UNITS, PHYSICAL QUANTITIES, AND VECTORS

1

? Being able to predict the path of a hurricane is essential for minimizing the damage it does to lives and property. If a hurricane is moving at 20 km/h in a direction 53° north of east, how far north does the hurricane move in one h?

LEARNING GOALS

By studying this chapter, you will learn:

- What the fundamental quantities of mechanics are, and the units physicists use to measure them.
- How to keep track of significant figures in your calculations.
- The difference between scalars and vectors, and how to add and subtract vectors graphically.
- What the components of a vector are, and how to use them in calculations.
- What unit vectors are, and how to use them with components to describe vectors.
- Two ways of multiplying vectors.

The study of physics is important because physics is one of the most fundamental of the sciences. Scientists of all disciplines make use of the ideas of physics, including chemists who study the structure of molecules, paleontologists who try to reconstruct how dinosaurs walked, and climatologists who study how human activities affect the atmosphere and oceans. Physics is also the foundation of all engineering and technology. No engineer could design a flat-screen TV, an interplanetary spacecraft, or even a better mousetrap without first understanding the basic laws of physics.

The study of physics is also an adventure. You will find it challenging, sometimes frustrating, occasionally painful, and often richly rewarding and satisfying. It will appeal to your sense of beauty as well as to your rational intelligence. If you've ever wondered why the sky is blue, how radio waves can travel through empty space, or how a satellite stays in orbit, you can find the answers by using fundamental physics. Above all, you will come to see physics as a towering achievement of the human intellect in its quest to understand our world and ourselves.

In this opening chapter, we'll go over some important preliminaries that we'll need throughout our study. We'll discuss the nature of physical theory and the use of idealized models to represent physical systems. We'll introduce the systems of units used to describe physical quantities and discuss ways to describe the accuracy of a number. We'll look at examples of problems for which we can't (or don't want to) find a precise answer, but for which rough estimates can be useful and interesting. Finally, we'll study several aspects of vectors and vector algebra. Vectors will be needed throughout our study of physics to describe and analyze physical quantities, such as velocity and force, that have direction as well as magnitude.

1.1 The Nature of Physics

Physics is an *experimental* science. Physicists observe the phenomena of nature and try to find patterns and principles that relate these phenomena. These patterns are called physical theories or, when they are very well established and of broad use, physical laws or principles.

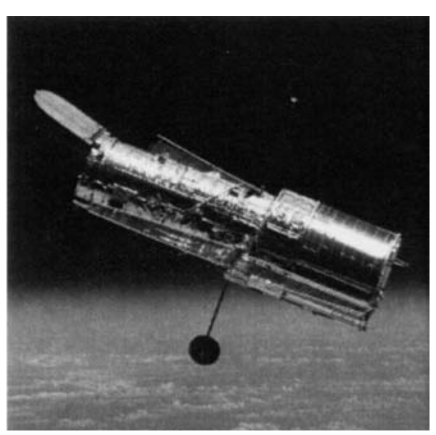
CAUTION **The meaning of the word "theory"** Calling an idea a theory does *not* mean that it's just a random thought or an unproven concept. Rather, a theory is an explanation of natural phenomena based on observation and accepted fundamental principles. An example is the well-established theory of biological evolution, which is the result of extensive research and observation by generations of biologists.

The development of physical theory requires creativity at every stage. The physicist has to learn to ask appropriate questions, design experiments to try to answer the questions, and draw appropriate conclusions from the results. Figure 1.1 shows two famous experimental facilities.

1.1 Two research laboratories. (a) According to legend, Galileo investigated falling bodies by dropping them from the Leaning Tower in Pisa, Italy, and he studied pendulum motion by observing the swinging of the chandelier in the adjacent cathedral. (b) The Hubble Space Telescope is the first major telescope to operate outside the earth's atmosphere. Measurements made with this telescope have helped determine the age and expansion rate of the universe.

(a)

(b)



Legend has it that Galileo Galilei (1564–1642) dropped light and heavy objects from the top of the Leaning Tower of Pisa (Fig. 1.1a) to find out whether their rates of fall were the same or different. Galileo recognized that only experimental investigation could answer this question. From examining the results of his experiments (which were actually much more sophisticated than in the legend), he made the inductive leap to the principle, or theory, that the acceleration of a falling body is independent of its weight.

The development of physical theories such as Galileo's is always a two-way process that starts and ends with observations or experiments. This development often takes an indirect path, with blind alleys, wrong guesses, and the discarding of unsuccessful theories in favor of more promising ones. Physics is not simply a collection of facts and principles; it is also the *process* by which we arrive at general principles that describe how the physical universe behaves.

No theory is ever regarded as the final or ultimate truth. The possibility always exists that new observations will require that a theory be revised or discarded. It is in the nature of physical theory that we can disprove a theory by finding behavior that is inconsistent with it, but we can never prove that a theory is always correct.

Getting back to Galileo, suppose we drop a feather and a cannonball. They certainly do *not* fall at the same rate. This does not mean that Galileo was wrong; it means that his theory was incomplete. If we drop the feather and the cannonball *in a vacuum* to eliminate the effects of the air, then they do fall at the same rate. Galileo's theory has a **range of validity:** It applies only to objects for which the force exerted by the air (due to air resistance and buoyancy) is much less than the weight. Objects like feathers or parachutes are clearly outside this range.

Every physical theory has a range of validity outside of which it is not applicable. Often a new development in physics extends a principle's range of validity. Galileo's analysis of falling bodies was greatly extended half a century later by Newton's laws of motion and law of gravitation.

1.2 Solving Physics Problems

At some point in their studies, almost all physics students find themselves thinking, "I understand the concepts, but I just can't solve the problems." But in physics, truly understanding a concept or principle is the same thing as being able to apply it to a variety of practical problems. Learning how to solve problems is absolutely essential; you don't *know* physics unless you can *do* physics.

How do you learn to solve physics problems? In every chapter of this book you will find *Problem-Solving Strategies* that offer techniques for setting up and solving problems efficiently and accurately. Following each *Problem-Solving Strategy* are one or more worked *Examples* that show these techniques in action.

(The *Problem-Solving Strategies* will also steer you away from some *incorrect* techniques that you may be tempted to use.) You'll also find additional examples that aren't associated with a particular *Problem-Solving Strategy.* Study these strategies and examples carefully, and work through each example for yourself on a piece of paper.

Different techniques are useful for solving different kinds of physics problems, which is why this book offers dozens of *Problem-Solving Strategies.* No matter what kind of problem you're dealing with, however, there are certain key steps that you'll always follow. (These same steps are equally useful for problems in math, engineering, chemistry, and many other fields.) In this book we've organized these steps into four stages of solving a problem.

All of the *Problem-Solving Strategies* and *Examples* in this book will follow these four steps. (In some cases we will combine the first two or three steps.) We encourage you to follow these same steps when you solve problems yourself. You may find it useful to remember the acronym ***I SEE***—short for *Identify, Set up, Execute,* and *Evaluate.*

Problem-Solving Strategy 1.1 Solving Physics Problems

IDENTIFY *the relevant concepts:* First, decide which physics ideas are relevant to the problem. Although this step doesn't involve any calculations, it's sometimes the most challenging part of solving the problem. Don't skip over this step, though; choosing the wrong approach at the beginning can make the problem more difficult than it has to be, or even lead you to an incorrect answer.

At this stage you must also identify the **target variable** of the problem—that is, is the quantity whose value you're trying to find. It could be the speed at which a projectile hits the ground, the intensity of a sound made by a siren, or the size of an image made by a lens. (Sometimes the goal will be to find a mathematical expression rather than a numerical value. Sometimes, too, the problem will have more than one target variable.) The target variable is the goal of the problem-solving process; don't lose sight of this goal as you work through the solution.

SET UP *the problem:* Based on the concepts you selected in the *Identify* step, choose the equations that you'll use to solve the problem and decide how you'll use them. If appropriate, draw a sketch of the situation described in the problem.

EXECUTE *the solution:* In this step, you "do the math." Before you launch into a flurry of calculations, make a list of all known and unknown quantities, and note which are the target variable or variables. Then solve the equations for the unknowns.

EVALUATE *your answer:* The goal of physics problem solving isn't just to get a number or a formula; it's to achieve better understanding. That means you must examine your answer to see what it's telling you. Be sure to ask yourself, "Does this answer make sense?" If your target variable was the radius of the earth and your answer is 6.38 centimeters (or if your answer is a negative number!), something went wrong in your problem-solving process. Go back and check your work, and revise your solution as necessary.

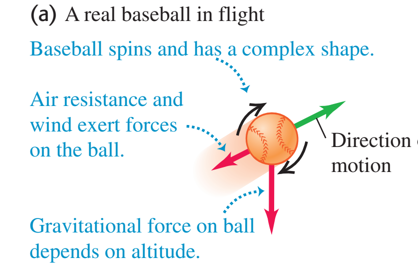
Idealized Models

In everyday conversation we use the word "model" to mean either a small-scale replica, such as a model railroad, or a person who displays articles of clothing (or the absence thereof). In physics a **model** is a simplified version of a physical system that would be too complicated to analyze in full detail.

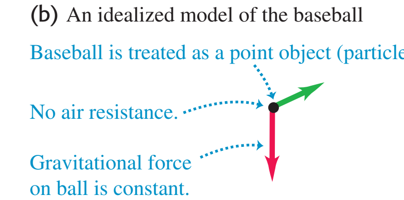
For example, suppose we want to analyze the motion of a thrown baseball (Fig. 1.2a). How complicated is this problem? The ball is not a perfect sphere (it has raised seams), and it spins as it moves through the air. Wind and air resistance influence its motion, the ball's weight varies a little as its distance from the center of the earth changes, and so on. If we try to include all these things, the analysis gets hopelessly complicated. Instead, we invent a simplified version of the problem. We neglect the size and shape of the ball by representing it as a point object, or **particle.** We neglect air resistance by making the ball move in a vacuum, and we make the weight constant. Now we have a problem that is simple enough to deal with (Fig. 1.2b). We will analyze this model in detail in Chapter 3.

1.2 To simplify the analysis of (a) a baseball in flight, we use (b) an idealized model.

(a) A real baseball in flight



(b) An idealized model of the baseball



To make an idealized model, we have to overlook quite a few minor effects to concentrate on the most important features of the system. Of course, we have to be careful not to neglect too much. If we ignore the effects of gravity completely, then our model predicts that when we throw the ball up, it will go in a straight line and disappear into space. We need to use some judgment and creativity to construct a model that simplifies a problem enough to make it manageable, yet keeps its essential features.

When we use a model to predict how a system will behave, the validity of our predictions is limited by the validity of the model. For example, Galileo's prediction about falling bodies (see Section 1.1) corresponds to an idealized model that does not include the effects of air resistance. This model works fairly well for a dropped cannonball, but not so well for a feather.

When we apply physical principles to complex systems in physical science and technology, we always use idealized models, and we have to be aware of the assumptions we are making. In fact, the principles of physics themselves are stated in terms of idealized models; we speak about point masses, rigid bodies, ideal insulators, and so on. Idealized models play a crucial role throughout this book. Watch for them in discussions of physical theories and their applications to specific problems.

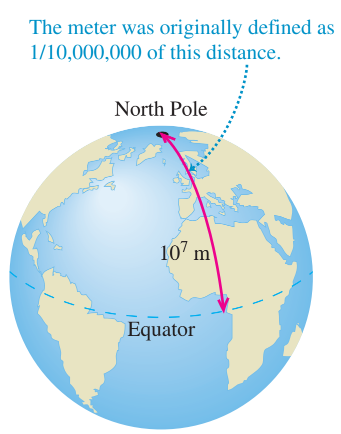
1.3 Standards and Units

As we learned in Section 1.1, physics is an experimental science. Experiments require measurements, and we generally use numbers to describe the results of measurements. Any number that is used to describe a physical phenomenon quantitatively is called a **physical quantity.** For example, two physical quantities that describe you are your weight and your height. Some physical quantities are so fundamental that we can define them only by describing how to measure them. Such a definition is called an **operational definition.** Two examples are measuring a distance by using a ruler and measuring a time interval by using a stopwatch. In other cases we define a physical quantity by describing how to calculate it from other quantities that we *can* measure. Thus we might define the average speed of a moving object as the distance traveled (measured with a ruler) divided by the time of travel (measured with a stopwatch).

When we measure a quantity, we always compare it with some reference standard. When we say that a Porsche Carrera GT is 4.61 meters long, we mean that it is 4.61 times as long as a meter stick, which we define to be 1 meter long. Such a standard defines a **unit** of the quantity. The meter is a unit of distance, and the second is a unit of time. When we use a number to describe a physical quantity, we must always specify the unit that we are using; to describe a distance as simply "4.61" wouldn't mean anything.

To make accurate, reliable measurements, we need units of measurement that do not change and that can be duplicated by observers in various locations. The system of units used by scientists and engineers around the world is commonly called "the metric system," but since 1960 it has been known officially as the **International System,** or **SI** (the abbreviation for its French name, *Système International*). A list of all SI units is given in Appendix A, as are definitions of the most fundamental units.

1.3 In 1791 the distance from the North Pole to the equator was defined to be exactly 10^7 m. With the modern definition of the meter, this distance is about 0.02% more than 10^7 m.



The definitions of the basic units of the metric system have evolved over the years. When the metric system was established in 1791 by the French Academy of Sciences, the meter was defined as one ten-millionth of the distance from the North Pole to the equator (Fig. 1.3). The second was defined as the time required for a pendulum one meter long to swing from one side to the other. These definitions were cumbersome and hard to duplicate precisely, and by international agreement they have been replaced with more refined definitions.

Time

From 1889 until 1967, the unit of time was defined as a certain fraction of the mean solar day, the average time between successive arrivals of the sun at its highest point in the sky. The present standard, adopted in 1967, is much more precise. It is based on an atomic clock, which uses the energy difference between the two lowest energy states of the cesium atom. When bombarded by microwaves of precisely the proper frequency, cesium atoms undergo a transition from one of these states to the other. One **second** (abbreviated s) is defined as the time required for 9,192,631,770 cycles of this microwave radiation.

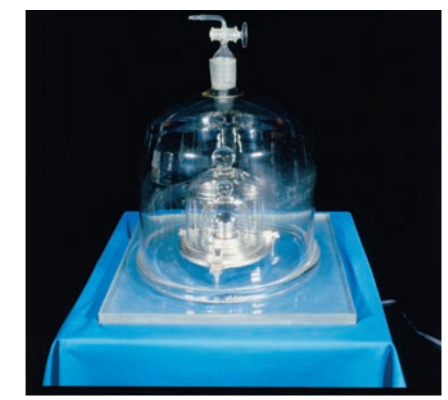
Length

In 1960 an atomic standard for the meter was also established, using the wavelength of the orange-red light emitted by atoms of krypton (^{86}Kr) in a glow discharge tube. Using this length standard, the speed of light in a vacuum was measured to be 299,792,458 m/s. In November 1983, the length standard was changed again so that the speed of light in a vacuum was *defined* to be precisely 299,792,458 m/s. The meter is defined to be consistent with this number and with the above definition of the second. Hence the new definition of the **meter** (abbreviated m) is the distance that light travels in a vacuum in 1/299,792,458 second. This provides a much more precise standard of length than the one based on a wavelength of light.

Mass

The standard of mass, the **kilogram** (abbreviated kg), is defined to be the mass of a particular cylinder of platinum–iridium alloy kept at the International Bureau of Weights and Measures at Sèvres, near Paris (Fig. 1.4). An atomic standard of mass would be more fundamental, but at present we cannot measure masses on an atomic scale with as much accuracy as on a macroscopic scale. The *gram* (which is not a fundamental unit) is 0.001 kilogram.

1.4 The metal object carefully enclosed within these nested glass containers is the international standard kilogram.



Unit Prefixes

Once we have defined the fundamental units, it is easy to introduce larger and smaller units for the same physical quantities. In the metric system these other units are related to the fundamental units (or, in the case of mass, to the gram) by multiples of 10 or $\frac{1}{10}$. Thus one kilometer (1 km) is 1000 meters, and one centimeter (1 cm) is $\frac{1}{100}$ meter. We usually express multiples of 10 or $\frac{1}{10}$ in exponential notation: $1000 = 10^3$, $\frac{1}{1000} = 10^{-3}$, and so on. With this notation, $1 \text{ km} = 10^3 \text{ m}$ and $1 \text{ cm} = 10^{-2} \text{ m}$.

The names of the additional units are derived by adding a **prefix** to the name of the fundamental unit. For example, the prefix "kilo-," abbreviated k, always means a unit larger by a factor of 1000; thus

$$1 \text{ kilometer} = 1 \text{ km} = 10^3 \text{ meters} = 10^3 \text{ m}$$
$$1 \text{ kilogram} = 1 \text{ kg} = 10^3 \text{ grams} = 10^3 \text{ g}$$
$$1 \text{ kilowatt} = 1 \text{ kW} = 10^3 \text{ watts} = 10^3 \text{ W}$$

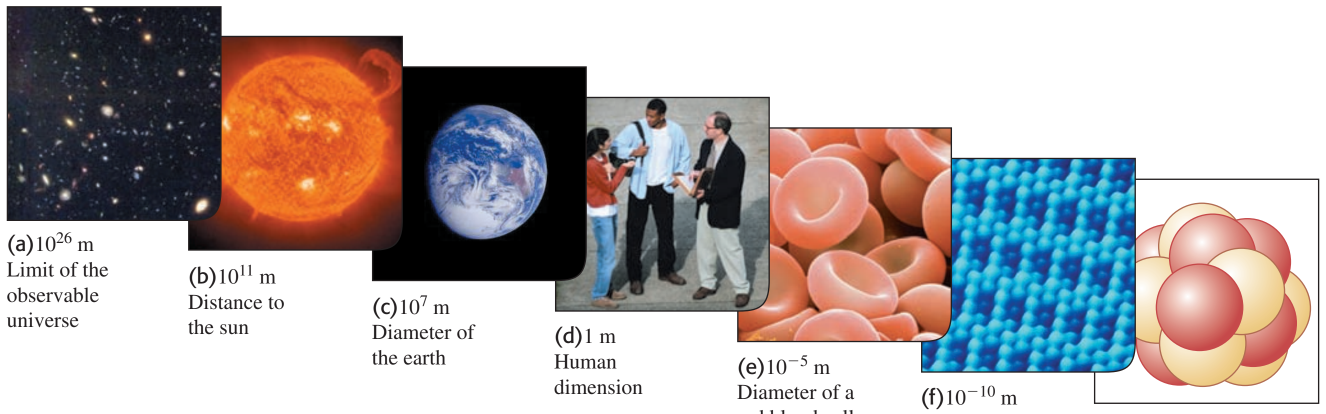
A table on the inside back cover of this book lists the standard SI prefixes, with their meanings and abbreviations.

Here are several examples of the use of multiples of 10 and their prefixes with the units of length, mass, and time. Figure 1.5 shows how these prefixes help describe both large and small distances.

Length

1 nanometer $= 1 \text{ nm} = 10^{-9} \text{ m}$ (a few times the size of the largest atom)
1 micrometer $= 1\ \mu\text{m} = 10^{-6} \text{ m}$ (size of some bacteria and living cells)
1 millimeter $= 1 \text{ mm} = 10^{-3} \text{ m}$ (diameter of the point of a ballpoint pen)
1 centimeter $= 1 \text{ cm} = 10^{-2} \text{ m}$ (diameter of your little finger)
1 kilometer $= 1 \text{ km} = 10^{3} \text{ m}$ (a 10-minute walk)

1.5 Some typical lengths in the universe. (a) The distance to the most remote galaxies we can see is about 10^{26} m, or 10^{23} km. (b) The sun is 1.50×10^{11} m, or 1.50×10^{8} km, from earth. (c) The diameter of the earth is 1.28×10^{7} m, or 12,800 km. (d) A typical human is about 1.7 m, or 170 cm, tall. (e) Human red blood cells are about 8×10^{-6} m (0.008 mm, or 8 μm) in diameter. (f) These oxygen atoms, shown arrayed on the surface of a crystal, are about 10^{-10} m, or 10^{-4} μm, in radius. (g) Typical atomic nuclei (shown in an artist's impression) have radii of about 10^{-14} m, or 10^{-5} nm.



(a) 10^{26} m Limit of the observable universe

(b) 10^{11} m Distance to the sun

(c) 10^{7} m Diameter of the earth

(d) 1 m Human dimension

(e) 10^{-5} m Diameter of a red blood cell

(f) 10^{-10} m Radius of an atom

(g) 10^{-14} m Radius of an atomic nucleus

Mass

1 microgram = 1 μg = 10^{-6} g = 10^{-9} kg (mass of a very small dust particle)
1 milligram = 1 mg = 10^{-3} g = 10^{-6} kg (mass of a grain of salt)
1 gram = 1 g = 10^{-3} kg (mass of a paper clip)

Time

1 nanosecond = 1 ns = 10^{-9} s (time for light to travel 0.3 m)
1 microsecond = 1 μs = 10^{-6} s (time for an orbiting space shuttle to travel 8 mm)
1 millisecond = 1 ms = 10^{-3} s (time for sound to travel 0.35 m)

The British System

Finally, we mention the British system of units. These units are used only in the United States and a few other countries, and in most of these they are being replaced by SI units. British units are now officially defined in terms of SI units, as follows:

Length: 1 inch = 2.54 cm (exactly)
Force: 1 pound = 4.448221615260 newtons (exactly)

The newton, abbreviated N, is the SI unit of force. The British unit of time is the second, defined the same way as in SI. In physics, British units are used only in mechanics and thermodynamics; there is no British system of electrical units.

In this book we use SI units for all examples and problems, but we occasionally give approximate equivalents in British units. As you do problems using SI units, you may also wish to convert to the approximate British equivalents if they are more familiar to you (Fig. 1.6). But you should try to *think* in SI units as much as you can.

1.6 Many everyday items make use of both SI and British units. An example is this speedometer from a U.S.-built automobile, which shows the speed in both kilometers per hour (inner scale) and miles per hour (outer scale).

1.4 Unit Consistency and Conversions

We use equations to express relationships among physical quantities, represented by algebraic symbols. Each algebraic symbol always denotes both a number and a unit. For example, d might represent a distance of 10 m, t a time of 5 s, and v a speed of 2 m/s.

An equation must always be **dimensionally consistent.** You can't add apples and automobiles; two terms may be added or equated only if they have the same units. For example, if a body moving with constant speed v travels a distance d in a time t, these quantities are related by the equation

$$d = vt$$

If d is measured in meters, then the product vt must also be expressed in meters. Using the above numbers as an example, we may write

$$10\text{ m} = \left(2\,\frac{\text{m}}{\cancel{\text{s}}}\right)(5\,\cancel{\text{s}})$$

Because the unit 1/s on the right side of the equation cancels the unit s, the product has units of meters, as it must. In calculations, units are treated just like algebraic symbols with respect to multiplication and division.

CAUTION **Always use units in calculations** When a problem requires calculations using numbers with units, *always* write the numbers with the correct units and carry the units through the calculation as in the example above. This provides a very useful check for calculations. If at some stage in a calculation you find that an equation or an expression has inconsistent units, you know you have made an error somewhere. In this book we will *always* carry units through all calculations, and we strongly urge you to follow this practice when you solve problems.

Problem-Solving Strategy 1.2 Unit Conversions

IDENTIFY *the relevant concepts:* Unit conversion is important, but it's also important to recognize when it's needed. In most cases, you're best off using the fundamental SI units (lengths in meters, masses in kilograms, and time in seconds) within a problem. If you need the answer to be in a different set of units (such as kilometers, grams, or hours), wait until the end of the problem to make the conversion. In the following examples, we'll concentrate on unit conversion alone, so we'll skip the *Identify* step.

SET UP *the problem* and **EXECUTE** *the solution:* Units are multiplied and divided just like ordinary algebraic symbols. This gives us an easy way to convert a quantity from one set of units to another. The key idea is to express the same physical quantity in two different units and form an equality.

For example, when we say that 1 min = 60 s, we don't mean that the number 1 is equal to the number 60; rather, we mean that 1 min represents the same physical time interval as 60 s. For this reason, the ratio (1 min)/(60 s) equals 1, as does its reciprocal (60 s)/(1 min). We may multiply a quantity by either of these factors without changing that quantity's physical meaning. For example, to find the number of seconds in 3 min, we write

$$3\text{ min} = (3\,\cancel{\text{min}})\left(\frac{60\text{ s}}{1\,\cancel{\text{min}}}\right) = 180\text{ s}$$

EVALUATE *your answer:* If you do your unit conversions correctly, unwanted units will cancel, as in the example above. If instead you had multiplied 3 min by (1 min)/(60 s), your result would have been $\frac{1}{20}$ min^2/s, which is a rather odd way of measuring time. To be sure you convert units properly, you must write down the units at *all* stages of the calculation.

Finally, check whether your answer is reasonable. Is the result 3 min = 180 s reasonable? The answer is yes; the second is a smaller unit than the minute, so there are more seconds than minutes in the same time interval.

Example 1.1 Converting speed units

The official world land speed record is 1228.0 km/h, set on October 15, 1997, by Andy Green in the jet engine car *Thrust SSC.* Express this speed in meters per second.

SOLUTION

IDENTIFY AND SET UP: We want to convert the units of a speed from km/h to m/s.

EXECUTE: The prefix k means 10^3, so the speed 1228.0 km/h = 1228.0×10^3 m/h. We also know that there are 3600 s in 1 h. So we must combine the speed of 1228.0×10^3 m/h and a factor of 3600. But should we multiply or divide by this factor? If we treat the factor as a pure number without units, we're forced to guess how to proceed.

The correct approach is to carry the units with each factor. We then arrange the factor so that the hour unit cancels:

$$1228.0\text{ km/h} = \left(1228.0 \times 10^3\,\frac{\text{m}}{\cancel{\text{h}}}\right)\left(\frac{1\,\cancel{\text{h}}}{3600\text{ s}}\right) = 341.11\text{ m/s}$$

If you multiplied by (3600 s)/(1 h) instead of (1 h)/(3600 s), the hour unit wouldn't cancel, and you would be able to easily

Continued

recognize your error. Again, the *only* way to be sure that you correctly convert units is to carry the units throughout the calculation.

EVALUATE: While you probably have a good intuition for speeds in kilometers per hour or miles per hour, speeds in meters per second are likely to be a bit more mysterious. It helps to remember that a typical walking speed is about 1 m/s: the length of an average person's stride is about one meter, and a good walking pace is about one stride per second. By comparison, a speed of 341.11 m/s is rapid indeed!

Example 1.2 Converting volume units

The world's largest cut diamond is the First Star of Africa (mounted in the British Royal Sceptre and kept in the Tower of London). Its volume is 1.84 cubic inches. What is its volume in cubic centimeters? In cubic meters?

SOLUTION

IDENTIFY AND SET UP: Here we are to convert the units of a volume from cubic inches (in.^3) to cubic centimeters (cm^3) and cubic meters (m^3).

EXECUTE: To convert cubic inches to cubic centimeters, we multiply by $[(2.54\text{ cm})/(1\text{ in.})]^3$, not just $(2.54\text{ cm})/(1\text{ in.})$. We find

$$1.84\text{ in.}^3 = (1.84\text{ in.}^3)\left(\frac{2.54\text{ cm}}{1\text{ in.}}\right)^3$$
$$= (1.84)(2.54)^3\frac{\cancel{\text{in.}^3}\text{ cm}^3}{\cancel{\text{in.}^3}} = 30.2\text{ cm}^3$$

Also, $1\text{ cm} = 10^{-2}\text{ m}$, and

$$30.2\text{ cm}^3 = (30.2\text{ cm}^3)\left(\frac{10^{-2}\text{ m}}{1\text{ cm}}\right)^3$$
$$= (30.2)(10^{-2})^3\frac{\cancel{\text{cm}^3}\text{ m}^3}{\cancel{\text{cm}^3}} = 30.2\times10^{-6}\text{ m}^3$$
$$= 3.02\times10^{-5}\text{ m}^3$$

EVALUATE: While 1 centimeter is 10^{-2} of a meter (that is, $1\text{ cm} = 10^{-2}\text{ m}$), our answer shows that a cubic centimeter (1 cm^3) is *not* 10^{-2} of a cubic meter. Rather, it is the volume of a cube whose sides are 1 cm long. So $1\text{ cm}^3 = (1\text{ cm})^3 = (10^{-2}\text{ m})^3 = (10^{-2})^3\text{ m}^3$, or $1\text{ cm}^3 = 10^{-6}\text{ m}^3$.

1.5 Uncertainty and Significant Figures

Measurements always have uncertainties. If you measure the thickness of the cover of this book using an ordinary ruler, your measurement is reliable only to the nearest millimeter, and your result will be 3 mm. It would be *wrong* to state this result as 3.00 mm; given the limitations of the measuring device, you can't tell whether the actual thickness is 3.00 mm, 2.85 mm, or 3.11 mm. But if you use a micrometer caliper, a device that measures distances reliably to the nearest 0.01 mm, the result will be 2.91 mm. The distinction between these two measurements is in their **uncertainty.** The measurement using the micrometer caliper has a smaller uncertainty; it's a more accurate measurement. The uncertainty is also called the **error** because it indicates the maximum difference there is likely to be between the measured value and the true value. The uncertainty or error of a measured value depends on the measurement technique used.

We often indicate the **accuracy** of a measured value—that is, how close it is likely to be to the true value—by writing the number, the symbol $\pm$, and a second number indicating the uncertainty of the measurement. If the diameter of a steel rod is given as 56.47 ± 0.02 mm, this means that the true value is unlikely to be less than 56.45 mm or greater than 56.49 mm. In a commonly used shorthand notation, the number 1.6454(21) means 1.6454 ± 0.0021. The numbers in parentheses show the uncertainty in the final digits of the main number.

We can also express accuracy in terms of the maximum likely **fractional error** or **percent error** (also called *fractional uncertainty* and *percent uncertainty*). A resistor labeled "47 ohms $\pm$ 10%" probably has a true resistance that differs from 47 ohms by no more than 10% of 47 ohms—that is, about 5 ohms. The resistance is probably between 42 and 52 ohms. For the diameter of the steel rod given above, the fractional error is $(0.02\text{ mm})/(56.47\text{ mm})$, or about 0.0004; the percent error is $(0.0004)(100\%)$, or about 0.04%. Even small percent errors can sometimes be very significant (Fig. 1.7).

1.7 This spectacular mishap was the result of a very small percent error—traveling a few meters too far in a journey of hundreds of thousands of meters.

In many cases the uncertainty of a number is not stated explicitly. Instead, the uncertainty is indicated by the number of meaningful digits, or **significant figures,** in the measured value. We gave the thickness of the cover of this book as 2.91 mm, which has three significant figures. By this we mean that the first two digits are known to be correct, while the third digit is uncertain. The last digit is in the hundredths place, so the uncertainty is about 0.01 mm. Two values with the *same* number of significant figures may have *different* uncertainties; a distance given as 137 km also has three significant figures, but the uncertainty is about 1 km.

When you use numbers having uncertainties to compute other numbers, the computed numbers are also uncertain. When numbers are multiplied or divided, the number of significant figures in the result can be no greater than in the factor with the fewest significant figures. For example, $3.1416 \times 2.34 \times 0.58 = 4.3$. When we add and subtract numbers, it's the location of the decimal point that matters, not the number of significant figures. For example, $123.62 + 8.9 = 132.5$. Although 123.62 has an uncertainty of about 0.01, 8.9 has an uncertainty of about 0.1. So their sum has an uncertainty of about 0.1 and should be written as 132.5, not 132.52. Table 1.1 summarizes these rules for significant figures.

Table 1.1 Using Significant Figures

Mathematical Operation	Significant Figures in Result
Multiplication or division	No more than in the number with the fewest significant figures *Example:* $(0.745 \times 2.2)/3.885 = 0.42$ *Example:* $(1.32578 \times 10^7) \times (4.11 \times 10^{-3}) = 5.45 \times 10^4$
Addition or subtraction	Determined by the number with the largest uncertainty (i.e., the fewest digits to the right of the decimal point) *Example:* $27.153 + 138.2 - 11.74 = 153.6$

Note: In this book we will usually give numerical values with three significant figures.

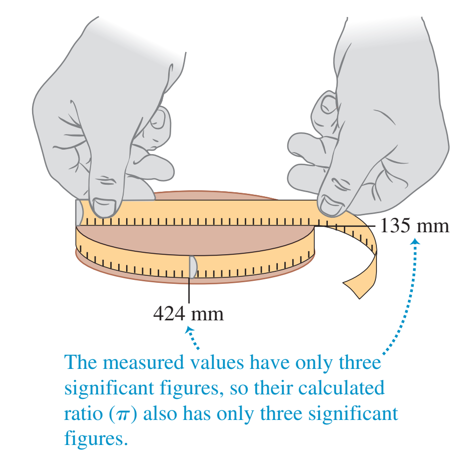
As an application of these ideas, suppose you want to verify the value of π, the ratio of the circumference of a circle to its diameter. The true value of this ratio to ten digits is 3.141592654. To test this, you draw a large circle and measure its circumference and diameter to the nearest millimeter, obtaining the values 424 mm and 135 mm (Fig. 1.8). You punch these into your calculator and obtain the quotient 3.140740741. This may seem to disagree with the true value of π, but keep in mind that each of your measurements has three significant figures, so your measured value of π, equal to $(424\text{ mm})/(135\text{ mm})$, can have only three significant figures. It should be stated simply as 3.14. Within the limit of three significant figures, your value does agree with the true value.

In the examples and problems in this book we usually give numerical values with three significant figures, so your answers should usually have no more than three significant figures. (Many numbers in the real world have even less accuracy. An automobile speedometer, for example, usually gives only two significant figures.) Even if you do the arithmetic with a calculator that displays ten digits, it would be wrong to give a ten-digit answer because it misrepresents the accuracy of the results. Always round your final answer to keep only the correct number of significant figures or, in doubtful cases, one more at most. In Example 1.1 it would have been wrong to state the answer as 341.11111 m/s. Note that when you reduce such an answer to the appropriate number of significant figures, you must *round,* not *truncate.* Your calculator will tell you that the ratio of 525 m to 311 m is 1.688102894; to three significant figures, this is 1.69, not 1.68.

When we calculate with very large or very small numbers, we can show significant figures much more easily by using **scientific notation,** sometimes called **powers-of-10 notation.** The distance from the earth to the moon is about 384,000,000 m, but writing the number in this form doesn't indicate the number of significant figures. Instead, we move the decimal point eight places to the left (corresponding to dividing by 10^8) and multiply by 10^8; that is,

$$384{,}000{,}000\text{ m} = 3.84 \times 10^8\text{ m}$$

1.8 Determining the value of π from the circumference and diameter of a circle.



In this form, it is clear that we have three significant figures. The number 4.00×10^{-7} also has three significant figures, even though two of them are zeros. Note that in scientific notation the usual practice is to express the quantity as a number between 1 and 10 multiplied by the appropriate power of 10.

When an integer or a fraction occurs in a general equation, we treat that number as having no uncertainty at all. For example, in the equation $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, which is Eq. (2.13) in Chapter 2, the coefficient 2 is *exactly* 2. We can consider this coefficient as having an infinite number of significant figures $(2.000000 \ldots)$. The same is true of the exponent 2 in v_x^2 and v_{0x}^2.

Finally, let's note that **precision** is not the same as *accuracy*. A cheap digital watch that gives the time as 10:35:17 A.M. is very *precise* (the time is given to the second), but if the watch runs several minutes slow, then this value isn't very *accurate*. On the other hand, a grandfather clock might be very accurate (that is, display the correct time), but if the clock has no second hand, it isn't very precise. A high-quality measurement, like those used to define standards (see Section 1.3), is both precise *and* accurate.

Example 1.3 Significant figures in multiplication

The rest energy E of an object with rest mass m is given by Einstein's equation

$$E = mc^2$$

where c is the speed of light in a vacuum. Find E for an object with $m = 9.11 \times 10^{-31}$ kg (to three significant figures, the mass of an electron). The SI unit for E is the joule (J); $1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$.

SOLUTION

IDENTIFY AND SET UP: Our target variable is the energy E. We are given the equation to use and the value of the mass m; from Section 1.3 the exact value of the speed of light is $c = 299{,}792{,}458 \text{ m/s} = 2.99792458 \times 10^8 \text{ m/s}$.

EXECUTE: Substituting the values of m and c into Einstein's equation, we find

$$\begin{aligned} E &= (9.11 \times 10^{-31} \text{ kg})(2.99792458 \times 10^8 \text{ m/s})^2 \\ &= (9.11)(2.99792458)^2(10^{-31})(10^8)^2 \text{ kg} \cdot \text{m}^2/\text{s}^2 \\ &= (81.87659678)(10^{[-31+(2\times 8)]}) \text{ kg} \cdot \text{m}^2/\text{s}^2 \\ &= 8.187659678 \times 10^{-14} \text{ kg} \cdot \text{m}^2/\text{s}^2 \end{aligned}$$

Since the value of m was given to only three significant figures, we must round this to

$$E = 8.19 \times 10^{-14} \text{ kg} \cdot \text{m}^2/\text{s}^2 = 8.19 \times 10^{-14} \text{ J}$$

Most calculators use scientific notation and add exponents automatically, but you should be able to do such calculations by hand when necessary.

EVALUATE: While the rest energy contained in an electron may seem ridiculously small, on the atomic scale it is tremendous. Compare our answer to 10^{-19} J, the energy gained or lost by a single atom during a typical chemical reaction; the rest energy of an electron is about 1,000,000 times larger! (We will discuss the significance of rest energy in Chapter 37.)

Test Your Understanding of Section 1.5 The density of a material is equal to its mass divided by its volume. What is the density (in kg/m^3) of a rock of mass 1.80 kg and volume $6.0 \times 10^{-4} \text{ m}^3$? (i) $3 \times 10^3 \text{ kg/m}^3$; (ii) $3.0 \times 10^3 \text{ kg/m}^3$; (iii) $3.00 \times 10^3 \text{ kg/m}^3$; (iv) $3.000 \times 10^3 \text{ kg/m}^3$; (v) any of these—all of these answers are mathematically equivalent.

MP

1.6 Estimates and Orders of Magnitude

We have stressed the importance of knowing the accuracy of numbers that represent physical quantities. But even a very crude estimate of a quantity often gives us useful information. Sometimes we know how to calculate a certain quantity, but we have to guess at the data we need for the calculation. Or the calculation might be too complicated to carry out exactly, so we make some rough approximations. In either case our result is also a guess, but such a guess can be useful even if it is uncertain by a factor of two, ten, or more. Such calculations are often called **order-of-magnitude estimates.** The great Italian-American nuclear physicist Enrico Fermi (1901–1954) called them "back-of-the-envelope calculations."

Exercises 1.18 through 1.29 at the end of this chapter are of the estimating, or "order-of-magnitude," variety. Some are silly, and most require guesswork for the needed input data. Don't try to look up a lot of data; make the best guesses you can. Even when they are off by a factor of ten, the results can be useful and interesting.

Example 1.4 An order-of-magnitude estimate

You are writing an adventure novel in which the hero escapes across the border with a billion dollars' worth of gold in his suitcase. Is this possible? Would that amount of gold fit in a suitcase? Would it be too heavy to carry?

SOLUTION

IDENTIFY, SET UP, AND EXECUTE: Gold sells for around \$400 an ounce. On a particular day the price might be \$200 or \$600, but never mind. An ounce is about 30 grams. Actually, an ordinary (avoirdupois) ounce is 28.35 g; an ounce of gold is a troy ounce, which is 9.45% more. Again, never mind. Ten dollars' worth of gold has a mass somewhere around one gram, so a billion (10^9) dollars' worth of gold is a hundred million (10^8) grams, or a hundred thousand (10^5) kilograms. This corresponds to a weight in British units of around 200,000 lb, or 100 tons. Whether the precise number is closer to 50 tons or 200 tons doesn't matter. Either way, the hero is not about to carry it across the border in a suitcase.

We can also estimate the *volume* of this gold. If its density were the same as that of water (1 g/cm^3), the volume would be 10^8 cm^3, or 100 m^3. But gold is a heavy metal; we might guess its density to be 10 times that of water. Gold is actually 19.3 times as dense as water. But by guessing 10, we find a volume of 10 m^3. Visualize 10 cubical stacks of gold bricks, each 1 meter on a side, and ask yourself whether they would fit in a suitcase!

EVALUATE: Clearly, your novel needs rewriting. Try the calculation again with a suitcase full of five-carat (1-gram) diamonds, each worth \$100,000. Would this work?

Test Your Understanding of Section 1.6 Can you estimate the total number of teeth in all the mouths of everyone (students, staff, and faculty) on your campus? (*Hint:* How many teeth are in your mouth? Count them!)

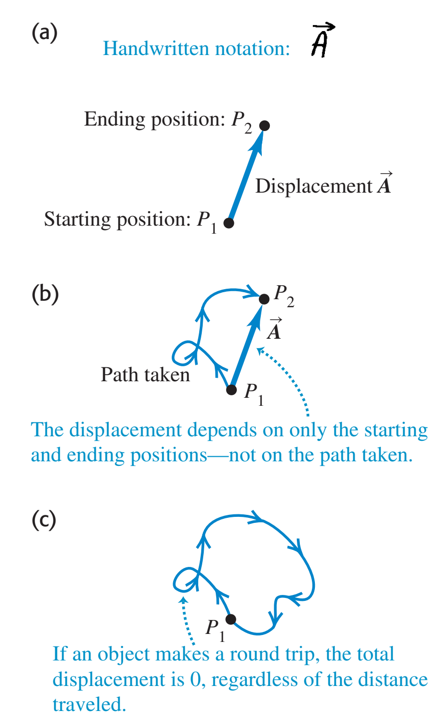
1.7 Vectors and Vector Addition

Some physical quantities, such as time, temperature, mass, and density, can be described completely by a single number with a unit. But many other important quantities in physics have a *direction* associated with them and cannot be described by a single number. A simple example is the motion of an airplane. To describe this motion completely, we must say not only how fast the plane is moving, but also in what direction. To fly from Chicago to New York, a plane has to head east, not south. The speed of the airplane combined with its direction of motion together constitute a quantity called *velocity*. Another example is *force*, which in physics means a push or pull exerted on a body. Giving a complete description of a force means describing both how hard the force pushes or pulls on the body and the direction of the push or pull.

When a physical quantity is described by a single number, we call it a **scalar quantity.** In contrast, a **vector quantity** has both a **magnitude** (the "how much" or "how big" part) and a direction in space. Calculations that combine scalar quantities use the operations of ordinary arithmetic. For example, $6 \text{ kg} + 3 \text{ kg} = 9 \text{ kg}$, or $4 \times 2 \text{ s} = 8 \text{ s}$. However, combining vectors requires a different set of operations.

To understand more about vectors and how they combine, we start with the simplest vector quantity, **displacement.** Displacement is simply a change in position of a point. (The point may represent a particle or a small body.) In Fig. 1.9a we represent the change of position from point P_1 to point P_2 by a line from P_1 to P_2, with an arrowhead at P_2 to represent the direction of motion. Displacement is a vector quantity because we must state not only how far the particle moves, but also in what direction. Walking 3 km north from your front door doesn't get you

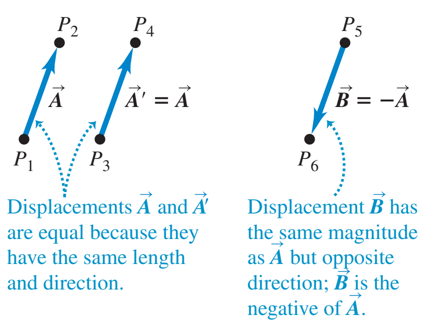
1.9 Displacement as a vector quantity. A displacement is always a straight-line segment directed from the starting point to the ending point, even if the path is curved.



to the same place as walking 3 km southeast; these two displacements have the same magnitude, but different directions.

We usually represent a vector quantity such as displacement by a single letter, such as $\vec{A}$ in Fig. 1.9a. In this book we always print vector symbols in ***boldface italic type with an arrow above them.*** We do this to remind you that vector quantities have different properties from scalar quantities; the arrow is a reminder that vectors have direction. In handwriting, vector symbols are usually underlined or written with an arrow above them (see Fig. 1.9a). When you write a symbol for a vector, *always* write it with an arrow on top. If you don't distinguish between scalar and vector quantities in your notation, you probably won't make the distinction in your thinking either, and hopeless confusion will result.

We always *draw* a vector as a line with an arrowhead at its tip. The length of the line shows the vector's magnitude, and the direction of the line shows the vector's direction. Displacement is always a straight-line segment, directed from the starting point to the ending point, even though the actual path of the particle may be curved. In Fig. 1.9b the particle moves along the curved path shown from P_1 to P_2, but the displacement is still the vector $\vec{A}$. Note that displacement is not related directly to the total *distance* traveled. If the particle were to continue on past P_2 and then return to P_1, the displacement for the entire trip would be *zero* (Fig. 1.9c).



1.10 The meaning of vectors that have the same magnitude and the same or opposite direction.

If two vectors have the same direction, they are **parallel.** If they have the same magnitude *and* the same direction, they are *equal,* no matter where they are located in space. The vector $\vec{A}'$ from point P_3 to point P_4 in Fig. 1.10 has the same length and direction as the vector $\vec{A}$ from P_1 to P_2. These two displacements are equal, even though they start at different points. We write this as $\vec{A}' = \vec{A}$ in Fig. 1.10; the boldface equals sign emphasizes that equality of two vector quantities is not the same relationship as equality of two scalar quantities. Two vector quantities are equal only when they have the same magnitude *and* the same direction.

The vector $\vec{B}$ in Fig. 1.10, however, is not equal to $\vec{A}$ because its direction is *opposite* to that of $\vec{A}$. We define the **negative of a vector** as a vector having the same magnitude as the original vector but the *opposite* direction. The negative of vector quantity $\vec{A}$ is denoted as $-\vec{A}$, and we use a boldface minus sign to emphasize the vector nature of the quantities. If $\vec{A}$ is 87 m south, then $-\vec{A}$ is 87 m north. Thus we can write the relationship between $\vec{A}$ and $\vec{B}$ in Fig. 1.10 as $\vec{A} = -\vec{B}$ or $\vec{B} = -\vec{A}$. When two vectors $\vec{A}$ and $\vec{B}$ have opposite directions, whether their magnitudes are the same or not, we say that they are **antiparallel.**

We usually represent the *magnitude* of a vector quantity (in the case of a displacement vector, its length) by the same letter used for the vector, but in *light italic type* with *no* arrow on top, rather than boldface italic with an arrow (which is reserved for vectors). An alternative notation is the vector symbol with vertical bars on both sides:

$$(\text{Magnitude of } \vec{A}) = A = |\vec{A}| \qquad (1.1)$$

By definition the magnitude of a vector quantity is a scalar quantity (a number) and is *always positive.* We also note that a vector can never be equal to a scalar because they are different kinds of quantities. The expression "$\vec{A} = 6$ m" is just as wrong as "2 oranges = 3 apples" or "6 lb = 7 km"!

When drawing diagrams with vectors, we'll generally use a scale similar to those used for maps. For example, a displacement of 5 km might be represented in a diagram by a vector 1 cm long, and a displacement of 10 km by a vector 2 cm long. In a diagram for velocity vectors, we might use a scale in which a vector that is 1 cm long represents a velocity of magnitude 5 meters per second (5 m/s). A velocity of 20 m/s would then be represented by a vector 4 cm long, with the appropriate direction.

Vector Addition

Suppose a particle undergoes a displacement $\vec{A}$ followed by a second displacement $\vec{B}$ (Fig. 1.11a). The final result is the same as if the particle had started at the same initial point and undergone a single displacement $\vec{C}$, as shown. We call displacement $\vec{C}$ the **vector sum,** or **resultant,** of displacements $\vec{A}$ and $\vec{B}$. We express this relationship symbolically as

$$\vec{C} = \vec{A} + \vec{B} \qquad (1.2)$$

The boldface plus sign emphasizes that adding two vector quantities requires a geometrical process and is not the same operation as adding two scalar quantities such as $2 + 3 = 5$. In vector addition we usually place the *tail* of the *second* vector at the *head,* or tip, of the *first* vector (Fig. 1.11a).

If we make the displacements $\vec{A}$ and $\vec{B}$ in reverse order, with $\vec{B}$ first and $\vec{A}$ second, the result is the same (Fig. 1.11b). Thus

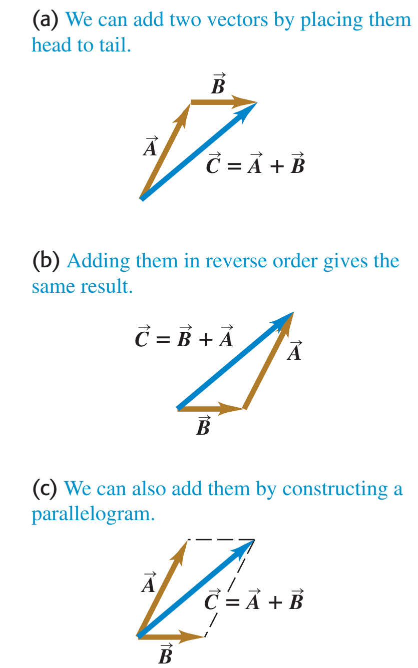
$$\vec{C} = \vec{B} + \vec{A} \quad \text{and} \quad \vec{A} + \vec{B} = \vec{B} + \vec{A} \qquad (1.3)$$

This shows that the order of terms in a vector sum doesn't matter. In other words, vector addition obeys the commutative law.

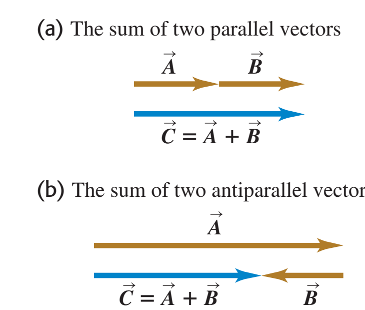
Figure 1.11c shows another way to represent the vector sum: If vectors $\vec{A}$ and $\vec{B}$ are both drawn with their tails at the same point, vector $\vec{C}$ is the diagonal of a parallelogram constructed with $\vec{A}$ and $\vec{B}$ as two adjacent sides.

CAUTION **Magnitudes in vector addition** It's a common error to conclude that if $\vec{C} = \vec{A} + \vec{B}$, then the magnitude C should just equal the magnitude A plus the magnitude B. In general, this conclusion is *wrong;* for the vectors shown in Fig. 1.11, you can see that $C < A + B$. The magnitude of $\vec{A} + \vec{B}$ depends on the magnitudes of $\vec{A}$ and $\vec{B}$ *and* on the angle between $\vec{A}$ and $\vec{B}$ (see Problem 1.92). Only in the special case in which $\vec{A}$ and $\vec{B}$ are *parallel* is the magnitude of $\vec{C} = \vec{A} + \vec{B}$ equal to the sum of the magnitudes of $\vec{A}$ and $\vec{B}$ (Fig. 1.12a). By contrast, when the vectors are *antiparallel* (Fig. 1.12b) the magnitude of $\vec{C}$ equals the *difference* of the magnitudes of $\vec{A}$ and $\vec{B}$. If you're careful about distinguishing between scalar and vector quantities, you'll avoid making errors about the magnitude of a vector sum.

1.11 Three ways to add two vectors. As shown in (b), the order in vector addition doesn't matter; vector addition is commutative.



1.12 (a) Only when two vectors $\vec{A}$ and $\vec{B}$ are parallel does the magnitude of their sum equal the sum of their magnitudes: $C = A + B$. (b) When $\vec{A}$ and $\vec{B}$ are antiparallel, the magnitude of their sum equals the *difference* of their magnitudes: $C = |A - B|$.



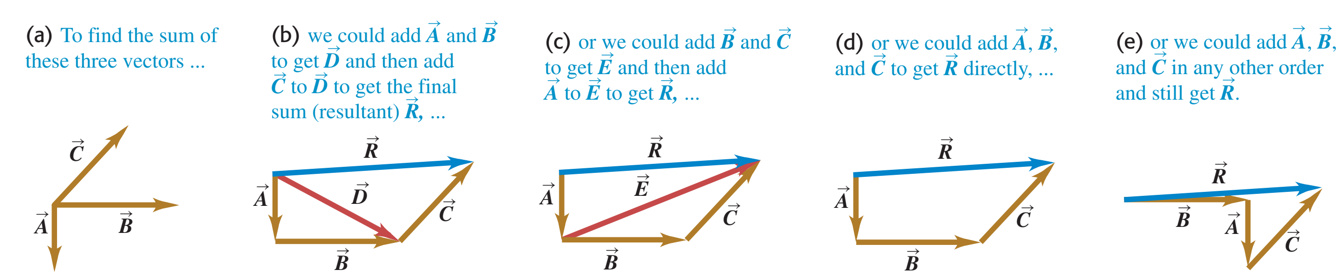
When we need to add more than two vectors, we may first find the vector sum of any two, add this vectorially to the third, and so on. Figure 1.13a shows three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$. In Fig. 1.13b, we first add $\vec{A}$ and $\vec{B}$ to give a vector sum $\vec{D}$; we then add vectors $\vec{C}$ and $\vec{D}$ by the same process to obtain the vector sum $\vec{R}$:

$$\vec{R} = (\vec{A} + \vec{B}) + \vec{C} = \vec{D} + \vec{C}$$

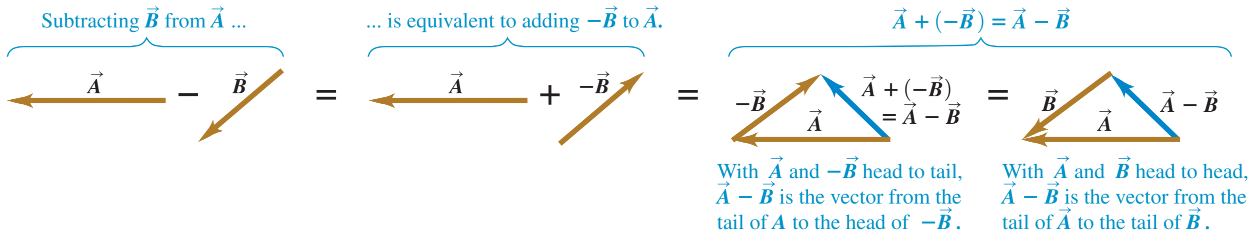
Alternatively, we can first add $\vec{B}$ and $\vec{C}$ to obtain vector $\vec{E}$ (Fig. 1.13c), and then add $\vec{A}$ and $\vec{E}$ to obtain $\vec{R}$:

$$\vec{R} = \vec{A} + (\vec{B} + \vec{C}) = \vec{A} + \vec{E}$$

1.13 Several constructions for finding the vector sum $\vec{A} + \vec{B} + \vec{C}$.



1.14 To construct the vector difference $\vec{A} - \vec{B}$, you can either place the tail of $-\vec{B}$ at the head of $\vec{A}$ or place the two vectors $\vec{A}$ and $\vec{B}$ head to head.



We don't even need to draw vectors $\vec{D}$ and $\vec{E}$; all we need to do is draw $\vec{A}$, $\vec{B}$, and $\vec{C}$ in succession, with the tail of each at the head of the one preceding it. The sum vector $\vec{R}$ extends from the tail of the first vector to the head of the last vector (Fig. 1.13d). The order makes no difference; Fig. 1.13e shows a different order, and we invite you to try others. We see that vector addition obeys the associative law.

We can *subtract* vectors as well as add them. To see how, recall that the vector $-\vec{A}$ has the same magnitude as $\vec{A}$ but the opposite direction. We define the difference $\vec{A} - \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ to be the vector sum of $\vec{A}$ and $-\vec{B}$:

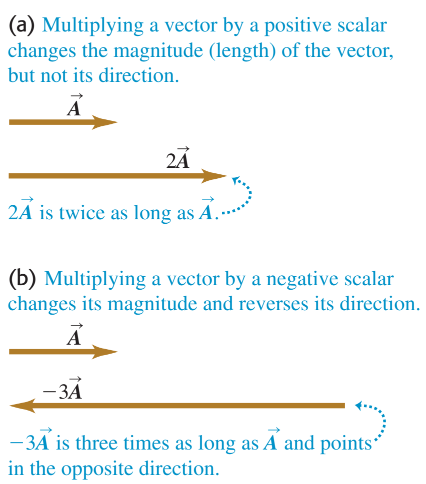
$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \tag{1.4}$$

Figure 1.14 shows an example of vector subtraction.

A vector quantity such as a displacement can be multiplied by a scalar quantity (an ordinary number). The displacement $2\vec{A}$ is a displacement (vector quantity) in the same direction as the vector $\vec{A}$ but twice as long; this is the same as adding $\vec{A}$ to itself (Fig. 1.15a). In general, when a vector $\vec{A}$ is multiplied by a scalar c, the result $c\vec{A}$ has magnitude $|c|A$ (the absolute value of c multiplied by the magnitude of the vector $\vec{A}$). If c is positive, $c\vec{A}$ is in the same direction as $\vec{A}$; if c is negative, $c\vec{A}$ is in the direction opposite to $\vec{A}$. Thus $3\vec{A}$ is parallel to $\vec{A}$, while $-3\vec{A}$ is antiparallel to $\vec{A}$ (Fig. 1.15b).

The scalar quantity used to multiply a vector may also be a physical quantity having units. For example, you may be familiar with the relationship $\vec{F} = m\vec{a}$; the net force $\vec{F}$ (a vector quantity) that acts on a body is equal to the product of the body's mass m (a positive scalar quantity) and its acceleration $\vec{a}$ (a vector quantity). The direction of $\vec{F}$ is the same as that of $\vec{a}$ because m is positive, and the magnitude of $\vec{F}$ is equal to the mass m (which is positive and equals its own absolute value) multiplied by the magnitude of $\vec{a}$. The unit of force is the unit of mass multiplied by the unit of acceleration.

1.15 Multiplying a vector (a) by a positive scalar and (b) by a negative scalar.



Example 1.5 Vector addition

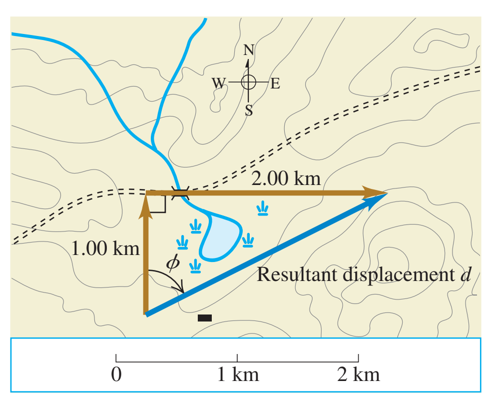
A cross-country skier skis 1.00 km north and then 2.00 km east on a horizontal snow field. How far and in what direction is she from the starting point?

SOLUTION

IDENTIFY: The problem involves combining displacements, so we can solve it using vector addition. The target variables are the skier's total distance and direction from her starting point. The distance is just the magnitude of her resultant displacement vector from the point of origin to where she stops, and the direction we want is the direction of the resultant displacement vector.

SET UP: Figure 1.16 is a scale diagram of the skier's displacements. We describe the direction from the starting point by the angle ϕ (the Greek letter phi). By careful measurement we find that the distance from the starting point to the ending point is about 2.2 km and that ϕ is about 63°. But we can *calculate* a much more accurate result by adding the 1.00-km and 2.00-km displacement vectors.

1.16 The vector diagram, drawn to scale, for a cross-country ski trip.



EXECUTE: The vectors in the diagram form a right triangle; the distance from the starting point to the ending point is equal to the length of the hypotenuse. We find this length by using the Pythagorean theorem:

$$\sqrt{(1.00\text{ km})^2 + (2.00\text{ km})^2} = 2.24\text{ km}$$

The angle ϕ can be found with a little simple trigonometry. If you need a review, the trigonometric functions and identities are summarized in Appendix B, along with other useful mathematical and geometrical relationships. By the definition of the tangent function,

$$\tan\phi = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{2.00\text{ km}}{1.00\text{ km}}$$
$$\phi = 63.4°$$

We can describe the direction as 63.4° east of north or 90° − 63.4° = 26.6° north of east. Take your choice!

EVALUATE: It's good practice to check the results of a vector-addition problem by making measurements on a drawing of the situation. Happily, the answers we found by calculation (2.24 km and ϕ = 63.4°) are very close to the cruder results we found by measurement (about 2.2 km and about 63°). If they were substantially different, we would have to go back and check for errors.

Test Your Understanding of Section 1.7 Two displacement vectors, $\vec{S}$ and $\vec{T}$, have magnitudes $S = 3$ m and $T = 4$ m. Which of the following could be the magnitude of the difference vector $\vec{S} - \vec{T}$? (There may be more than one correct answer.) (i) 9 m; (ii) 7 m; (iii) 5 m; (iv) 1 m; (v) 0 m; (vi) −1 m.

1.8 Components of Vectors

In Section 1.7 we added vectors by using a scale diagram and by using properties of right triangles. Measuring a diagram offers only very limited accuracy, and calculations with right triangles work only when the two vectors are perpendicular. So we need a simple but general method for adding vectors. This is called the method of *components.*

To define what we mean by the components of a vector $\vec{A}$, we begin with a rectangular (Cartesian) coordinate system of axes (Fig. 1.17a). We then draw the vector with its tail at O, the origin of the coordinate system. We can represent any vector lying in the xy-plane as the sum of a vector parallel to the x-axis and a vector parallel to the y-axis. These two vectors are labeled $\vec{A}_x$ and $\vec{A}_y$ in Fig. 1.17a; they are called the **component vectors** of vector $\vec{A}$, and their vector sum is equal to $\vec{A}$. In symbols,

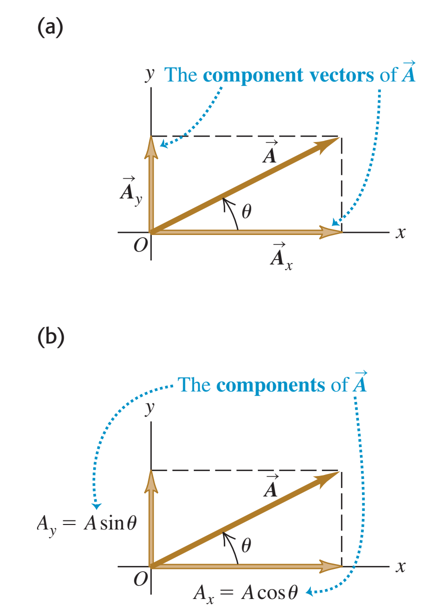
$$\vec{A} = \vec{A}_x + \vec{A}_y \tag{1.5}$$

Since each component vector lies along a coordinate-axis direction, we need only a single number to describe each one. When the component vector $\vec{A}_x$ points in the positive x-direction, we define the number A_x to be equal to the magnitude of $\vec{A}_x$. When the component vector $\vec{A}_x$ points in the negative x-direction, we define the number A_x to be equal to the negative of that magnitude (the magnitude of a vector quantity is itself never negative). We define the number A_y in the same way. The two numbers A_x and A_y are called the **components** of $\vec{A}$ (Fig. 1.17b).

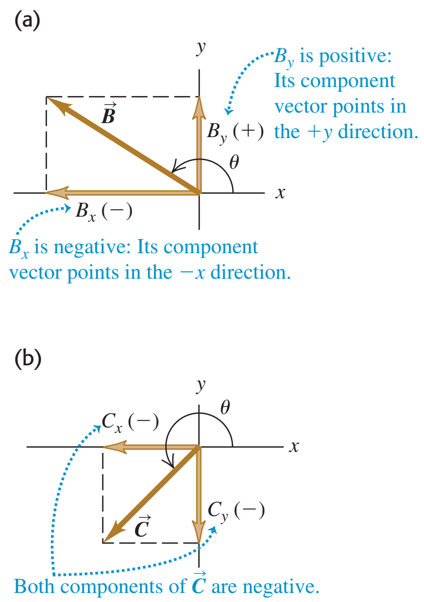
CAUTION **Components are not vectors** The components A_x and A_y of a vector $\vec{A}$ are just numbers; they are *not* vectors themselves. This is why we print the symbols for components in light italic type with *no* arrow on top instead of the boldface italic with an arrow, which is reserved for vectors.

We can calculate the components of the vector $\vec{A}$ if we know its magnitude A and its direction. We'll describe the direction of a vector by its angle relative to some reference direction. In Fig. 1.17b this reference direction is the

1.17 Representing a vector $\vec{A}$ in terms of (a) component vectors $\vec{A}_x$ and $\vec{A}_y$ and (b) components A_x and A_y (which in this case are both positive).



1.18 The components of a vector may be positive or negative numbers.



positive x-axis, and the angle between vector $\vec{A}$ and the positive x-axis is θ (the Greek letter theta). Imagine that the vector $\vec{A}$ originally lies along the $+x$-axis and that you then rotate it to its correct direction, as indicated by the arrow in Fig. 1.17b on the angle θ. If this rotation is from the $+x$-axis toward the $+y$-axis, as shown in Fig. 1.17b, then θ is *positive;* if the rotation is from the $+x$-axis toward the $-y$-axis, θ is *negative.* Thus the $+y$-axis is at an angle of 90°, the $-x$-axis at 180°, and the $-y$-axis at 270° (or −90°). If θ is measured in this way, then from the definition of the trigonometric functions,

$$\frac{A_x}{A} = \cos\theta \quad \text{and} \quad \frac{A_y}{A} = \sin\theta$$
$$A_x = A\cos\theta \quad \text{and} \quad A_y = A\sin\theta \qquad (1.6)$$
$$(\theta \text{ measured from the } +x\text{-axis, rotating toward the } +y\text{-axis})$$

In Fig. 1.17b, A_x is positive because its direction is along the positive x-axis, and A_y is positive because its direction is along the positive y-axis. This is consistent with Eqs. (1.6); θ is in the first quadrant (between 0° and 90°), and both the cosine and the sine of an angle in this quadrant are positive. But in Fig. 1.18a the component B_x is negative; its direction is opposite to that of the positive x-axis. Again, this agrees with Eqs. (1.6); the cosine of an angle in the second quadrant is negative. The component B_y is positive ($\sin\theta$ is positive in the second quadrant). In Fig. 1.18b, both C_x and C_y are negative (both $\cos\theta$ and $\sin\theta$ are negative in the third quadrant).

CAUTION **Relating a vector's magnitude and direction to its components** Equations (1.6) are correct *only* when the angle θ is measured from the positive x-axis as described above. If the angle of the vector is given from a different reference direction or using a different sense of rotation, the relationships are different. Be careful! Example 1.6 illustrates this point.

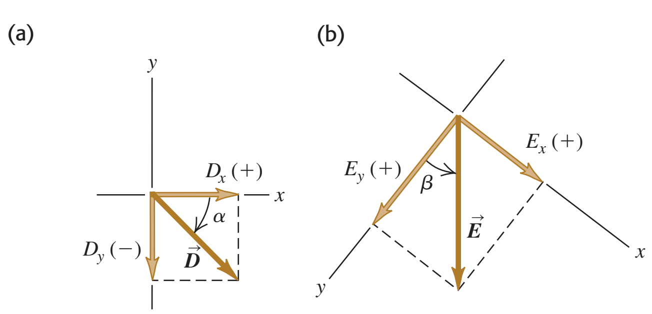
Example 1.6 Finding components

(a) What are the x- and y-components of vector $\vec{D}$ in Fig. 1.19a? The magnitude of the vector is $D = 3.00$ m and the angle $\alpha = 45°$. (b) What are the x- and y-components of vector $\vec{E}$ in Fig. 1.19b? The magnitude of the vector is $E = 4.50$ m and the angle $\beta = 37.0°$.

SOLUTION

IDENTIFY: In each case we are given the magnitude and direction of a vector, and we are asked to find its components.

1.19 Calculating the x- and y-components of vectors.



SET UP: It would seem that all we need is Eqs. (1.6). However, we need to be careful because the angles in Fig. 1.19 are *not* measured from the $+x$-axis toward the $+y$-axis.

EXECUTE: (a) The angle between $\vec{D}$ and the positive x-axis is α (the Greek letter alpha), but this angle is measured toward the *negative* y-axis. So the angle we must use in Eqs. (1.6) is $\theta = -\alpha = -45°$. We find

$$D_x = D\cos\theta = (3.00\text{ m})(\cos(-45°)) = +2.1\text{ m}$$
$$D_y = D\sin\theta = (3.00\text{ m})(\sin(-45°)) = -2.1\text{ m}$$

The vector has a positive x-component and a negative y-component, as shown in the figure. Had you been careless and substituted +45° for θ in Eqs. (1.6), you would have gotten the wrong sign for D_y.

(b) The x-axis isn't horizontal in Fig. 1.19b, nor is the y-axis vertical. Don't worry, though: *Any* orientation of the x- and y-axes is permissible, just so the axes are mutually perpendicular. (In Chapter 5 we'll use axes like these to study an object sliding on an incline; one axis will lie along the incline and the other will be perpendicular to the incline.)

Here the angle β (the Greek letter beta) is the angle between $\vec{E}$ and the positive y-axis, *not* the positive x-axis, so we *cannot* use this angle in Eqs. (1.6). Instead, note that $\vec{E}$ defines the hypotenuse of a right triangle; the other two sides of the triangle are the magnitudes of E_x and E_y, the x- and y-components of $\vec{E}$. The sine of β is the opposite side (the magnitude of E_x) divided by the hypotenuse (the magnitude E), and the cosine of β is the adjacent side (the magnitude of E_y) divided by the hypotenuse (again, the magnitude E). Both components of $\vec{E}$ are positive, so

$$E_x = E\sin\beta = (4.50\text{ m})(\sin 37.0°) = +2.71\text{ m}$$
$$E_y = E\cos\beta = (4.50\text{ m})(\cos 37.0°) = +3.59\text{ m}$$

Had you used Eqs. (1.6) directly and written $E_x = E\cos 37.0°$ and $E_y = E\sin 37.0°$, your answers for E_x and E_y would have been reversed!

If you insist on using Eqs. (1.6), you must first find the angle between $\vec{E}$ and the positive x-axis, measured toward the positive y-axis; this is $\theta = 90.0° - \beta = 90.0° - 37.0° = 53.0°$. Then $E_x = E\cos\theta$ and $E_y = E\sin\theta$. You can substitute the values of E and θ into Eqs. (1.6) to show that the results for E_x and E_y are the same as those given above.

EVALUATE: Notice that the answers to part (b) have three significant figures, but the answers to part (a) have only two. Can you see why?

Doing Vector Calculations Using Components

Using components makes it relatively easy to do various calculations involving vectors. Let's look at three important examples.

1. Finding a vector's magnitude and direction from its components. We can describe a vector completely by giving either its magnitude and direction or its x- and y-components. Equations (1.6) show how to find the components if we know the magnitude and direction. We can also reverse the process: We can find the magnitude and direction if we know the components. By applying the Pythagorean theorem to Fig. 1.17b, we find that the magnitude of vector $\vec{A}$ is

$$A = \sqrt{A_x^2 + A_y^2} \qquad (1.7)$$

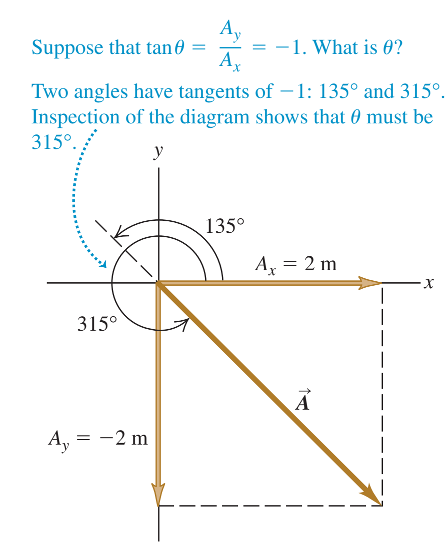
(We always take the positive root.) Equation (1.7) is valid for any choice of x-axis and y-axis, as long as they are mutually perpendicular. The expression for the vector direction comes from the definition of the tangent of an angle. If θ is measured from the positive x-axis, and a positive angle is measured toward the positive y-axis (as in Fig. 1.17b), then

$$\tan\theta = \frac{A_y}{A_x} \quad \text{and} \quad \theta = \arctan\frac{A_y}{A_x} \qquad (1.8)$$

We will always use the notation arctan for the inverse tangent function. The notation $\tan^{-1}$ is also commonly used, and your calculator may have an INV or 2ND button to be used with the TAN button.

CAUTION **Finding the direction of a vector from its components** There is one slight complication in using Eqs. (1.8) to find θ. Suppose $A_x = 2$ m and $A_y = -2$ m as in Fig. 1.20; then $\tan\theta = -1$. But there are two angles that have tangents of −1—namely, 135° and 315° (or −45°). In general, any two angles that differ by 180° have the same tangent. To decide which is correct, we have to look at the individual components. Because A_x is positive and A_y is negative, the angle must be in the fourth quadrant; thus $\theta = 315°$ (or −45°) is the correct value. Most pocket calculators give $\arctan(-1) = -45°$. In this case that is correct; but if instead we have $A_x = -2$ m and $A_y = 2$ m, then the correct angle is 135°. Similarly, when A_x and A_y are both negative, the tangent is positive, but the angle is in the third quadrant. You should *always* draw a sketch like Fig. 1.20 to check which of the two possibilities is the correct one.

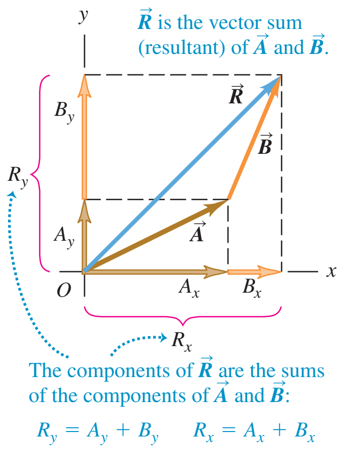
1.20 Drawing a sketch of a vector reveals the signs of its x- and y-components.



2. Multiplying a vector by a scalar. If we multiply a vector $\vec{A}$ by a scalar c, each component of the product $\vec{D} = c\vec{A}$ is just the product of c and the corresponding component of $\vec{A}$:

$$D_x = cA_x \qquad D_y = cA_y \qquad (\text{components of } \vec{D} = c\vec{A}) \qquad (1.9)$$

For example, Eq. (1.9) says that each component of the vector $2\vec{A}$ is twice as great as the corresponding component of the vector $\vec{A}$, so $2\vec{A}$ is in the same direction as $\vec{A}$ but has twice the magnitude. Each component of the vector $-3\vec{A}$ is three times as great as the corresponding component of the vector $\vec{A}$ but has the opposite sign, so $-3\vec{A}$ is in the opposite direction from $\vec{A}$ and has three times the magnitude. Hence Eqs. (1.9) are consistent with our discussion in Section 1.7 of multiplying a vector by a scalar (see Fig. 1.15).



1.21 Finding the vector sum (resultant) of $\vec{A}$ and $\vec{B}$ using components.

3. Using components to calculate the vector sum (resultant) of two or more vectors. Figure 1.21 shows two vectors $\vec{A}$ and $\vec{B}$ and their vector sum $\vec{R}$, along with the x- and y-components of all three vectors. You can see from the diagram that the x-component R_x of the vector sum is simply the sum $(A_x + B_x)$ of the x-components of the vectors being added. The same is true for the y-components. In symbols,

$$R_x = A_x + B_x \qquad R_y = A_y + B_y \qquad \left(\text{components of } \vec{R} = \vec{A} + \vec{B}\right) \tag{1.10}$$

Figure 1.21 shows this result for the case in which the components A_x, A_y, B_x, and B_y are all positive. You should draw additional diagrams to verify for yourself that Eqs. (1.10) are valid for *any* signs of the components of $\vec{A}$ and $\vec{B}$.

If we know the components of any two vectors $\vec{A}$ and $\vec{B}$, perhaps by using Eqs. (1.6), we can compute the components of the vector sum $\vec{R}$. Then if we need the magnitude and direction of $\vec{R}$, we can obtain them from Eqs. (1.7) and (1.8) with the A's replaced by R's.

We can extend this procedure to find the sum of any number of vectors. If $\vec{R}$ is the vector sum of $\vec{A}, \vec{B}, \vec{C}, \vec{D}, \vec{E}, \ldots$, the components of $\vec{R}$ are

$$\begin{aligned} R_x &= A_x + B_x + C_x + D_x + E_x + \cdots \\ R_y &= A_y + B_y + C_y + D_y + E_y + \cdots \end{aligned} \tag{1.11}$$

We have talked only about vectors that lie in the xy-plane, but the component method works just as well for vectors having any direction in space. We introduce a z-axis perpendicular to the xy-plane; then in general a vector $\vec{A}$ has components A_x, A_y, and A_z in the three coordinate directions. The magnitude A is given by

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \tag{1.12}$$

Again, we always take the positive root. Also, Eqs. (1.11) for the components of the vector sum $\vec{R}$ have an additional member:

$$R_z = A_z + B_z + C_z + D_z + E_z + \cdots$$

Finally, while our discussion of vector addition has centered on combining *displacement* vectors, the method is applicable to all other vector quantities as well. When we study the concept of force in Chapter 4, we'll find that forces are vectors that obey the same rules of vector addition that we've used with displacement. Other vector quantities will make their appearance in later chapters.

Problem-Solving Strategy 1.3 Vector Addition

IDENTIFY *the relevant concepts:* Decide what your target variable is. It may be the magnitude of the vector sum, the direction, or both.

SET UP *the problem:* Draw the individual vectors being summed and the coordinate axes being used. In your drawing, place the tail of the first vector at the origin of coordinates; place the tail of the second vector at the head of the first vector; and so on. Draw the vector sum $\vec{R}$ from the tail of the first vector to the head of the last vector. Use your drawing to make rough estimates of the magnitude and direction of $\vec{R}$; you'll use these estimates later to check your calculations.

EXECUTE *the solution* as follows:

1. Find the x- and y-components of each individual vector and record your results in a table. If a vector is described by its magnitude A and its angle θ, measured from the $+x$-axis toward the $+y$-axis, then the components are given by

$$A_x = A\cos\theta \qquad A_y = A\sin\theta$$

Some components may be positive and some may be negative, depending on how the vector is oriented (that is, what quadrant θ lies in). You can use this sign table as a check:

Quadrant	I	II	III	IV
A_x	+	−	−	+
A_y	+	+	−	−

If the angles of the vectors are given in some other way, perhaps using a different reference direction, convert them to angles measured from the $+x$-axis as described above. Be particularly careful with signs.

2. Add the individual x-components algebraically, including signs, to find R_x, the x-component of the vector sum. Do the same for the y-components to find R_y.

3. Then the magnitude R and direction θ of the vector sum are given by

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \arctan\frac{R_y}{R_x}$$

EVALUATE *your answer:* Check your results for the magnitude and direction of the vector sum by comparing them with the rough estimates you made from your drawing. Remember that the magnitude R is *always* positive and that θ is measured from the positive x-axis. The value of θ that you find with a calculator may be the correct one, or it may be off by 180°. You can decide by examining your drawing.

If your calculations disagree totally with the estimates from your drawing, check whether your calculator is set in "radians" or "degrees" mode. If it's in "radians" mode, entering angles in degrees will give nonsensical answers.

Example 1.7 Adding vectors with components

Three players on a reality TV show are brought to the center of a large, flat field. Each is given a meter stick, a compass, a calculator, a shovel, and (in a different order for each contestant) the following three displacements:

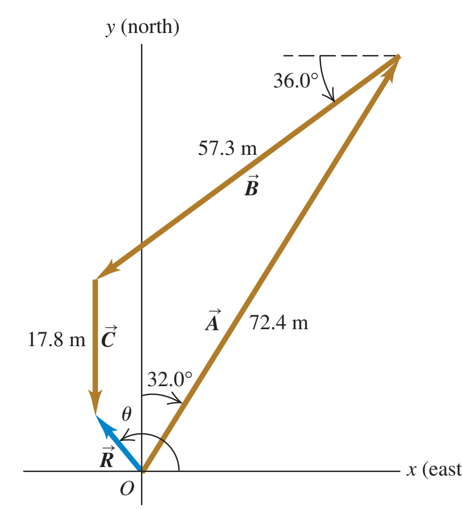
72.4 m, 32.0° east of north
57.3 m, 36.0° south of west
17.8 m straight south

The three displacements lead to the point where the keys to a new Porsche are buried. Two players start measuring immediately, but the winner first *calculates* where to go. What does she calculate?

SOLUTION

IDENTIFY: The goal is to find the sum (resultant) of the three displacements, so this is a problem in vector addition.

SET UP: Figure 1.22 shows the situation. We have chosen the $+x$-axis as east and the $+y$-axis as north, the usual choice for maps. Let $\vec{A}$ be the first displacement, $\vec{B}$ the second, and $\vec{C}$ the third. We can estimate from the diagram that the vector sum $\vec{R}$ is about 10 m, 40° west of north.



1.22 Three successive displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$ and the resultant (vector sum) displacement $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.

EXECUTE: The angles of the vectors, measured from the $+x$-axis toward the $+y$-axis, are $(90.0° - 32.0°) = 58.0°$, $(180.0° + 36.0°) = 216.0°$, and 270.0°. We have to find the components of each. Because of our choice of axes, we may use Eqs. (1.6), and so the components of $\vec{A}$ are

$$A_x = A\cos\theta_A = (72.4\text{ m})(\cos 58.0°) = 38.37\text{ m}$$
$$A_y = A\sin\theta_A = (72.4\text{ m})(\sin 58.0°) = 61.40\text{ m}$$

Note that we have kept one too many significant figures in the components; we will wait until the end to round to the correct number of significant figures. The table shows the components of all the displacements, the addition of the components, and the other calculations. Always arrange your component calculations systematically like this.

Distance	Angle	x-component	y-component
$A = 72.4$ m	58.0°	38.37 m	61.40 m
$B = 57.3$ m	216.0°	−46.36 m	−33.68 m
$C = 17.8$ m	270.0°	0.00 m	−17.80 m
		$R_x = -7.99$ m	$R_y = 9.92$ m

$$R = \sqrt{(-7.99\text{ m})^2 + (9.92\text{ m})^2} = 12.7\text{ m}$$

$$\theta = \arctan\frac{9.92\text{ m}}{-7.99\text{ m}} = 129° = 39° \text{ west of north}$$

The losers try to measure three angles and three distances totaling 147.5 m, one meter at a time. The winner measured only one angle and one much shorter distance.

EVALUATE: Our calculated answers for R and θ are not too different from our estimates of 10 m and 40° west of north; that's good! Notice that $\theta = -51°$, or 51° south of east, also satisfies the equation for θ. But since the winner has made a drawing of the displacement vectors (Fig. 1.22), she knows that $\theta = 129°$ is the only correct solution for the angle.

Example 1.8 A vector in three dimensions

After an airplane takes off, it travels 10.4 km west, 8.7 km north, and 2.1 km up. How far is it from the takeoff point?

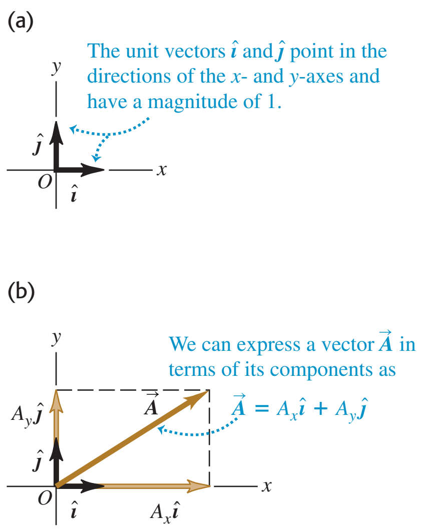
SOLUTION

Let the $+x$-axis be east, the $+y$-axis north, and the $+z$-axis up. Then $A_x = -10.4$ km, $A_y = 8.7$ km, and $A_z = 2.1$ km; Eq. (1.12) gives

$$A = \sqrt{(-10.4\ \text{km})^2 + (8.7\ \text{km})^2 + (2.1\ \text{km})^2} = 13.7\ \text{km}$$

Test Your Understanding of Section 1.8 Two vectors $\vec{A}$ and $\vec{B}$ both lie in the xy-plane. (a) Is it possible for $\vec{A}$ to have the same magnitude as $\vec{B}$ but different components? (b) Is it possible for $\vec{A}$ to have the same components as $\vec{B}$ but a different magnitude?

1.9 Unit Vectors



1.23 (a) The unit vectors $\hat{\imath}$ and $\hat{\jmath}$. (b) Expressing a vector $\vec{A}$ in terms of its components.

A **unit vector** is a vector that has a magnitude of 1, with no units. Its only purpose is to *point*—that is, to describe a direction in space. Unit vectors provide a convenient notation for many expressions involving components of vectors. We will always include a caret or "hat" $(\hat{\ })$ in the symbol for a unit vector to distinguish it from ordinary vectors whose magnitude may or may not be equal to 1.

In an x-y coordinate system we can define a unit vector $\hat{\imath}$ that points in the direction of the positive x-axis and a unit vector $\hat{\jmath}$ that points in the direction of the positive y-axis (Fig. 1.23a). Then we can express the relationship between component vectors and components, described at the beginning of Section 1.8, as follows:

$$\vec{A}_x = A_x\hat{\imath}$$
$$\vec{A}_y = A_y\hat{\jmath} \qquad (1.13)$$

Similarly, we can write a vector $\vec{A}$ in terms of its components as

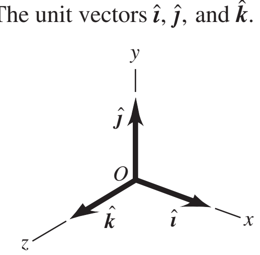
$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} \qquad (1.14)$$

Equations (1.13) and (1.14) are vector equations; each term, such as $A_x\hat{\imath}$, is a vector quantity (Fig. 1.23b). The boldface equals and plus signs denote vector equality and addition.

When two vectors $\vec{A}$ and $\vec{B}$ are represented in terms of their components, we can express the vector sum $\vec{R}$ using unit vectors as follows:

$$\begin{aligned}\vec{A} &= A_x\hat{\imath} + A_y\hat{\jmath}\\ \vec{B} &= B_x\hat{\imath} + B_y\hat{\jmath}\\ \vec{R} &= \vec{A} + \vec{B}\\ &= (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath})\\ &= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}\\ &= R_x\hat{\imath} + R_y\hat{\jmath}\end{aligned} \qquad (1.15)$$

Equation (1.15) restates the content of Eqs. (1.10) in the form of a single vector equation rather than two component equations.



1.24 The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

If the vectors do not all lie in the xy-plane, then we need a third component. We introduce a third unit vector $\hat{k}$ that points in the direction of the positive z-axis (Fig. 1.24). Then Eqs. (1.14) and (1.15) become

$$\begin{aligned}\vec{A} &= A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\\ \vec{B} &= B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\end{aligned} \qquad (1.16)$$

$$\begin{aligned}\vec{R} &= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}\\ &= R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}\end{aligned} \qquad (1.17)$$

Example 1.9 Using unit vectors

Given the two displacements

$$\vec{D} = (6\hat{\imath} + 3\hat{\jmath} - \hat{k})\ \text{m} \qquad \text{and} \qquad \vec{E} = (4\hat{\imath} - 5\hat{\jmath} + 8\hat{k})\ \text{m}$$

find the magnitude of the displacement $2\vec{D} - \vec{E}$.

SOLUTION

IDENTIFY: We are to multiply the vector $\vec{D}$ by 2 (a scalar) and then subtract the vector $\vec{E}$ from the result.

SET UP: Equation (1.9) says that to multiply $\vec{D}$ by 2, we simply multiply each of its components by 2. Then Eq. (1.17) tells us that to subtract $\vec{E}$ from $2\vec{D}$, we simply subtract the components of $\vec{E}$ from the components of $2\vec{D}$. (Recall from Section 1.7 that subtracting a vector is the same as adding the negative of that vector.) In each of these mathematical operations, the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ remain unchanged.

EXECUTE: Letting $\vec{F} = 2\vec{D} - \vec{E}$, we have

$$\begin{aligned}\vec{F} &= 2(6\hat{\imath} + 3\hat{\jmath} - \hat{k})\ \text{m} - (4\hat{\imath} - 5\hat{\jmath} + 8\hat{k})\ \text{m}\\ &= [(12 - 4)\hat{\imath} + (6 + 5)\hat{\jmath} + (-2 - 8)\hat{k}]\ \text{m}\\ &= (8\hat{\imath} + 11\hat{\jmath} - 10\hat{k})\ \text{m}\end{aligned}$$

The units of the vectors $\vec{D}$, $\vec{E}$, and $\vec{F}$ are meters, so the components of these vectors are also in meters. From Eq. (1.12),

$$\begin{aligned}F &= \sqrt{F_x^2 + F_y^2 + F_z^2}\\ &= \sqrt{(8\ \text{m})^2 + (11\ \text{m})^2 + (-10\ \text{m})^2} = 17\ \text{m}\end{aligned}$$

EVALUATE: Working with unit vectors makes vector addition and subtraction no more complicated than adding and subtracting ordinary numbers. Still, be sure to check for simple arithmetic errors.

Test Your Understanding of Section 1.9 Arrange the following vectors in order of their magnitude, with the vector of largest magnitude first. (i) $\vec{A} = (3\hat{\imath} + 5\hat{\jmath} - 2\hat{k})$ m; (ii) $\vec{B} = (-3\hat{\imath} + 5\hat{\jmath} - 2\hat{k})$ m; (iii) $\vec{C} = (3\hat{\imath} - 5\hat{\jmath} - 2\hat{k})$ m; (iv) $\vec{D} = (3\hat{\imath} + 5\hat{\jmath} + 2\hat{k})$ m.

MP

1.10 Products of Vectors

We have seen how addition of vectors develops naturally from the problem of combining displacements, and we will use vector addition for calculating many other vector quantities later. We can also express many physical relationships concisely by using *products* of vectors. Vectors are not ordinary numbers, so ordinary multiplication is not directly applicable to vectors. We will define two different kinds of products of vectors. The first, called the *scalar product,* yields a result that is a scalar quantity. The second, the *vector product,* yields another vector.

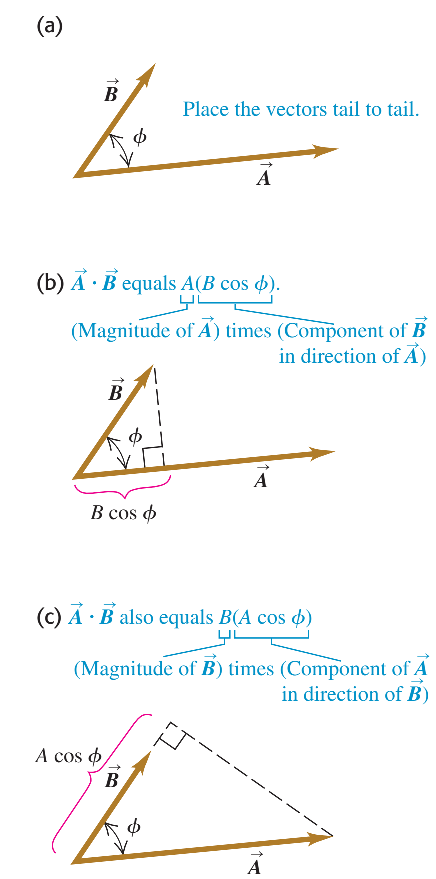
Scalar Product

The **scalar product** of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A}\cdot\vec{B}$. Because of this notation, the scalar product is also called the **dot product.** Although $\vec{A}$ and $\vec{B}$ are vectors, the quantity $\vec{A}\cdot\vec{B}$ is a scalar.

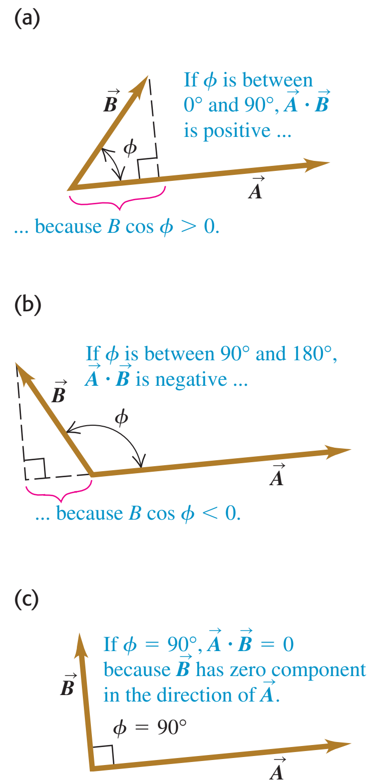
To define the scalar product $\vec{A}\cdot\vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$, we draw the two vectors with their tails at the same point (Fig. 1.25a). The angle ϕ (the Greek letter phi) between their directions ranges from 0° to 180°. Figure 1.25b shows the projection of the vector $\vec{B}$ onto the direction of $\vec{A}$; this projection is the component of $\vec{B}$ in the direction of $\vec{A}$ and is equal to $B\cos\phi$. (We can take components along any direction that's convenient, not just the x- and y-axes.) We define $\vec{A}\cdot\vec{B}$ to be the magnitude of $\vec{A}$ multiplied by the component of $\vec{B}$ in the direction of $\vec{A}$. Expressed as an equation,

$$\vec{A}\cdot\vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi \qquad \text{(definition of the scalar (dot) product)} \qquad (1.18)$$

1.25 Calculating the scalar product of two vectors, $\vec{A}\cdot\vec{B} = AB\cos\phi$.



1.26 The scalar product $\vec{A}\cdot\vec{B} = AB\cos\phi$ can be positive, negative, or zero, depending on the angle between $\vec{A}$ and $\vec{B}$.



Alternatively, we can define $\vec{A}\cdot\vec{B}$ to be the magnitude of $\vec{B}$ multiplied by the component of $\vec{A}$ in the direction of $\vec{B}$, as in Fig. 1.25c. Hence $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{B} = B(A\cos\phi) = AB\cos\phi$, which is the same as Eq. (1.18).

The scalar product is a scalar quantity, not a vector, and it may be positive, negative, or zero. When ϕ is between 0° and 90°, $\cos\phi > 0$ and the scalar product is positive (Fig. 1.26a). When ϕ is between 90° and 180° so that $\cos\phi < 0$, the component of $\vec{B}$ in the direction of $\vec{A}$ is negative, and $\vec{A}\cdot\vec{B}$ is negative (Fig. 1.26b). Finally, when $\phi = 90°$, $\vec{A}\cdot\vec{B} = 0$ (Fig. 1.26c). *The scalar product of two perpendicular vectors is always zero.*

For any two vectors $\vec{A}$ and $\vec{B}$, $AB\cos\phi = BA\cos\phi$. This means that $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$. The scalar product obeys the commutative law of multiplication; the order of the two vectors does not matter.

We will use the scalar product in Chapter 6 to describe work done by a force. When a constant force $\vec{F}$ is applied to a body that undergoes a displacement $\vec{s}$, the work W (a scalar quantity) done by the force is given by

$$W = \vec{F}\cdot\vec{s}$$

The work done by the force is positive if the angle between $\vec{F}$ and $\vec{s}$ is between 0° and 90°, negative if this angle is between 90° and 180°, and zero if $\vec{F}$ and $\vec{s}$ are perpendicular. (This is another example of a term that has a special meaning in physics; in everyday language, "work" isn't something that can be positive or negative.) In later chapters we'll use the scalar product for a variety of purposes, from calculating electric potential to determining the effects that varying magnetic fields have on electric circuits.

Calculating the Scalar Product Using Components

We can calculate the scalar product $\vec{A}\cdot\vec{B}$ directly if we know the x-, y-, and z-components of $\vec{A}$ and $\vec{B}$. To see how this is done, let's first work out the scalar products of the unit vectors. This is easy, since $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ all have magnitude 1 and are perpendicular to each other. Using Eq. (1.18), we find

$$\begin{aligned} \hat{\imath}\cdot\hat{\imath} &= \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = (1)(1)\cos 0° = 1 \\ \hat{\imath}\cdot\hat{\jmath} &= \hat{\imath}\cdot\hat{k} = \hat{\jmath}\cdot\hat{k} = (1)(1)\cos 90° = 0 \end{aligned} \tag{1.19}$$

Now we express $\vec{A}$ and $\vec{B}$ in terms of their components, expand the product, and use these products of unit vectors:

$$\begin{aligned} \vec{A}\cdot\vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})\cdot(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\ &= A_x\hat{\imath}\cdot B_x\hat{\imath} + A_x\hat{\imath}\cdot B_y\hat{\jmath} + A_x\hat{\imath}\cdot B_z\hat{k} \\ &\quad + A_y\hat{\jmath}\cdot B_x\hat{\imath} + A_y\hat{\jmath}\cdot B_y\hat{\jmath} + A_y\hat{\jmath}\cdot B_z\hat{k} \\ &\quad + A_z\hat{k}\cdot B_x\hat{\imath} + A_z\hat{k}\cdot B_y\hat{\jmath} + A_z\hat{k}\cdot B_z\hat{k} \\ &= A_xB_x\hat{\imath}\cdot\hat{\imath} + A_xB_y\hat{\imath}\cdot\hat{\jmath} + A_xB_z\hat{\imath}\cdot\hat{k} \\ &\quad + A_yB_x\hat{\jmath}\cdot\hat{\imath} + A_yB_y\hat{\jmath}\cdot\hat{\jmath} + A_yB_z\hat{\jmath}\cdot\hat{k} \\ &\quad + A_zB_x\hat{k}\cdot\hat{\imath} + A_zB_y\hat{k}\cdot\hat{\jmath} + A_zB_z\hat{k}\cdot\hat{k} \end{aligned} \tag{1.20}$$

From Eqs. (1.19) we see that six of these nine terms are zero, and the three that survive give simply

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z \quad \text{(scalar (dot) product in terms of components)} \tag{1.21}$$

Thus *the scalar product of two vectors is the sum of the products of their respective components.*

The scalar product gives a straightforward way to find the angle ϕ between any two vectors $\vec{A}$ and $\vec{B}$ whose components are known. In this case, Eq. (1.21) can be used to find the scalar product of $\vec{A}$ and $\vec{B}$. From Eq. (1.18) the scalar product is also equal to $AB\cos\phi$ The vector magnitudes A and B can be found from the vector components with Eq. (1.12), so $\cos\phi$ and hence the angle ϕ can be determined (see Example 1.11).

Example 1.10 Calculating a scalar product

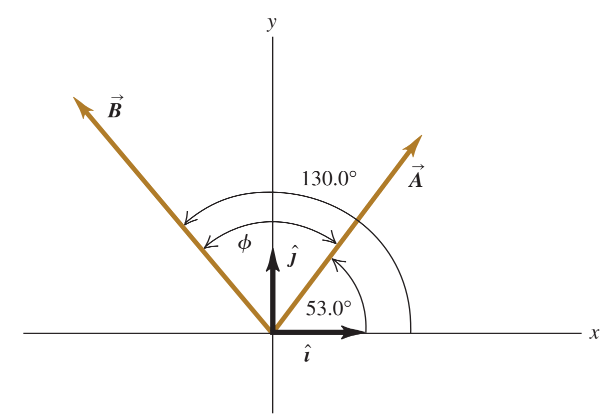
Find the scalar product $\vec{A}\cdot\vec{B}$ of the two vectors in Fig. 1.27. The magnitudes of the vectors are $A = 4.00$ and $B = 5.00$.

SOLUTION

IDENTIFY: We are given the magnitudes and directions of $\vec{A}$ and $\vec{B}$, and we wish to calculate their scalar product.

SET UP: We will calculate the scalar product in two ways: using the magnitudes of the vectors and the angle between them (Eq. 1.18), and using the components of the two vectors (Eq. 1.21).

1.27 Two vectors in two dimensions.



EXECUTE: With the first approach, the angle between the two vectors is $\phi = 130.0° - 53.0° = 77.0°$, so

$$\vec{A}\cdot\vec{B} = AB\cos\phi = (4.00)(5.00)\cos 77.0° = 4.50$$

This is positive because the angle between $\vec{A}$ and $\vec{B}$ is between 0° and 90°.

To use the second approach, we first need to find the components of the two vectors. Since the angles of $\vec{A}$ and $\vec{B}$ are given with respect to the $+x$-axis, and these angles are measured in the sense from the $+x$-axis to the $+y$-axis, we can use Eqs. (1.6):

$$\begin{aligned} A_x &= (4.00)\cos 53.0° = 2.407 \\ A_y &= (4.00)\sin 53.0° = 3.195 \\ A_z &= 0 \\ B_x &= (5.00)\cos 130.0° = -3.214 \\ B_y &= (5.00)\sin 130.0° = 3.830 \\ B_z &= 0 \end{aligned}$$

The z-components are zero because both vectors lie in the xy-plane. As in Example 1.7, we are keeping one too many significant figures in the components; we'll round to the correct number at the end. From Eq. (1.21) the scalar product is

$$\begin{aligned} \vec{A}\cdot\vec{B} &= A_xB_x + A_yB_y + A_zB_z \\ &= (2.407)(-3.214) + (3.195)(3.830) + (0)(0) = 4.50 \end{aligned}$$

EVALUATE: We get the same result for the scalar product with both methods, as we should.

Example 1.11 Finding angles with the scalar product

Find the angle between the two vectors

$$\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k} \qquad \text{and} \qquad \vec{B} = -4\hat{\imath} + 2\hat{\jmath} - \hat{k}$$

SOLUTION

IDENTIFY: We are given the x-, y-, and z-components of two vectors. Our target variable is the angle ϕ between them.

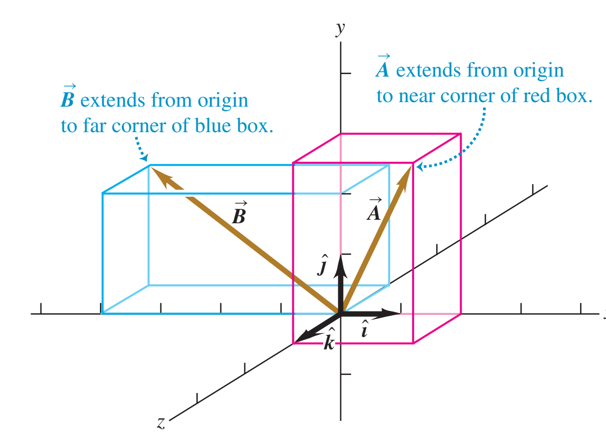
SET UP: Figure 1.28 shows the two vectors. The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is related to the angle ϕ between them and to the magnitudes A and B by Eq. (1.18). The scalar product is also related to the components of the two vectors by Eq. (1.21). If we are given the components of the vectors (as we are in this example), we first determine the scalar product $\vec{A}\cdot\vec{B}$ and the values of A and B, and then determine the target variable ϕ.

EXECUTE: We set our two expressions for the scalar product, Eq. (1.18) and Eq. (1.121), equal to each other. Rearranging, we obtain

$$\cos\phi = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

This formula can be used to find the angle between *any* two vectors $\vec{A}$ and $\vec{B}$. For our example the components of $\vec{A}$ are $A_x = 2$,

1.28 Two vectors in three dimensions.



Continued

$A_y = 3$, and $A_z = 1$, and the components of $\vec{B}$ are $B_x = -4$, $B_y = 2$, and $B_z = -1$. Thus

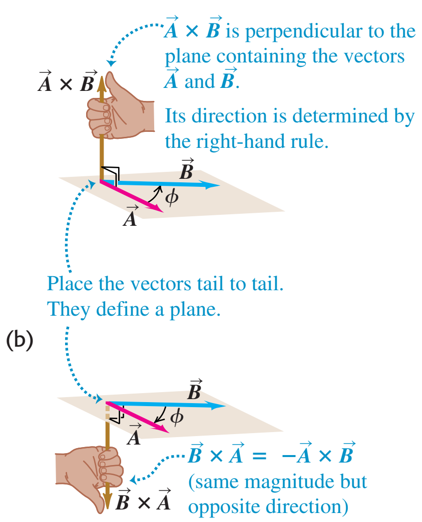
$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$
$$= (2)(-4) + (3)(2) + (1)(-1) = -3$$
$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$$
$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(-4^2) + 2^2 + (-1)^2} = \sqrt{21}$$

$$\cos\phi = \frac{A_xB_x + A_yB_y + A_zB_z}{AB} = \frac{-3}{\sqrt{14}\sqrt{21}} = -0.175$$
$$\phi = 100°$$

EVALUATE: As a check on this result, note that the scalar product $\vec{A}\cdot\vec{B}$ is negative. This means that ϕ is between 90° and 180° (see Fig. 1.26), in agreement with our answer.

1.29 (a) The vector product $\vec{A}\times\vec{B}$ determined by the right-hand rule. (b) $\vec{B}\times\vec{A} = -\vec{A}\times\vec{B}$; the vector product is anticommutative.

(a)



Vector Product

The **vector product** of two vectors $\vec{A}$ and $\vec{B}$, also called the **cross product,** is denoted by $\vec{A}\times\vec{B}$. As the name suggests, the vector product is itself a vector. We will use this product in Chapter 10 to describe torque and angular momentum; in Chapters 27 and 28 we will use it extensively to describe magnetic fields and forces.

To define the vector product $\vec{A}\times\vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ we again draw the two vectors with their tails at the same point (Fig. 1.29a). The two vectors then lie in a plane. We define the vector product to be a vector quantity with a direction perpendicular to this plane (that is, perpendicular to both $\vec{A}$ and $\vec{B}$) and a magnitude equal to $AB\sin\phi$. That is, if $\vec{C} = \vec{A}\times\vec{B}$, then

$$C = AB\sin\phi \quad \text{(magnitude of the vector (cross) product of } \vec{A} \text{ and } \vec{B}) \quad (1.22)$$

We measure the angle ϕ from $\vec{A}$ toward $\vec{B}$ and take it to be the smaller of the two possible angles, so ϕ ranges from 0° to 180°. Then $\sin\phi \geq 0$ and C in Eq. (1.22) is never negative, as must be the case for a vector magnitude. Note also that when $\vec{A}$ and $\vec{B}$ are parallel or antiparallel, $\phi = 0$ or 180° and $C = 0$. That is, *the vector product of two parallel or antiparallel vectors is always zero.* In particular, *the vector product of any vector with itself is zero.*

CAUTION **Vector product vs. scalar product** Be careful not to confuse the expression $AB\sin\phi$ for the magnitude of the vector product $\vec{A}\times\vec{B}$ with the similar expression $AB\cos\phi$ for the scalar product $\vec{A}\cdot\vec{B}$. To see the contrast between these two expressions, imagine that we vary the angle between $\vec{A}$ and $\vec{B}$ while keeping their magnitudes constant. When $\vec{A}$ and $\vec{B}$ are parallel, the magnitude of the vector product will be zero and the scalar product will be maximum. When $\vec{A}$ and $\vec{B}$ are perpendicular, the magnitude of the vector product will be maximum and the scalar product will be zero.

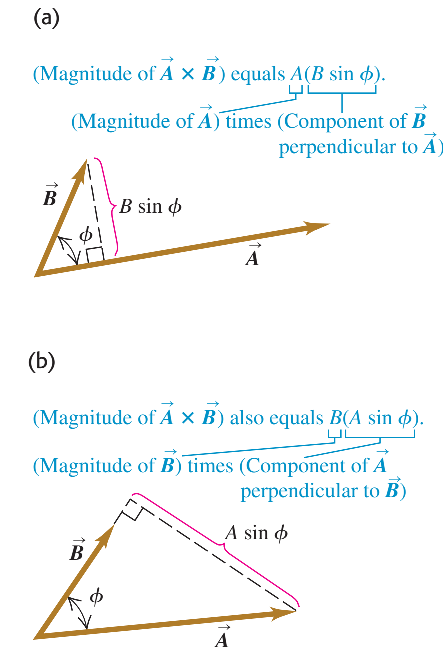
There are always *two* directions perpendicular to a given plane, one on each side of the plane. We choose which of these is the direction of $\vec{A}\times\vec{B}$ as follows. Imagine rotating vector $\vec{A}$ about the perpendicular line until it is aligned with $\vec{B}$, choosing the smaller of the two possible angles between $\vec{A}$ and $\vec{B}$. Curl the fingers of your right hand around the perpendicular line so that the fingertips point in the direction of rotation; your thumb will then point in the direction of $\vec{A}\times\vec{B}$. Figure 1.29a shows this **right-hand rule.**

Similarly, we determine the direction of $\vec{B}\times\vec{A}$ by rotating $\vec{B}$ into $\vec{A}$ as in Fig. 1.29b. The result is a vector that is *opposite* to the vector $\vec{A}\times\vec{B}$. The vector product is *not* commutative! In fact, for any two vectors $\vec{A}$ and $\vec{B}$,

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A} \quad (1.23)$$

Just as we did for the scalar product, we can give a geometrical interpretation of the magnitude of the vector product. In Fig. 1.30a, $B\sin\phi$ is the component of vector $\vec{B}$ that is *perpendicular* to the direction of vector $\vec{A}$ From Eq. (1.22) the magnitude of $\vec{A}\times\vec{B}$ equals the magnitude of $\vec{A}$ multiplied by the component of $\vec{B}$ perpendicular to $\vec{A}$. Figure 1.30b shows that the magnitude of $\vec{A}\times\vec{B}$ also equals the magnitude of $\vec{B}$ multiplied by the component of $\vec{A}$ perpendicular to $\vec{B}$. Note that Fig. 1.30 shows the case in which ϕ is between 0° and 90°; you should draw a similar diagram for ϕ between 90° and 180° to show that the same geometrical interpretation of the magnitude of $\vec{A}\times\vec{B}$ still applies.

1.30 Calculating the magnitude $AB\sin\phi$ of the vector product of two vectors, $\vec{A}\times\vec{B}$.



Calculating the Vector Product Using Components

If we know the components of $\vec{A}$ and $\vec{B}$, we can calculate the components of the vector product using a procedure similar to that for the scalar product. First we work out the multiplication table for the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, all three of which are perpendicular to each other (Fig. 1.31a). The vector product of any vector with itself is zero, so

$$\hat{\imath}\times\hat{\imath} = \hat{\jmath}\times\hat{\jmath} = \hat{k}\times\hat{k} = \mathbf{0}$$

The boldface zero is a reminder that each product is a zero *vector*—that is, one with all components equal to zero and an undefined direction. Using Eqs. (1.22) and (1.23) and the right-hand rule, we find

$$\begin{aligned}\hat{\imath}\times\hat{\jmath} &= -\hat{\jmath}\times\hat{\imath} = \hat{k}\\ \hat{\jmath}\times\hat{k} &= -\hat{k}\times\hat{\jmath} = \hat{\imath}\\ \hat{k}\times\hat{\imath} &= -\hat{\imath}\times\hat{k} = \hat{\jmath}\end{aligned} \quad (1.24)$$

You can verify these equations by referring to Fig. 1.31a.

Next we express $\vec{A}$ and $\vec{B}$ in terms of their components and the corresponding unit vectors, and we expand the expression for the vector product:

$$\begin{aligned}\vec{A}\times\vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})\times(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})\\ &= A_x\hat{\imath}\times B_x\hat{\imath} + A_x\hat{\imath}\times B_y\hat{\jmath} + A_x\hat{\imath}\times B_z\hat{k}\\ &\quad + A_y\hat{\jmath}\times B_x\hat{\imath} + A_y\hat{\jmath}\times B_y\hat{\jmath} + A_y\hat{\jmath}\times B_z\hat{k}\\ &\quad + A_z\hat{k}\times B_x\hat{\imath} + A_z\hat{k}\times B_y\hat{\jmath} + A_z\hat{k}\times B_z\hat{k}\end{aligned} \quad (1.25)$$

We can also rewrite the individual terms in Eq. (1.25) as $A_x\hat{\imath}\times B_y\hat{\jmath} = (A_xB_y)\hat{\imath}\times\hat{\jmath}$, and so on. Evaluating these by using the multiplication table for the unit vectors in Eqs. (1.24) and then grouping the terms, we find

$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k} \quad (1.26)$$

Thus the components of $\vec{C} = \vec{A}\times\vec{B}$ are given by

$$C_x = A_yB_z - A_zB_y \qquad C_y = A_zB_x - A_xB_z \qquad C_z = A_xB_y - A_yB_x$$
$$\text{(components of } \vec{C} = \vec{A}\times\vec{B}) \quad (1.27)$$

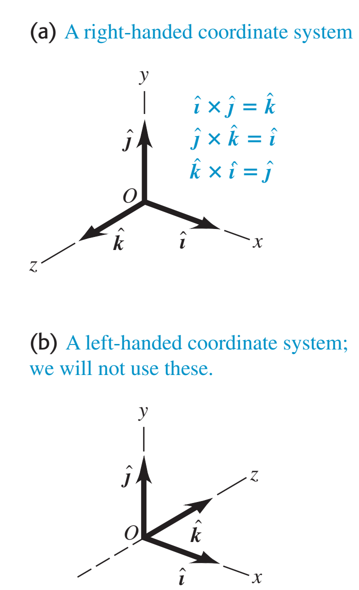
The vector product can also be expressed in determinant form as

$$\vec{A}\times\vec{B} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

If you aren't familiar with determinants, don't worry about this form.

With the axis system of Fig. 1.31a, if we reverse the direction of the z-axis, we get the system shown in Fig. 1.31b. Then, as you may verify, the definition of the vector product gives $\hat{\imath}\times\hat{\jmath} = -\hat{k}$ instead of $\hat{\imath}\times\hat{\jmath} = \hat{k}$. In fact, all vector products of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ would have signs opposite to those in Eqs. (1.24). We see that there are two kinds of coordinate systems, differing in the signs of the vector products of unit vectors. An axis system in which $\hat{\imath}\times\hat{\jmath} = \hat{k}$, as in Fig. 1.31a, is called a **right-handed system.** The usual practice is to use *only* right-handed systems, and we will follow that practice throughout this book.

1.31 (a) We will always use a right-handed coordinate system, like this one. (b) We will never use a left handed coordinate system (in which $\hat{\imath}\times\hat{\jmath} = -\hat{k}$, and so on).



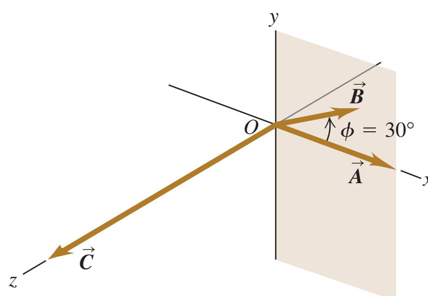
Example 1.12 Calculating a vector product

Vector $\vec{A}$ has magnitude 6 units and is in the direction of the $+x$-axis. Vector $\vec{B}$ has magnitude 4 units and lies in the xy-plane, making an angle of 30° with the $+x$-axis (Fig. 1.32). Find the vector product $\vec{A} \times \vec{B}$.

SOLUTION

IDENTIFY: We are given the magnitude and direction for each vector, and we want to find their vector product.

SET UP: We can find the vector product in one of two ways. The first way is to use Eq. (1.22) to determine the magnitude of $\vec{A} \times \vec{B}$ and then use the right-hand rule to find the direction of the vector product. The second way is to use the components of $\vec{A}$ and $\vec{B}$ to find the components of the vector product $\vec{C} = \vec{A} \times \vec{B}$ using Eqs. (1.27).



1.32 Vectors $\vec{A}$ and $\vec{B}$ and their vector product $\vec{C} = \vec{A} \times \vec{B}$. The vector $\vec{B}$ lies in the xy-plane.

EXECUTE: With the first approach, from Eq. (1.22) the magnitude of the vector product is

$$AB\sin\phi = (6)(4)(\sin 30°) = 12$$

From the right-hand rule, the direction of $\vec{A} \times \vec{B}$ is along the $+z$-axis, so we have $\vec{A} \times \vec{B} = 12\hat{k}$.

To use the second approach, we first write the components of $\vec{A}$ and $\vec{B}$:

$$A_x = 6 \qquad A_y = 0 \qquad A_z = 0$$
$$B_x = 4\cos 30° = 2\sqrt{3} \qquad B_y = 4\sin 30° = 2 \qquad B_z = 0$$

Defining $\vec{C} = \vec{A} \times \vec{B}$, we have from Eqs. (1.27) that

$$C_x = (0)(0) - (0)(2) = 0$$
$$C_y = (0)(2\sqrt{3}) - (6)(0) = 0$$
$$C_z = (6)(2) - (0)(2\sqrt{3}) = 12$$

The vector product $\vec{C}$ has only a z-component, and it lies along the $+z$-axis. The magnitude agrees with the result we obtained with the first approach, as it should.

EVALUATE: For this example the first approach was more direct because we knew the magnitudes of each vector and the angle between them, and furthermore, both vectors lay in one of the planes of the coordinate system. But often you will need to find the vector product of two vectors that are not so conveniently oriented or for which only the components are given. In such a case the second approach, using components, is more direct.

Test Your Understanding of Section 1.10 Vector $\vec{A}$ has magnitude 2 and vector $\vec{B}$ has magnitude 3. The angle ϕ between $\vec{A}$ and $\vec{B}$ is known to be either 0°, 90°, or 180°. For each of the following situations, state what the value of ϕ must be. (In each situation there may be more than one correct answer.) (a) $\vec{A} \cdot \vec{B} = 0$; (b) $\vec{A} \times \vec{B} = \mathbf{0}$; (c) $\vec{A} \cdot \vec{B} = 6$; (d) $\vec{A} \cdot \vec{B} = -6$; (e) (magnitude of $\vec{A} \times \vec{B}$) = 6.

CHAPTER 1 SUMMARY

Physical quantities and units: The fundamental physical quantities of mechanics are mass, length, and time. The corresponding basic SI units are the kilogram, the meter, and the second. Derived units for other physical quantities are products or quotients of the basic units. Equations must be dimensionally consistent; two terms can be added only when they have the same units. (See Examples 1.1 and 1.2.)

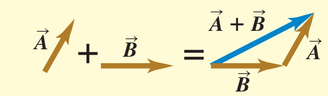
Significant figures: The accuracy of a measurement can be indicated by the number of significant figures or by a stated uncertainty. The result of a calculation usually has no more significant figures than the input data. When only crude estimates are available for input data, we can often make useful order-of-magnitude estimates. (See Examples 1.3 and 1.4.)

Significant figures in magenta

$$\pi = \frac{C}{2r} = \frac{0.424\ \text{m}}{2(0.06750\ \text{m})} = 3.14$$

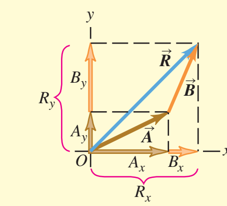
$$123.62 + 8.9 = 132.5$$

Scalars, vectors, and vector addition: Scalar quantities are numbers and combine with the usual rules of arithmetic. Vector quantities have direction as well as magnitude and combine according to the rules of vector addition. The negative of a vector has the same magnitude but points in the opposite direction. (See Example 1.5.)



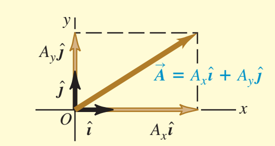
Vector components and vector addition: Vector addition can be carried out using components of vectors. The x-component of $\vec{R} = \vec{A} + \vec{B}$ is the sum of the x-components of $\vec{A}$ and $\vec{B}$, and likewise for the y- and z-components. (See Examples 1.6–1.8.)

$$R_x = A_x + B_x$$
$$R_y = A_y + B_y \qquad (1.10)$$
$$R_z = A_z + B_z$$



Unit vectors: Unit vectors describe directions in space. A unit vector has a magnitude of one, with no units. The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, aligned with the x-, y-, and z-axes of a rectangular coordinate system, are especially useful. (See Example 1.9.)

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \qquad (1.16)$$

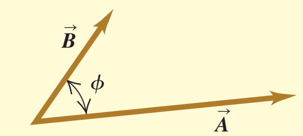


Scalar product: The scalar product $C = \vec{A} \cdot \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ is a scalar quantity. It can be expressed in terms of the magnitudes of $\vec{A}$ and $\vec{B}$ and the angle ϕ between the two vectors, or in terms of the components of $\vec{A}$ and $\vec{B}$. The scalar product is commutative; $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$. The scalar product of two perpendicular vectors is zero. (See Examples 1.10 and 1.11.)

$$\vec{A} \cdot \vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi \qquad (1.18)$$

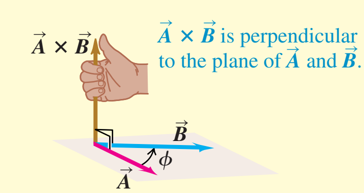
$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z \qquad (1.21)$$

Scalar product $\vec{A} \cdot \vec{B} = AB\cos\phi$



Vector product: The vector product $\vec{C} = \vec{A} \times \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ is another vector $\vec{C}$. The magnitude of $\vec{A} \times \vec{B}$ depends on the magnitudes of $\vec{A}$ and $\vec{B}$ and the angle ϕ between the two vectors. The direction of $\vec{A} \times \vec{B}$ is perpendicular to the plane of the two vectors being multiplied, as given by the right-hand rule. The components of $\vec{C} = \vec{A} \times \vec{B}$ can be expressed in terms of the components of $\vec{A}$ and $\vec{B}$. The vector product is not commutative; $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$. The vector product of two parallel or antiparallel vectors is zero. (See Example 1.12.)

$$C = AB\sin\phi \qquad (1.22)$$
$$C_x = A_yB_z - A_zB_y$$
$$C_y = A_zB_x - A_xB_z \qquad (1.27)$$
$$C_z = A_xB_y - A_yB_x$$



(Magnitude of $\vec{A} \times \vec{B}$) = $AB\sin\phi$

Key Terms

range of validity, *2*
target variable, *3*
model, *3*
particle, *3*
physical quantity, *4*
operational definition, *4*
unit, *4*
International System (SI), *4*
second, 5
meter, *5*
kilogram, *5*
prefix, *5*
dimensionally consistent, *7*
uncertainty (error), *8*
accuracy, *8*
fractional error, *8*
percent error, *8*
significant figures, *9*
scientific (powers-of-10) notation, *9*
precision, *10*
order-of-magnitude estimates, *11*
scalar quantity, *11*
vector quantity, *11*
magnitude of a vector, *11*
displacement, *11*
parallel vectors, *12*
negative of a vector, *12*
antiparallel vectors, *12*
vector sum (resultant), *13*
component vectors, *15*
components, *15*
unit vector, *20*
scalar (dot) product, *21*
vector (cross) product, *24*
right-hand rule, *24*
right-handed system, *25*

Answer to Chapter Opening Question

Take the $+x$-axis to point east and the $+y$-axis to point north. Then what we are trying to find is the y-component of the velocity vector, which has magnitude $v = 20$ km/h and is at an angle $\theta = 53°$ measured from the $+x$-axis toward the $+y$-axis. From Eqs. (1.6) we have $v_y = v\sin\theta = (20\text{ km/h})\sin 53° = 16$ km/h. So the hurricane moves 16 km north in 1 h.

Answers to Test Your Understanding Questions

1.5 Answer: (ii) Density $= (1.80\text{ kg})/(6.0 \times 10^{-4}\text{ m}^3) = 3.0 \times 10^3\text{ kg/m}^3$. When we multiply or divide, the number with the fewest significant figures controls the number of significant figures in the result.

1.6 The answer depends on how many students are enrolled at your campus.

1.7 Answers: (ii), (iii), and (iv) The vector $-\vec{T}$ has the same magnitude as the vector $\vec{T}$, so $\vec{S} - \vec{T} = \vec{S} + (-\vec{T})$ is the *sum* of one vector of magnitude 3 m and one of magnitude 4 m. This sum has magnitude 7 m if $\vec{S}$ and $-\vec{T}$ are parallel and magnitude 1 m if $\vec{S}$ and $-\vec{T}$ are antiparallel. The magnitude of $\vec{S} - \vec{T}$ is 5 m if $\vec{S}$ and $-\vec{T}$ are perpendicular, so that the vectors $\vec{S}$, $\vec{T}$, and $\vec{S} - \vec{T}$ form a 3-4-5 right triangle. Answer (i) is impossible because the magnitude of the sum of two vectors cannot be greater than the sum of the magnitudes; answer (v) is impossible because the sum of two vectors can be zero only if the two vectors are antiparallel and have the same magnitude; and answer (vi) is impossible because the magnitude of a vector cannot be negative.

1.8 Answers: (a) yes, (b) no Vectors $\vec{A}$ and $\vec{B}$ can have the same magnitude but different components if they point in different directions. If they have the same components, however, they are the same vector $(\vec{A} = \vec{B})$ and so must have the same magnitude.

1.9 Answer: all have the same magnitude The four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ all point in different directions, but all have the same magnitude:

$$A = B = C = D = \sqrt{(\pm 3\text{ m})^2 + (\pm 5\text{ m})^2 + (\pm 2\text{ m})^2}$$
$$= \sqrt{9\text{ m}^2 + 25\text{ m}^2 + 4\text{ m}^2} = \sqrt{38\text{ m}^2} = 6.2\text{ m}$$

1.10 Answers: (a) $\phi = 90°$, (b) $\phi = 0°$ or $\phi = 180°$, (c) $\phi = 0°$, (d) $\phi = 180°$, (e) $\phi = 90°$ (a) The scalar product is zero only if $\vec{A}$ and $\vec{B}$ are perpendicular. (b) The vector product is zero only if $\vec{A}$ and $\vec{B}$ are either parallel or antiparallel. (c) The scalar product is equal to the product of the magnitudes $(\vec{A}\cdot\vec{B} = AB)$ only if $\vec{A}$ and $\vec{B}$ are parallel. (d) The scalar product is equal to the negative of the product of the magnitudes $(\vec{A}\cdot\vec{B} = -AB)$ only if $\vec{A}$ and $\vec{B}$ are antiparallel. (e) The magnitude of the vector product is equal to the product of the magnitudes $[(\text{magnitude of } \vec{A} \times \vec{B}) = AB]$ only if $\vec{A}$ and $\vec{B}$ are perpendicular.

PROBLEMS

For instructor-assigned homework, go to **www.masteringphysics.com**

Discussion Questions

Q1.1. How many correct experiments do we need to disprove a theory? How many to prove a theory? Explain.

Q1.2. A guidebook describes the rate of climb of a mountain trail as 120 meters per kilometer. How can you express this as a number with no units?

Q1.3. Suppose you are asked to compute the tangent of 5.00 meters. Is this possible? Why or why not?

Q1.4. A highway contractor stated that in building a bridge deck he poured 250 yards of concrete. What do you think he meant?

Q1.5. What is your height in centimeters? What is your weight in newtons?

Q1.6. The U.S. National Institute of Science and Technology (NIST) maintains several accurate copies of the international standard kilogram. Even after careful cleaning, these national standard kilograms are gaining mass at an average rate of about 1 μg/y (1 y = 1 year) when compared every ten years or so to the standard international kilogram. Does this apparent change have any importance? Explain.

Q1.7. What physical phenomena (other than a pendulum or cesium clock) could you use to define a time standard?

Q1.8. Describe how you could measure the thickness of a sheet of paper with an ordinary ruler.

Q1.9. The quantity $\pi = 3.14159\ldots$ is a number with no dimensions, since it is a ratio of two lengths. Describe two or three other geometrical or physical quantities that are dimensionless.

Q1.10. What are the units of volume? Suppose another student tells you that a cylinder of radius r and height h has volume given by $\pi r^3 h$. Explain why this cannot be right.

Q1.11. Three archers each fire four arrows at a target. Joe's four arrows hit at points 10 cm above, 10 cm below, 10 cm to the left, and 10 cm to the right of the center of the target. All four of Moe's arrows hit within 1 cm of a point 20 cm from the center, and Flo's four arrows all hit within 1 cm of the center. The contest judge says that one of the archers is precise but not accurate, another archer is accurate but not precise, and the third archer is both accurate and precise. Which description goes with which archer? Explain your reasoning.

Q1.12. A circular racetrack has a radius of 500 m. What is the displacement of a bicyclist when she travels around the track from the north side to the south side? When she makes one complete circle around the track? Explain your reasoning.

Q1.13. Can you find two vectors with different lengths that have a vector sum of zero? What length restrictions are required for three vectors to have a vector sum of zero? Explain your reasoning.

Q1.14. One sometimes speaks of the "direction of time," evolving from past to future. Does this mean that time is a vector quantity? Explain your reasoning.

Q1.15. Air traffic controllers give instructions to airline pilots telling them in which direction they are to fly. These instructions are called "vectors." If these are the only instructions given, is the name "vector" used correctly? Why or why not?

Q1.16. Can you find a vector quantity that has a magnitude of zero but components that are different from zero? Explain. Can the magnitude of a vector be less than the magnitude of any of its components? Explain.

Q1.17. (a) Does it make sense to say that a vector is *negative?* Why? (b) Does it make sense to say that one vector is the negative of another? Why? Does your answer here contradict what you said in part (a)?

Q1.18. If $\vec{C}$ is the vector sum of $\vec{A}$ and $\vec{B}$, $\vec{C} = \vec{A} + \vec{B}$, what must be true if $C = A + B$? What must be true if $C = 0$?

Q1.19. If $\vec{A}$ and $\vec{B}$ are nonzero vectors, is it possible for $\vec{A}\cdot\vec{B}$ and $\vec{A} \times \vec{B}$ *both* to be zero? Explain.

Q1.20. What does $\vec{A}\cdot\vec{A}$, the scalar product of a vector with itself, give? What about $\vec{A} \times \vec{A}$, the vector product of a vector with itself?

Q1.21. Let $\vec{A}$ represent any nonzero vector. Why is $\vec{A}/A$ a unit vector and what is its direction? If θ is the angle that $\vec{A}$ makes with the $+x$-axis, explain why $(\vec{A}/A)\cdot\hat{\imath}$ is called the *direction cosine* for that axis.

Q1.22. Which of the following are legitimate mathematical operations: (a) $\vec{A}\cdot(\vec{B} - \vec{C})$; (b) $(\vec{A} - \vec{B}) \times \vec{C}$; (c) $\vec{A}\cdot(\vec{B} \times \vec{C})$; (d) $\vec{A} \times (\vec{B} \times \vec{C})$; (e) $\vec{A} \times (\vec{B}\cdot\vec{C})$? In each case, give the reason for your answer.

Q1.23. Consider the two repeated vector products $\vec{A} \times (\vec{B} \times \vec{C})$ and $(\vec{A} \times \vec{B}) \times \vec{C}$. Give an example that illustrates the general rule that these two vector products do not have the same magnitude or direction. Can you choose the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ such that these two vector products *are* equal? If so, give an example.

Q1.24. Show that, no matter what $\vec{A}$ and $\vec{B}$ are, $\vec{A}\cdot(\vec{A} \times \vec{B}) = 0$. (*Hint:* Do not look for an elaborate mathematical proof. Rather look at the definition of the direction of the cross product.)

Q1.25. (a) If $\vec{A}\cdot\vec{B} = 0$, does it necessary follow that $A = 0$ or $B = 0$? Explain. (b) If $\vec{A} \times \vec{B} = \mathbf{0}$, does it necessary follow that $A = 0$ or $B = 0$? Explain.

Q1.26. If $\vec{A} = \mathbf{0}$ for a vector in the xy plane, does it follow that $A_x = -A_y$? What *can* you say about A_x and A_y?

Exercises

Section 1.3 Standards and Units
Section 1.4 Unit Consistency and Conversions

1.1. Starting with the definition 1 in. = 2.54 cm, find the number of (a) kilometers in 1.00 mile and (b) feet in 1.00 km.

1.2. According to the label on a bottle of salad dressing, the volume of the contents is 0.473 liter (L). Using only the conversions 1 L = 1000 cm^3 and 1 in. = 2.54 cm, express this volume in cubic inches.

1.3. How many nanoseconds does it take light to travel 1.00 ft in vacuum? (This result is a useful quantity to remember.)

1.4. The density of lead is 11.3 g/cm^3. What is this value in kilograms per cubic meter?

1.5. The most powerful engine available for the classic 1963 Chevrolet Corvette Sting Ray developed 360 horsepower and had a displacement of 327 cubic inches. Express this displacement in liters (L) by using only the conversions 1 L = 1000 cm^3 and 1 in. = 2.54 cm.

1.6. A square field measuring 100.0 m by 100.0 m has an area of 1.00 hectare. An acre has an area of 43,600 ft^2. If a country lot has an area of 12.0 acres, what is the area in hectares?

1.7. How many years older will you be 1.00 billion seconds from now? (Assume a 365-day year.)

1.8. While driving in an exotic foreign land you see a speed limit sign on a highway that reads 180,000 furlongs per fortnight. How many miles per hour is this? (One furlong is $\frac{1}{8}$ mile, and a fortnight is 14 days. A furlong originally referred to the length of a plowed furrow.)

1.9. A certain fuel-efficient hybrid car gets gasoline mileage of 55.0 mpg (miles per gallon). (a) If you are driving this car in Europe and want to compare its mileage with that of other European cars, express this mileage in km/L (L = liter). Use the conversion factors in Appendix E. (b) If this car's gas tank holds 45 L, how many tanks of gas will you use to drive 1500 km?

1.10. The following conversions occur frequently in physics and are very useful. (a) Use 1 mi = 5280 ft and 1 h = 3600 s to convert 60 mph to units of ft/s. (b) The acceleration of a freely falling object is 32 ft/s^2. Use 1 ft = 30.48 cm to express this acceleration in units of m/s^2. (c) The density of water is 1.0 g/cm^3. Convert this density to units of kg/m^3.

1.11. Neptunium. In the fall of 2002, a group of scientists at Los Alamos National Laboratory determined that the critical mass of neptunium-237 is about 60 kg. The critical mass of a fissionable material is the minimum amount that must be brought together to start a chain reaction. This element has a density of 19.5 g/cm^3. What would be the radius of a sphere of this material that has a critical mass?

Section 1.5 Uncertainty and Significant Figures

1.12. A useful and easy-to-remember approximate value for the number of seconds in a year is $\pi \times 10^7$. Determine the percent error in this approximate value. (There are 365.24 days in one year.)

1.13. Figure 1.7 shows the result of unacceptable error in the stopping position of a train. (a) If a train travels 890 km from Berlin to Paris and then overshoots the end of the track by 10 m, what is the percent error in the total distance covered? (b) Is it correct to write the total distance covered by the train as 890,010 m? Explain.

1.14. With a wooden ruler you measure the length of a rectangular piece of sheet metal to be 12 mm. You use micrometer calipers to measure the width of the rectangle and obtain the value 5.98 mm. Give your answers to the following questions to the correct number of significant figures. (a) What is the area of the rectangle? (b) What is the ratio of the rectangle's width to its length? (c) What is the perimeter of the rectangle? (d) What is the difference between the length and width? (e) What is the ratio of the length to the width?
1.15. Estimate the percent error in measuring (a) a distance of about 75 cm with a meter stick; (b) a mass of about 12 g with a chemical balance; (c) a time interval of about 6 min with a stopwatch.
1.16. A rectangular piece of aluminum is 5.10 ± 0.01 cm long and 1.90 ± 0.01 cm wide. (a) Find the area of the rectangle and the uncertainty in the area. (b) Verify that the fractional uncertainty in the area is equal to the sum of the fractional uncertainties in the length and in the width. (This is a general result; see Challenge Problem 1.98.)
1.17. As you eat your way through a bag of chocolate chip cookies, you observe that each cookie is a circular disk with a diameter of 8.50 ± 0.02 cm and a thickness of 0.050 ± 0.005 cm. (a) Find the average volume of a cookie and the uncertainty in the volume. (b) Find the ratio of the diameter to the thickness and the uncertainty in this ratio.

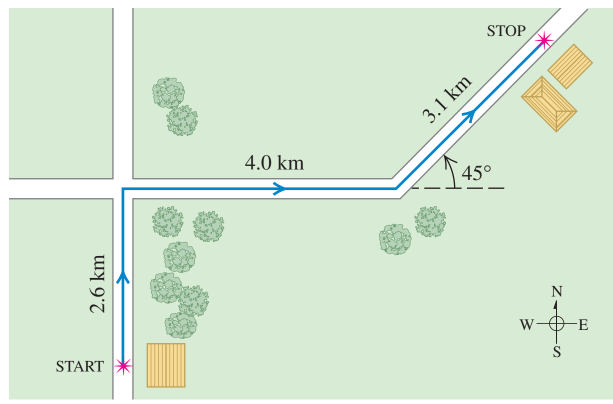
Section 1.6 Estimates and Orders of Magnitude

1.18. How many gallons of gasoline are used in the United States in one day? Assume two cars for every three people, that each car is driven an average of 10,000 mi per year, and that the average car gets 20 miles per gallon.
1.19. A rather ordinary middle-aged man is in the hospital for a routine check-up. The nurse writes the quantity 200 on his medical chart but forgets to include the units. Which of the following quantities could the 200 plausibly represent? (a) his mass in kilograms; (b) his height in meters; (c) his height in centimeters; (d) his height in millimeters; (e) his age in months.
1.20. How many kernels of corn does it take to fill a 2-L soft drink bottle?
1.21. How many words are there in this book?
1.22. Four astronauts are in a spherical space station. (a) If, as is typical, each of them breathes about 500 cm^3 of air with each breath, approximately what volume of air (in cubic meters) do these astronauts breathe in a year? (b) What would the diameter (in meters) of the space station have to be to contain all this air?
1.23. How many times does a typical person blink her eyes in a lifetime?
1.24. How many times does a human heart beat during a lifetime? How many gallons of blood does it pump? (Estimate that the heart pumps 50 cm^3 of blood with each beat.)
1.25. In Wagner's opera *Das Rheingold,* the goddess Freia is ransomed for a pile of gold just tall enough and wide enough to hide her from sight. Estimate the monetary value of this pile. The density of gold is 19.3 g/cm^3, and its value is about \$10 per gram (although this varies).
1.26. You are using water to dilute small amounts of chemicals in the laboratory, drop by drop. How many drops of water are in a 1.0 L bottle? (*Hint:* Start by estimating the diameter of a drop of water.)
1.27. How many pizzas are consumed each academic year by students at your school?
1.28. How many dollar bills would you have to stack to reach the moon? Would that be cheaper than building and launching a spacecraft? (*Hint:* Start by folding a dollar bill to see how many thicknesses make 1.0 mm.)
1.29. How much would it cost to paper the entire United States (including Alaska and Hawaii) with dollar bills? What would be the cost to each person in the United States?

Section 1.7 Vectors and Vector Addition

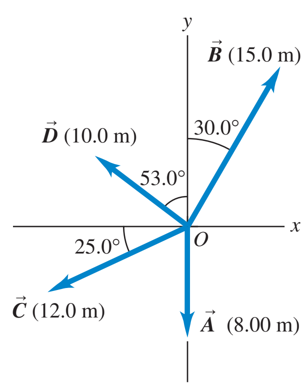
1.30. Hearing rattles from a snake, you make two rapid displacements of magnitude 1.8 m and 2.4 m. In sketches (roughly to scale), show how your two displacements might add up to give a resultant of magnitude (a) 4.2 m; (b) 0.6 m; (c) 3.0 m.
1.31. A postal employee drives a delivery truck along the route shown in Fig. 1.33. Determine the magnitude and direction of the resultant displacement by drawing a scale diagram. (See also Exercise 1.38 for a different approach to this same problem.)

Figure 1.33 Exercises 1.31 and 1.38.



1.32. For the vectors $\vec{A}$ and $\vec{B}$ in Fig. 1.34, use a scale drawing to find the magnitude and direction of (a) the vector sum $\vec{A} + \vec{B}$ and (b) the vector difference $\vec{A} - \vec{B}$. Use your answers to find the magnitude and direction of (c) $-\vec{A} - \vec{B}$ and (d) $\vec{B} - \vec{A}$. (See also Exercise 1.39 for a different approach to this problem.)

Figure 1.34 Exercises 1.32, 1.35, 1.39, 1.47, 1.53, and 1.57, and Problem 1.72.



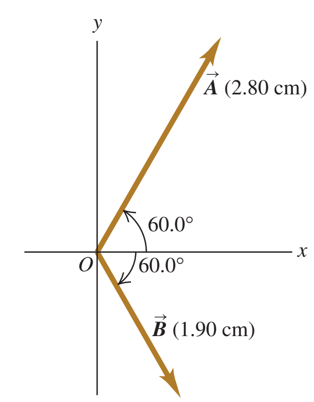
1.33. A spelunker is surveying a cave. She follows a passage 180 m straight west, then 210 m in a direction 45° east of south, and then 280 m at 30° east of north. After a fourth unmeasured displacement, she finds herself back where she started. Use a scale drawing to determine the magnitude and direction of the fourth displacement. (See also Problem 1.73 for a different approach to this problem.)

Section 1.8 Components of Vectors

1.34. Use a scale drawing to find the *x*- and *y*-components of the following vectors. For each vector the numbers given are the magnitude of the vector and the angle, measured in the sense from the $+x$-axis toward the $+y$-axis, that it makes with the $+x$-axis: (a) magnitude 9.30 m, angle 60.0°; (b) magnitude 22.0 km, angle 135°; (c) magnitude 6.35 cm, angle 307°.

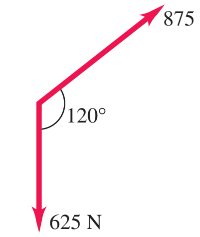
1.35. Compute the *x*- and *y*-components of the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ in Fig. 1.34.
1.36. Let the angle θ be the angle that the vector $\vec{A}$ makes with the $+x$-axis, measured counterclockwise from that axis. Find the angle θ for a vector that has the following components: (a) $A_x = 2.00$ m, $A_y = -1.00$ m; (b) $A_x = 2.00$ m, $A_y = 1.00$ m; (c) $A_x = -2.00$ m, $A_y = 1.00$ m; (d) $A_x = -2.00$ m, $A_y = -1.00$ m.
1.37. A rocket fires two engines simultaneously. One produces a thrust of 725 N directly forward, while the other gives a 513-N thrust at 32.4° above the forward direction. Find the magnitude and direction (relative to the forward direction) of the resultant force that these engines exert on the rocket.
1.38. A postal employee drives a delivery truck over the route shown in Fig. 1.33. Use the method of components to determine the magnitude and direction of her resultant displacement. In a vector-addition diagram (roughly to scale), show that the resultant displacement found from your diagram is in qualitative agreement with the result you obtained using the method of components.
1.39. For the vectors $\vec{A}$ and $\vec{B}$ in Fig. 1.34, use the method of components to find the magnitude and direction of (a) the vector sum $\vec{A} + \vec{B}$; (b) the vector sum $\vec{B} + \vec{A}$; (c) the vector difference $\vec{A} - \vec{B}$; (d) the vector difference $\vec{B} - \vec{A}$.
1.40. Find the magnitude and direction of the vector represented by the following pairs of components: (a) $A_x = -8.60$ cm, $A_y = 5.20$ cm; (b) $A_x = -9.70$ m, $A_y = -2.45$ m; (c) $A_x = 7.75$ km, $A_y = -2.70$ km.
1.41. A disoriented physics professor drives 3.25 km north, then 4.75 km west, and then 1.50 km south. Find the magnitude and direction of the resultant displacement, using the method of components. In a vector addition diagram (roughly to scale), show that the resultant displacement found from your diagram is in qualitative agreement with the result you obtained using the method of components.
1.42. Vector $\vec{A}$ has components $A_x = 1.30$ cm, $A_y = 2.25$ cm; vector $\vec{B}$ has components $B_x = 4.10$ cm, $B_y = -3.75$ cm. Find (a) the components of the vector sum $\vec{A} + \vec{B}$; (b) the magnitude and direction of $\vec{A} + \vec{B}$; (c) the components of the vector difference $\vec{B} - \vec{A}$; (d) the magnitude and direction of $\vec{B} - \vec{A}$.
1.43. Vector $\vec{A}$ is 2.80 cm long and is 60.0° above the *x*-axis in the first quadrant. Vector $\vec{B}$ is 1.90 cm long and is 60.0° below the *x*-axis in the fourth quadrant (Fig. 1.35). Use components to find the magnitude and direction of (a) $\vec{A} + \vec{B}$; (b) $\vec{A} - \vec{B}$; (c) $\vec{B} - \vec{A}$. In each case, sketch the vector addition or subtraction and show that your numerical answers are in qualitative agreement with your sketch.

Figure 1.35 Exercises 1.43 and 1.59.



1.44. A river flows from south to north at 5.0 km/h. On this river, a boat is heading east to west perpendicular to the current at 7.0 km/h. As viewed by an eagle hovering at rest over the shore, how fast and in what direction is this boat traveling?
1.45. Use vector components to find the magnitude and direction of the vector needed to balance the two vectors shown in Figure 1.36. Let the 625-N vector be along the $-y$-axis and let the $+x$-axis be perpendicular to it toward the right.

Figure 1.36 Exercise 1.45.

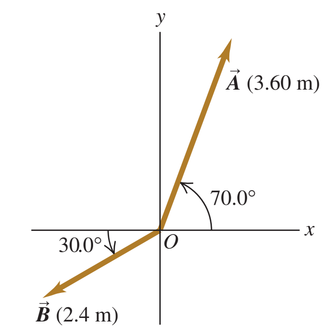


1.46. Two ropes in a vertical plane exert equal magnitude forces on a hanging weight but pull with an angle of 86.0° between them. What pull does each one exert if their resultant pull is 372 N directly upward?

Section 1.9 Unit Vectors

1.47. Write each vector in Fig. 1.34 in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.
1.48. In each case, find the *x*- and *y*-components of vector $\vec{A}$: (a) $\vec{A} = 5.0\hat{\imath} - 6.3\hat{\jmath}$; (b) $\vec{A} = 11.2\hat{\jmath} - 9.91\hat{\imath}$; (c) $\vec{A} = -15.0\hat{\imath} + 22.4\hat{\jmath}$; (d) $\vec{A} = 5.0\vec{B}$, where $\vec{B} = 4\hat{\imath} - 6\hat{\jmath}$.
1.49. (a) Write each vector in Fig. 1.37 in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$. (b) Use unit vectors to express the vector $\vec{C}$, where $\vec{C} = 3.00\vec{A} - 4.00\vec{B}$. (c) Find the magnitude and direction of $\vec{C}$.

Figure 1.37 Exercise 1.49 and Problem 1.86.



1.50. Given two vectors $\vec{A} = 4.00\hat{\imath} + 3.00\hat{\jmath}$ and $\vec{B} = 5.00\hat{\imath} - 2.00\hat{\jmath}$, (a) find the magnitude of each vector; (b) write an expression for the vector difference $\vec{A} - \vec{B}$ using unit vectors; (c) find the magnitude and direction of the vector difference $\vec{A} - \vec{B}$. (d) In a vector diagram show $\vec{A}$, $\vec{B}$, and $\vec{A} - \vec{B}$, and also show that your diagram agrees qualitatively with your answer in part (c).
1.51. (a) Is the vector $(\hat{\imath} + \hat{\jmath} + \hat{k})$ a unit vector? Justify your answer. (b) Can a unit vector have any components with magnitude greater than unity? Can it have any negative components? In each case justify your answer. (c) If $\vec{A} = a(3.0\hat{\imath} + 4.0\hat{\jmath})$, where a is a constant, determine the value of a that makes $\vec{A}$ a unit vector.

Section 1.10 Products of Vectors

1.52. (a) Use vector components to prove that two vectors commute for both addition and the scalar product. (b) Prove that two vectors *anticommute* for the vector product; that is, prove that $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$.
1.53. For the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Fig. 1.34, find the scalar products (a) $\vec{A} \cdot \vec{B}$; (b) $\vec{B} \cdot \vec{C}$; (c) $\vec{A} \cdot \vec{C}$.
1.54. (a) Find the scalar product of the two vectors $\vec{A}$ and $\vec{B}$ given in Exercise 1.50. (b) Find the angle between these two vectors.
1.55. Find the angle between each of the following pairs of vectors:
(a) $\vec{A} = -2.00\hat{\imath} + 6.00\hat{\jmath}$ and $\vec{B} = 2.00\hat{\imath} - 3.00\hat{\jmath}$
(b) $\vec{A} = 3.00\hat{\imath} + 5.00\hat{\jmath}$ and $\vec{B} = 10.00\hat{\imath} + 6.00\hat{\jmath}$
(c) $\vec{A} = -4.00\hat{\imath} + 2.00\hat{\jmath}$ and $\vec{B} = 7.00\hat{\imath} + 14.00\hat{\jmath}$
1.56. By making simple sketches of the appropriate vector products, show that (a) $\vec{A} \cdot \vec{B}$ can be interpreted as the product of the magnitude of $\vec{A}$ times the component of $\vec{B}$ along $\vec{A}$, or the magnitude of $\vec{B}$ times the component of $\vec{A}$ along $\vec{B}$; (b) $|\vec{A} \times \vec{B}|$ can be interpreted as the product of the magnitude of $\vec{A}$ times the component of $\vec{B}$ perpendicular to $\vec{A}$, or the magnitude of $\vec{B}$ times the component of $\vec{A}$ perpendicular to $\vec{B}$.

1.57. For the vectors $\vec{A}$ and $\hat{D}$ in Fig. 1.34, (a) find the magnitude and direction of the vector product $\vec{A} \times \hat{D}$; (b) find the magnitude and direction of $\hat{D} \times \vec{A}$.

1.58. Find the vector product $\vec{A} \times \vec{B}$ (expressed in unit vectors) of the two vectors given in Exercise 1.50. What is the magnitude of the vector product?

1.59. For the two vectors in Fig. 1.35, (a) find the magnitude and direction of the vector product $\vec{A} \times \vec{B}$; (b) find the magnitude and direction of $\vec{B} \times \vec{A}$.

Problems

1.60. An acre, a unit of land measurement still in wide use, has a length of one furlong $\left(\frac{1}{8}\text{ mi}\right)$ and a width one-tenth of its length. (a) How many acres are in a square mile? (b) How many square feet are in an acre? See Appendix E. (c) An acre-foot is the volume of water that would cover 1 acre of flat land to a depth of 1 foot. How many gallons are in 1 acre-foot?

1.61. An Earthlike Planet. In January 2006, astronomers reported the discovery of a planet comparable in size to the earth orbiting another star and having a mass of about 5.5 times the earth's mass. It is believed to consist of a mixture of rock and ice, similar to Neptune. If this planet has the same density as Neptune $\left(1.76\text{ g/cm}^3\right)$, what is its radius expressed (a) in kilometers and (b) as a multiple of earth's radius? Consult Appendix F for astronomical data.

1.62. The Hydrogen Maser. You can use the radio waves generated by a hydrogen maser as a standard of frequency. The frequency of these waves is 1,420,405,751.786 hertz. (A hertz is another name for one cycle per second.) A clock controlled by a hydrogen maser is off by only 1 s in 100,000 years. For the following questions, use only three significant figures. (The large number of significant figures given for the frequency simply illustrates the remarkable accuracy to which it has been measured.) (a) What is the time for one cycle of the radio wave? (b) How many cycles occur in 1 h? (c) How many cycles would have occurred during the age of the earth, which is estimated to be 4.6×10^9 years? (d) By how many seconds would a hydrogen maser clock be off after a time interval equal to the age of the earth?

1.63. Estimate the number of atoms in your body. (*Hint:* Based on what you know about biology and chemistry, what are the most common types of atom in your body? What is the mass of each type of atom? Appendix D gives the atomic masses for different elements, measured in atomic mass units; you can find the value of an atomic mass unit, or 1 u, in Appendix F.)

1.64. Biological tissues are typically made up of 98% water. Given that the density of water is $1.0 \times 10^3\text{ kg/m}^3$, estimate the mass of (a) the heart of an adult human; (b) a cell with a diameter of $0.5\ \mu\text{m}$; (c) a honey bee.

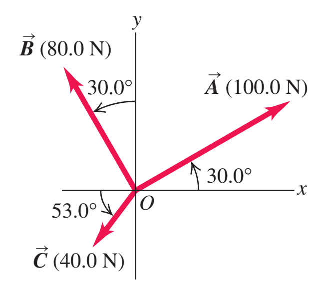
1.65. Iron has a property such that a 1.00-m^3 volume has a mass of 7.86×10^3 kg (density equals $7.86 \times 10^3\text{ kg/m}^3$). You want to manufacture iron into cubes and spheres. Find (a) the length of the side of a cube of iron that has a mass of 200.0 g and (b) the radius of a solid sphere of iron that has a mass of 200.0 g.

1.66. Stars in the Universe Astronomers frequently say that there are more stars in the universe than there are grains of sand on all the beaches on the earth. (a) Given that a typical grain of sand is about 0.2 mm in diameter, estimate the number of grains of sand on all the earth's beaches, and hence the approximate number of stars in the universe. It would be helpful to consult an atlas and do some measuring. (b) Given that a typical galaxy contains about 100 billion stars and there are more than 100 billion galaxies in the known universe, estimate the number of stars in the universe and compare this number with your result from part (a).

1.67. Physicists, mathematicians, and others often deal with large numbers. The number 10^{100} has been given the whimsical name *googol* by mathematicians. Let us compare some large numbers in physics with the googol. (*Note:* This problem requires numerical values that you can find in the appendices of the book, with which you should become familiar.) (a) Approximately how many atoms make up our planet? For simplicity, assume the average atomic mass of the atoms is 14 g/mol. Avogadro's number gives the number of atoms in a mole. (b) Approximately how many neutrons are in a neutron star? Neutron stars are composed almost entirely of neutrons and have approximately twice the mass of the sun. (c) In the leading theory of the origin of the universe, the entire universe that we can now observe occupied, at a very early time, a sphere whose radius was approximately equal to the present distance of the earth to the sun. At that time the universe had a density (mass divided by volume) of 10^{15} g/cm^3. Assuming that one-third of the particles were protons, one-third of the particles were neutrons, and the remaining one-third were electrons, how many particles then made up the universe?

1.68. Three horizontal ropes pull on a large stone stuck in the ground, producing the vector forces $\vec{A}$, $\vec{B}$, and $\vec{C}$ shown in Fig. 1.38. Find the magnitude and direction of a fourth force on the stone that will make the vector sum of the four forces zero.

Figure 1.38 Problem 1.68.



1.69. Two workers pull horizontally on a heavy box, but one pulls twice as hard as the other. The larger pull is directed at 25.0° west of north, and the resultant of these two pulls is 350.0 N directly northward. Use vector components to find the magnitude of each of these pulls and the direction of the smaller pull.

1.70. Emergency Landing. A plane leaves the airport in Galisteo and flies 170 km at 68° east of north and then changes direction to fly 230 km at 48° south of east, after which it makes an immediate emergency landing in a pasture. When the airport sends out a rescue crew, in which direction and how far should this crew fly to go directly to this plane?

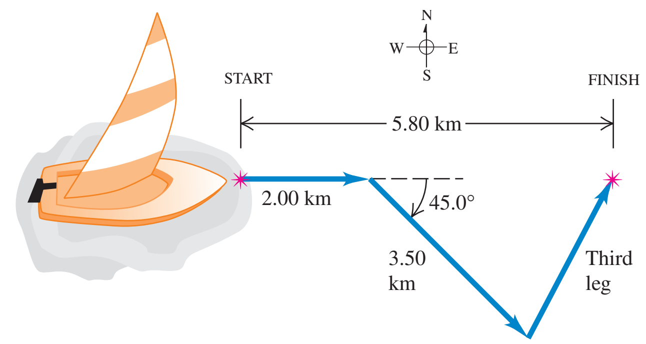
1.71. You are to program a robotic arm on an assembly line to move in the *xy*-plane. Its first displacement is $\vec{A}$; its second displacement is $\vec{B}$, of magnitude 6.40 cm and direction 63.0° measured in the sense from the $+x$-axis toward the $-y$-axis. The resultant $\vec{C} = \vec{A} + \vec{B}$ of the two displacements should also have a magnitude of 6.40 cm, but a direction 22.0° measured in the sense from the $+x$-axis toward the $+y$-axis. (a) Draw the vector addition diagram for these vectors, roughly to scale. (b) Find the components of $\vec{A}$. (c) Find the magnitude and direction of $\vec{A}$.

1.72. (a) Find the magnitude and direction of the vector $\vec{R}$ that is the sum of the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Fig. 1.34. In a diagram, show how $\vec{R}$ is formed from these three vectors. (b) Find the magnitude and direction of the vector $\vec{S} = \vec{C} - \vec{A} - \vec{B}$. In a diagram, show how $\vec{S}$ is formed from these three vectors.

1.73. As noted in Exercise 1.33, a spelunker is surveying a cave. She follows a passage 180 m straight west, then 210 m in a direction 45° east of south, and then 280 m at 30° east of north. After a fourth unmeasured displacement she finds herself back where she started. Use the method of components to determine the magnitude and direction of the fourth displacement. Draw the vector addition diagram and show that it is in qualitative agreement with your numerical solution.

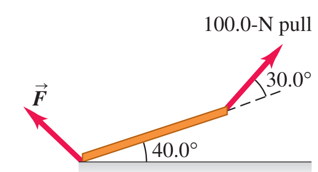
1.74. A sailor in a small sailboat encounters shifting winds. She sails 2.00 km east, then 3.50 km southeast, and then an additional distance in an unknown direction. Her final position is 5.80 km directly east of the starting point (Fig. 1.39). Find the magnitude and direction of the third leg of the journey. Draw the vector addition diagram and show that it is in qualitative agreement with your numerical solution.

Figure 1.39 Problem 1.74.



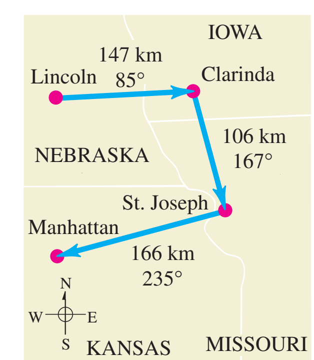
1.75. Equilibrium. We say an object is in *equilibrium* if all the forces on it balance (add up to zero). Figure 1.40 shows a beam weighing 124 N that is supported in equilibrium by a 100.0-N pull and a force $\vec{F}$ at the floor. The third force on the beam is the 124-N weight that acts vertically downward. (a) Use vector components to find the magnitude and direction of $\vec{F}$. (b) Check the reasonableness of your answer in part (a) by doing a graphical solution approximately to scale.

Figure 1.40 Problem 1.75.



1.76. On a training flight, a student pilot flies from Lincoln, Nebraska to Clarinda, Iowa, then to St. Joseph, Missouri, and then to Manhattan, Kansas (Fig. 1.41). The directions are shown relative to north: 0° is north, 90° is east, 180° is south, and 270° is west. Use the method of components to find (a) the distance she has to fly from Manhattan to get back to Lincoln, and (b) the direction (relative to north) she must fly to get there. Illustrate your solutions with a vector diagram.

Figure 1.41 Problem 1.76.



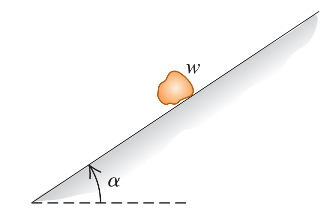
1.77. A graphic artist is creating a new logo for her company's website. In the graphics program she is using, each pixel in an image file has coordinates (x, y), where the origin $(0, 0)$ is at the upper left corner of the image, the $+x$-axis points to the right, and the $+y$-axis points down. Distances are measured in pixels. (a) The artist draws a line from the pixel location $(10, 20)$ to the location $(210, 200)$. She wishes to draw a second line that starts at $(10, 20)$, is 250 pixels long, and is at angle of 30° measured clockwise from the first line. At which pixel location should this second line end? Give your answer to the nearest pixel. (b) The artist now draws an arrow that connects the lower right end of the first line to the lower right end of the second line. Find the length and direction of this arrow. Draw a diagram showing all three lines.

1.78. Getting Back. An explorer in the dense jungles of equatorial Africa leaves his hut. He takes 40 steps northeast, then 80 steps 60° north of west, then 50 steps due south. Assume his steps all have equal length. (a) Sketch, roughly to scale, the three vectors and their resultant. (b) Save the explorer from becoming hopelessly lost in the jungle by giving him the displacement, calculated using the method of components, that will return him to his hut.

1.79. A ship leaves the island of Guam and sails 285 km at 40.0° north of west. In which direction must it now head and how far must it sail so that its resultant displacement will be 115 km directly east of Guam?

1.80. A boulder of weight w rests on a hillside that rises at a constant angle α above the horizontal, as shown in Fig. 1.42. Its weight is a force on the boulder that has direction vertically downward. (a) In terms of α and w, what is the component of the weight of the boulder in the direction parallel to the surface of the hill? (b) What is the component of the weight in the direction perpendicular to the surface of the hill? (c) An air conditioner unit is fastened to a roof that slopes upward at an angle of 35.0°. In order that the unit not slide down the roof, the component of the unit's weight parallel to the roof cannot exceed 550 N. What is the maximum allowed weight of the unit?

Figure 1.42 Problem 1.80.



1.81. Bones and Muscles. A patient in therapy has a forearm that weighs 20.5 N and that lifts a 112.0-N weight. These two forces have direction vertically downward. The only other significant forces on his forearm come from the biceps muscle (which acts perpendicularly to the forearm) and the force at the elbow. If the biceps produces a pull of 232 N when the forearm is raised 43° above the horizontal, find the magnitude and direction of the force that the elbow exerts on the forearm. (The sum of the elbow force and the biceps force must balance the weight of the arm and the weight it is carrying, so their vector sum must be 132.5 N, upward.)

1.82. You are hungry and decide to go to your favorite neighborhood fast-food restaurant. You leave your apartment and take the elevator 10 flights down (each flight is 3.0 m) and then go 15 m south to the apartment exit. You then proceed 0.2 km east, turn north, and go 0.1 km to the entrance of the restaurant. (a) Determine the displacement from your apartment to the restaurant. Use unit vector notation for your answer, being sure to make clear your choice of coordinates. (b) How far did you travel along the path you took from your apartment to the restaurant, and what is the magnitude of the displacement you calculated in part (a)?

1.83. While following a treasure map, you start at an old oak tree. You first walk 825 m directly south, then turn and walk 1.25 km at 30.0° west of north, and finally walk 1.00 km at 40.0° north of east, where you find the treasure: a biography of Isaac Newton! (a) To return to the old oak tree, in what direction should you head and how far will you walk? Use components to solve this problem.

(b) To see whether your calculation in part (a) is reasonable, check it with a graphical solution drawn roughly to scale.

1.84. You are camping with two friends, Joe and Karl. Since all three of you like your privacy, you don't pitch your tents close together. Joe's tent is 21.0 m from yours, in the direction 23.0° south of east. Karl's tent is 32.0 m from yours, in the direction 37.0° north of east. What is the distance between Karl's tent and Joe's tent?

1.85. Vectors $\vec{A}$ and $\vec{B}$ are drawn from a common point. Vector $\vec{A}$ has magnitude A and angle θ_A measured in the sense from the $+x$-axis to the $+y$-axis. The corresponding quantities for vector $\vec{B}$ are B and θ_B. Then $\vec{A} = A\cos\theta_A\hat{\imath} + A\sin\theta_A\hat{\jmath}$, $\vec{B} = B\cos\theta_B\hat{\imath} + B\sin\theta_B\hat{\jmath}$, and $\phi = |\theta_B - \theta_A|$ is the angle between $\vec{A}$ and $\vec{B}$. (a) Derive Eq. (1.18) from Eq. (1.21). (b) Derive Eq. (1.22) from Eqs. (1.27).

1.86. For the two vectors $\vec{A}$ and $\vec{B}$ in Fig. 1.37, (a) find the scalar product $\vec{A}\cdot\vec{B}$, and (b) find the magnitude and direction of the vector product $\vec{A}\times\vec{B}$.

1.87. Figure 1.11c shows a parallelogram based on the two vectors $\vec{A}$ and $\vec{B}$. (a) Show that the magnitude of the cross product of these two vectors is equal to the area of the parallelogram. (*Hint:* Area = base × height.) (b) What is the angle between the cross product and the plane of the parallelogram?

1.88. The vector $\vec{A}$ is 3.50 cm long and is directed into this page. Vector $\vec{B}$ points from the lower right corner of this page to the upper left corner of this page. Define an appropriate right-handed coordinate system and find the three components of the vector product $\vec{A}\times\vec{B}$, measured in cm^2. In a diagram, show your coordinate system and the vectors $\vec{A}$, $\vec{B}$, and $\vec{A}\times\vec{B}$.

1.89. Given two vectors $\vec{A} = -2.00\hat{\imath} + 3.00\hat{\jmath} + 4.00\hat{k}$ and $\vec{B} = 3.00\hat{\imath} + 1.00\hat{\jmath} - 3.00\hat{k}$, do the following. (a) Find the magnitude of each vector. (b) Write an expression for the vector difference $\vec{A} - \vec{B}$, using unit vectors. (c) Find the magnitude of the vector difference $\vec{A} - \vec{B}$. Is this the same as the magnitude of $\vec{B} - \vec{A}$? Explain.

1.90. Bond Angle in Methane. In the methane molecule, CH_4, each hydrogen atom is at a corner of a regular tetrahedron with the carbon atom at the center. In coordinates where one of the C—H bonds is in the direction of $\hat{\imath} + \hat{\jmath} + \hat{k}$, an adjacent C—H bond is in the $\hat{\imath} - \hat{\jmath} - \hat{k}$ direction. Calculate the angle between these two bonds.

1.91. The two vectors $\vec{A}$ and $\vec{B}$ are drawn from a common point, and $\vec{C} = \vec{A} + \vec{B}$. (a) Show that if $C^2 = A^2 + B^2$, the angle between the vectors $\vec{A}$ and $\vec{B}$ is 90°. (b) Show that if $C^2 < A^2 + B^2$, the angle between the vectors $\vec{A}$ and $\vec{B}$ is greater than 90°. (c) Show that if $C^2 > A^2 + B^2$, the angle between the vectors $\vec{A}$ and $\vec{B}$ is between 0° and 90°.

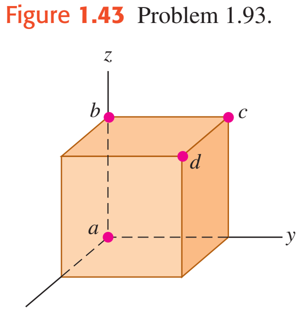
1.92. When two vectors $\vec{A}$ and $\vec{B}$ are drawn from a common point, the angle between them is ϕ. (a) Using vector techniques, show that the magnitude of their vector sum is given by

$$\sqrt{A^2 + B^2 + 2AB\cos\phi}$$

(b) If $\vec{A}$ and $\vec{B}$ have the same magnitude, for which value of ϕ will their vector sum have the same magnitude as $\vec{A}$ or $\vec{B}$?

1.93. A cube is placed so that one corner is at the origin and three edges are along the x-, y-, and z-axes of a coordinate system (Fig. 1.43). Use vectors to compute (a) the angle between the edge along the z-axis (line ab) and the diagonal from the origin to the opposite corner (line ad), and (b) the angle between line ac (the diagonal of a face) and line ad.

Figure 1.43 Problem 1.93.



1.94. Obtain a *unit vector* perpendicular to the two vectors given in Problem 1.89.

1.95. You are given vectors $\vec{A} = 5.0\hat{\imath} - 6.5\hat{\jmath}$ and $\vec{B} = -3.5\hat{\imath} + 7.0\hat{\jmath}$. A third vector $\vec{C}$ lies in the xy-plane. Vector $\vec{C}$ is perpendicular to vector $\vec{A}$, and the scalar product of $\vec{C}$ with $\vec{B}$ is 15.0. From this information, find the components of vector $\vec{C}$.

1.96. Two vectors $\vec{A}$ and $\vec{B}$ have magnitude $A = 3.00$ and $B = 3.00$. Their vector product is $\vec{A}\times\vec{B} = -5.00\hat{k} + 2.00\hat{\imath}$. What is the angle between $\vec{A}$ and $\vec{B}$?

1.97. Later in our study of physics we will encounter quantities represented by $(\vec{A}\times\vec{B})\cdot\vec{C}$. (a) Prove that for any three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$, $\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C}$. (b) Calculate $(\vec{A}\times\vec{B})\cdot\vec{C}$ for the three vectors $\vec{A}$ with magnitude $A = 5.00$ and angle $\theta_A = 26.0°$ measured in the sense from the $+x$-axis toward the $+y$-axis, $\vec{B}$ with $B = 4.00$ and $\theta_B = 63.0°$, and $\vec{C}$ with magnitude 6.00 and in the $+z$-direction. Vectors $\vec{A}$ and $\vec{B}$ are in the xy-plane.

Challenge Problems

1.98. The length of a rectangle is given as $L \pm l$ and its width as $W \pm w$. (a) Show that the uncertainty in its area A is $a = Lw + lW$. Assume that the uncertainties l and w are small, so that the product lw is very small and you can ignore it. (b) Show that the fractional uncertainty in the area is equal to the sum of the fractional uncertainty in length and the fractional uncertainty in width. (c) A rectangular solid has dimensions $L \pm l$, $W \pm w$, and $H \pm h$. Find the fractional uncertainty in the volume, and show that it equals the sum of the fractional uncertainties in the length, width, and height.

1.99. Completed Pass. At Enormous State University (ESU), the football team records its plays using vector displacements, with the origin taken to be the position of the ball before the play starts. In a certain pass play, the receiver starts at $+1.0\hat{\imath} - 5.0\hat{\jmath}$, where the units are yards, $\hat{\imath}$ is to the right, and $\hat{\jmath}$ is downfield. Subsequent displacements of the receiver are $+9.0\hat{\imath}$ (in motion before the snap), $+11.0\hat{\jmath}$ (breaks downfield), $-6.0\hat{\imath} + 4.0\hat{\jmath}$ (zigs), and $+12.0\hat{\imath} + 18.0\hat{\jmath}$ (zags). Meanwhile, the quarterback has dropped straight back to a position $-7.0\hat{\jmath}$. How far and in which direction must the quarterback throw the ball? (Like the coach, you will be well advised to diagram the situation before solving it numerically.)

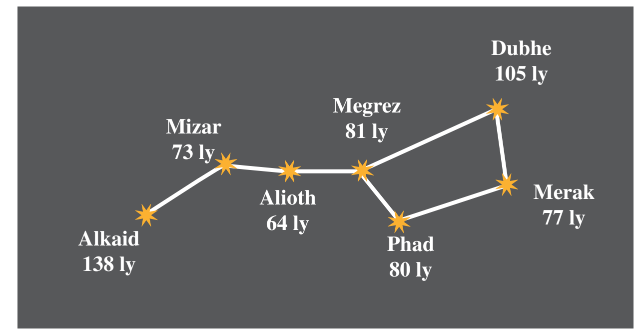
1.100. Navigating in the Solar System. The *Mars Polar Lander* spacecraft was launched on January 3, 1999. On December 3, 1999, the day that *Mars Polar Lander* touched down on the Martian surface, the positions of the earth and Mars were given by these coordinates:

	x	y	z
Earth	0.3182 AU	0.9329 AU	0.0000 AU
Mars	1.3087 AU	−0.4423 AU	−0.0414 AU

In these coordinates, the sun is at the origin and the plane of the earth's orbit is the xy-plane. The earth passes through the $+x$-axis once a year on the autumnal equinox, the first day of autumn in the northern hemisphere (on or about September 22). One AU, or *astronomical unit,* is equal to 1.496×10^8 km, the average distance from the earth to the sun. (a) In a diagram, show the positions of the sun, the earth, and Mars on December 3, 1999. (b) Find the following distances in AU on December 3, 1999: (i) from the sun to the earth; (ii) from the sun to Mars; (iii) from the earth to Mars. (c) As seen from the earth, what was the angle between the direction to the sun and the direction to Mars on December 3, 1999? (d) Explain whether Mars was visible from your location at midnight on December 3, 1999. (When it is midnight at your location, the sun is on the opposite side of the earth from you.)

1.101. Navigating in the Big Dipper. All the stars of the Big Dipper (part of the constellation Ursa Major) may appear to be the same distance from the earth, but in fact they are very far from each other. Figure 1.44 shows the distances from the earth to each of these stars. The distances are given in light years (ly), the distance that light travels in one year. One light year equals 9.461×10^{15} m. (a) Alkaid and Merak are 25.6° apart in the earth's sky. In a diagram, show the relative positions of Alkaid, Merak, and our sun. Find the distance in light years from Alkaid to Merak. (b) To an inhabitant of a planet orbiting Merak, how many degrees apart in the sky would Alkaid and our sun be?

Figure 1.44 Challenge Problem 1.101.



1.102. The vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, called the *position vector,* points from the origin $(0, 0, 0)$ to an arbitrary point in space with coordinates (x, y, z). Use what you know about vectors to prove the following: All points (x, y, z) that satisfy the equation $Ax + By + Cz = 0$, where A, B, and C are constants, lie in a plane that passes through the origin and that is perpendicular to the vector $A\hat{\imath} + B\hat{\jmath} + C\hat{k}$. Sketch this vector and the plane.