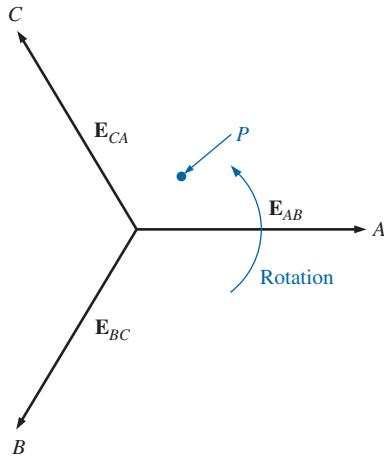
### 23.4 PHASE SEQUENCE (Y-CONNECTED GENERATOR)

The **phase sequence** can be determined by the order in which the phasors representing the phase voltages pass through a fixed point on the phasor diagram if the phasors are rotated in a counterclockwise direction. For example, in Fig. 23.9 the phase sequence is *ABC*. However, since the fixed point can be chosen anywhere on the phasor diagram, the sequence can



**FIG. 23.9**

Determining the phase sequence from the phase voltages of a three-phase generator.



**FIG. 23.10**

Determining the phase sequence from the line voltages of a three-phase generator.

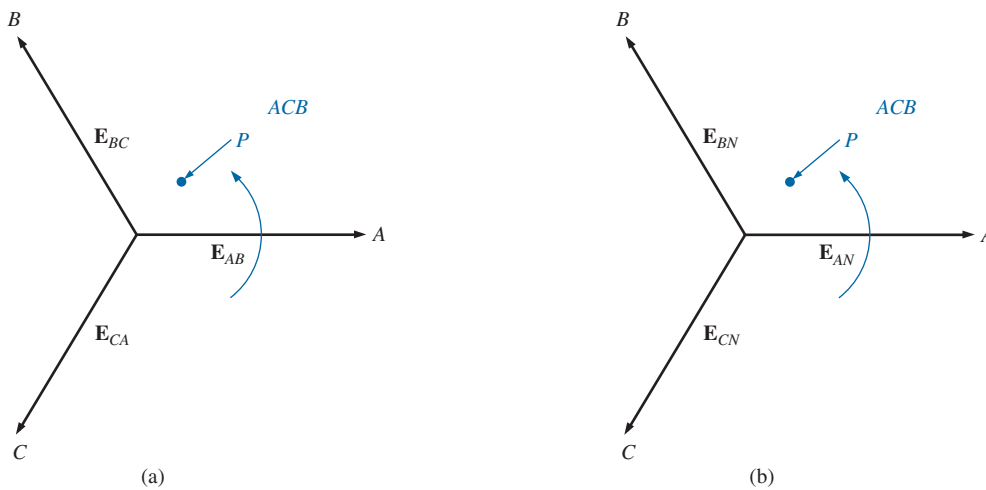
also be written as *BCA* or *CAB*. The phase sequence is quite important in the three-phase distribution of power. In a three-phase motor, for example, if two phase voltages are interchanged, the sequence will change, and the direction of rotation of the motor will be reversed. Other effects will be described when we consider the loaded three-phase system.

The phase sequence can also be described in terms of the line voltages. Drawing the line voltages on a phasor diagram in Fig. 23.10, we are able to determine the phase sequence by again rotating the phasors in the counterclockwise direction. In this case, however, the sequence can be determined by noting the order of the passing first or second subscripts. In the system in Fig. 23.10, for example, the phase sequence of the first subscripts passing point *P* is *ABC*, and the phase sequence of the second subscripts is *BCA*. But we know that *BCA* is equivalent to *ABC*, so the sequence is the same for each. Note that the phase sequence is the same as that of the phase voltages described in Fig. 23.9.

If the sequence is given, the phasor diagram can be drawn by simply picking a reference voltage, placing it on the reference axis, and then drawing the other voltages at the proper angular position. For a sequence of *ACB*, for example, we might choose  $E_{AB}$  to be the reference [Fig. 23.11(a)] if we wanted the phasor diagram of the line voltages, or  $E_{AN}$  for the phase voltages [Fig. 23.11(b)]. For the sequence indicated, the phasor diagrams would be as in Fig. 23.11. In phasor notation,

$$\text{Line voltages} \begin{cases} \mathbf{E}_{AB} = E_{AB} \angle 0^\circ & (\text{reference}) \\ \mathbf{E}_{CA} = E_{CA} \angle -120^\circ \\ \mathbf{E}_{BC} = E_{BC} \angle +120^\circ \end{cases}$$

$$\text{Phase voltages} \begin{cases} \mathbf{E}_{AN} = E_{AN} \angle 0^\circ & (\text{reference}) \\ \mathbf{E}_{CN} = E_{CN} \angle -120^\circ \\ \mathbf{E}_{BN} = E_{BN} \angle +120^\circ \end{cases}$$



**FIG. 23.11**

Drawing the phasor diagram from the phase sequence.





### 23.5 Y-CONNECTED GENERATOR WITH A Y-CONNECTED LOAD

Loads connected to three-phase supplies are of two types: the Y and the  $\Delta$ . If a Y-connected load is connected to a Y-connected generator, the system is symbolically represented by Y-Y. The physical setup of such a system is shown in Fig. 23.12.

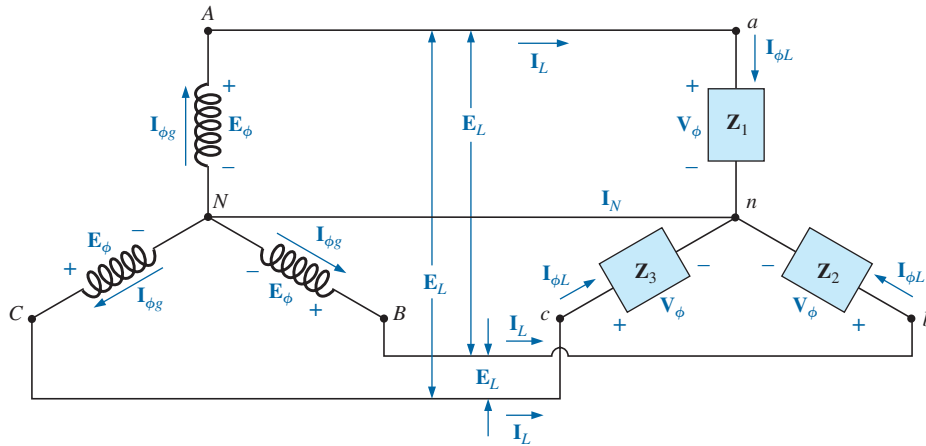


FIG. 23.12

*Y-connected generator with a Y-connected load.*

If the load is balanced, the **neutral connection** can be removed without affecting the circuit in any manner; that is, if

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3$$

then  $I_N$  will be zero. (This will be demonstrated in Example 23.1.) Note that in order to have a balanced load, the phase angle must also be the same for each impedance—a condition that was unnecessary in dc circuits when we considered balanced systems.

In practice, if a factory, for example, had only balanced, three-phase loads, the absence of the neutral would have no effect since, ideally, the system would always be balanced. The cost would therefore be less since the number of required conductors would be reduced. However, lighting and most other electrical equipment use only one of the phase voltages, and even if the loading is designed to be balanced (as it should be), there is never perfect continuous balancing since lights and other electrical equipment are turned on and off, upsetting the balanced condition. The neutral is therefore necessary to carry the resulting current away from the load and back to the Y-connected generator. This is demonstrated when we consider unbalanced Y-connected systems.

We shall now examine the *four-wire Y-Y-connected system*. The current passing through each phase of the generator is the same as its corresponding line current, which in turn for a Y-connected load is equal to the current in the phase of the load to which it is attached:

$$\mathbf{I}_{\phi g} = \mathbf{I}_L = \mathbf{I}_{\phi L} \quad (23.6)$$

For a balanced or an unbalanced load, since the generator and load have a common neutral point, then



$$\boxed{V_\phi = E_\phi} \tag{23.7}$$

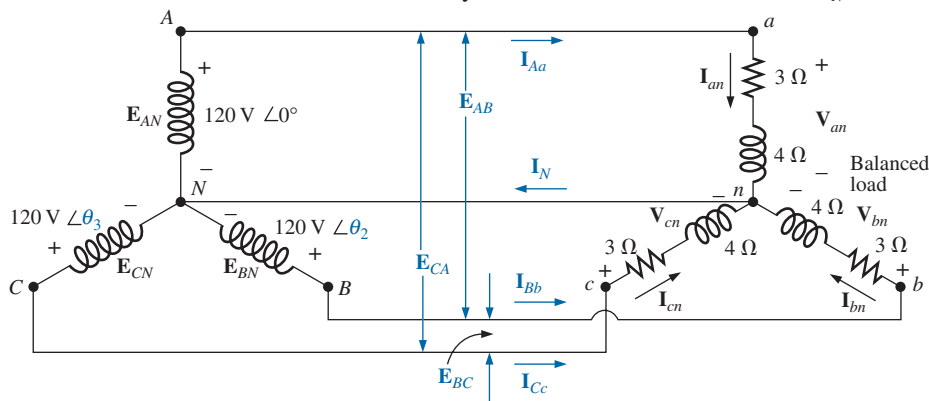
In addition, since  $I_{\phi L} = V_\phi / Z_\phi$ , the magnitude of the current in each phase is equal for a balanced load and unequal for an unbalanced load. Recall that for the Y-connected generator, the magnitude of the line voltage is equal to  $\sqrt{3}$  times the phase voltage. This same relationship can be applied to a balanced or an unbalanced four-wire Y-connected load:

$$\boxed{E_L = \sqrt{3}V_\phi} \tag{23.8}$$

For a voltage drop across a load element, the first subscript refers to that terminal through which the current enters the load element, and the second subscript refers to the terminal from which the current leaves. In other words, the first subscript is, by definition, positive with respect to the second for a voltage drop. Note Fig. 23.13, in which the standard double subscripts for a source of voltage and a voltage drop are indicated.

**EXAMPLE 23.1** The phase sequence of the Y-connected generator in Fig. 23.13 is *ABC*.

- Find the phase angles  $\theta_2$  and  $\theta_3$ .
- Find the magnitude of the line voltages.
- Find the line currents.
- Verify that, since the load is balanced,  $I_N = 0$ .



**FIG. 23.13**  
Example 23.1.

**Solutions:**

- a. For an *ABC* phase sequence,

$$\theta_2 = -120^\circ \quad \text{and} \quad \theta_3 = +120^\circ$$

- b.  $E_L = \sqrt{3}E_\phi = (1.73)(120 \text{ V}) = 208 \text{ V}$ . Therefore,

$$E_{AB} = E_{BC} = E_{CA} = 208 \text{ V}$$

- c.  $V_\phi = E_\phi$ . Therefore,

$$V_{an} = E_{AN} \quad V_{bn} = E_{BN} \quad V_{cn} = E_{CN}$$

$$I_{\phi L} = I_{an} = \frac{V_{an}}{Z_{an}} = \frac{120 \text{ V} \angle 0^\circ}{3 \Omega + j4 \Omega} = \frac{120 \text{ V} \angle 0^\circ}{5 \Omega \angle 53.13^\circ} = 24 \text{ A} \angle -53.13^\circ$$





$$\mathbf{I}_{bn} = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_{bn}} = \frac{120 \text{ V} \angle -120^\circ}{5 \Omega \angle 53.13^\circ} = 24 \text{ A} \angle -173.13^\circ$$

$$\mathbf{I}_{cn} = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_{cn}} = \frac{120 \text{ V} \angle +120^\circ}{5 \Omega \angle 53.13^\circ} = 24 \text{ A} \angle 66.87^\circ$$

and, since  $\mathbf{I}_L = \mathbf{I}_{\phi L}$ ,

$$\mathbf{I}_{Aa} = \mathbf{I}_{an} = 24 \text{ A} \angle -53.13^\circ$$

$$\mathbf{I}_{Bb} = \mathbf{I}_{bn} = 24 \text{ A} \angle -173.13^\circ$$

$$\mathbf{I}_{Cc} = \mathbf{I}_{cn} = 24 \text{ A} \angle 66.87^\circ$$

d. Applying Kirchoff's current law, we have

$$\mathbf{I}_N = \mathbf{I}_{Aa} + \mathbf{I}_{Bb} + \mathbf{I}_{Cc}$$

In rectangular form,

$$\mathbf{I}_{Aa} = 24 \text{ A} \angle -53.13^\circ = 14.40 \text{ A} - j 19.20 \text{ A}$$

$$\mathbf{I}_{Bb} = 24 \text{ A} \angle -173.13^\circ = -22.83 \text{ A} - j 2.87 \text{ A}$$

$$\mathbf{I}_{Cc} = 24 \text{ A} \angle 66.87^\circ = 9.43 \text{ A} + j 22.07 \text{ A}$$

$$\Sigma(\mathbf{I}_{Aa} + \mathbf{I}_{Bb} + \mathbf{I}_{Cc}) = 0 + j 0$$

and  $\mathbf{I}_N$  is in fact equals to **zero**, as required for a balanced load.

### 23.6 Y-Δ SYSTEM

There is no neutral connection for the Y-Δ system in Fig. 23.14. Any variation in the impedance of a phase that produces an unbalanced system simply varies the line and phase currents of the system.

For a balanced load,

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 \tag{23.9}$$

The voltage across each phase of the load is equal to the line voltage of the generator for a balanced or an unbalanced load:

$$\mathbf{V}_\phi = \mathbf{E}_L \tag{23.10}$$

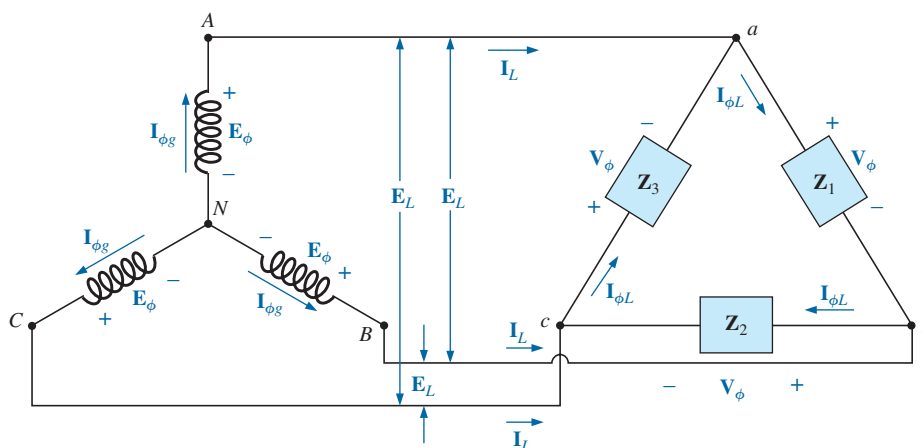


FIG. 23.14

Y-connected generator with a Δ-connected load.



The relationship between the line currents and phase currents of a balanced  $\Delta$  load can be found using an approach very similar to that used in Section 23.3 to find the relationship between the line voltages and phase voltages of a Y-connected generator. For this case, however, Kirchhoff's current law is used instead of Kirchhoff's voltage law.

The results obtained are

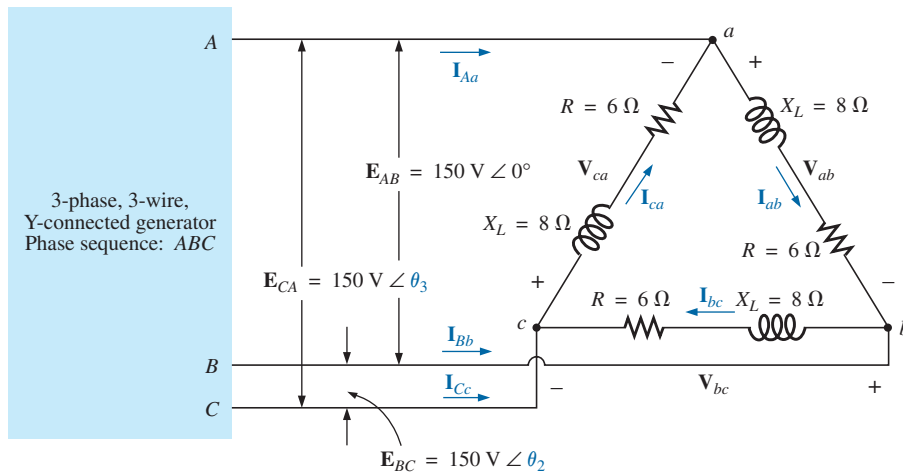
$$I_L = \sqrt{3}I_\phi \tag{23.11}$$

and the phase angle between a line current and the nearest phase current is  $30^\circ$ . A more detailed discussion of this relationship between the line and phase currents of a  $\Delta$ -connected system can be found in Section 23.7.

For a balanced load, the line currents will be equal in magnitude, as will the phase currents.

**EXAMPLE 23.2** For the three-phase system in Fig. 23.15:

- Find the phase angles  $\theta_2$  and  $\theta_3$ .
- Find the current in each phase of the load.
- Find the magnitude of the line currents.



**FIG. 23.15**  
Example 23.2.

**Solutions:**

- For an ABC sequence,

$$\theta_2 = -120^\circ \quad \text{and} \quad \theta_3 = +120^\circ$$

- $V_\phi = E_L$ . Therefore,

$$V_{ab} = E_{AB} \quad V_{ca} = E_{CA} \quad V_{bc} = E_{BC}$$

The phase currents are

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{150 \text{ V } \angle 0^\circ}{6 \Omega + j8 \Omega} = \frac{150 \text{ V } \angle 0^\circ}{10 \Omega \angle 53.13^\circ} = 15 \text{ A } \angle -53.13^\circ$$

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{150 \text{ V } \angle -120^\circ}{10 \Omega \angle 53.13^\circ} = 15 \text{ A } \angle -173.13^\circ$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{150 \text{ V } \angle +120^\circ}{10 \Omega \angle 53.13^\circ} = 15 \text{ A } \angle 66.87^\circ$$



c.  $I_L = \sqrt{3}I_\phi = (1.73)(15 \text{ A}) = 25.95 \text{ A}$ . Therefore,

$$I_{Aa} = I_{Bb} = I_{Cc} = \mathbf{25.95 \text{ A}}$$

### 23.7 Δ-CONNECTED GENERATOR

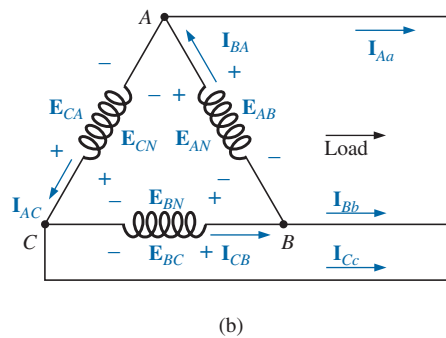
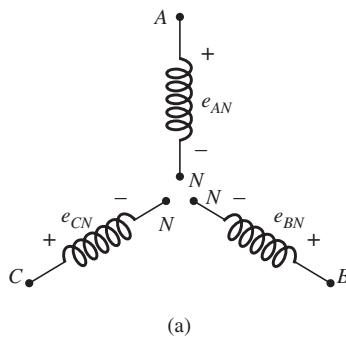
If we rearrange the coils of the generator in Fig. 23.16(a) as shown in Fig. 23.16(b), the system is referred to as a *three-phase, three-wire, Δ-connected ac generator*. In this system, the phase and line voltages are equivalent and equal to the voltage induced across each coil of the generator; that is,

$$\left. \begin{aligned} \mathbf{E}_{AB} = \mathbf{E}_{AN} \quad \text{and} \quad e_{AN} &= \sqrt{2}E_{AN} \sin \omega t \\ \mathbf{E}_{BC} = \mathbf{E}_{BN} \quad \text{and} \quad e_{BN} &= \sqrt{2}E_{BN} \sin(\omega t - 120^\circ) \\ \mathbf{E}_{CA} = \mathbf{E}_{CN} \quad \text{and} \quad e_{CN} &= \sqrt{2}E_{CN} \sin(\omega t + 120^\circ) \end{aligned} \right\} \begin{array}{l} \text{Phase} \\ \text{sequence} \\ ABC \end{array}$$

or

$$\mathbf{E}_L = \mathbf{E}_{\phi g}$$

(23.12)



**FIG. 23.16**  
*Δ-connected generator.*

Note that only one voltage (magnitude) is available instead of the two available in the Y-connected system.

Unlike the line current for the Y-connected generator, the line current for the Δ-connected system is not equal to the phase current. The relationship between the two can be found by applying Kirchhoff's current law at one of the nodes and solving for the line current in terms of the phase currents; that is, at node A,

$$\mathbf{I}_{BA} = \mathbf{I}_{Aa} + \mathbf{I}_{AC}$$

or

$$\mathbf{I}_{Aa} = \mathbf{I}_{BA} - \mathbf{I}_{AC} = \mathbf{I}_{BA} + \mathbf{I}_{CA}$$

The phasor diagram is shown in Fig. 23.17 for a balanced load.

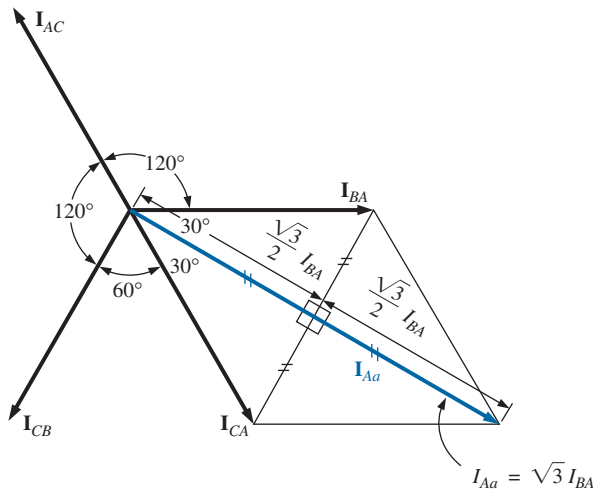
Using the same procedure to find the line current as was used to find the line voltage of a Y-connected generator produces the following:

$$\begin{aligned} I_{Aa} &= \sqrt{3}I_{BA} \angle -30^\circ \\ I_{Bb} &= \sqrt{3}I_{CB} \angle -150^\circ \\ I_{Cc} &= \sqrt{3}I_{AC} \angle 90^\circ \end{aligned}$$

In general:

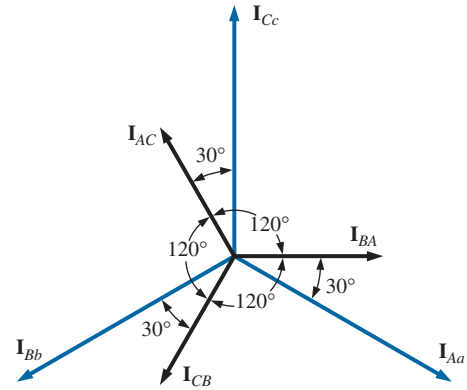
$$\mathbf{I}_L = \sqrt{3}\mathbf{I}_{\phi g}$$

(23.13)



**FIG. 23.17**

Determining a line current from the phase currents of a  $\Delta$ -connected, three-phase generator.



**FIG. 23.18**

The phasor diagram of the currents of a three-phase,  $\Delta$ -connected generator.

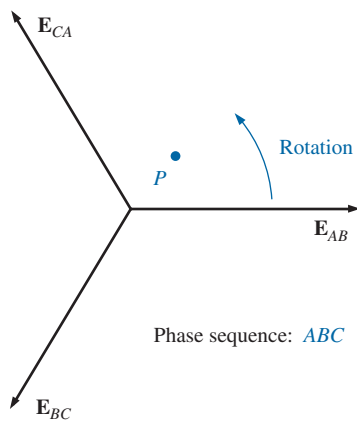
with the phase angle between a line current and the nearest phase current at  $30^\circ$ . The phasor diagram of the currents is shown in Fig. 23.18.

Just as for the voltages of a Y-connected generator, the phasor sum of the line currents or phase currents for  $\Delta$ -connected systems with balanced loads is zero.

### 23.8 PHASE SEQUENCE ( $\Delta$ -CONNECTED GENERATOR)

Even though the line and phase voltages of a  $\Delta$ -connected system are the same, it is standard practice to describe the phase sequence in terms of the line voltages. The method used is the same as that described for the line voltages of the Y-connected generator. For example, the phasor diagram of the line voltages for a phase sequence  $ABC$  is shown in Fig. 23.19. In drawing such a diagram, one must take care to have the sequence of the first and second subscripts the same. In phasor notation,

$$\begin{aligned} E_{AB} &= E_{AB} \angle 0^\circ \\ E_{BC} &= E_{BC} \angle -120^\circ \\ E_{CA} &= E_{CA} \angle 120^\circ \end{aligned}$$



**FIG. 23.19**

Determining the phase sequence for a  $\Delta$ -connected, three-phase generator.

### 23.9 $\Delta$ - $\Delta$ , $\Delta$ -Y THREE-PHASE SYSTEMS

The basic equations necessary to analyze either of the two systems ( $\Delta$ - $\Delta$ ,  $\Delta$ -Y) have been presented at least once in this chapter. Following are two descriptive examples, one with a  $\Delta$ -connected load and one with a Y-connected load.

**EXAMPLE 23.3** For the  $\Delta$ - $\Delta$  system shown in Fig. 23.20:

- Find the phase angles  $\theta_2$  and  $\theta_3$  for the specified phase sequence.
- Find the current in each phase of the load.
- Find the magnitude of the line currents.

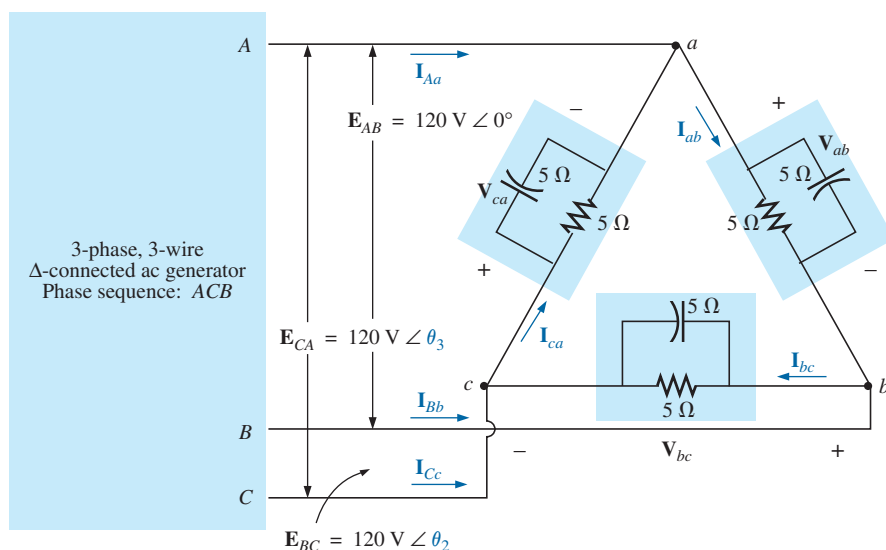


FIG. 23.20

Example 23.3: Δ-Δ system.

**Solutions:**

a. For an ACB phase sequence,

$$\theta_2 = 120^\circ \quad \text{and} \quad \theta_3 = -120^\circ$$

b.  $V_\phi = E_L$ . Therefore,

$$V_{ab} = E_{AB} \quad V_{ca} = E_{CA} \quad V_{bc} = E_{BC}$$

The phase currents are

$$\begin{aligned} \mathbf{I}_{ab} &= \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} = \frac{120 \text{ V } \angle 0^\circ}{\frac{(5 \Omega \angle 0^\circ)(5 \Omega \angle -90^\circ)}{5 \Omega - j 5 \Omega}} = \frac{120 \text{ V } \angle 0^\circ}{7.071 \angle -45^\circ} \\ &= \frac{120 \text{ V } \angle 0^\circ}{3.54 \Omega \angle -45^\circ} = \mathbf{33.9 \text{ A } \angle 45^\circ} \end{aligned}$$

$$\mathbf{I}_{bc} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{bc}} = \frac{120 \text{ V } \angle 120^\circ}{3.54 \Omega \angle -45^\circ} = \mathbf{33.9 \text{ A } \angle 165^\circ}$$

$$\mathbf{I}_{ca} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{ca}} = \frac{120 \text{ V } \angle -120^\circ}{3.54 \Omega \angle -45^\circ} = \mathbf{33.9 \text{ A } \angle -75^\circ}$$

c.  $I_L = \sqrt{3}I_\phi = (1.73)(34 \text{ A}) = 58.82 \text{ A}$ . Therefore,

$$I_{Aa} = I_{Bb} = I_{Cc} = \mathbf{58.82 \text{ A}}$$

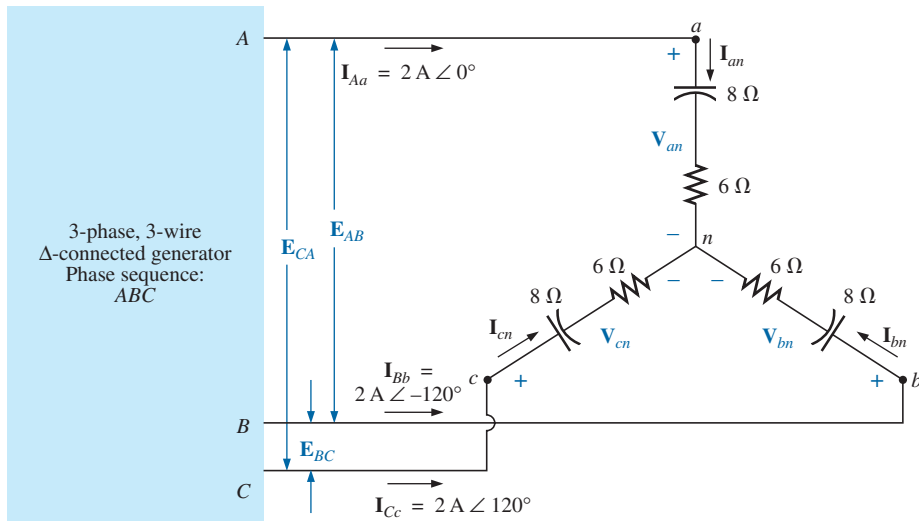
**EXAMPLE 23.4** For the Δ-Y system shown in Fig. 23.21:

- Find the voltage across each phase of the load.
- Find the magnitude of the line voltages.

**Solutions:**

a.  $I_{\phi L} = I_L$ . Therefore,

$$\begin{aligned} \mathbf{I}_{an} &= \mathbf{I}_{Aa} = 2 \text{ A } \angle 0^\circ \\ \mathbf{I}_{bn} &= \mathbf{I}_{Bb} = 2 \text{ A } \angle -120^\circ \\ \mathbf{I}_{cn} &= \mathbf{I}_{Cc} = 2 \text{ A } \angle 120^\circ \end{aligned}$$



**FIG. 23.21**

Example 23.4: Δ-Y system.

The phase voltages are

$$V_{an} = I_{an}Z_{an} = (2 \text{ A } \angle 0^\circ)(10 \Omega \angle -53.13^\circ) = 20 \text{ V } \angle -53.13^\circ$$

$$V_{bn} = I_{bn}Z_{bn} = (2 \text{ A } \angle -120^\circ)(10 \Omega \angle -53.13^\circ) = 20 \text{ V } \angle -173.13^\circ$$

$$V_{cn} = I_{cn}Z_{cn} = (2 \text{ A } \angle 120^\circ)(10 \Omega \angle -53.13^\circ) = 20 \text{ V } \angle 66.87^\circ$$

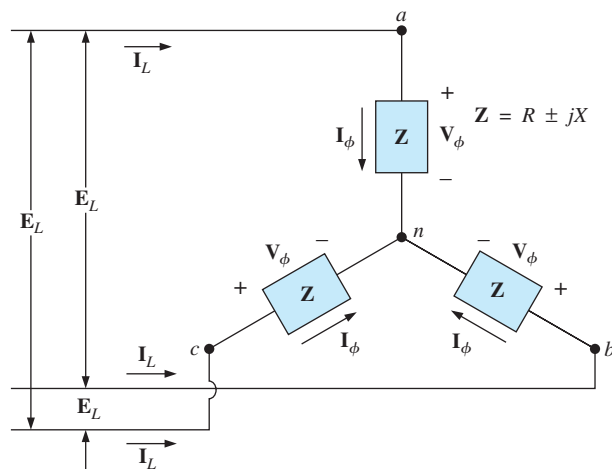
b.  $E_L = \sqrt{3}V_\phi = (1.73)(20 \text{ V}) = 34.6 \text{ V}$ . Therefore,

$$E_{BA} = E_{CB} = E_{AC} = 34.6 \text{ V}$$

## 23.10 POWER

### Y-Connected Balanced Load

Please refer to Fig. 23.22 for the following discussion.



**FIG. 23.22**

Y-connected balanced load.





**Average Power** The average power delivered to each phase can be determined by Eq. (23.14).

$$P_{\phi} = V_{\phi} I_{\phi} \cos \theta_{I_{\phi}}^V = I_{\phi}^2 R_{\phi} = \frac{V_R^2}{R_{\phi}} \quad (\text{watts, W}) \quad (23.14)$$

where  $\theta_{I_{\phi}}^V$  indicates that  $\theta$  is the phase angle between  $V_{\phi}$  and  $I_{\phi}$ .

The total power delivered can be determined by Eq. (23.15) or Eq. (23.16).

$$P_T = 3P_{\phi} \quad (\text{W}) \quad (23.15)$$

or, since  $V_{\phi} = \frac{E_L}{\sqrt{3}}$  and  $I_{\phi} = I_L$

then  $P_T = 3 \frac{E_L}{\sqrt{3}} I_L \cos \theta_{I_{\phi}}^V$

But  $\left(\frac{3}{\sqrt{3}}\right)(1) = \left(\frac{3}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{3\sqrt{3}}{3} = \sqrt{3}$

Therefore,

$$P_T = \sqrt{3} E_L I_L \cos \theta_{I_{\phi}}^V = 3 I_L^2 R_{\phi} \quad (\text{W}) \quad (23.16)$$

**Reactive Power** The reactive power of each phase (in volt-amperes reactive) is

$$Q_{\phi} = V_{\phi} I_{\phi} \sin \theta_{I_{\phi}}^V = I_{\phi}^2 X_{\phi} = \frac{V_X^2}{X_{\phi}} \quad (\text{VAR}) \quad (23.17)$$

The total reactive power of the load is

$$Q_T = 3Q_{\phi} \quad (\text{VAR}) \quad (23.18)$$

or, proceeding in the same manner as above, we have

$$Q_T = \sqrt{3} E_L I_L \sin \theta_{I_{\phi}}^V = 3 I_L^2 X_{\phi} \quad (\text{VAR}) \quad (23.19)$$

**Apparent Power** The apparent power of each phase is

$$S_{\phi} = V_{\phi} I_{\phi} \quad (\text{VA}) \quad (23.20)$$

The total apparent power of the load is

$$S_T = 3S_{\phi} \quad (\text{VA}) \quad (23.21)$$

or, as before,

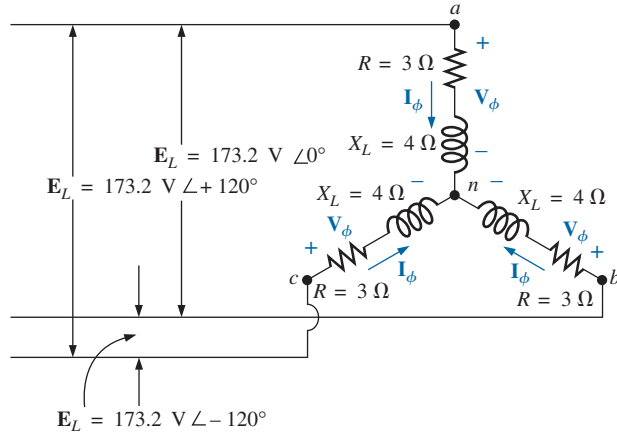
$$S_T = \sqrt{3} E_L I_L \quad (\text{VA}) \quad (23.22)$$



**Power Factor** The power factor of the system is given by

$$F_p = \frac{P_T}{S_T} = \cos \theta_{I_\phi}^V \quad (\text{leading or lagging}) \quad (23.23)$$

**EXAMPLE 23.5** For the Y-connected load in Fig. 23.23:



**FIG. 23.23**  
Example 23.5.

- Find the average power to each phase and the total load.
- Determine the reactive power to each phase and the total reactive power.
- Find the apparent power to each phase and the total apparent power.
- Find the power factor of the load.

**Solutions:**

- The average power is

$$P_\phi = V_\phi I_\phi \cos \theta_{I_\phi}^V = (100 \text{ V})(20 \text{ A}) \cos 53.13^\circ = (2000)(0.6) = \mathbf{1200 \text{ W}}$$

$$P_\phi = I_\phi^2 R_\phi = (20 \text{ A})^2 (3 \Omega) = (400)(3) = \mathbf{1200 \text{ W}}$$

$$P_\phi = \frac{V_R^2}{R_\phi} = \frac{(60 \text{ V})^2}{3 \Omega} = \frac{3600}{3} = \mathbf{1200 \text{ W}}$$

$$P_T = 3P_\phi = (3)(1200 \text{ W}) = \mathbf{3600 \text{ W}}$$

or

$$P_T = \sqrt{3} E_L I_L \cos \theta_{I_\phi}^V = (1.732)(173.2 \text{ V})(20 \text{ A})(0.6) = \mathbf{3600 \text{ W}}$$

- The reactive power is

$$Q_\phi = V_\phi I_\phi \sin \theta_{I_\phi}^V = (100 \text{ V})(20 \text{ A}) \sin 53.13^\circ = (2000)(0.8) = \mathbf{1600 \text{ VAR}}$$

$$\text{or} \quad Q_\phi = I_\phi^2 X_\phi = (20 \text{ A})^2 (4 \Omega) = (400)(4) = \mathbf{1600 \text{ VAR}}$$

$$Q_T = 3Q_\phi = (3)(1600 \text{ VAR}) = \mathbf{4800 \text{ VAR}}$$

or

$$Q_T = \sqrt{3} E_L I_L \sin \theta_{I_\phi}^V = (1.732)(173.2 \text{ V})(20 \text{ A})(0.8) = \mathbf{4800 \text{ VAR}}$$



c. The *apparent power* is

$$S_\phi = V_\phi I_\phi = (100 \text{ V})(20 \text{ A}) = \mathbf{2000 \text{ VA}}$$

$$S_T = 3S_\phi = (3)(2000 \text{ VA}) = \mathbf{6000 \text{ VA}}$$

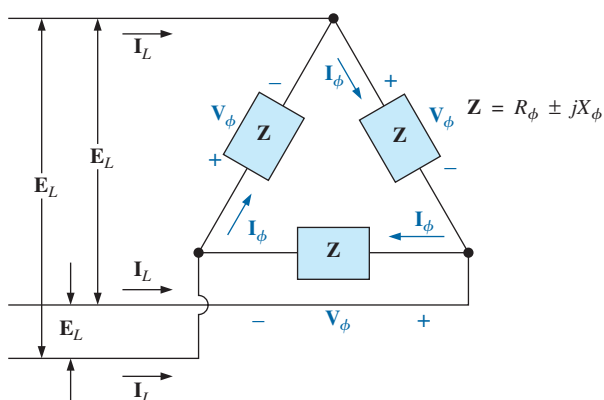
or  $S_T = \sqrt{3}E_L I_L = (1.732)(173.2 \text{ V})(20 \text{ A}) = \mathbf{6000 \text{ VA}}$

d. The *power factor* is

$$F_p = \frac{P_T}{S_T} = \frac{3600 \text{ W}}{6000 \text{ VA}} = \mathbf{0.6 \text{ lagging}}$$

## $\Delta$ -Connected Balanced Load

Please refer to Fig. 23.24 for the following discussion.



**FIG. 23.24**

$\Delta$ -connected balanced load.

### Average Power

$$P_\phi = V_\phi I_\phi \cos \theta_{I_\phi}^{V_\phi} = I_\phi^2 R_\phi = \frac{V_\phi^2}{R_\phi} \quad (\text{W}) \quad (23.24)$$

$$P_T = 3P_\phi \quad (\text{W}) \quad (23.25)$$

### Reactive Power

$$Q_\phi = V_\phi I_\phi \sin \theta_{I_\phi}^{V_\phi} = I_\phi^2 X_\phi = \frac{V_\phi^2}{X_\phi} \quad (\text{VAR}) \quad (23.26)$$

$$Q_T = 3Q_\phi \quad (\text{VAR}) \quad (23.27)$$

### Apparent Power

$$S_\phi = V_\phi I_\phi \quad (\text{VA}) \quad (23.28)$$

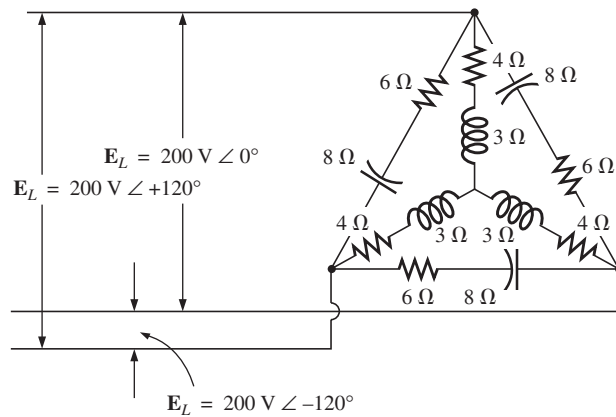


$$S_T = 3S_\phi = \sqrt{3}E_L I_L \quad (\text{VA}) \quad (23.29)$$

**Power Factor**

$$F_p = \frac{P_T}{S_T} \quad (23.30)$$

**EXAMPLE 23.6** For the  $\Delta$ -Y connected load in Fig. 23.25, find the total average, reactive, and apparent power. In addition, find the power factor of the load.



**FIG. 23.25**  
Example 23.6.

**Solution:** Consider the  $\Delta$  and Y separately.

**For the  $\Delta$ :**

$$Z_\Delta = 6 \Omega - j 8 \Omega = 10 \Omega \angle -53.13^\circ$$

$$I_\phi = \frac{E_L}{Z_\Delta} = \frac{200 \text{ V}}{10 \Omega} = 20 \text{ A}$$

$$P_{T_\Delta} = 3I_\phi^2 R_\phi = (3)(20 \text{ A})^2(6 \Omega) = \mathbf{7200 \text{ W}}$$

$$Q_{T_\Delta} = 3I_\phi^2 X_\phi = (3)(20 \text{ A})^2(8 \Omega) = \mathbf{9600 \text{ VAR (C)}}$$

$$S_{T_\Delta} = 3V_\phi I_\phi = (3)(200 \text{ V})(20 \text{ A}) = \mathbf{12,000 \text{ VA}}$$

**For the Y:**

$$Z_Y = 4 \Omega + j 3 \Omega = 5 \Omega \angle 36.87^\circ$$

$$I_\phi = \frac{E_L/\sqrt{3}}{Z_Y} = \frac{200 \text{ V}/\sqrt{3}}{5 \Omega} = \frac{116 \text{ V}}{5 \Omega} = 23.12 \text{ A}$$

$$P_{T_Y} = 3I_\phi^2 R_\phi = (3)(23.12 \text{ A})^2(4 \Omega) = \mathbf{6414.41 \text{ W}}$$

$$Q_{T_Y} = 3I_\phi^2 X_\phi = (3)(23.12 \text{ A})^2(3 \Omega) = \mathbf{4810.81 \text{ VAR (L)}}$$

$$S_{T_Y} = 3V_\phi I_\phi = (3)(116 \text{ V})(23.12 \text{ A}) = \mathbf{8045.76 \text{ VA}}$$



For the total load:

$$P_T = P_{T\Delta} + P_{TY} = 7200 \text{ W} + 6414.41 \text{ W} = \mathbf{13,614.41 \text{ W}}$$

$$Q_T = Q_{T\Delta} - Q_{TY} = 9600 \text{ VAR (C)} - 4810.81 \text{ VAR (I)}$$

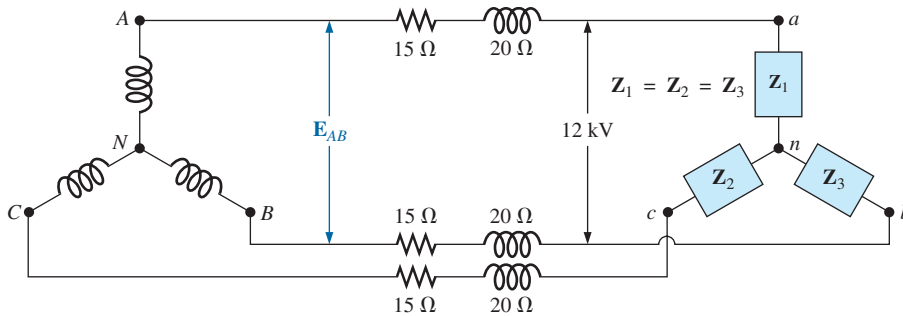
$$= \mathbf{4789.19 \text{ VAR (C)}}$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(13,614.41 \text{ W})^2 + (4789.19 \text{ VAR})^2}$$

$$= \mathbf{14,432.2 \text{ VA}}$$

$$F_p = \frac{P_T}{S_T} = \frac{13,614.41 \text{ W}}{14,432.20 \text{ VA}} = \mathbf{0.943 \text{ leading}}$$

**EXAMPLE 23.7** Each transmission line of the three-wire, three-phase system in Fig. 23.26 has an impedance of  $15 \Omega + j20 \Omega$ . The system delivers a total power of 160 kW at 12,000 V to a balanced three-phase load with a lagging power factor of 0.86.



**FIG. 23.26**  
Example 23.7.

- Determine the magnitude of the line voltage  $E_{AB}$  of the generator.
- Find the power factor of the total load applied to the generator.
- What is the efficiency of the system?

**Solutions:**

$$a. \quad V_\phi(\text{load}) = \frac{V_L}{\sqrt{3}} = \frac{12,000 \text{ V}}{1.73} = 6936.42 \text{ V}$$

$$P_T(\text{load}) = 3V_\phi I_\phi \cos \theta$$

and

$$I_\phi = \frac{P_T}{3V_\phi \cos \theta} = \frac{160,000 \text{ W}}{3(6936.42 \text{ V})(0.86)}$$

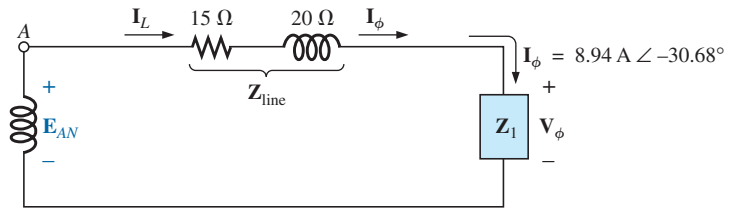
$$= \mathbf{8.94 \text{ A}}$$

Since  $\theta = \cos^{-1} 0.86 = 30.68^\circ$ , assigning  $\mathbf{V}_\phi$  an angle of  $0^\circ$  or  $\mathbf{V}_\phi = V_\phi \angle 0^\circ$ , a lagging power factor results in

$$\mathbf{I}_\phi = 8.94 \text{ A} \angle -30.68^\circ$$

For each phase, the system will appear as shown in Fig. 23.27, where

$$\mathbf{E}_{AN} - \mathbf{I}_\phi \mathbf{Z}_{\text{line}} - \mathbf{V}_\phi = 0$$



**FIG. 23.27**

The loading on each phase of the system in Fig. 23.26.

or

$$\begin{aligned}
 E_{AN} &= I_{\phi} Z_{\text{line}} + V_{\phi} \\
 &= (8.94 \text{ A } \angle -30.68^{\circ})(25 \Omega \angle 53.13^{\circ}) + 6936.42 \text{ V } \angle 0^{\circ} \\
 &= 223.5 \text{ V } \angle 22.45^{\circ} + 6936.42 \text{ V } \angle 0^{\circ} \\
 &= 206.56 \text{ V} + j 85.35 \text{ V} + 6936.42 \text{ V} \\
 &= 7142.98 \text{ V} + j 85.35 \text{ V} \\
 &= 7143.5 \text{ V } \angle 0.68^{\circ}
 \end{aligned}$$

Then 
$$E_{AB} = \sqrt{3} E_{\phi g} = (1.73)(7143.5 \text{ V}) = \mathbf{12,358.26 \text{ V}}$$

b. 
$$\begin{aligned}
 P_T &= P_{\text{load}} + P_{\text{lines}} \\
 &= 160 \text{ kW} + 3(I_L)^2 R_{\text{line}} \\
 &= 160 \text{ kW} + 3(8.94 \text{ A})^2 15 \Omega \\
 &= 160,000 \text{ W} + 3596.55 \text{ W} \\
 &= 163,596.55 \text{ W}
 \end{aligned}$$

and 
$$P_T = \sqrt{3} V_L I_L \cos \theta_T$$

or 
$$\cos \theta_T = \frac{P_T}{\sqrt{3} V_L I_L} = \frac{163,596.55 \text{ W}}{(1.73)(12,358.26 \text{ V})(8.94 \text{ A})}$$

and 
$$F_p = \mathbf{0.856} < 0.86 \text{ of load}$$

c. 
$$\begin{aligned}
 \eta &= \frac{P_o}{P_i} = \frac{P_o}{P_o + P_{\text{losses}}} = \frac{160 \text{ kW}}{160 \text{ kW} + 3596.55 \text{ W}} = 0.978 \\
 &= \mathbf{97.8\%}
 \end{aligned}$$

### 23.11 THREE-WATTMETER METHOD

The power delivered to a balanced or an unbalanced four-wire, Y-connected load can be found by the **three-wattmeter method**, that is, by using three wattmeters in the manner shown in Fig. 23.28. Each wattmeter measures the power delivered to each phase. The potential coil of each wattmeter is connected parallel with the load, while the current coil is in series with the load. The total average power of the system can be found by summing the three wattmeter readings; that is,

$$P_{T_Y} = P_1 + P_2 + P_3 \tag{23.31}$$

For the load (balanced or unbalanced), the wattmeters are connected as shown in Fig. 23.29. The total power is again the sum of the three wattmeter readings:

$$P_{T_{\Delta}} = P_1 + P_2 + P_3 \tag{23.32}$$



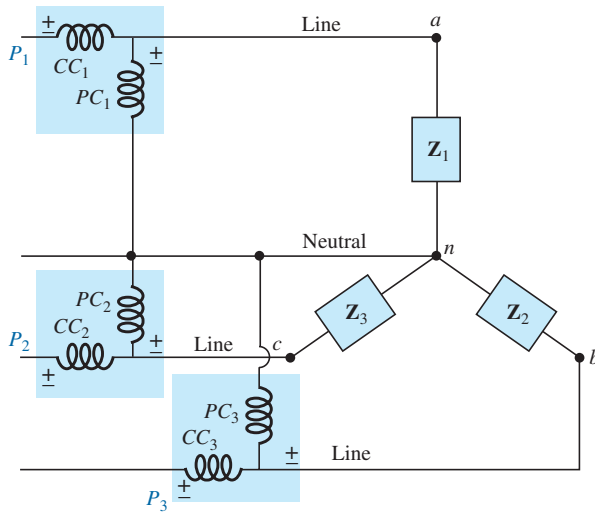


FIG. 23.28

Three-wattmeter method for a Y-connected load.

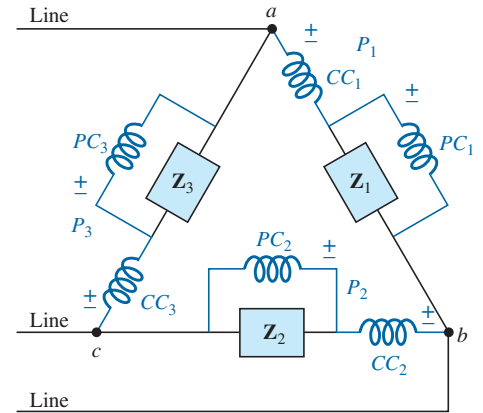


FIG. 23.29

Three-wattmeter method for a Δ-connected load.

If in either of the cases just described the load is balanced, the power delivered to each phase will be the same. The total power is then just three times any one wattmeter reading.

### 23.12 TWO-WATTMETER METHOD

The power delivered to a three-phase, three-wire, Δ- or Y-connected, balanced or unbalanced local can be found using only two wattmeters if the proper connection is employed and if the wattmeter readings are interpreted properly. The basic connections of this **two-wattmeter method** are shown in Fig. 23.30. One end of each potential coil is connected to the same line. The current coils are then placed in the remaining lines.

The connection shown in Fig. 23.31 also satisfies the requirements. A third hookup is also possible, but this is left to the reader as an exercise.

The total power delivered to the load is the algebraic sum of the two wattmeter readings. For a *balanced* load, we now consider two methods of determining whether the total power is the sum or the difference of the two wattmeter readings. The first method to be described requires that we know or are able to find the power factor (leading or lagging) of any one

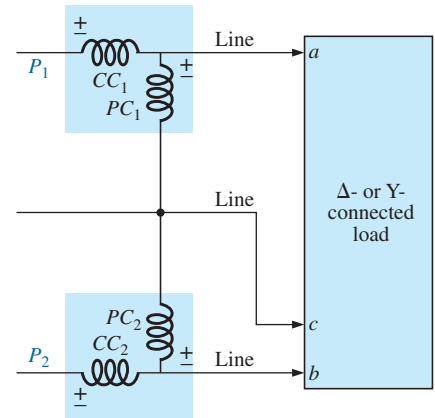


FIG. 23.30

Two-wattmeter method for a Δ- or a Y-connected load.

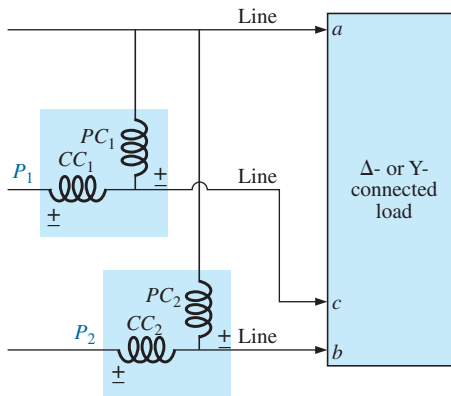
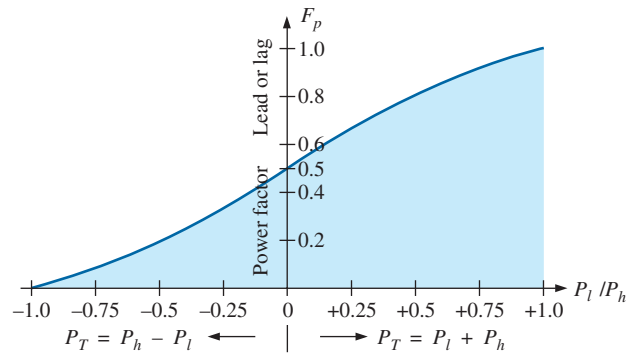


FIG. 23.31

Alternative hookup for the two-wattmeter method.



**FIG. 23.32**

*Determining whether the readings obtained using the two-wattmeter method should be added or subtracted.*

phase of the load. When this information has been obtained, it can be applied directly to the curve in Fig. 23.32.

The curve in Fig. 23.32 is a plot of the power factor of the load (phase) versus the ratio  $P_l/P_h$ , where  $P_l$  and  $P_h$  are the magnitudes of the lower- and higher-reading wattmeters, respectively. Note that for a power factor (leading or lagging) greater than 0.5, the ratio has a positive value. This indicates that both wattmeters are reading positive, and the total power is the sum of the two wattmeter readings; that is,  $P_T = P_l + P_h$ . For a power factor less than 0.5 (leading or lagging), the ratio has a negative value. This indicates that the smaller-reading wattmeter is reading negative, and the total power is the difference of the two wattmeter readings; that is,  $P_T = P_h - P_l$ .

A closer examination reveals that, when the power factor is 1 ( $\cos 0^\circ = 1$ ), corresponding to a purely resistive load,  $P_l/P_h = 1$  or  $P_l = P_h$ , and both wattmeters have the same wattage indication. At a power factor equal to 0 ( $\cos 90^\circ = 0$ ), corresponding to a purely reactive load,  $P_l/P_h = -1$  or  $P_l = -P_h$ , and both wattmeters again have the same wattage indication but with opposite signs. The transition from a negative to a positive ratio occurs when the power factor of the load is 0.5 or  $\theta = \cos^{-1} 0.5 = 60^\circ$ . At this power factor,  $P_l/P_h = 0$ , so that  $P_l = 0$ , while  $P_h$  reads the total power delivered to the load.

The second method for determining whether the total power is the sum or difference of the two wattmeter readings involves a simple laboratory test. For the test to be applied, both wattmeters must first have an up-scale deflection. If one of the wattmeters has a below-zero indication, an up-scale deflection can be obtained by simply reversing the leads of the current coil of the wattmeter. To perform the test:

1. Take notice of which line does not have a current coil sensing the line current.
2. For the lower-reading wattmeter, disconnect the lead of the potential coil connected to the line without the current coil.
3. Take the disconnected lead of the lower-reading wattmeter's potential coil, and touch a connection point on the line that has the current coil of the higher-reading wattmeter.
4. If the pointer deflects downward (below zero watts), the wattage reading of the lower-reading wattmeter should be subtracted from that of the higher-reading wattmeter. Otherwise, the readings should be added.



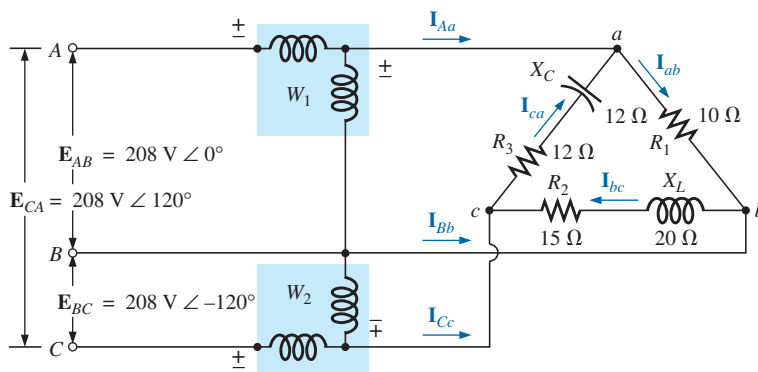
For a balanced system, since

$$P_T = P_h \pm P_1 = \sqrt{3}E_L I_L \cos \theta_{I_\phi}^V$$

the power factor of the load (phase) can be found from the wattmeter readings and the magnitude of the line voltage and current:

$$F_p = \cos \theta_{I_\phi}^V = \frac{P_h \pm P_1}{\sqrt{3}E_L I_L} \quad (23.33)$$

**EXAMPLE 23.8** For the unbalanced  $\Delta$ -connected load in Fig. 23.33 with two properly connected wattmeters:



**FIG. 23.33**  
Example 23.8.

- Determine the magnitude and angle of the phase currents.
- Calculate the magnitude and angle of the line currents.
- Determine the power reading of each wattmeter.
- Calculate the total power absorbed by the load.
- Compare the result of part (d) with the total power calculated using the phase currents and the resistive elements.

**Solutions:**

- $$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{E_{AB}}{Z_{ab}} = \frac{208 \text{ V } \angle 0^\circ}{10 \Omega \angle 0^\circ} = 20.8 \text{ A } \angle 0^\circ$$

$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{E_{BC}}{Z_{bc}} = \frac{208 \text{ V } \angle -120^\circ}{15 \Omega + j20 \Omega} = \frac{208 \text{ V } \angle -120^\circ}{25 \Omega \angle 53.13^\circ}$$

$$= 8.32 \text{ A } \angle -173.13^\circ$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{E_{CA}}{Z_{ca}} = \frac{208 \text{ V } \angle +120^\circ}{12 \Omega + j12 \Omega} = \frac{208 \text{ V } \angle +120^\circ}{16.97 \Omega \angle -45^\circ}$$

$$= 12.26 \text{ A } \angle 165^\circ$$
- $$I_{Aa} = I_{ab} - I_{ca}$$

$$= 20.8 \text{ A } \angle 0^\circ - 12.26 \text{ A } \angle 165^\circ$$

$$= 20.8 \text{ A} - (-11.84 \text{ A} + j 3.17 \text{ A})$$

$$= 20.8 \text{ A} + 11.84 \text{ A} - j 3.17 \text{ A} = 32.64 \text{ A} - j 3.17 \text{ A}$$

$$= 32.79 \text{ A } \angle -5.55^\circ$$



$$\begin{aligned}
 \mathbf{I}_{Bb} &= \mathbf{I}_{bc} - \mathbf{I}_{ab} \\
 &= 8.32 \text{ A } \angle -173.13^\circ - 20.8 \text{ A } \angle 0^\circ \\
 &= (-8.26 \text{ A} - j 1 \text{ A}) - 20.8 \text{ A} \\
 &= -8.26 \text{ A} - 20.8 \text{ A} - j 1 \text{ A} = -29.06 \text{ A} - j 1 \text{ A} \\
 &= \mathbf{29.08 \text{ A } \angle -178.03^\circ}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{I}_{Cc} &= \mathbf{I}_{ca} - \mathbf{I}_{bc} \\
 &= 12.26 \text{ A } \angle 165^\circ - 8.32 \text{ A } \angle -173.13^\circ \\
 &= (-11.84 \text{ A} + j 3.17 \text{ A}) - (-8.26 \text{ A} - j 1 \text{ A}) \\
 &= -11.84 \text{ A} + 8.26 \text{ A} + j(3.17 \text{ A} + 1 \text{ A}) = -3.58 \text{ A} + j 4.17 \text{ A} \\
 &= \mathbf{5.5 \text{ A } \angle 130.65^\circ}
 \end{aligned}$$

$$\begin{aligned}
 \text{c. } P_1 &= V_{ab} I_{Aa} \cos \theta_{I_{Aa}}^{V_{ab}} \quad V_{ab} = 208 \text{ V } \angle 0^\circ \\
 &\quad \mathbf{I_{Aa} = 32.79 \text{ A } \angle -5.55^\circ} \\
 &= (208 \text{ V})(32.79 \text{ A}) \cos 5.55^\circ \\
 &= \mathbf{6788.35 \text{ W}}
 \end{aligned}$$

$$V_{bc} = \mathbf{E_{BC}} = 208 \text{ V } \angle -120^\circ$$

$$\begin{aligned}
 \text{but} \quad V_{cb} &= \mathbf{E_{CB}} = 208 \text{ V } \angle -120^\circ + 180^\circ \\
 &= 208 \text{ V } \angle 60^\circ
 \end{aligned}$$

$$\text{with} \quad \mathbf{I_{Cc} = 5.5 \text{ A } \angle 130.65^\circ}$$

$$\begin{aligned}
 P_2 &= V_{cb} I_{Cc} \cos \theta_{I_{Cc}}^{V_{cb}} \\
 &= (208 \text{ V})(5.5 \text{ A}) \cos 70.65^\circ \\
 &= \mathbf{379.1 \text{ W}}
 \end{aligned}$$

$$\begin{aligned}
 \text{d. } P_T &= P_1 + P_2 = 6788.35 \text{ W} + 379.1 \text{ W} \\
 &= \mathbf{7167.45 \text{ W}}
 \end{aligned}$$

$$\begin{aligned}
 \text{e. } P_T &= (I_{ab})^2 R_1 + (I_{bc})^2 R_2 + (I_{ca})^2 R_3 \\
 &= (20.8 \text{ A})^2 10 \Omega + (8.32 \text{ A})^2 15 \Omega + (12.26 \text{ A})^2 12 \Omega \\
 &= 4326.4 \text{ W} + 1038.34 \text{ W} + 1803.69 \text{ W} \\
 &= \mathbf{7168.43 \text{ W}}
 \end{aligned}$$

(The slight difference is due to the level of accuracy carried through the calculations.)

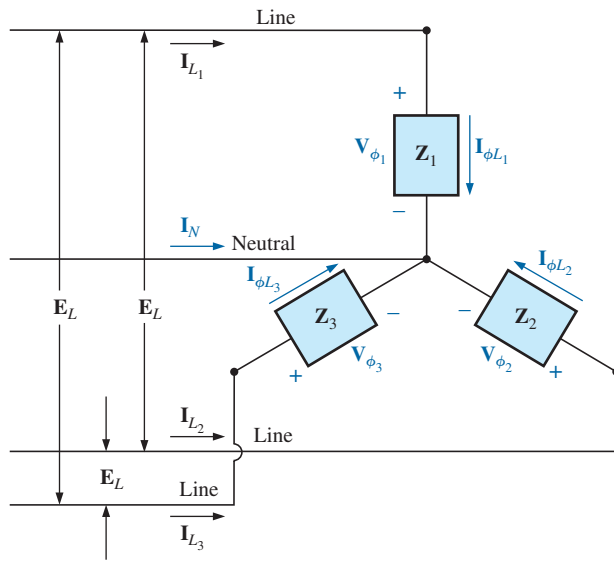
### 23.13 UNBALANCED, THREE-PHASE, FOUR-WIRE, Y-CONNECTED LOAD

For the three-phase, four-wire, Y-connected load in Fig. 23.34, conditions are such that *none* of the load impedances are equal—hence we have an **unbalanced polyphase load**. Since the neutral is a common point between the load and source, no matter what the impedance of each phase of the load and source, the voltage across each phase is the phase voltage of the generator:

$$\boxed{V_\phi = E_\phi} \quad (23.34)$$

The phase currents can therefore be determined by Ohm's law:

$$\boxed{I_{\phi_1} = \frac{V_{\phi_1}}{Z_1} = \frac{E_{\phi_1}}{Z_1} \quad \text{and so on}} \quad (23.35)$$

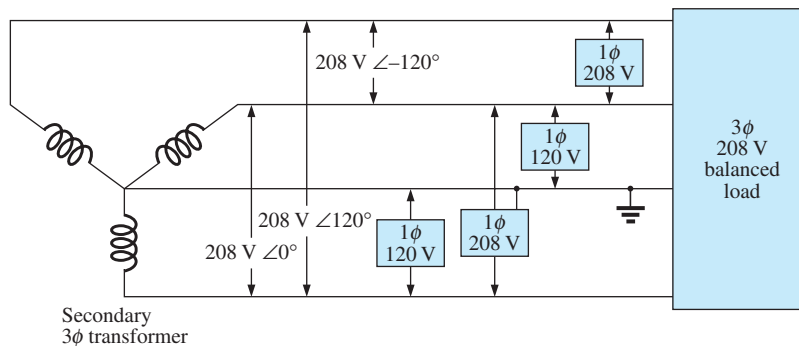


**FIG. 23.34**  
*Unbalanced Y-connected load.*

The current in the neutral for any unbalanced system can then be found by applying Kirchhoff’s current law at the common point  $n$ :

$$\mathbf{I}_N = \mathbf{I}_{\phi 1} + \mathbf{I}_{\phi 2} + \mathbf{I}_{\phi 3} = \mathbf{I}_{L1} + \mathbf{I}_{L2} + \mathbf{I}_{L3} \quad (23.36)$$

Because of the variety of equipment found in an industrial environment, both three-phase power and single-phase power are usually provided with the single-phase obtained off the three-phase system. In addition, since the load on each phase is continually changing, a four-wire system (with a neutral) is normally used to ensure steady voltage levels and to provide a path for the current resulting from an unbalanced load. The system in Fig. 23.35 has a three-phase transformer dropping the line voltage from 13,800 V to 208 V. All the lower-power-demand loads such as lighting, wall outlets, security, etc., use the single-phase, 120 V line to neutral voltage. Higher power loads, such as air conditioners, electric ovens or dryers, etc., use the single-phase, 208 V available from line to line. For larger motors and special high-demand equipment, the full three-phase power can be taken directly off the system, as shown in Fig. 23.35. In the design and construction of a commercial establishment, the



**FIG. 23.35**  
*3 $\phi$ /1 $\phi$ , 208 V/120 V industrial supply.*



National Electric Code requires that every effort be made to ensure that the expected loads, whether they be single- or multiphase, result in a total load that is as balanced as possible between the phases, thus ensuring the highest level of transmission efficiency.

### 23.14 UNBALANCED, THREE-PHASE, THREE-WIRE, Y-CONNECTED LOAD

For the system shown in Fig. 23.36, the required equations can be derived by first applying Kirchhoff’s voltage law around each closed loop to produce

$$\begin{aligned} E_{AB} - V_{an} + V_{bn} &= 0 \\ E_{BC} - V_{bn} + V_{cn} &= 0 \\ E_{CA} - V_{cn} + V_{an} &= 0 \end{aligned}$$

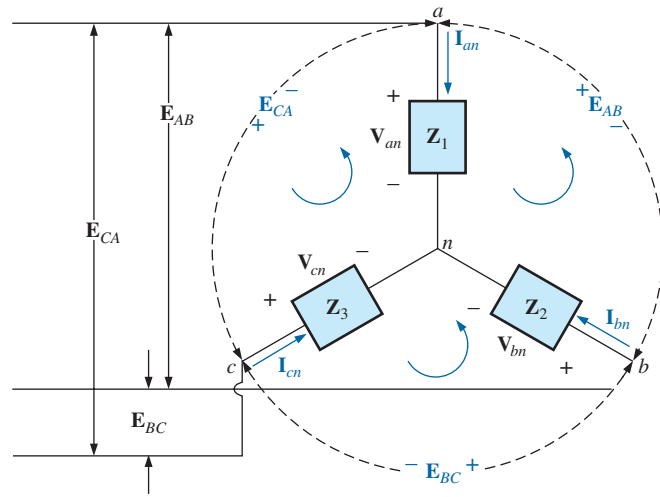


FIG. 23.36

Unbalanced, three-phase, three-wire, Y-connected load.

Substituting, we have

$$V_{an} = I_{an}Z_1 \quad V_{bn} = I_{bn}Z_2 \quad V_{cn} = I_{cn}Z_3$$

$$E_{AB} = I_{an}Z_1 - I_{bn}Z_2 \tag{23.37a}$$

$$E_{BC} = I_{bn}Z_2 - I_{cn}Z_3 \tag{23.37b}$$

$$E_{CA} = I_{cn}Z_3 - I_{an}Z_1 \tag{23.37c}$$

Applying Kirchhoff’s current law at node *n* results in

$$I_{an} + I_{bn} + I_{cn} = 0 \quad \text{and} \quad I_{bn} = -I_{an} - I_{cn}$$

Substituting for *I<sub>bn</sub>* in Eqs. (23.37a) and (23.37b) yields

$$E_{AB} = I_{an}Z_1 - [-(I_{an} + I_{cn})]Z_2$$

$$E_{BC} = -(I_{an} + I_{cn})Z_2 - I_{cn}Z_3$$

which are rewritten as

$$E_{AB} = I_{an}(Z_1 + Z_2) + I_{cn}Z_2$$

$$E_{BC} = I_{an}(-Z_2) + I_{cn}[-(Z_2 + Z_3)]$$

Using determinants, we have





$$\begin{aligned} \mathbf{I}_{an} &= \frac{\begin{vmatrix} \mathbf{E}_{AB} & \mathbf{Z}_2 \\ \mathbf{E}_{BC} & -(\mathbf{Z}_2 + \mathbf{Z}_3) \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & \mathbf{Z}_2 \\ -\mathbf{Z}_2 & -(\mathbf{Z}_2 + \mathbf{Z}_3) \end{vmatrix}} \\ &= \frac{-(\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{E}_{AB} - \mathbf{E}_{BC}\mathbf{Z}_2}{-\mathbf{Z}_1\mathbf{Z}_2 - \mathbf{Z}_1\mathbf{Z}_3 - \mathbf{Z}_2\mathbf{Z}_3 - \mathbf{Z}_2^2 + \mathbf{Z}_2^2} \\ \mathbf{I}_{an} &= \frac{-\mathbf{Z}_2(\mathbf{E}_{AB} + \mathbf{E}_{BC}) - \mathbf{Z}_3\mathbf{E}_{AB}}{-\mathbf{Z}_1\mathbf{Z}_2 - \mathbf{Z}_1\mathbf{Z}_3 - \mathbf{Z}_2\mathbf{Z}_3} \end{aligned}$$

Applying Kirchhoff's voltage law to the line voltages:

$$\mathbf{E}_{AB} + \mathbf{E}_{CA} + \mathbf{E}_{BC} = 0 \quad \text{or} \quad \mathbf{E}_{AB} + \mathbf{E}_{BC} = -\mathbf{E}_{CA}$$

Substituting for  $(\mathbf{E}_{AB} + \mathbf{E}_{BC})$  in the above equation for  $\mathbf{I}_{an}$ :

$$\mathbf{I}_{an} = \frac{-\mathbf{Z}_2(-\mathbf{E}_{CA}) - \mathbf{Z}_3\mathbf{E}_{AB}}{-\mathbf{Z}_1\mathbf{Z}_2 - \mathbf{Z}_1\mathbf{Z}_3 - \mathbf{Z}_2\mathbf{Z}_3}$$

and

$$\mathbf{I}_{an} = \frac{\mathbf{E}_{AB}\mathbf{Z}_3 - \mathbf{E}_{CA}\mathbf{Z}_2}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3} \quad (23.38)$$

In the same manner, it can be shown that

$$\mathbf{I}_{cn} = \frac{\mathbf{E}_{CA}\mathbf{Z}_2 - \mathbf{E}_{BC}\mathbf{Z}_1}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3} \quad (23.39)$$

Substituting Eq. (23.39) for  $\mathbf{I}_{cn}$  in the right-hand side of Eq. (23.37b), we obtain

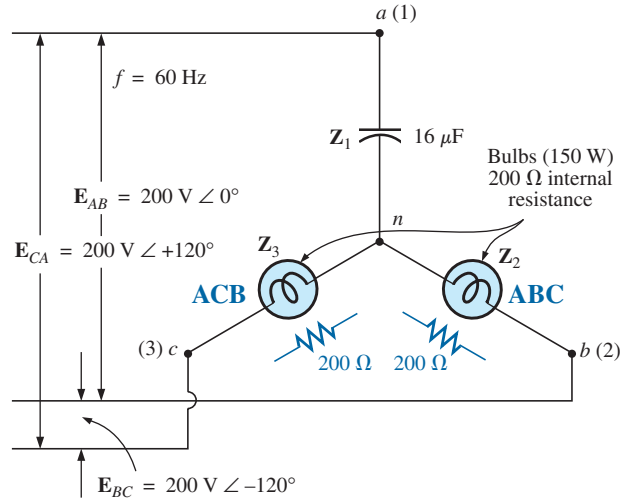
$$\mathbf{I}_{bn} = \frac{\mathbf{E}_{BC}\mathbf{Z}_1 - \mathbf{E}_{AB}\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3} \quad (23.40)$$

**EXAMPLE 23.9** A *phase-sequence indicator* is an instrument that can display the phase sequence of a polyphase circuit. A network that performs this function appears in Fig. 23.37. The applied phase sequence is  $ABC$ . The bulb corresponding to this phase sequence burns more brightly than the bulb indicating the  $ACB$  sequence because a greater current is passing through the  $ABC$  bulb. Calculating the phase currents demonstrates that this situation does in fact exist:

$$Z_1 = X_C = \frac{1}{\omega C} = \frac{1}{(377 \text{ rad/s})(16 \times 10^{-6} \text{ F})} = 166 \Omega$$

By Eq. (23.39),

$$\begin{aligned} \mathbf{I}_{cn} &= \frac{\mathbf{E}_{CA}\mathbf{Z}_2 - \mathbf{E}_{BC}\mathbf{Z}_1}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3} \\ &= \frac{(200 \text{ V} \angle 120^\circ)(200 \Omega \angle 0^\circ) - (200 \text{ V} \angle -120^\circ)(166 \Omega \angle -90^\circ)}{(166 \Omega \angle -90^\circ)(200 \Omega \angle 0^\circ) + (166 \Omega \angle -90^\circ)(200 \Omega \angle 0^\circ) + (200 \Omega \angle 0^\circ)(200 \Omega \angle 0^\circ)} \end{aligned}$$



**FIG. 23.37**  
Example 23.9.

$$I_{cn} = \frac{40,000 \text{ V } \angle 120^\circ + 33,200 \text{ V } \angle -30^\circ}{33,200 \Omega \angle -90^\circ + 33,200 \Omega \angle -90^\circ + 40,000 \Omega \angle 0^\circ}$$

Dividing the numerator and denominator by 1000 and converting both to the rectangular domain yields

$$\begin{aligned} I_{cn} &= \frac{(-20 + j 34.64) + (28.75 - j 16.60)}{40 - j 66.4} \\ &= \frac{8.75 + j 18.04}{77.52 \angle -58.93^\circ} = \frac{20.05 \angle 64.13^\circ}{77.52 \angle -58.93^\circ} \\ I_{cn} &= \mathbf{0.259 \text{ A } \angle 123.06^\circ} \end{aligned}$$

By Eq. (23.40),

$$\begin{aligned} I_{bn} &= \frac{E_{BC}Z_1 - E_{AB}Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \\ &= \frac{(200 \text{ V } \angle -120^\circ)(166 \angle -90^\circ) - (200 \text{ V } \angle 0^\circ)(200 \angle 0^\circ)}{77.52 \times 10^3 \Omega \angle -58.93^\circ} \\ I_{bn} &= \frac{33,200 \text{ V } \angle -210^\circ - 40,000 \text{ V } \angle 0^\circ}{77.52 \times 10^3 \Omega \angle -58.93^\circ} \end{aligned}$$

Dividing by 1000 and converting to the rectangular domain yields

$$\begin{aligned} I_{bn} &= \frac{-28.75 + j 16.60 - 40.0}{77.52 \angle -58.93^\circ} = \frac{-68.75 + j 16.60}{77.52 \angle -58.93^\circ} \\ &= \frac{70.73 \angle 166.43^\circ}{77.52 \angle -58.93^\circ} = \mathbf{0.91 \text{ A } \angle 225.36^\circ} \end{aligned}$$

and  $I_{bn} > I_{cn}$  by a factor of more than 3 : 1. Therefore, the bulb indicating an *ABC* sequence will burn more brightly due to the greater current. If the phase sequence were *ACB*, the reverse would be true.



## PROBLEMS

### SECTION 23.5 Y-Connected Generator with a Y-Connected Load

- A balanced Y load having a  $10\ \Omega$  resistance in each leg is connected to a three-phase, four-wire, Y-connected generator having a line voltage of 208 V. Calculate the magnitude of

  - the phase voltage of the generator.
  - the phase voltage of the load.
  - the phase current of the load.
  - the line current.
- Repeat Problem 1 if each phase impedance is changed to a  $12\ \Omega$  resistor in series with a  $16\ \Omega$  capacitive reactance.
- Repeat Problem 1 if each phase impedance is changed to a  $10\ \Omega$  resistor in parallel with a  $10\ \Omega$  capacitive reactance.
- The phase sequence for the Y-Y system in Fig. 23.38 is  $ABC$ .

  - Find the angles  $\theta_2$  and  $\theta_3$  for the specified phase sequence.
  - Find the voltage across each phase impedance in phasor form.
  - Find the current through each phase impedance in phasor form.
  - Draw the phasor diagram of the currents found in part (c), and show that their phasor sum is zero.
  - Find the magnitude of the line currents.
  - Find the magnitude of the line voltages.
- Repeat Problem 4 if the phase impedances are changed to a  $9\ \Omega$  resistor in series with a  $12\ \Omega$  inductive reactance.
- Repeat Problem 4 if the phase impedances are changed to a  $6\ \Omega$  resistance in parallel with an  $8\ \Omega$  capacitive reactance.
- For the system in Fig. 23.39, find the magnitude of the unknown voltages and currents.

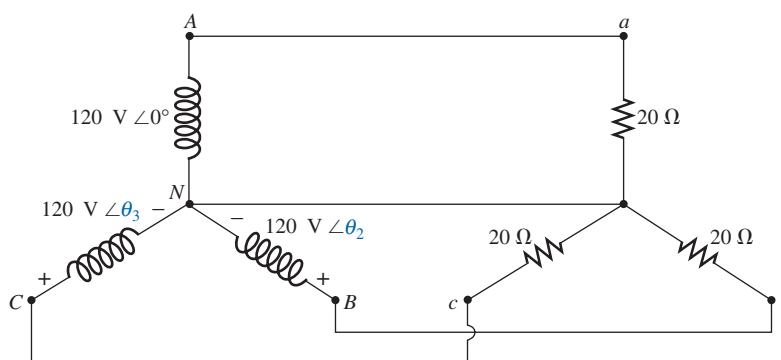


FIG. 23.38

Problems 4, 5, 6, and 31.

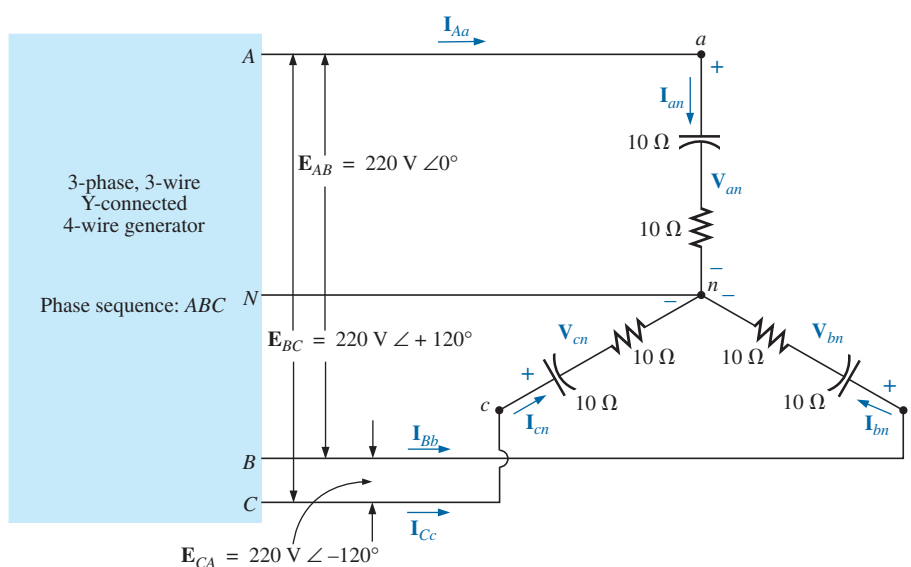


FIG. 23.39

Problems 7, 32, and 44.



\*8. Compute the magnitude of the voltage  $E_{AB}$  for the balanced three-phase system in Fig. 23.40.

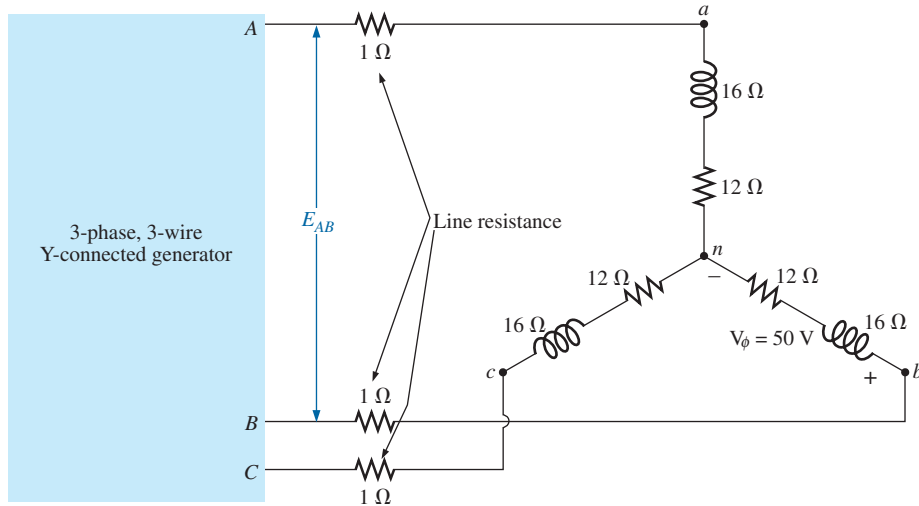


FIG. 23.40  
Problem 8.

\*9. For the Y-Y system in Fig. 23.41:

- Find the magnitude and angle associated with the voltages  $E_{AN}$ ,  $E_{BN}$ , and  $E_{CN}$ .
- Determine the magnitude and angle associated with each phase current of the load:  $I_{an}$ ,  $I_{bn}$ , and  $I_{cn}$ .
- Find the magnitude and phase angle of each line current:  $I_{Aa}$ ,  $I_{Bb}$ , and  $I_{Cc}$ .
- Determine the magnitude and phase angle of the voltage across each phase of the load:  $V_{an}$ ,  $V_{bn}$ , and  $V_{cn}$ .

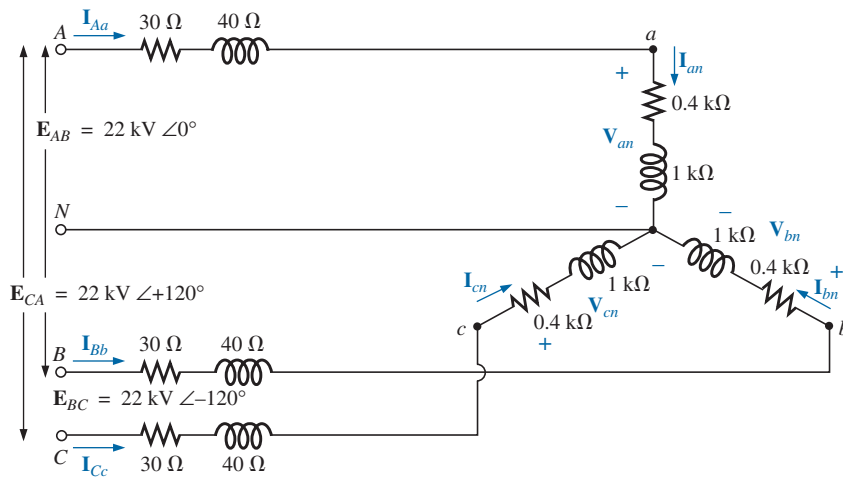
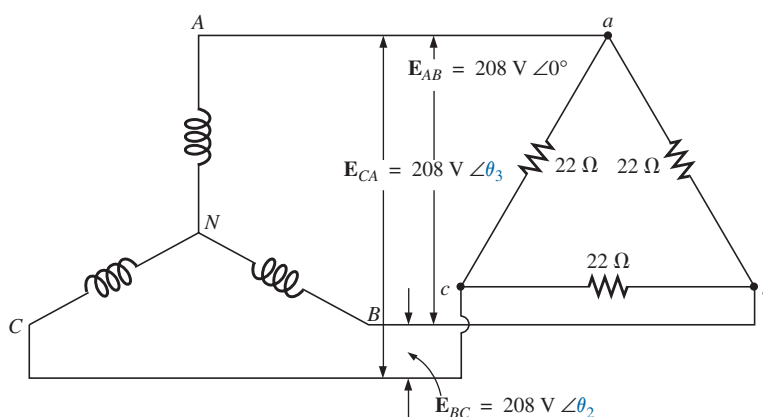


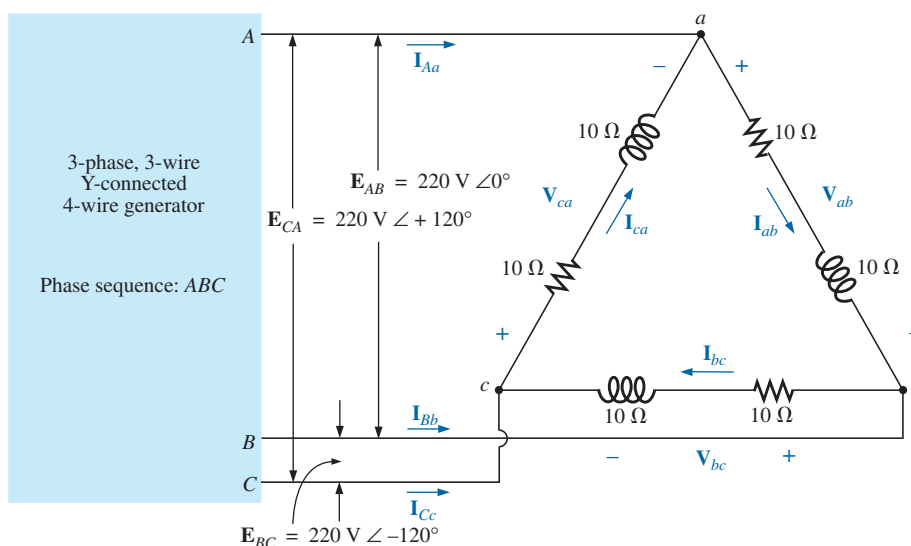
FIG. 23.41  
Problem 9.


**SECTION 23.6 Y-Δ System**

10. A balanced  $\Delta$  load having a  $20\ \Omega$  resistance in each leg is connected to a three-phase, three-wire, Y-connected generator having a line voltage of  $208\ \text{V}$ . Calculate the magnitude of
- the phase voltage of the generator.
  - the phase voltage of the load.
  - the phase current of the load.
  - the line current.
11. Repeat Problem 10 if each phase impedance is changed to a  $6.8\ \Omega$  resistor in series with a  $14\ \Omega$  inductive reactance.
12. Repeat Problem 10 if each phase impedance is changed to an  $18\ \Omega$  resistance in parallel with an  $18\ \Omega$  capacitive reactance.
13. The phase sequence for the Y- $\Delta$  system in Fig. 23.42 is  $ABC$ .
- Find the angles  $\theta_2$  and  $\theta_3$  for the specified phase sequence.
  - Find the voltage across each phase impedance in phasor form.
  - Draw the phasor diagram of the voltages found in part (b), and show that their sum is zero around the closed loop of the  $\Delta$  load.
  - Find the current through each phase impedance in phasor form.
  - Find the magnitude of the line currents.
  - Find the magnitude of the generator phase voltages.
14. Repeat Problem 13 if the phase impedances are changed to a  $100\ \Omega$  resistor in series with a capacitive reactance of  $100\ \Omega$ .
15. Repeat Problem 13 if the phase impedances are changed to a  $3\ \Omega$  resistor in parallel with an inductive reactance of  $4\ \Omega$ .
16. For the system in Fig. 23.43, find the magnitude of the unknown voltages and currents.


**FIG. 23.42**

Problems 13, 14, 15, 34, and 45.


**FIG. 23.43**

Problems 16, 35, and 47.



- \*17. For the  $\Delta$ -connected load in Fig. 23.44:
- Find the magnitude and angle of each phase current  $I_{ab}$ ,  $I_{bc}$ , and  $I_{ca}$ .
  - Calculate the magnitude and angle of each line current  $I_{Aa}$ ,  $I_{Bb}$ , and  $I_{Cc}$ .
  - Determine the magnitude and angle of the voltages  $E_{AB}$ ,  $E_{BC}$ , and  $E_{CA}$ .

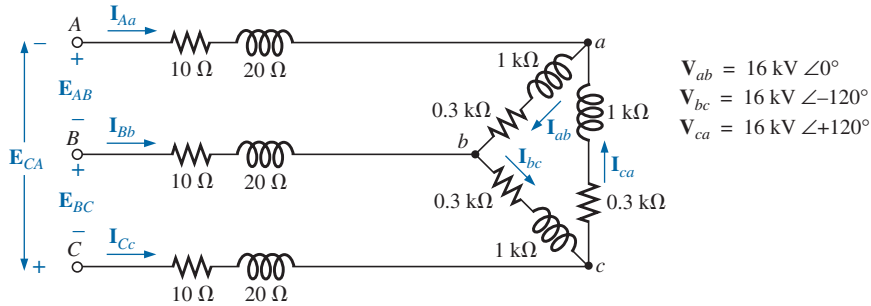


FIG. 23.44  
Problem 17.

**SECTION 23.9**  $\Delta$ - $\Delta$ ,  $\Delta$ -Y Three-Phase Systems

- A balanced Y load having a  $30 \Omega$  resistance in each leg is connected to a three-phase,  $\Delta$ -connected generator having a line voltage of 208 V. Calculate the magnitude of
  - the phase voltage of the generator.
  - the phase voltage of the load.
  - the phase current of the load.
  - the line current.
- Repeat Problem 18 if each phase impedance is changed to a  $12 \Omega$  resistor in series with a  $12 \Omega$  inductive reactance.
- Repeat Problem 18 if each phase impedance is changed to a  $15 \Omega$  resistor in parallel with a  $20 \Omega$  capacitive reactance.
- For the system in Fig. 23.45, find the magnitude of the unknown voltages and currents.

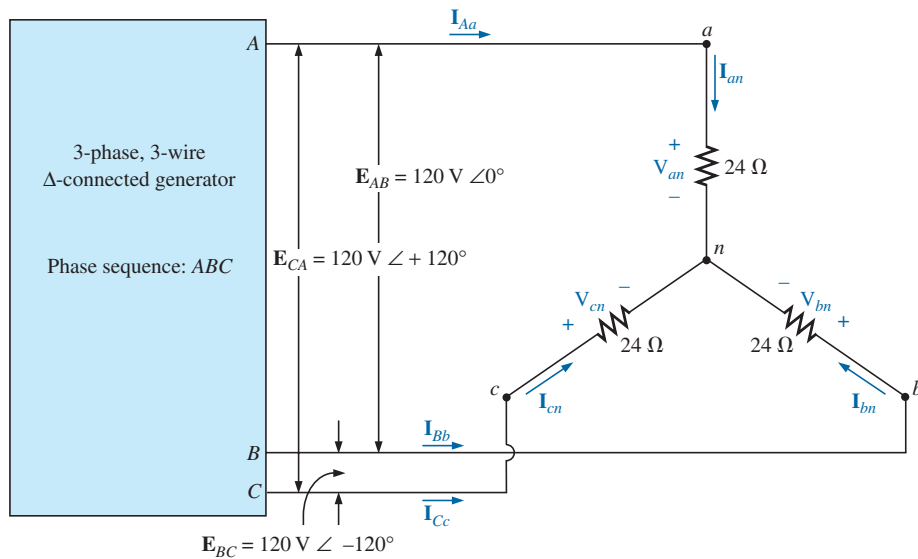
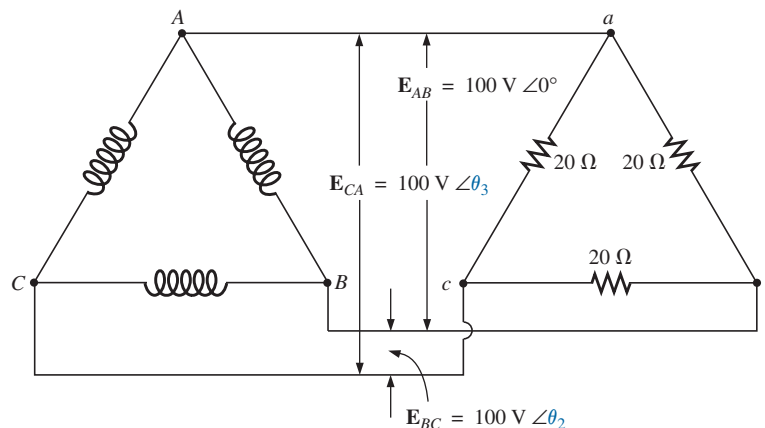


FIG. 23.45  
Problems 21, 22, 23, and 37.





22. Repeat Problem 21 if each phase impedance is changed to a  $10\ \Omega$  resistor in series with a  $20\ \Omega$  inductive reactance.
23. Repeat Problem 21 if each phase impedance is changed to a  $20\ \Omega$  resistor in parallel with a  $15\ \Omega$  capacitive reactance.
24. A balanced  $\Delta$  load having a  $220\ \Omega$  resistance in each leg is connected to a three-phase,  $\Delta$ -connected generator having a line voltage of  $440\ \text{V}$ . Calculate the magnitude of
  - a. the phase voltage of the generator.
  - b. the phase voltage of the load.
  - c. the phase current of the load.
  - d. the line current.
25. Repeat Problem 24 if each phase impedance is changed to a  $12\ \Omega$  resistor in series with a  $9\ \Omega$  capacitive reactance.
26. Repeat Problem 24 if each phase impedance is changed to a  $22\ \Omega$  resistor in parallel with a  $22\ \Omega$  inductive reactance.
27. The phase sequence for the  $\Delta$ - $\Delta$  system in Fig. 23.46 is  $ABC$ .
  - a. Find the angles  $\theta_2$  and  $\theta_3$  for the specified phase sequence.
  - b. Find the voltage across each phase impedance in phasor form.
  - c. Draw the phasor diagram of the voltages found in part (b), and show that their phasor sum is zero around the closed loop of the  $\Delta$  load.
  - d. Find the current through each phase impedance in phasor form.
  - e. Find the magnitude of the line currents.



**FIG. 23.46**  
Problem 27.

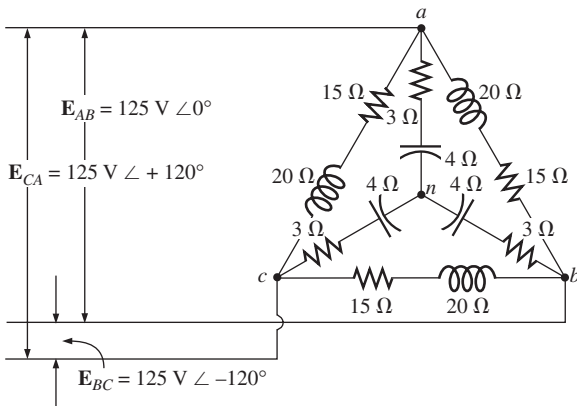
28. Repeat Problem 25 if each phase impedance is changed to a  $12\ \Omega$  resistor in series with a  $16\ \Omega$  inductive reactance.
29. Repeat Problem 25 if each phase impedance is changed to a  $20\ \Omega$  resistor in parallel with a  $20\ \Omega$  capacitive reactance.

### SECTION 23.10 Power

30. Find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the three-phase system in Problem 2.
31. Find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the three-phase system in Problem 4.
32. Find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the three-phase system in Problem 7.

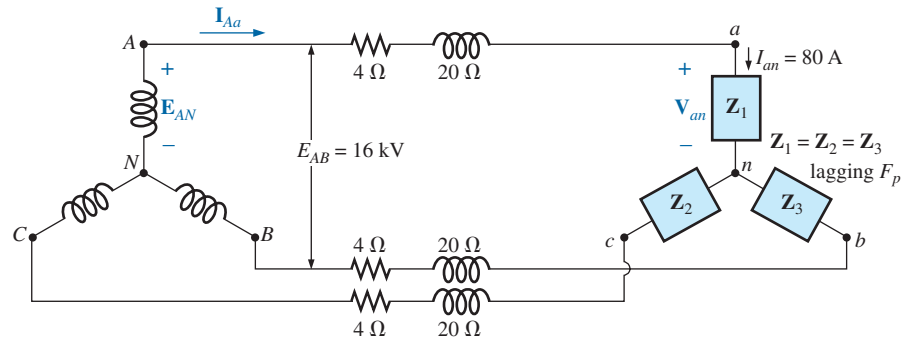


- 33. Find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the three-phase system in Problem 12.
- 34. Find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the three-phase system in Problem 14.
- 35. Find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the three-phase system in Problem 16.
- 36. Find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the three-phase system in Problem 20.
- 37. Find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the three-phase system in Problem 22.
- 38. Find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the three-phase system in Problem 26.
- 39. Find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the three-phase system in Problem 28.
- 40. A balanced, three-phase,  $\Delta$ -connected load has a line voltage of 200 and a total power consumption of 4800 W at a lagging power factor of 0.8. Find the impedance of each phase in rectangular coordinates.
- 41. A balanced, three-phase, Y-connected load has a line voltage of 208 and a total power consumption of 1200 W at a leading power factor of 0.6. Find the impedance of each phase in rectangular coordinates.
- \*42. Find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the system in Fig. 23.47.



**FIG. 23.47**  
Problem 42.

- \*43. The Y-Y system in Fig. 23.48 has a balanced load and a line impedance  $Z_{line} = 4 \Omega + j 20 \Omega$ . If the line voltage at the generator is 16,000 V and the total power delivered to the load is 1200 kW at 80 A, determine each of the following:
  - a. The magnitude of each phase voltage of the generator.
  - b. The magnitude of the line currents.
  - c. The total power delivered by the source.
  - d. The power factor angle of the entire load “seen” by the source.
  - e. The magnitude and angle of the current  $I_{Aa}$  if  $E_{AN} = E_{AN} \angle 0^\circ$ .
  - f. The magnitude and angle of the phase voltage  $V_{an}$ .
  - g. The impedance of the load of each phase in rectangular coordinates.



**FIG. 23.48**  
Problem 43.

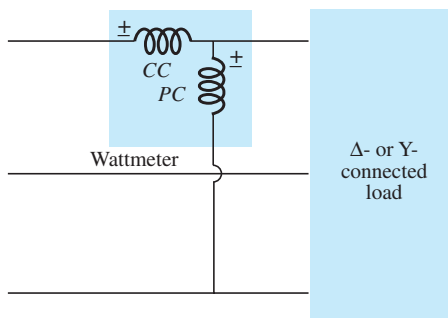
- h. The difference between the power factor of the load and the power factor of the entire system (including  $Z_{\text{line}}$ ).
- i. The efficiency of the system.

### SECTION 23.11 Three-Wattmeter Method

44. a. Sketch the connections required to measure the total watts delivered to the load in Fig. 23.39 using three wattmeters.  
b. Determine the total wattage dissipation and the reading of each wattmeter.
45. Repeat Problem 44 for the network in Fig. 23.42.

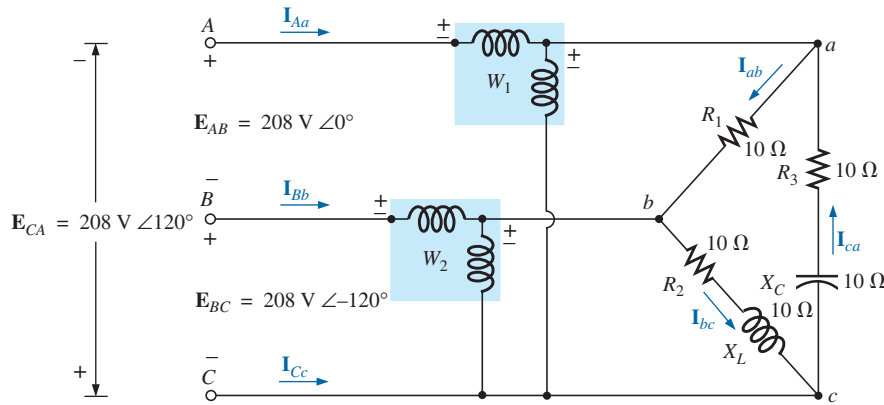
### SECTION 23.12 Two-Wattmeter Method

46. a. For the three-wire system in Fig. 23.49, properly connect a second wattmeter so that the two measure the total power delivered to the load.  
b. If one wattmeter has a reading of 200 W and the other a reading of 85 W, what is the total dissipation in watts if the total power factor is 0.8 leading?  
c. Repeat part (b) if the total power factor is 0.2 lagging and  $P_l = 100$  W.



**FIG. 23.49**  
Problem 46.

47. Sketch three different ways that two wattmeters can be connected to measure the total power delivered to the load in Problem 16.

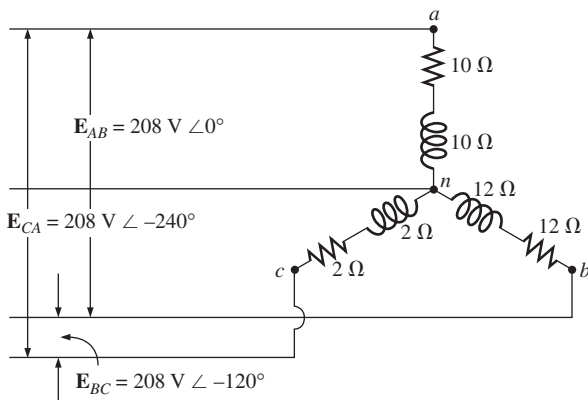


**FIG. 23.50**  
Problem 48.

- \*48. For the Y-Δ system in Fig. 23.50:
- Determine the magnitude and angle of the phase currents.
  - Find the magnitude and angle of the line currents.
  - Determine the reading of each wattmeter.
  - Find the total power delivered to the load.

**SECTION 23.13 Unbalanced, Three-Phase, Four-Wire, Y-Connected Load**

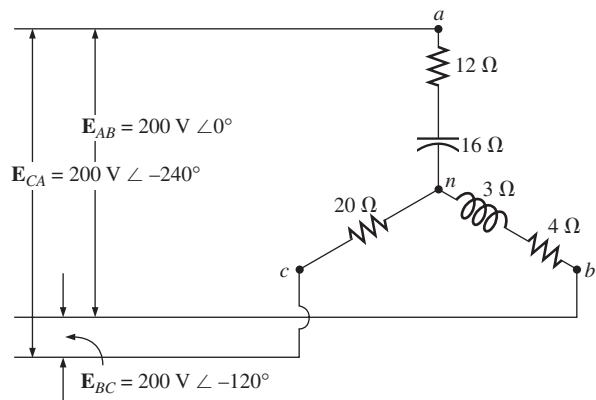
- \*49. For the system in Fig. 23.51:
- Calculate the magnitude of the voltage across each phase of the load.
  - Find the magnitude of the current through each phase of the load.
  - Find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the system.
  - Find the phase currents in phasor form.
  - Using the results of part (c), determine the current  $I_N$ .



**FIG. 23.51**  
Problem 49.

**SECTION 23.14 Unbalanced, Three-Phase, Three-Wire, Y-Connected Load**

- \*50. For the three-phase, three-wire system in Fig. 23.52, find the magnitude of the current through each phase of the load, and



**FIG. 23.52**  
Problem 50.

find the total watts, volt-amperes reactive, volt-amperes, and  $F_p$  of the load.

**GLOSSARY**

- Δ-connected ac generator** A three-phase generator having the three phases connected in the shape of the capital Greek letter delta ( $\Delta$ ).
- Line current** The current that flows from the generator to the load of a single-phase or polyphase system.
- Line voltage** The potential difference that exists between the lines of a single-phase or polyphase system.
- Neutral connection** The connection between the generator and the load that, under balanced conditions, will have zero current associated with it.
- Phase current** The current that flows through each phase of a single-phase (or polyphase) generator or load.
- Phase sequence** The order in which the generated sinusoidal voltages of a polyphase generator will affect the load to which they are applied.
- Phase voltage** The voltage that appears between the line and neutral of a Y-connected generator and from line to line in a Δ-connected generator.



**Polyphase ac generator** An electromechanical source of ac power that generates more than one sinusoidal voltage per rotation of the rotor. The frequency generated is determined by the speed of rotation and the number of poles of the rotor.

**Single-phase ac generator** An electromechanical source of ac power that generates a single sinusoidal voltage having a frequency determined by the speed of rotation and the number of poles of the rotor.

**Three-wattmeter method** A method for determining the total power delivered to a three-phase load using three wattmeters.

**Two-wattmeter method** A method for determining the total power delivered to a  $\Delta$ - or Y-connected three-phase load using only two wattmeters and considering the power factor of the load.

**Unbalanced polyphase load** A load not having the same impedance in each phase.

**Y-connected three-phase generator** A three-phase source of ac power in which the three phases are connected in the shape of the letter Y.

