Chapter 11 - Nuclear Structure

11-1: ${}^{6}_{3}\text{Li}$: $Z = 3$ protons, $A - Z = 6 - 3 = 3$ neutrons.
${}^{22}_{10}\text{Ne}$: $Z = 10$ protons, $A - Z = 22 - 10 = 12$ neutrons.
${}^{94}_{40}\text{Zr}$: $Z = 40$ protons, $A - Z = 94 - 40 = 54$ neutrons.
${}^{180}_{72}\text{Hf}$: $Z = 72$ protons, $A - Z = 180 - 72 = 108$ neutrons.

11-3: The radius of a gold nucleus is, from Equation (11.1),

$$R = R_0\, A^{1/3} = \left(1.2 \times 10^{-15}\ \text{m}\right)(197)^{1/3} = 6.98 \times 10^{-15}\ \text{m}.$$

The momentum of an electron with this wavelength is $p = h/\lambda$, and the kinetic energy is

$$\begin{aligned}\text{KE} = E - mc^2 &= \sqrt{(pc)^2 + \left(mc^2\right)^2} - mc^2 = \sqrt{\left(\frac{hc}{\lambda}\right)^2 + \left(mc^2\right)^2} - mc^2 \\ &= \sqrt{\left(\frac{1.240 \times 10^{-6}\ \text{eV}\cdot\text{m}}{6.98 \times 10^{-15}\ \text{m}}\right)^2 + (0.511\ \text{MeV})^2} - (0.511\ \text{MeV}) \\ &= 177\ \text{MeV}.\end{aligned}$$

11-5: From Equation (11.1), the radius of such a nucleus would be

$$R = R_0\, A^{1/3} = \left(1.2 \times 10^{-15}\ \text{m}\right)(294)^{1/3} = 8.0 \times 10^{-15}\ \text{m} = 8.0\ \text{fm}.$$

11-7: For the electron, the magnetic potential energy is

$$U = \mu_B\, B = \left(5.788 \times 10^{-5}\ \text{eV/T}\right)(0.10\ \text{T}) = 5.8 \times 10^{-6}\ \text{eV}.$$

For the proton, the magnetic potential energy is

$$U = \mu_p\, B = (2.793)\left(3.152 \times 10^{-8}\ \text{eV/T}\right)(0.10\ \text{T}) = 8.8 \times 10^{-9}\ \text{eV}.$$

11-9: (a) The ratio

$$\frac{\mu_p B}{kT} = \frac{(2.793)\left(3.152 \times 10^{-8}\ \text{eV/T}\right)(1.0\ \text{T})}{(8.617 \times 10^{-5}\ \text{eV/K})(293\ \text{K})} = 3.49 \times 10^{-6}$$

is so small that the difference in populations of the two levels will be small. That is, each state can be assumed to have approximately $N/2$ protons, with the number of spin-up protons being $(N/2)e^{\mu_p B/kT}$ and the number of spin-down protons $(N/2)e^{-\mu_p B/kT}$. The difference is

$$\begin{aligned}\Delta N = N_- - N_+ &= \frac{N}{2}\left(e^{\mu_p B/kT} - e^{-\mu_p B/kT}\right)\\ &= N \sinh\left(\frac{\mu_p B}{kT}\right) = \left(10^6\right)\sinh(3.49 \times 10^{-6}) = 3.5.\end{aligned}$$

In the above, the approximation $\sinh(x) \approx x$ is certainly valid. A more rigorous algebraic treatment, maintaining the same ratio of N_- to N_+ but requiring the sum to be exactly N is possible, leading to $\Delta N = N \tanh(\mu_p B/kT)$, but gives the same result.

(b) Repeating the above with $T = 20$ K gives $\Delta N = N \sinh(5.1 \times 10^{-5}) = 51$.

(c) Because the populations are so close, induced emission will nearly equal induced absorption, so there will be very little net absorption of the radiation.

(d) This is a two-level system, and could not be used as the basis for a laser.

11-11: The strong nuclear interaction, unlike the Coulombic or gravitational interactions, is short-range; the limited range limits the size of nuclei. (An explanation of why nuclear forces are short-range is given in Section 11.7 of the text.)

11-13: The nucleus ${}^{8}_{3}\text{Li}$ has three protons and five neutrons, and hence is an odd-odd nucleus, and is unstable, so ${}^{7}_{3}\text{Li}$ is the more stable of the two. The nucleus ${}^{15}_{6}\text{C}$ has three more neutrons (9) than protons (6); for a nucleus this small (in atomic number), that many excess neutrons do not serve to hold the nucleus together, and ${}^{13}_{6}\text{C}$ is more stable.

11-15: Using the values for the atomic masses and the constituent masses from the Appendix, the binding energy per nucelon of ${}^{20}_{10}\text{Ne}$ is

$$\begin{aligned}&\frac{1}{20}\left[10\,(m_{\text{H}}) + 10\,(m_n) - m\left({}^{20}_{10}\text{Ne}\right)\right]\\ &= \frac{1}{20}\left[10\,(1.007825\ \text{u}) + 10\,(1.008665\ \text{u}) - 19.992439\ \text{u}\right](931.49\ \text{MeV/u})\\ &= 8.03\ \text{MeV}.\end{aligned}$$

For ${}^{56}_{23}\mathrm{Fe}$, the binding energy per nucleon is

$$\frac{1}{56}\,[26\,(1.007825\ \mathrm{u}) + 30\,(1.008665\ \mathrm{u}) - 55.934939\ \mathrm{u}]\,(931.49\ \mathrm{MeV/u}) = 8.79\ \mathrm{MeV}.$$

11-17: To remove a neutron from the ${}^{4}_{2}\mathrm{He}$ nucleus, the energy needed is

$$\begin{aligned} &m\big({}^{3}_{3}\mathrm{He}\big) + m_n - m\big({}^{4}_{2}\mathrm{He}\big) \\ &\quad = [3.016029\ \mathrm{u} + 1.008665\ \mathrm{u} - 4.002603\ \mathrm{u}]\,(931.49\ \mathrm{MeV/u}) = 20.58\ \mathrm{MeV}. \end{aligned}$$

Then, to remove a proton, the energy needed is

$$\begin{aligned} &m\big({}^{2}_{1}\mathrm{H}\big) + m_{\mathrm{H}} - m\big({}^{3}_{2}\mathrm{He}\big) \\ &\quad = [2.014102\ \mathrm{u} + 1.007825\ \mathrm{u} - 3.016029\ \mathrm{u}]\,(931.49\ \mathrm{MeV/u}) = 5.49\ \mathrm{MeV}. \end{aligned}$$

To separate the remaining proton and neutron, the energy needed is

$$\begin{aligned} &m_n + m_{\mathrm{H}} - m\big({}^{2}_{1}\mathrm{H}\big) \\ &\quad = [1.008665\ \mathrm{u} + 1.007825\ \mathrm{u} - 2.014102\ \mathrm{u}]\,(931.49\ \mathrm{MeV/u}) = 2.24\ \mathrm{MeV}. \end{aligned}$$

The sum of these energies, to three significant figures, is 28.3 MeV.

The binding energy of ${}^{4}_{2}\mathrm{He}$ is

$$\begin{aligned} &2\,m_{\mathrm{H}} + 2\,m_n - m\big({}^{4}_{2}\mathrm{He}\big) \\ &\quad = [2\,(1.007825\ \mathrm{u}) + 2\,(1.008665\ \mathrm{u}) - 4.002603\ \mathrm{u}]\,(931.49\ \mathrm{MeV/u}) = 28.3\ \mathrm{MeV}, \end{aligned}$$

the same as found above. Algebraically, the answers must be the same.

11-19: The electric potential energy of two protons separated by a distance 1.7 fm = 1.7×10^{-15} m is

$$\begin{aligned} \frac{e^2}{4\pi\,\epsilon_0\,r} &= \frac{\big(8.988 \times 10^{9}\ \mathrm{N{\cdot}m^2/C^2}\big)\,\big(1.602 \times 10^{-19}\ \mathrm{C}\big)^2}{(1.7 \times 10^{-15}\ \mathrm{m})} \\ &= 1.357 \times 10^{-13}\ \mathrm{J} = 0.85\ \mathrm{MeV} \end{aligned}$$

to the given two significant figures.

The difference in binding energies is

$$\big[2\,m_{\mathrm{H}} + m_n - m\big({}^{3}_{2}\mathrm{He}\big)\big] - \big[m_{\mathrm{H}} + 2\,m_n - m\big({}^{3}_{1}\mathrm{H}\big)\big] = m_{\mathrm{H}} - m_n - m\big({}^{3}_{2}\mathrm{He}\big) + m\big({}^{3}_{1}\mathrm{H}\big)\,.$$

Using the atomic masses from the Appendix,

$$\begin{aligned} \Delta E &= [1.007825\ \mathrm{u} - 1.008665\ \mathrm{u} - 3.016029\ \mathrm{u} + 3.016050\ \mathrm{u}]\,(931.49\ \mathrm{MeV/u}) \\ &= -0.763\ \mathrm{MeV}, \end{aligned}$$

or -0.76 MeV to two significant figures, with the minus sign indicating that the tritium nucleus ${}^3_1\text{H}$ is more tightly bound than the ${}^3_2\text{He}$ nucleus. The magnitudes of the binding energy and the electric potential energy of the protons in ${}^3_2\text{He}$ are roughly the same, indicating that the most important contribution to the difference in binding energies is the mutual repulsion of the protons, an effect that is not present in ${}^3_1\text{H}$. The closeness of magnitudes of the energies found is an indication that the nuclear forces must be very nearly independent of charge.

11-21: Using $A = 40$ and $Z = 20$ in Equation (11.18) (which makes the asymmetry term vanish) and the + sign (even-even) for the pairing term, the predicted binding energy is

$$\begin{aligned} E_b &= (14.1\text{ MeV})\,(40) - (13.0\text{ MeV})\,(40)^{2/3} - (0.595\text{ MeV})\frac{(20)(19)}{(40)^{1/3}} + \frac{(33.5\text{ MeV})}{(40)^{3/4}} \\ &= 347.95\text{ MeV}. \end{aligned}$$

The actual binding energy is

$$\begin{aligned} &20\,m_{\text{H}} + 20\,m_n - m\left({}^{40}_{20}\text{Ca}\right) \\ &= [20\,(1.007825\text{ u}) + 20\,(1.008665\text{ u}) - 39.962591\text{ u}]\,(931.49\text{ MeV/u}) \\ &= 342.05\text{ MeV}, \end{aligned}$$

and the discrepancy is

$$\frac{347.95\text{ MeV} - 342.05\text{ MeV}}{342.05\text{ MeV}} = 0.017 = 17\%.$$

11-23: (a) For mirror isobars of the form ${}^{2Z+1}_{Z}\text{X}$ and ${}^{2Z+1}_{Z+1}\text{Y}$, the difference in binding energy is (apart from a factor of c^2)

$$\begin{aligned} E_{Z+1} - E_Z &= [(Z+1)\,m_{\text{H}} + Z\,m_n - M_{Z+1}] - [Z\,m_{\text{H}} + (Z+1)\,m_n - M_Z] \\ &= -\Delta M - \Delta m, \end{aligned}$$

where ΔM is the difference between the atomic masses of ${}^{2Z+1}_{Z}\text{X}$ and ${}^{2Z+1}_{Z+1}\text{Y}$, and $\Delta m = m_n - m_{\text{H}}$.

The difference between the coulomb energies is

$$\Delta E_c = \frac{3}{5}\,\frac{e^2}{4\pi\,\epsilon_0 R}\,[(Z+1)Z - Z(Z-1)] = \frac{3}{5}\,\frac{e^2}{4\pi\,\epsilon_0 R}\,2Z = \frac{3\,Z\,e^2}{10\pi\,\epsilon_0\,R}.$$

If this difference is equal to the *negative* of the difference in binding energies,

$$(\Delta M + \Delta m)\, c^2 = \frac{3}{10}\, \frac{Z\, e^2}{\pi \epsilon_0\, R}.$$

Solving for R,

$$R = \frac{3}{10}\, \frac{Z\, e^2}{\pi \epsilon_0}\, \frac{1}{(\Delta M + \Delta m)\, c^2}.$$

(b) For the mirror isobars ${}^{15}_{7}\text{N}$ and ${}^{15}_{8}\text{O}$, $Z = 7$ and

$$\begin{aligned}
&(\Delta M + \Delta m)\, c^2 \\
&= [15.003065 \text{ u} - 15.000109 \text{ u} + 1.008665 \text{ u} - 1.007825 \text{ u}]\, (931.49 \text{ MeV/u}) \\
&= 3.536 \text{ MeV} = 5.665 \times 10^{-13} \text{ J}.
\end{aligned}$$

Using this in the expression for R found in part (a),

$$R = \frac{3}{10}\, \frac{(7)\left(1.602 \times 10^{-19} \text{ C}\right)^2}{\pi\, (8.854 \times 10^{-12} \text{ C}^2/\, (\text{N}\cdot\text{m}^2))}\, \frac{1}{5.665 \times 10^{-13} \text{ J}} = 3.42 \text{ fm}.$$

11-25: (a) Removing a neutron from an isotope of krypton leaves an isotope of krypton with mass number one less than that of the original isotope. For the given isotopes, the energy equivalents are

$$\begin{aligned}
&m_n + m\left({}^{80}_{36}\text{Kr}\right) - m\left({}^{81}_{36}\text{Kr}\right) \\
&= [1.008665 \text{ u} + 79.916375 \text{ u} - 80.916578 \text{ u}]\, (931.49 \text{ MeV/u}) = 7.88 \text{ MeV} \\
&m_n + m\left({}^{81}_{36}\text{Kr}\right) - m\left({}^{82}_{36}\text{Kr}\right) \\
&= [1.008665 \text{ u} + 80.916578 \text{ u} - 81.913483 \text{ u}]\, (931.49 \text{ MeV/u}) = 10.95 \text{ MeV} \\
&m_n + m\left({}^{82}_{36}\text{Kr}\right) - m\left({}^{83}_{36}\text{Kr}\right) \\
&= [1.008665 \text{ u} + 81.913483 \text{ u} - 82.914134 \text{ u}]\, (931.49 \text{ MeV/u}) = 7.46 \text{ MeV}.
\end{aligned}$$

(b) ${}^{82}_{36}\text{Kr}$ has 36 protons and 46 neutrons, and so the neutrons are paired; the tendency of neutrons to pair together means removing a neutron from a ${}^{82}_{36}\text{Kr}$ nucleus requires more energy.

11-27: In Equation (11.18), with $A = 127$ for each isobar, the coulomb energy term and the assymetry term will be different for the two nuclei. For ${}^{127}_{53}\text{I}$, $Z(Z - 1) = (53)(52) = 2756$ and $(A{-}2Z)^2 = 441$. For ${}^{127}_{52}\text{Te}$, $Z(Z{-}1) = (52)(51) =$

2652 and $(A-2Z)^2 = 529$. The difference in binding energies predicted by the liquid drop model is

$$\begin{aligned}\Delta E = E\left({}^{127}_{53}\mathrm{I}\right) - \left({}^{127}_{52}\mathrm{Te}\right) &= -\frac{a_3}{A^{1/3}}(2756-2652) - \frac{a_4}{A}(361-529)\\ &= -\frac{(0.595\ \mathrm{MeV})(104)}{(127)^{1/3}} - \frac{(19.0\ \mathrm{MeV})(-88)}{(127)}\\ &= 0.855\ \mathrm{MeV},\end{aligned}$$

and so ${}^{127}_{53}\mathrm{I}$ is more stable, and ${}^{127}_{52}\mathrm{Te}$ decays into ${}^{127}_{53}\mathrm{I}$ by negative beta decay (electron emission).

11-29: A nucleon confined to a region of size $\Delta x = 2$ fm will have an uncertainty in momentum at least as large as $\dfrac{\hbar}{2\,\Delta x} = 2.63 \times 10^{-20}$ kg·m/s. The minimum kinetic energy a nucleon with this momentum would have is

$$\frac{(\Delta p)^2}{2\,m} = \frac{\left(2.63 \times 10^{-20}\ \mathrm{kg\cdot m/s}\right)^2}{2\,(1.6736 \times 10^{-27}\ \mathrm{kg})} = 2.1 \times 10^{-13}\ \mathrm{J} = 1.3\ \mathrm{MeV},$$

which is consistent with a potential well 35 MeV deep. Note that the nonrelativistic expression for kinetic energy is sufficient, and that the result is not changed if the mass of a neutron is used instead of the mass of a hydrogen atom.