ELECTROMAGNETIC INDUCTION

29

? When a credit card is "swiped" through a card reader, the information coded in a magnetic pattern on the back of the card is transmitted to the cardholder's bank. Why is it necessary to swipe the card rather than holding it motionless in the card reader's slot?

LEARNING GOALS

By studying this chapter, you will learn:

- The experimental evidence that a changing magnetic field induces an emf.
- How Faraday's law relates the induced emf in a loop to the change in magnetic flux through the loop.
- How to determine the direction of an induced emf.
- How to calculate the emf induced in a conductor moving through a magnetic field.
- How a changing magnetic flux generates an electric field that is very different from that produced by an arrangement of charges.
- The four fundamental equations that completely describe both electricity and magnetism.

Almost every modern device or machine, from a computer to a washing machine to a power drill, has electric circuits at its heart. We learned in Chapter 25 that an electromotive force (emf) is required for a current to flow in a circuit; in Chapter 25 and 26 we almost always took the source of emf to be a battery. But for the vast majority of electric devices that are used in industry and in the home (including any device that you plug into a wall socket), the source of emf is *not* a battery but an electrical generating station. Such a station produces electric energy by converting other forms of energy: gravitational potential energy at a hydroelectric plant, chemical energy in a coal- or oil-fired plant, nuclear energy at a nuclear plant. But how is this energy conversion done? In other words, what is the physics behind the production of almost all of our electric energy needs?

The answer is a phenomenon known as *electromagnetic induction:* If the magnetic flux through a circuit changes, an emf and a current are induced in the circuit. In a power-generating station, magnets move relative to coils of wire to produce a changing magnetic flux in the coils and hence an emf. Other key components of electric power systems, such as transformers, also depend on magnetically induced emfs. Indeed, thanks to its key role in electric power generation, electromagnetic induction is one of the foundations of our technological society.

The central principle of electromagnetic induction, and the keystone of this chapter, is *Faraday's law.* This law relates induced emf to changing magnetic flux in any loop, including a closed circuit. We also discuss Lenz's law, which helps us to predict the directions of induced emfs and currents. This chapter provides the principles we need to understand electrical energy-conversion devices such as motors, generators, and transformers.

Electromagnetic induction tells us that a time-varying magnetic field can act as a source of electric field. We will also see how a time-varying *electric* field can act as a source of *magnetic* field. These remarkable results form part of a neat package of formulas, called *Maxwell's equations,* that describe the behavior of electric and magnetic fields in *any* situation. Maxwell's equations pave the way toward an understanding of electromagnetic waves, the topic of Chapter 32.

29.1 Induction Experiments

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13.9 Electomagnetic Induction

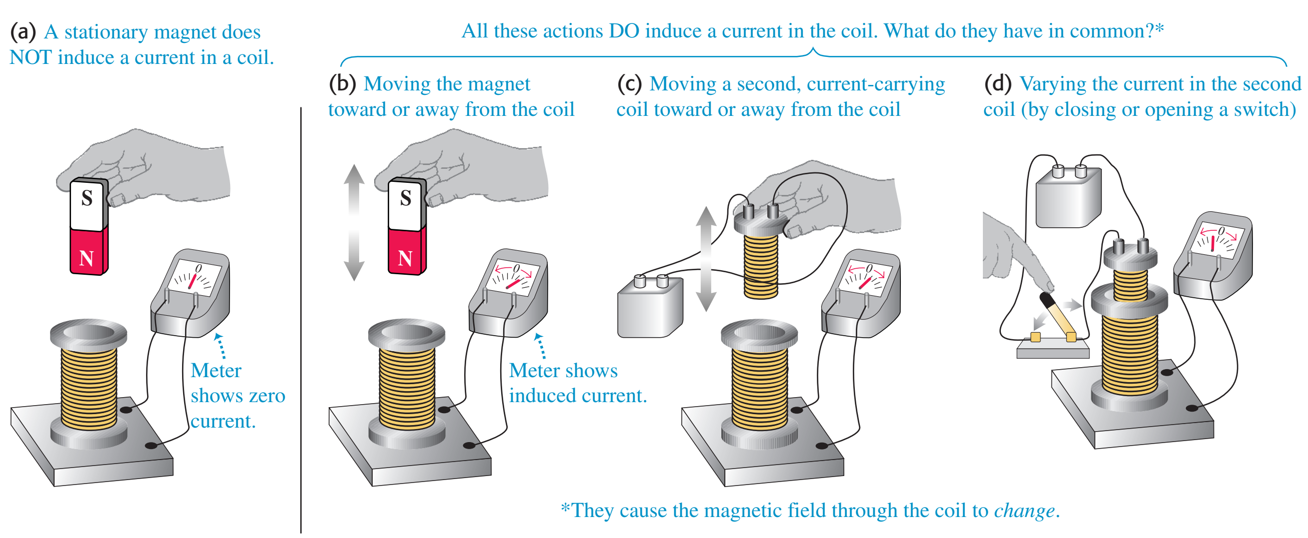
During the 1830s, several pioneering experiments with magnetically induced emf were carried out in England by Michael Faraday and in the United States by Joseph Henry (1797–1878), later the first director of the Smithsonian Institution. Figure 29.1 shows several examples. In Figure 29.1a, a coil of wire is connected to a galvanometer. When the nearby magnet is stationary, the meter shows no current. This isn't surprising; there is no source of emf in the circuit. But when we *move* the magnet either toward or away from the coil, the meter shows current in the circuit, but *only* while the magnet is moving (Fig. 29.1b). If we keep the magnet stationary and move the coil, we again detect a current during the motion. We call this an **induced current,** and the corresponding emf required to cause this current is called an **induced emf.**

In Fig. 29.1c we replace the magnet with a second coil connected to a battery. When the second coil is stationary, there is no current in the first coil. However, when we move the second coil toward or away from the first or move the first toward or away from the second, there is current in the first coil, but again *only* while one coil is moving relative to the other.

Finally, using the two-coil setup in Fig. 29.1d, we keep both coils stationary and vary the current in the second coil, either by opening and closing the switch or by changing the resistance of the second coil with the switch closed (perhaps by changing the second coil's temperature). We find that as we open or close the switch, there is a momentary current pulse in the first circuit. When we vary the resistance (and thus the current) in the second coil, there is an induced current in the first circuit, but only while the current in the second circuit is changing.

To explore further the common elements in these observations, let's consider a more detailed series of experiments with the situation shown in Figure 29.2. We connect a coil of wire to a galvanometer, then place the coil between the

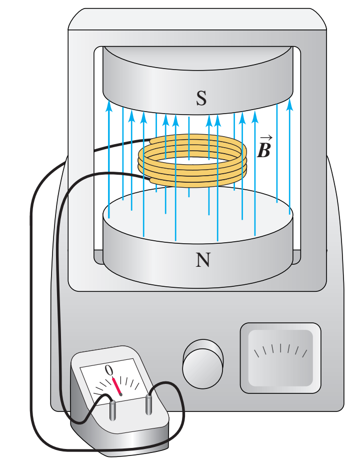
29.1 Demonstrating the phenomenon of induced current.



poles of an electromagnet whose magnetic field we can vary. Here's what we observe:

1. When there is no current in the electromagnet, so that $\vec{B} = 0$, the galvanometer shows no current.
2. When the electromagnet is turned on, there is a momentary current through the meter as $\vec{B}$ increases.
3. When $\vec{B}$ levels off at a steady value, the current drops to zero, no matter how large $\vec{B}$ is.
4. With the coil in a horizontal plane, we squeeze it so as to decrease the cross-sectional area of the coil. The meter detects current only *during* the deformation, not before or after. When we increase the area to return the coil to its original shape, there is current in the opposite direction, but only while the area of the coil is changing.
5. If we rotate the coil a few degrees about a horizontal axis, the meter detects current during the rotation, in the same direction as when we decreased the area. When we rotate the coil back, there is a current in the opposite direction during this rotation.
6. If we jerk the coil out of the magnetic field, there is a current during the motion, in the same direction as when we decreased the area.
7. If we decrease the number of turns in the coil by unwinding one or more turns, there is a current during the unwinding, in the same direction as when we decreased the area. If we wind more turns onto the coil, there is a current in the opposite direction during the winding.
8. When the magnet is turned off, there is a momentary current in the direction opposite to the current when it was turned on.
9. The faster we carry out any of these changes, the greater the current.
10. If all these experiments are repeated with a coil that has the same shape but different material and different resistance, the current in each case is inversely proportional to the total circuit resistance. This shows that the induced emfs that are causing the current do not depend on the material of the coil but only on its shape and the magnetic field.

29.2 A coil in a magnetic field. When the $\vec{B}$ field is constant and the shape, location, and orientation of the coil do not change, no current is induced in the coil. A current is induced when any of these factors change.



The common element in all these experiments is changing *magnetic flux* Φ_B through the coil connected to the galvanometer. In each case the flux changes either because the magnetic field changes with time or because the coil is moving through a nonuniform magnetic field. Check back through the list to verify this statement. Faraday's law of induction, the subject of the next section, states that in all of these situations the induced emf is proportional to the *rate of change* of magnetic flux Φ_B through the coil. The *direction* of the induced emf depends on whether the flux is increasing or decreasing. If the flux is constant, there is no induced emf.

Induced emfs are not mere laboratory curiosities but have a tremendous number of practical applications. If you are reading these words indoors, you are making use of induced emfs right now! At the power plant that supplies your neighborhood, an electric generator produces an emf by varying the magnetic flux through coils of wire. (In the next section we'll see in detail how this is done.) This emf supplies the voltage between the terminals of the wall sockets in your home, and this voltage supplies the power to your reading lamp. Indeed, any appliance that you plug into a wall socket makes use of induced emfs.

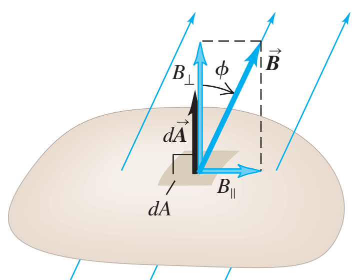
Magnetically induced emfs, just like the emfs discussed in Section 25.4, are always the result of the action of *nonelectrostatic* forces. When these forces are the result of additional electric fields induced by changing magnetic fields, we have to distinguish carefully between electric fields produced by charges (according to Coulomb's law) and those produced by changing magnetic fields. We'll denote these by $\vec{E}_c$ (where c stands for Coulomb or conservative) and $\vec{E}_n$ (where n stands for non-Coulomb or nonconservative), respectively. We'll return to this distinction later in this chapter and the next.

29.2 Faraday's Law

The common element in all induction effects is changing magnetic flux through a circuit. Before stating the simple physical law that summarizes all of the kinds of experiments described in Section 29.1, let's first review the concept of magnetic flux Φ_B (which we introduced in Section 27.3). For an infinitesimal-area element $d\vec{A}$ in a magnetic field $\vec{B}$ (Figure 29.3), the magnetic flux $d\Phi_B$ through the area is

$$d\Phi_B = \vec{B}\cdot d\vec{A} = B_\perp\, dA = B\, dA\cos\phi$$

29.3 Calculating the magnetic flux through an area element.



where $B_\perp$ is the component of $\vec{B}$ perpendicular to the surface of the area element and ϕ is the angle between $\vec{B}$ and $d\vec{A}$. (As in Chapter 27, be careful to distinguish between two quantities named "phi," ϕ and Φ_B.) The total magnetic flux Φ_B through a finite area is the integral of this expression over the area:

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\, dA\cos\phi \qquad (29.1)$$

If $\vec{B}$ is uniform over a flat area $\vec{A}$, then

$$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\phi \qquad (29.2)$$

Figure 29.4 reviews the rules for using Eq. (29.2).

CAUTION **Choosing the direction of $d\vec{A}$ or $\vec{A}$** In Eqs. (29.1) and (29.2) we have to be careful to define the direction of the vector area $d\vec{A}$ or $\vec{A}$ unambiguously. There are always two directions perpendicular to any given area, and the sign of the magnetic flux through the area depends on which one we choose to be positive. For example, in Fig. 29.3 we chose $d\vec{A}$ to point upward so ϕ is less than 90° and $\vec{B}\cdot d\vec{A}$ is positive. We could have chosen instead to have $d\vec{A}$ point downward, in which case ϕ would have been greater than 90° and $\vec{B}\cdot d\vec{A}$ would have been negative. Either choice is equally good, but once we make a choice we must stick with it. ❚

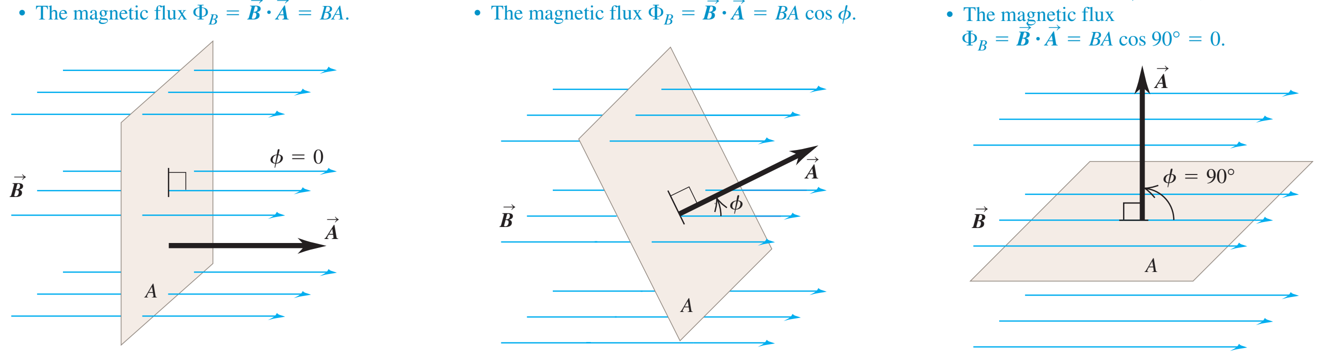
Faraday's law of induction states:

The induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop.

In symbols, Faraday's law is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law of induction)} \qquad (29.3)$$

29.4 Calculating the flux of a uniform magnetic field through a flat area. (Compare to Fig. 22.6, which shows the rules for calculating the flux of a uniform *electric* field.)



To understand the negative sign, we have to introduce a sign convention for the induced emf $\mathcal{E}$. But first let's look at a simple example of this law in action.

Example 29.1 Emf and current induced in a loop

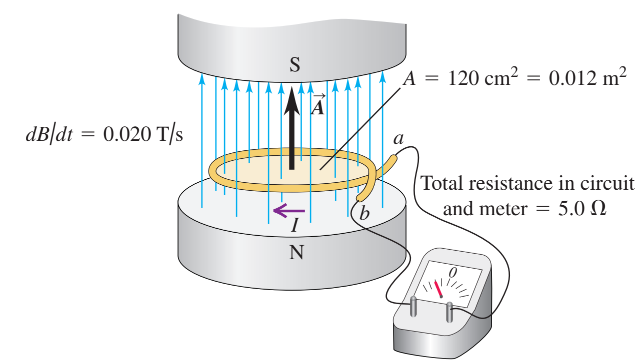
The magnetic field between the poles of the electromagnet in Figure 29.5 is uniform at any time, but its magnitude is increasing at the rate of 0.020 T/s. The area of the conducting loop in the field is 120 cm², and the total circuit resistance, including the meter, is 5.0 Ω. (a) Find the induced emf and the induced current in the circuit. (b) If the loop is replaced by one made of an insulator, what effect does this have on the induced emf and induced current?

SOLUTION

IDENTIFY: The magnetic flux through the loop changes as the magnetic field changes. Hence there will be an induced emf in the loop, and we can find its value (one of our target variables) using Faraday's law. We can determine the current produced in the loop by this emf (our other target variable) using the same techniques as in Chapter 25.

SET UP: We calculate the magnetic flux using Eq. (29.2) and then use Faraday's law given by Eq. (29.3) to determine the resulting induced emf $\mathcal{E}$. Then we calculate the induced current produced by this emf using the relationship $\mathcal{E} = IR$, where R is the total resistance of the circuit that includes the loop.

29.5 A stationary conducting loop in an increasing magnetic field.



EXECUTE: (a) The vector area of the loop is perpendicular to the plane of the loop; we choose it to be vertically upward. Then the vectors $\vec{A}$ and $\vec{B}$ are parallel. Since $\vec{B}$ is uniform, the magnetic flux through the loop is $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos 0 = BA$. The area $A = 0.012\ \text{m}^2$ is constant, so the rate of change of magnetic flux is

$$\frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = \frac{dB}{dt}A = (0.020\ \text{T/s})(0.012\ \text{m}^2)$$
$$= 2.4\times10^{-4}\ \text{V} = 0.24\ \text{mV}$$

This, apart from a sign that we haven't discussed yet, is the induced emf $\mathcal{E}$. The corresponding induced current is

$$I = \frac{\mathcal{E}}{R} = \frac{2.4\times10^{-4}\ \text{V}}{5.0\ \Omega} = 4.8\times10^{-5}\ \text{A} = 0.048\ \text{mA}$$

(b) By changing to a loop made of insulator, we've made the resistance of the loop very high. Faraday's law, Eq. (29.3), does not involve the resistance of the circuit in any way, so the induced *emf* does not change. But the *current* will be smaller, as given by the equation $I = \mathcal{E}/R$. If the loop is made of a perfect insulator with infinite resistance, the induced current is zero even though an emf is present. This situation is analogous to an isolated battery whose terminals aren't connected to anything: There is an emf present, but no current flows.

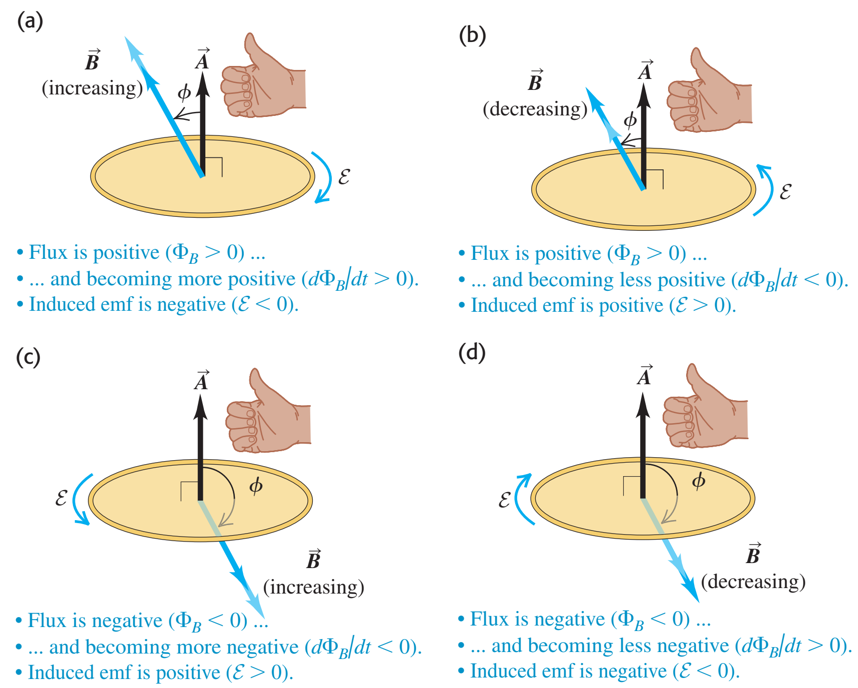
EVALUATE: It's worthwhile to verify unit consistency in this calculation. There are many ways to do this; one is to note that because of the magnetic force relationship $\vec{F} = q\vec{v}\times\vec{B}$, the units of magnetic field are the units of force divided by the units of (charge times velocity): $1\ \text{T} = (1\ \text{N})/(1\ \text{C}\cdot\text{m/s})$. The units of magnetic flux can then be expressed as $(1\ \text{T})(1\ \text{m}^2) = 1\ \text{N}\cdot\text{s}\cdot\text{m/C}$, and the rate of change of magnetic flux as $1\ \text{N}\cdot\text{m/C} = 1\ \text{J/C} = 1\ \text{V}$. Thus the unit of $d\Phi_B/dt$ is the volt, as required by Eq. (29.3). Also recall that the unit of magnetic flux is the weber (Wb): $1\ \text{T}\cdot\text{m}^2 = 1\ \text{Wb}$, so $1\ \text{V} = 1\ \text{Wb/s}$.

Direction of Induced EMF

We can find the direction of an induced emf or current by using Eq. (29.3) together with some simple sign rules. Here's the procedure:

1. Define a positive direction for the vector area $\vec{A}$.
2. From the directions of $\vec{A}$ and the magnetic field $\vec{B}$, determine the sign of the magnetic flux Φ_B and its rate of change $d\Phi_B/dt$. Figure 29.6 shows several examples.
3. Determine the sign of the induced emf or current. If the flux is increasing, so $d\Phi_B/dt$ is positive, then the induced emf or current is negative; if the flux is decreasing, $d\Phi_B/dt$ is negative and the induced emf or current is positive.

29.6 The magnetic flux is becoming (a) more positive, (b) less positive, (c) more negative, and (d) less negative. Therefore Φ_B is increasing in (a) and (d) and decreasing in (b) and (c). In (a) and (d) the emfs are negative (they are opposite to the direction of the curled fingers of your right hand when your right thumb points along $\vec{A}$). In (b) and (c) the emfs are positive (in the same direction as the curled fingers).



4. Finally, determine the direction of the induced emf or current using your right hand. Curl the fingers of your right hand around the $\vec{A}$ vector, with your right thumb in the direction of $\vec{A}$. If the induced emf or current in the circuit is *positive,* it is in the same direction as your curled fingers; if the induced emf or current is *negative,* it is in the opposite direction.

In Example 29.1, in which $\vec{A}$ is upward, a positive $\mathcal{E}$ would be directed counterclockwise around the loop, as seen from above. Both $\vec{A}$ and $\vec{B}$ are upward in this example, so Φ_B is positive; the magnitude B is increasing, so $d\Phi_B/dt$ is positive. Hence by Eq. (29.3), $\mathcal{E}$ in Example 29.1 is *negative.* Its actual direction is thus *clockwise* around the loop, as seen from above.

If the loop in Fig. 29.5 is a conductor, an induced current results from this emf; this current is also clockwise, as Fig. 29.5 shows. This induced current produces an additional magnetic field through the loop, and the right-hand rule described in Section 28.6 shows that this field is *opposite* in direction to the increasing field produced by the electromagnet. This is an example of a general rule called *Lenz's law,* which says that any induction effect tends to oppose the change that caused it; in this case the change is the increase in the flux of the electromagnet's field through the loop. (We'll study this law in detail in the next section.)

You should check out the signs of the induced emfs and currents for the list of experiments in Section 29.1. For example, when the loop in Fig. 29.2 is in a constant field and we tilt it or squeeze it to *decrease* the flux through it, the induced emf and current are counterclockwise, as seen from above.

CAUTION **Induced emfs are caused by changes in flux** Since magnetic flux plays a central role in Faraday's law, it's tempting to think that *flux* is the cause of induced emf and that an induced emf will appear in a circuit whenever there is a magnetic field in the region bordered by the circuit. But Eq. (29.3) shows that only a *change* in flux through a circuit, not flux itself, can induce an emf in a circuit. If the flux through a circuit has a constant value, whether positive, negative, or zero, there is no induced emf.

If we have a coil with N identical turns, and if the flux varies at the same rate through each turn, the *total* rate of change through all the turns is N times as large as for a single turn. If Φ_B is the flux through each turn, the total emf in a coil with N turns is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \qquad (29.4)$$

As we discussed in this chapter's introduction, induced emfs play an essential role in the generation of electric power for commercial use. Several of the following examples explore different methods of producing emfs by the motion of a conductor relative to a magnetic field, giving rise to a changing flux through a circuit.

Problem-Solving Strategy 29.1 Faraday's Law

IDENTIFY: *the relevant concepts:* Faraday's law applies when there is a changing magnetic flux. To use the law, make sure you can identify an area through which there is a flux of magnetic field. This will usually be the area enclosed by a loop, usually made of a conducting material (though not always—see part (b) of Example 29.1). As always, identify the target variable(s).

SET UP *the problem* using the following steps:
1. Faraday's law relates the induced emf to the rate of change of magnetic flux. To calculate this rate of change, you first have to understand what is making the flux change. Is the conductor moving? Is it changing orientation? Is the magnetic field changing? Remember that it's not the flux itself that counts, but its *rate of change.*
2. Choose a direction for the area vector $\vec{A}$ or $d\vec{A}$. The direction must always be perpendicular to the plane of the area. Note that you always have two choices of direction. For instance, if the plane of the area is horizontal, $\vec{A}$ could point straight up or straight down. It's like choosing which direction is the positive one in a problem involving motion in a straight line; it doesn't matter which direction you choose, just so you use it consistently throughout the problem.

EXECUTE *the solution* as follows:
1. Calculate the magnetic flux using Eq. (29.2) if $\vec{B}$ is uniform over the area of the loop or Eq. (29.1) if it isn't uniform, being mindful of the direction you chose for the area vector.
2. Calculate the induced emf using Eq. (29.3) or (29.4). If your conductor has N turns in a coil, don't forget to multiply by N. Remember the sign rule for the positive direction of emf and use it consistently.
3. If the circuit resistance is known, you can calculate the magnitude of the induced current I using $\mathcal{E} = IR$.

EVALUATE *your answer:* Check your results for the proper units, and double-check that you have properly implemented the sign rules for calculating magnetic flux and induced emf.

Example 29.2 Magnitude and direction of an induced emf

A coil of wire containing 500 circular loops with radius 4.00 cm is placed between the poles of a large electromagnet, where the magnetic field is uniform and at an angle of 60° with the plane of the coil. The field decreases at a rate of 0.200 T/s. What are the magnitude and direction of the induced emf?

SOLUTION

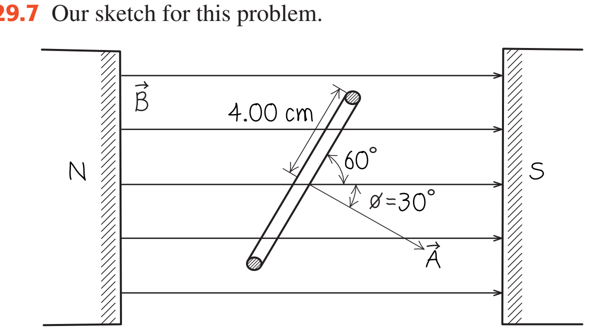
IDENTIFY: Our target variable is the emf induced by a varying magnetic flux through the coil. The flux varies because the magnetic field decreases in amplitude.

SET UP: We choose the area vector $\vec{A}$ to be in the direction shown in Figure 29.7. With this choice, the geometry is very similar to Fig. 29.6b. That figure will help us determine the direction of the induced emf.

EXECUTE: The magnetic field is uniform over the loop, so we can calculate the flux using Eq. (29.2): $\Phi_B = BA\cos\phi$, where $\phi = 30°$. In this expression, the only quantity that changes with time is the magnitude B of the field.

CAUTION **Remember how ϕ is defined** You may have been tempted to say that $\phi = 60°$ in this problem. If so, remember that ϕ is the angle between $\vec{A}$ and $\vec{B}$, *not* the angle between $\vec{B}$ and the plane of the loop.

29.7 Our sketch for this problem.



Continued

The rate of change of the flux is $d\Phi_B/dt = (dB/dt)A\cos\phi$. In our problem, $dB/dt = -0.200\ \text{T/s}$ and $A = \pi(0.0400\ \text{m})^2 = 0.00503\ \text{m}^2$, so

$$\frac{d\Phi_B}{dt} = \frac{dB}{dt}A\cos 30^\circ$$
$$= (-0.200\ \text{T/s})(0.00503\ \text{m}^2)(0.866)$$
$$= -8.71\times10^{-4}\ \text{T}\cdot\text{m}^2/\text{s} = -8.71\times10^{-4}\ \text{Wb/s}$$

From Eq. (29.4), the induced emf in the coil of $N = 500$ turns is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$
$$= -(500)(-8.71\times10^{-4}\ \text{Wb/s}) = 0.435\ \text{V}$$

Note that the answer is positive. This means that when you point your right thumb in the direction of the area vector $\vec{A}$ (30° above the magnetic field $\vec{B}$), the positive direction for $\mathcal{E}$ is in the direction of the curled fingers of your right hand. Hence the emf in this example is in this same direction (compare Fig. 29.6b). If you were viewing the coil from the left side in Fig. 29.7a and looking in the direction of $\vec{A}$, the emf would be clockwise.

EVALUATE: If the ends of the wire are connected together, the direction of current in the coil is in the same direction as the emf—that is, clockwise as seen from the left side of the coil. A clockwise current gives added magnetic flux through the coil in the same direction as the flux from the electromagnet, and therefore tends to oppose the decrease in total flux. We'll see more examples of this in Section 29.3.

Conceptual Example 29.3 The search coil

One practical way to measure magnetic field strength uses a small, closely wound coil with N turns called a *search coil.* The coil, of area A, is initially held so that its area vector $\vec{A}$ is aligned with a magnetic field with magnitude B. The coil is then either quickly rotated a quarter-turn about a diameter or quickly pulled out of the field. Explain how this device can be used to measure the value of B.

SOLUTION

Initially, the flux through the coil is $\Phi_B = NBA$; when the coil is rotated or pulled from the field, the flux decreases rapidly from NBA to zero. While the flux is decreasing, there is a momentary induced emf, and a momentary induced current occurs in an external circuit connected to the coil. The rate of change of flux through the coil is proportional to the current, or rate of flow of charge, so it is easy to show that the *total* flux change is proportional to the total charge that flows around the circuit. We can build an instrument that measures this total charge, and from this we can compute B. We leave the details as a problem (see Exercise 29.3). Strictly speaking, this method gives only the *average* field over the area of the coil. But if the area is small, this average field is very nearly equal to the field at the center of the coil.

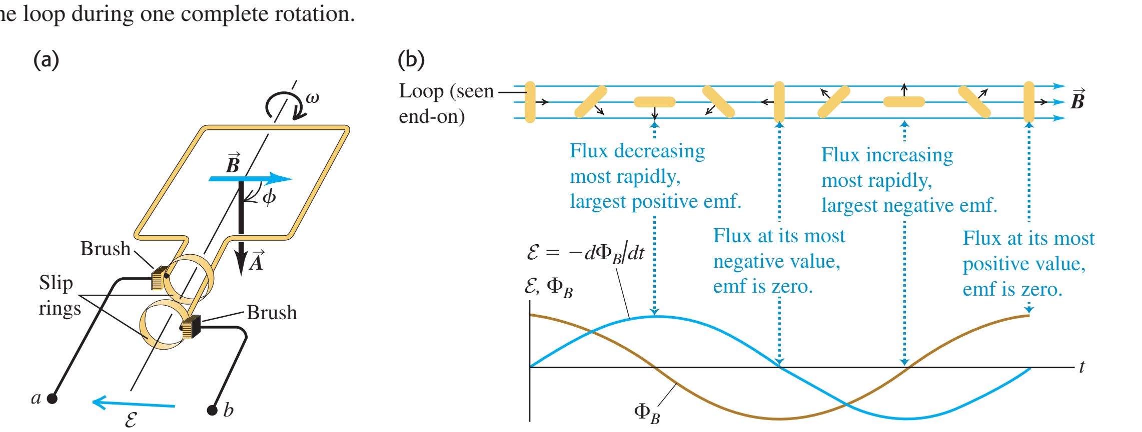
Example 29.4 Generator I: A simple alternator

Figure 29.8a shows a simple version of an *alternator,* a device that generates an emf. A rectangular loop is made to rotate with constant angular speed ω about the axis shown. The magnetic field $\vec{B}$ is uniform and constant. At time $t = 0$, $\phi = 0$. Determine the induced emf.

SOLUTION

IDENTIFY: Again the emf (our target variable) is produced by a varying magnetic flux. In this situation, however, the magnetic field $\vec{B}$ is constant; the flux changes because the direction of $\vec{A}$ changes as the loop rotates.

29.8 (a) Schematic diagram of an alternator. A conducting loop rotates in a magnetic field, producing an emf. Connections from each end of the loop to the external circuit are made by means of that end's slip ring. The system is shown at the time when the angle $\phi = \omega t = 90°$. (b) Graph of the flux through the loop and the resulting emf at terminals ab, along with corresponding positions of the loop during one complete rotation.



SET UP: Figure 29.8a shows the direction of the area vector $\vec{A}$. Note that as the loop rotates, the angle ϕ between $\vec{A}$ and $\vec{B}$ increases at a constant rate.

EXECUTE: Again the magnetic field is uniform over the loop, so the magnetic flux is easy to calculate. The rate of change of the angle ϕ is equal to ω, the angular speed of the loop, so we can write $\phi = \omega t$. Hence

$$\Phi_B = BA\cos\phi = BA\cos\omega t$$

The derivative of $\cos\omega t$ is $(d/dt)\cos\omega t = -\omega\sin\omega t$. Hence, by Faraday's law [Eq. (29.3)] the induced emf is

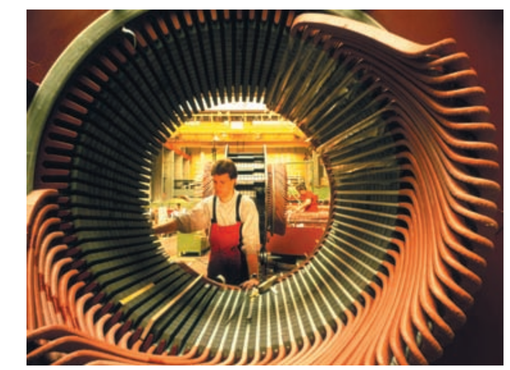
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \omega BA\sin\omega t$$

EVALUATE: The induced emf $\mathcal{E}$ varies sinusoidally with time (Fig. 29.8b). When the plane of the loop is perpendicular to $\vec{B}$ ($\phi = 0$ or $180°$), Φ_B reaches its maximum and minimum values. At these times, its instantaneous rate of change is zero and $\mathcal{E}$ is zero. Also, $\mathcal{E}$ is greatest in absolute value when the plane of the loop is parallel to $\vec{B}$ ($\phi = 90°$ or $270°$) and Φ_B is changing most rapidly. Finally, we note that the induced emf does not depend on the *shape* of the loop, but only on its area. Because $\mathcal{E}$ is directly proportional to ω and B, some tachometers use the emf in a rotating coil to measure rotational speed. Other devices use an emf of this kind to measure magnetic field.

We can use the alternator as a source of emf in an external circuit by use of two *slip rings,* which rotate with the loop, as shown in Fig. 29.8a. The rings slide against stationary contacts called *brushes,* which are connected to the output terminals a and b. Since the emf varies sinusoidally, the current that results in the circuit is an *alternating* current that also varies sinusoidally in magnitude and direction. An alternator is also called an *alternating-current* (ac) *generator* for this reason. The amplitude of the emf can be increased by increasing the rotation speed, the field magnitude, or the loop area or by using N loops instead of one, as in Eq. (29.4).

Alternators are used in automobiles to generate the currents in the ignition, the lights, and the entertainment system. The arrangement is a little different than in this example; rather than having a rotating loop in a magnetic field, the loop stays fixed and an electromagnet rotates. (The rotation is provided by a mechanical connection between the alternator and the engine.) But the result is the same; the flux through the loop varies sinusoidally, producing a sinusoidally varying emf. Larger alternators of this same type are used in electric power plants (Figure 29.9).

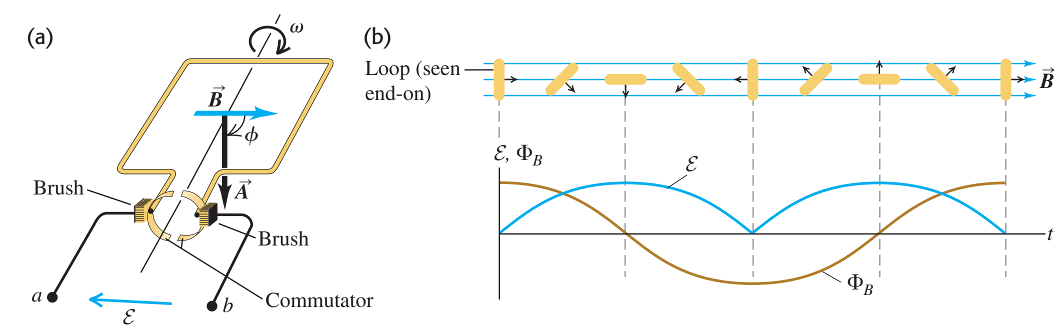
29.9 A commercial alternator uses many loops of wire wound around a barrel-like structure called an armature. The armature and wire remain stationary while electromagnets rotate on a shaft (not shown) through the center of the armature. The resulting induced emf is far larger than would be possible with a single loop of wire.



Example 29.5 Generator II: A DC generator and back emf in a motor

The alternator in Example 29.4 produces a sinusoidally varying emf and hence an alternating current. We can use a similar scheme to make a *direct-current* (dc) *generator* that produces an emf that always has the same sign. A prototype dc generator is shown in Fig. 29.10a. The arrangement of split rings is called a *commutator;* it reverses the connections to the external circuit at angular positions where the emf reverses. The resulting emf is shown in Fig. 29.10b. Commercial dc generators have a large number of coils and commutator segments; this arrangement smooths out the bumps in the emf, so the terminal voltage is not only one-directional but also practically constant. This brush-and-commutator arrangement is the same as that in the direct-current motor we discussed in Section 27.8. The motor's *back emf* is just the emf induced by the changing magnetic flux through its rotating coil. Consider a motor with a square coil 10.0 cm on a side, with 500 turns of wire. If the magnetic field has magnitude 0.200 T, at what rotation speed is the *average* back emf of the motor equal to 112 V?

29.10 (a) Schematic diagram of a dc generator, using a split-ring commutator. The ring halves are attached to the loop and rotate with it. (b) Graph of the resulting induced emf at terminals ab. Compare to Fig. 29.8b.



Continued

SOLUTION

IDENTIFY: As far as the rotating loop is concerned, the situation is the same as in Example 29.4 except that we now have N turns of wire. Without the commutator, the emf would alternate between positive and negative values and have an average value of zero (Fig. 29.8b). But with the commutator added, the emf is never negative and its average value is positive (Fig. 29.10b). Using our result from Example 29.4, we'll determine an expression for this average value and solve that expression for the rotational speed ω (our target variable).

SET UP: The setup is the same as in Example 29.4.

EXECUTE: Comparing Figs. 29.8b and 29.10b shows that the back emf of the motor is just the absolute value of the emf found for an alternator in Example 29.4, multiplied by the number of turns N in the coil as in Eq. (29.4):

$$|\mathcal{E}| = N\omega BA|\sin\omega t|$$

To find the *average* back emf, we replace $|\sin\omega t|$ by its average value. The average value of the sine function is found by integrating $\sin\omega t$ over half a cycle, from $t = 0$ to $t = T/2 = \pi/\omega$, and then dividing by the elapsed time π/ω. During this half cycle, the sine function is positive, so $|\sin\omega t| = \sin\omega t$, and we find

$$(|\sin\omega t|)_{av} = \frac{\int_0^{\pi/\omega}\sin\omega t\,dt}{\pi/\omega} = \frac{2}{\pi}$$

or about 0.64. The average back emf is then

$$\mathcal{E}_{av} = \frac{2N\omega BA}{\pi}$$

The back emf is proportional to the rotation speed ω, as was stated without proof in Section 27.8. Solving for ω, we obtain

$$\omega = \frac{\pi\mathcal{E}_{av}}{2NBA} = \frac{\pi(112\text{ V})}{2(500)(0.200\text{ T})(0.100\text{ m})^2} = 176\text{ rad/s}$$

We used the relationships $1\text{ V} = 1\text{ Wb/s} = 1\text{ T}\cdot\text{m}^2/\text{s}$ from Example 29.1. We were able to add "radians" to the units of the answer because it is a dimensionless quantity, as we discussed in Chapter 9. The rotation speed can also be written as

$$\omega = 176\text{ rad/s}\frac{1\text{ rev}}{2\pi\text{ rad}}\frac{60\text{ s}}{1\text{ min}} = 1680\text{ rev/min}$$

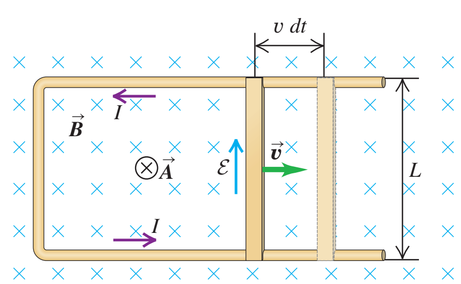
EVALUATE: The average back emf is directly proportional to ω. Hence the slower the rotation speed, the less the back emf and the greater the possibility of burning out the motor, as we described in Example 27.11 (Section 27.8).

While we have used a very simple model of a generator in this and the preceding example, the same principles apply to the operation of commercial generators.

Example 29.6 Generator III: The slidewire generator

Figure 29.11 shows a U-shaped conductor in a uniform magnetic field $\vec{B}$ perpendicular to the plane of the figure, directed *into* the page. We lay a metal rod with length L across the two arms of the conductor, forming a circuit, and move the rod to the right with constant velocity $\vec{v}$. This induces an emf and a current, which is why this device is called a *slidewire generator.* Find the magnitude and direction of the resulting induced emf.

29.11 A slidewire generator. The magnetic field $\vec{B}$ and the vector area $\vec{A}$ are both directed into the figure. The increase in magnetic flux (caused by an increase in area) induces the emf and current.



SOLUTION

IDENTIFY: The magnetic flux changes because the area of the loop—bounded on the right by the moving rod—is increasing. Our target variable is the emf $\mathcal{E}$ induced in this expanding loop.

SET UP: The magnetic field is uniform over the area of the loop, so we can again calculate the magnetic flux using $\Phi_B = BA\cos\phi$. We choose the area vector $\vec{A}$ to point straight into the plane of the picture, in the same direction as $\vec{B}$. With this choice a positive emf will be one that is directed clockwise around the loop. (You can check this with the right-hand rule. Using your right hand, point your thumb into the page and curl your fingers as in Fig. 29.6.)

EXECUTE: Since $\vec{B}$ and $\vec{A}$ point in the same direction, the angle $\phi = 0$ and $\Phi_B = BA$. The magnetic field magnitude B is constant, so the induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt}$$

To calculate dA/dt, note that in a time dt the sliding rod moves a distance $v\,dt$ (Fig. 29.11) and the loop area increases by an amount $dA = Lv\,dt$. Hence the induced emf is

$$\mathcal{E} = -B\frac{Lv\,dt}{dt} = -BLv$$

The minus sign tells us that the emf is directed *counterclockwise* around the loop. The induced current is also counterclockwise, as shown in the figure.

EVALUATE: Note that the emf is constant if the velocity $\vec{v}$ of the rod is constant. In this case the slidewire generator acts as a *direct-current* generator. It's not a very practical device because the rod eventually moves beyond the U-shaped conductor and loses contact, after which the current stops.

Example 29.7 Work and power in the slidewire generator

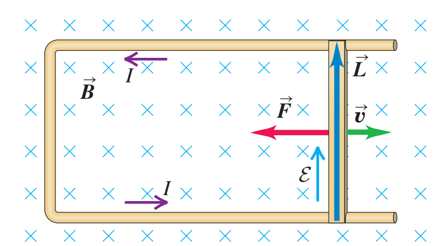
In the slidewire generator of Example 29.6, energy is dissipated in the circuit owing to its resistance. Let the resistance of the circuit (made up of the moving slidewire and the U-shaped conductor that connects the ends of the slidewire) at a given point in the slidewire's motion be R. Show that the rate at which energy is dissipated in the circuit is exactly equal to the rate at which work must be done to move the rod through the magnetic field.

SOLUTION

IDENTIFY: Our target variables are the *rates* at which energy is dissipated and at which work is done. This means that we'll be working with the concept of power (recall Section 6.4). Energy is dissipated in the circuit because there is resistance; to describe this we'll need the ideas of Section 25.5. It takes work to move the rod because there is an induced current flowing through it. The magnetic field exerts a force on this current-carrying rod, and whoever is pushing the rod has to do work against this force.

SET UP: We found the induced emf $\mathcal{E}$ in this circuit in Example 29.6. The current I in the circuit equals the absolute value of $\mathcal{E}$ divided by the resistance R, and the rate at which energy is dissipated in the rod is $P_{dissipated} = I^2R$. The magnetic force on the rod is $\vec{F} = I\vec{L}\times\vec{B}$; the vector $\vec{L}$ points along the rod in the direction of the current. Figure 29.12 shows that this force is opposite to the velocity of the rod, and so to maintain the motion a force of equal magnitude must be applied in the direction of the rod's motion (that is, in the direction of $\vec{v}$). The rate of doing work is equal to the product of the applied force and the speed of the rod: $P_{applied} = Fv$.

29.12 The magnetic force $\vec{F} = I\vec{L}\times\vec{B}$ that acts on the rod due to the induced current is to the left, opposite to $\vec{v}$.



EXECUTE: First we'll calculate $P_{dissipated}$. From Example 29.6, $\mathcal{E} = -BLv$. Hence the current in the rod is

$$I = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}$$

and the rate of energy dissipation is

$$P_{dissipated} = I^2R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R}$$

To calculate $P_{applied}$, we first calculate the magnitude of $\vec{F} = I\vec{L}\times\vec{B}$. Since $\vec{L}$ and $\vec{B}$ are perpendicular, this magnitude is

$$F = ILB = \frac{BLv}{R}LB = \frac{B^2L^2v}{R}$$

Hence the rate at which work is done by this applied force is

$$P_{applied} = Fv = \frac{B^2L^2v^2}{R}$$

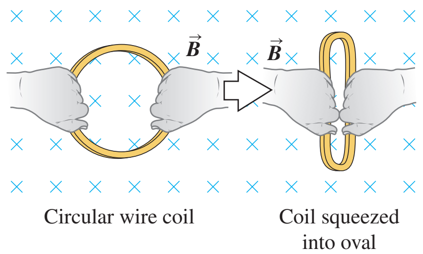
EVALUATE: The rate at which work is done is just equal to the rate at which energy is dissipated in the resistance.

CAUTION **You can't violate energy conservation** You might think that reversing the direction of $\vec{B}$ or of $\vec{v}$ might make it possible to have the magnetic force $\vec{F} = I\vec{L}\times\vec{B}$ be in the *same* direction as $\vec{v}$. This would be a pretty neat trick. Once the rod was moving, the changing magnetic flux would induce an emf and a current, and the magnetic force on the rod would make it move even faster, increasing the emf and current; this would go on until the rod was moving at tremendous speed and producing electric power at a prodigious rate. If this seems too good to be true, not to mention a violation of energy conservation, that's because it is. Reversing $\vec{B}$ also reverses the sign of the induced emf and current and hence the direction of $\vec{L}$, so the magnetic force still opposes the motion of the rod; a similar result holds true if we reverse $\vec{v}$. This behavior is part of Lenz's law, to be discussed in Section 29.3.

Generators As Energy Converters

Example 29.7 shows that the slidewire generator doesn't produce electric energy out of nowhere; the energy is supplied by whatever body exerts the force that keeps the rod moving. All that the generator does is to *convert* that energy into a different form. The equality between the rate at which *mechanical* energy is supplied to a generator and the rate at which *electric* energy is generated holds for all types of generators. This is true in particular for the alternator described in Example 29.4. (We are neglecting the effects of friction in the bearings of an alternator or between the rod and the U-shaped conductor of a slidewire generator. If these are included, the conservation of energy demands that the energy lost to friction is not available for conversion to electric energy. In real generators the friction is kept to a minimum to keep the energy-conversion process as efficient as possible.)

In Chapter 27 we stated that the magnetic force on moving charges can never do work. But you might think that the magnetic force $\vec{F} = I\vec{L}\times\vec{B}$ in Example 29.7 *is* doing (negative) work on the current-carrying rod as it moves, contradicting our earlier statement. In fact, the work done by the magnetic force is actually zero. The moving charges that make up the current in the rod in Fig. 29.12 have a vertical component of velocity, causing a horizontal component of force on these charges. As a result, there is a horizontal displacement of charge within the rod, the left side acquiring a net positive charge and the right side a net negative charge. The result is a horizontal component of electric field, perpendicular to the length of the rod (analogous to the Hall effect, described in Section 27.9). It is this field, in the direction of motion of the rod, that does work on the mobile charges in the rod and hence indirectly on the atoms making up the rod.



Test Your Understanding of Section 29.2 The figure at left shows a wire coil being squeezed in a uniform magnetic field. (a) While the coil is being squeezed, is the induced emf in the coil (i) clockwise, (ii) counterclockwise, or (iii) zero? (b) Once the coil has reached its final squeezed shape, is the induced emf in the coil (i) clockwise, (ii) counterclockwise, or (iii) zero?

29.3 Lenz's Law

Lenz's law is a convenient alternative method for determining the direction of an induced current or emf. Lenz's law is not an independent principle; it can be derived from Faraday's law. It always gives the same results as the sign rules we introduced in connection with Faraday's law, but it is often easier to use. Lenz's law also helps us gain intuitive understanding of various induction effects and of the role of energy conservation. H. F. E. Lenz (1804–1865) was a Russian scientist who duplicated independently many of the discoveries of Faraday and Henry. **Lenz's law** states:

The direction of any magnetic induction effect is such as to oppose the cause of the effect.

The "cause" may be changing flux through a stationary circuit due to a varying magnetic field, changing flux due to motion of the conductors that make up the circuit, or any combination. If the flux in a stationary circuit changes, as in Examples 29.1 and 29.2, the induced current sets up a magnetic field of its own. Within the area bounded by the circuit, this field is *opposite* to the original field if the original field is *increasing* but is in the *same* direction as the original field if the latter is *decreasing*. That is, the induced current opposes the *change in flux* through the circuit (*not* the flux itself).

If the flux change is due to motion of the conductors, as in Examples 29.3 through 29.7, the direction of the induced current in the moving conductor is such that the direction of the magnetic-field force on the conductor is opposite in direction to its motion. Thus the motion of the conductor, which caused the induced current, is opposed. We saw this explicitly for the slidewire generator in Example 29.7. In all these cases the induced current tries to preserve the *status quo* by opposing motion or a change of flux.

Lenz's law is also directly related to energy conservation. If the induced current in Example 29.7 were in the direction opposite to that given by Lenz's law, the magnetic force on the rod would accelerate it to ever-increasing speed with no external energy source, even though electric energy is being dissipated in the circuit. This would be a clear violation of energy conservation and doesn't happen in nature.

Conceptual Example 29.8 The slidewire generator, revisited

In Fig. 29.11, the induced current in the loop causes an additional magnetic field in the area bounded by the loop. The direction of the induced current is counterclockwise. From the discussion of Section 28.2, we see that the direction of the additional magnetic field caused by this current is *out of* the plane of the figure. Its direction is opposite that of the original magnetic field, so it tends to cancel the effect of that field. This is consistent with the prediction of Lenz's law.

Conceptual Example 29.9 Finding the direction of induced current

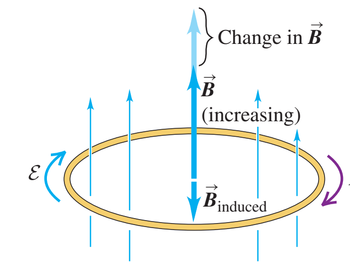
In Fig. 29.13 there is a uniform magnetic field $\vec{B}$ through the coil. The magnitude of the field is increasing, and the resulting induced emf causes an induced current. Use Lenz's law to determine the direction of the induced current.

SOLUTION

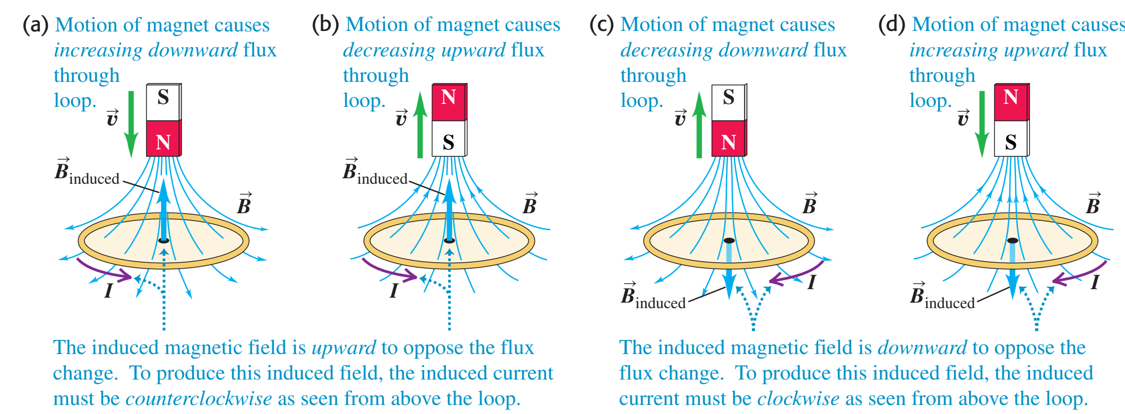
This situation is the same as in Example 29.1 (Section 29.2). By Lenz's law the induced current must produce a magnetic field $\vec{B}_{induced}$ inside the coil that is downward, opposing the change in flux. Using the right-hand rule we described in Section 28.5 for the direction of the magnetic field produced by a circular loop, $\vec{B}_{induced}$ will be in the desired direction if the induced current flows as shown in Fig. 29.13.

Figure 29.14 shows several applications of Lenz's law to the similar situation of a magnet moving near a conducting loop. In each of the four cases shown, the induced current produces a magnetic field of its own, in a direction that opposes the change in flux through the loop due to the magnet's motion.

29.13 The induced current due to the change in $\vec{B}$ is clockwise, as seen from above the loop. The added field $\vec{B}_{induced}$ that it causes is downward, opposing the change in the upward field $\vec{B}$.



29.14 Directions of induced currents as a bar magnet moves along the axis of a conducting loop. If the bar magnet is stationary, there is no induced current.



Lenz's Law and the Response to Flux Changes

Since an induced current always opposes any change in magnetic flux through a circuit, how is it possible for the flux to change at all? The answer is that Lenz's law gives only the *direction* of an induced current; the *magnitude* of the current depends on the resistance of the circuit. The greater the circuit resistance, the less the induced current that appears to oppose any change in flux and the easier it is for a flux change to take effect. If the loop in Fig. 29.14 were made out of wood (an insulator), there would be almost no induced current in response to changes in the flux through the loop.

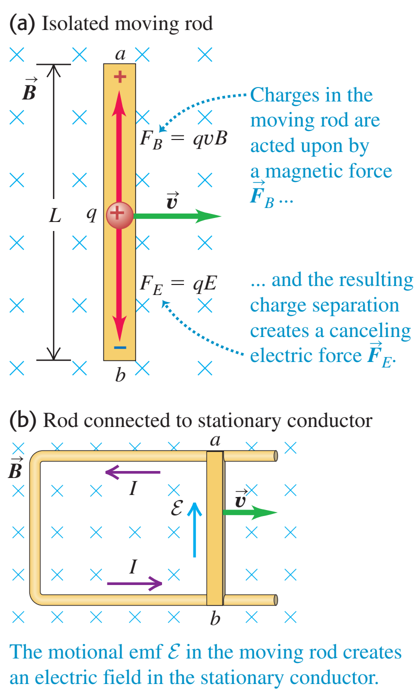
Conversely, the less the circuit resistance, the greater the induced current and the more difficult it is to change the flux through the circuit. If the loop in Fig. 29.14 is a good conductor, an induced current flows as long as the magnet moves relative to the loop. Once the magnet and loop are no longer in relative motion, the induced current very quickly decreases to zero because of the nonzero resistance in the loop.

The extreme case occurs when the resistance of the circuit is *zero*. Then the induced current in Fig. 29.14 will continue to flow even after the induced emf has disappeared—that is, even after the magnet has stopped moving relative to the loop. Thanks to this *persistent current,* it turns out that the flux through the loop is exactly the same as it was before the magnet started to move, so the flux through a loop of zero resistance *never* changes. Exotic materials called *superconductors* do indeed have zero resistance; we discuss these further in Section 29.8.

Test Your Understanding of Section 29.3 (a) Suppose the magnet in Fig. 29.14a were stationary and the loop of wire moved upward. Would the induced current in the loop be (i) in the same direction as shown in Fig. 29.14a, (ii) in the direction opposite to that shown in Fig. 29.14a, or (iii) zero? (b) Suppose the magnet and loop of wire in Fig. 29.14a both moved downward at the same velocity. Would the induced current in the loop be (i) in the same direction as shown in Fig. 29.14a, (ii) in the direction opposite to that shown in Fig. 29.14a, or (iii) zero?

29.4 Motional Electromotive Force

29.15 A conducting rod moving in a uniform magnetic field. **(a)** The rod, the velocity, and the field are mutually perpendicular. **(b)** Direction of induced current in the circuit.



We've seen several situations in which a conductor moves in a magnetic field, as in the generators discussed in Examples 29.4 through 29.7. We can gain additional insight into the origin of the induced emf in these situations by considering the magnetic forces on mobile charges in the conductor. Figure 29.15a shows the same moving rod that we discussed in Example 29.6, separated for the moment from the U-shaped conductor. The magnetic field $\vec{B}$ is uniform and directed into the page, and we move the rod to the right at a constant velocity $\vec{v}$. A charged particle q in the rod then experiences a magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ with magnitude $F = |q|vB$. We'll assume in the following discussion that q is positive; in that case the direction of this force is upward along the rod, from b toward a.

This magnetic force causes the free charges in the rod to move, creating an excess of positive charge at the upper end a and negative charge at the lower end b. This in turn creates an electric field $\vec{E}$ within the rod, in the direction from a toward b (opposite to the magnetic force). Charge continues to accumulate at the ends of the rod until $\vec{E}$ becomes large enough for the downward electric force (with magnitude qE) to cancel exactly the *upward* magnetic force (with magnitude qvB). Then $qE = qvB$ and the charges are in equilibrium.

The magnitude of the potential difference $V_{ab} = V_a - V_b$ is equal to the electric field magnitude E multiplied by the length L of the rod. From the above discussion, $E = vB$, so

$$V_{ab} = EL = vBL \qquad (29.5)$$

with point a at higher potential than point b.

Now suppose the moving rod slides along a stationary U-shaped conductor, forming a complete circuit (Fig. 29.15b). No *magnetic* force acts on the charges in the stationary U-shaped conductor, but the charge that was near points a and b redistributes itself along the stationary conductor, creating an *electric* field within it. This field establishes a current in the direction shown. The moving rod has become a source of electromotive force; within it, charge moves from lower to higher potential, and in the remainder of the circuit, charge moves from higher to lower potential. We call this emf a **motional electromotive force,** denoted by $\mathcal{E}$. From the above discussion, the magnitude of this emf is

$$\mathcal{E} = vBL \qquad \text{(motional emf; length and velocity perpendicular to uniform } \vec{B}\text{)} \qquad (29.6)$$

corresponding to a force per unit charge of magnitude vB acting for a distance L along the moving rod. If the total circuit resistance of the U-shaped conductor and the sliding rod is R, the induced current I in the circuit is given by $vBL = IR$. This is the same result we obtained in Section 29.2 using Faraday's law, and indeed motional emf is a particular case of Faraday's law, one of the several examples described in Section 29.2.

The emf associated with the moving rod in Fig. 29.15 is analogous to that of a battery with its positive terminal at a and its negative terminal at b, although the origins of the two emfs are quite different. In each case a nonelectrostatic force acts on the charges in the device, in the direction from b to a, and the emf is the work per unit charge done by this force when a charge moves from b to a in the device. When the device is connected to an external circuit, the direction of current is from b to a in the device and from a to b in the external circuit. While we have discussed motional emf in terms of a closed circuit like that in Fig. 29.15b, a motional emf is also present in the isolated moving rod in Fig. 29.15a, in the same way that a battery has an emf even when it's not part of a circuit.

ActivPhysics Online 13.10 Motional EMF

The direction of the induced emf in Fig. 29.15 can be deduced by using Lenz's law, even if (as in Fig. 29.15a) the conductor does not form a complete circuit. In this case we can mentally complete the circuit between the ends of the conductor and use Lenz's law to determine the direction of the current. From this we can deduce the polarity of the ends of the open-circuit conductor. The direction from the − end to the + end within the conductor is the direction the current would have if the circuit were complete.

You should verify that if we express v in meters per second, B in teslas, and L in meters, then $\mathcal{E}$ is in volts. (Recall that 1 V = 1 J/C.)

Motional emf: General Form

We can generalize the concept of motional emf for a conductor with *any* shape, moving in any magnetic field, uniform or not (assuming that the magnetic field at each point does not vary with time). For an element $d\vec{l}$ of conductor, the contribution $d\mathcal{E}$ to the emf is the magnitude dl multiplied by the component of $\vec{v} \times \vec{B}$ (the magnetic force per unit charge) parallel to $d\vec{l}$; that is,

$$d\mathcal{E} = (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

For any closed conducting loop, the total emf is

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} \qquad \text{(motional emf: closed conducting loop)} \qquad (29.7)$$

This expression looks very different from our original statement of Faraday's law, Eq. (29.3), which stated that $\mathcal{E} = -d\Phi_B/dt$. In fact, though, the two statements are equivalent. It can be shown that the rate of change of magnetic flux through a moving conducting loop is always given by the negative of the expression in Eq. (29.7). Thus this equation gives us an alternative formulation of Faraday's law. This alternative is often more convenient than the original one in problems with *moving* conductors. But when we have *stationary* conductors in changing magnetic fields, Eq. (29.7) *cannot* be used; in this case, $\mathcal{E} = -d\Phi_B/dt$ is the only correct way to express Faraday's law.

Example 29.10 Calculating motional emf

Suppose the moving rod in Fig. 29.15b is 0.10 m long, the velocity v is 2.5 m/s, the total resistance of the loop is 0.030 Ω, and B is 0.60 T. Find $\mathcal{E}$, the induced current, and the force acting on the rod.

SOLUTION

IDENTIFY: The first target variable is the *motional* emf $\mathcal{E}$ due to the rod's motion. We'll find the current from the values of $\mathcal{E}$ and the resistance R. The force on the rod is actually a magnetic force exerted by $\vec{B}$ on the current in the rod.

SET UP: We'll use the motional emf expression developed in this section, the familiar relationship $\mathcal{E} = IR$, and the formula $\vec{F} = I\vec{L} \times \vec{B}$ for the magnetic force on a current-carrying rod of length $L = 0.10$ m.

EXECUTE: From Eq. (29.6) the emf is

$$\mathcal{E} = vBL = (2.5\ \text{m/s})(0.60\ \text{T})(0.10\ \text{m}) = 0.15\ \text{V}$$

The resulting induced current in the loop is

$$I = \frac{\mathcal{E}}{R} = \frac{0.15\ \text{V}}{0.030\ \Omega} = 5.0\ \text{A}$$

The magnetic force on the rod carrying this current is directed *opposite* to the rod's motion. You can see this by applying the right-hand rule for vector products to the formula $\vec{F} = I\vec{L} \times \vec{B}$. The vector $\vec{L}$ points from b to a in Fig. 29.15, in the same direction as the induced current in the rod. Since $\vec{L}$ and $\vec{B}$ are perpendicular, this force has magnitude

$$F = ILB = (5.0\ \text{A})(0.10\ \text{m})(0.60\ \text{T}) = 0.30\ \text{N}$$

EVALUATE: We can check our answer for the direction of $\vec{F}$ by using Lenz's law. If we take the area vector $\vec{A}$ to point into the plane of the loop, the magnetic flux is positive and increasing as the rod moves to the right and increases the area of the loop. Lenz's law tells us that a force appears to oppose this increase in flux. Hence the force on the rod is to the left, opposite its motion.

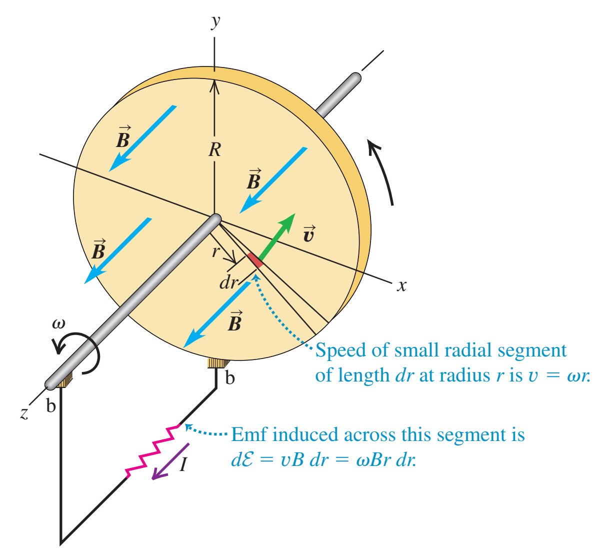
Example 29.11 The Faraday disk dynamo

A conducting disk with radius R, shown in Fig. 29.16, lies in the xy-plane and rotates with constant angular velocity ω about the z-axis. The disk is in a uniform, constant $\vec{B}$ field parallel to the z-axis. Find the induced emf between the center and the rim of the disk.

SOLUTION

IDENTIFY: A motional emf is present because the conducting disk moves relative to the $\vec{B}$ field. The complication is that different parts of the disk move at different speeds v, depending on their distance from the rotation axis. We'll address this by considering small segments of the disk and adding (actually integrating) their contributions to determine our target variable, the emf between the center and the rim.

29.16 A conducting disk with radius R rotating at an angular speed ω in a magnetic field $\vec{B}$. The emf is induced along radial lines of the disk and is applied to an external circuit through the two sliding contacts labeled b.



SET UP: Consider the small segment of the disk labeled by its velocity vector $\vec{v}$. The magnetic force per unit charge on this segment is $\vec{v} \times \vec{B}$, which points radially outward from the center of the disk. Hence the induced emf tends to make a current flow radially outward, which tells us that the moving conducting path to think about here is a straight line from the center to the rim. We can find the emf from each small disk segment along this line using the expression $d\mathcal{E} = (\vec{v} \times \vec{B}) \cdot d\vec{l}$ and then integrate to find the total emf.

CAUTION Speed in a rotating disk You might be tempted to use Eq. (29.5) and simply multiply vB times the length of the moving conducting path, which is just the radius R. That wouldn't be right, because v has different values at different points along the path.

EXECUTE: Let's consider the motional emf $d\mathcal{E}$ due to a small radial segment at a distance r from the rotation axis. The associated length vector $d\vec{l}$ (of length dr) points radially outward, in the same direction as $\vec{v} \times \vec{B}$. The vectors $\vec{v}$ and $\vec{B}$ are perpendicular, and the magnitude of $\vec{v}$ is $v = \omega r$. Hence the total emf between center and rim is the sum of all such contributions:

$$\mathcal{E} = \int_0^R \omega B r\, dr = \frac{1}{2}\omega B R^2$$

EVALUATE: We can use this device as a source of emf in a circuit by completing the circuit through stationary brushes (b in the figure) that contact the disk and its conducting shaft as shown. The emf in such a disk was studied by Faraday; the device is called a *Faraday disk dynamo* or a *homopolar generator.* Unlike the alternator in Example 29.4, the Faraday disk dynamo is a direct-current generator; it produces an emf that is constant in time. Can you use Lenz's law to show that for the direction of rotation in Fig. 29.16, the current in the external circuit must be in the direction shown?

Test Your Understanding of Section 29.4 The earth's magnetic field points toward (magnetic) north. For simplicity, assume that the field has no vertical component (as is the case near the earth's equator). (a) If you hold a metal rod in your hand and walk toward the east, how should you orient the rod to get the maximum motional emf between its ends? (i) east-west; (ii) north-south; (iii) up-down; (iv) you get the same motional emf with all of these orientations. (b) How should you hold it to get *zero* emf as you walk toward the east? (i) east-west; (ii) north-south; (iii) up-down; (iv) none of these. (c) In which direction should you travel so that the motional emf across the rod is zero no matter how the rod is oriented? (i) west; (ii) north; (iii) south; (iv) straight up; (v) straight down.

29.5 Induced Electric Fields

When a conductor moves in a magnetic field, we can understand the induced emf on the basis of magnetic forces on the charges in the conductor, as described in Section 29.4. But an induced emf also occurs when there is a changing flux through a stationary conductor. What is it that pushes the charges around the circuit in this type of situation?

As an example, let's consider the situation shown in Fig. 29.17. A long, thin solenoid with cross-sectional area A and n turns per unit length is encircled at its center by a circular conducting loop. The galvanometer G measures the current in the loop. A current I in the winding of the solenoid sets up a magnetic field $\vec{B}$ along the solenoid axis, as shown, with magnitude B as calculated in Example 28.9 (Section 28.7): $B = \mu_0 nI$, where n is the number of turns per unit length. If we neglect the small field outside the solenoid and take the area vector $\vec{A}$ to point in the same direction as $\vec{B}$, then the magnetic flux Φ_B through the loop is

$$\Phi_B = BA = \mu_0 nIA$$

When the solenoid current I changes with time, the magnetic flux Φ_B also changes, and according to Faraday's law the induced emf in the loop is given by

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\mu_0 nA\frac{dI}{dt} \qquad (29.8)$$

If the total resistance of the loop is R, the induced current in the loop, which we may call I', is $I' = \mathcal{E}/R$.

But what *force* makes the charges move around the loop? It can't be a magnetic force because the conductor isn't moving in a magnetic field and in fact isn't even *in* a magnetic field. We are forced to conclude that there has to be an **induced electric field** in the conductor *caused by the changing magnetic flux.* This may be a little jarring; we are accustomed to thinking about electric field as being caused by electric charges, and now we are saying that a changing magnetic field somehow acts as a source of electric field. Furthermore, it's a strange sort of electric field. When a charge q goes once around the loop, the total work done on it by the electric field must be equal to q times the emf $\mathcal{E}$. That is, the electric field in the loop *is not conservative,* as we used the term in Chapter 23, because the line integral of $\vec{E}$ around a closed path is not zero. Indeed, this line integral, representing the work done by the induced $\vec{E}$ field per unit charge, is equal to the induced emf $\mathcal{E}$:

$$\oint \vec{E} \cdot d\vec{l} = \mathcal{E} \qquad (29.9)$$

From Faraday's law the emf $\mathcal{E}$ is also the negative of the rate of change of magnetic flux through the loop. Thus for this case we can restate Faraday's law as

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \text{(stationary integration path)} \qquad (29.10)$$

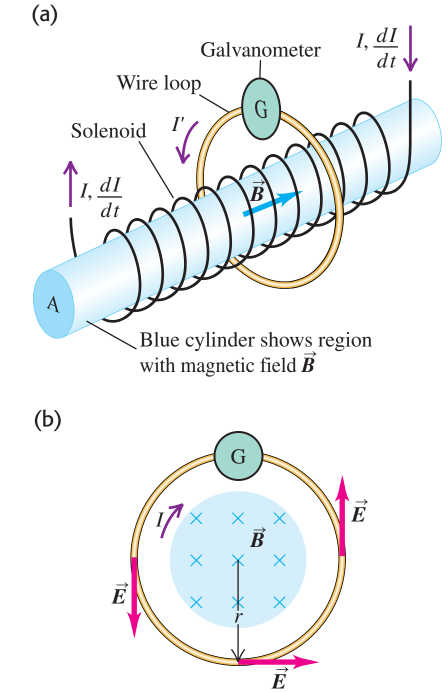
Note that Faraday's law is always true in the form $\mathcal{E} = -d\Phi_B/dt$; the form given in Eq. (29.10) is valid *only* if the path around which we integrate is *stationary.*

As an example of a situation to which Eq. (29.10) can be applied, consider the stationary circular loop in Fig. 29.17b, which we take to have radius r. Because of cylindrical symmetry, the electric field $\vec{E}$ has the same magnitude at every point on the circle and is tangent to it at each point. (Symmetry would also permit the field to be *radial,* but then Gauss's law would require the presence of a net charge inside the circle, and there is none.) The line integral in Eq. (29.10) becomes simply the magnitude E times the circumference $2\pi r$ of the loop, $\oint \vec{E} \cdot d\vec{l} = 2\pi r E$, and Eq. (29.10) gives

$$E = \frac{1}{2\pi r}\left|\frac{d\Phi_B}{dt}\right| \qquad (29.11)$$

The directions of $\vec{E}$ at points on the loop are shown in Fig. 29.17b. We know that $\vec{E}$ has to have the direction shown when $\vec{B}$ in the solenoid is increasing, because

29.17 (a) The windings of a long solenoid carry a current I that is increasing at a rate dI/dt. The magnetic flux in the solenoid is increasing at a rate $d\Phi_B/dt$, and this changing flux passes through a wire loop. An emf $\mathcal{E} = -d\Phi_B/dt$ is induced in the loop, inducing a current I' that is measured by the galvanometer G. (b) Cross-sectional view.



$\oint \vec{E} \cdot d\vec{l}$ has to be negative when $d\Phi_B/dt$ is positive. The same approach can be used to find the induced electric field *inside* the solenoid when the solenoid $\vec{B}$ field is changing; we leave the details to you (see Exercise 29.29).

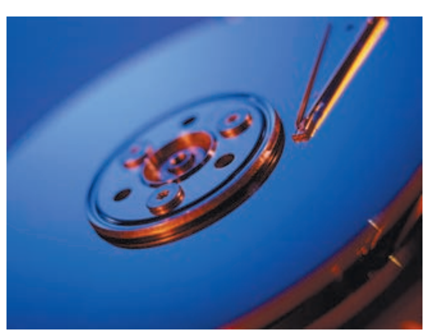
Nonelectrostatic Electric Fields

Now let's summarize what we've learned. Faraday's law, Eq. (29.3), is valid for two rather different situations. In one, an emf is induced by magnetic forces on charges when a conductor moves through a magnetic field. In the other, a time-varying magnetic field induces an electric field in a stationary conductor and hence induces an emf; in fact, the $\vec{E}$ field is induced even when no conductor is present. This $\vec{E}$ field differs from an electro*static* field in an important way. It is *nonconservative;* the line integral $\oint \vec{E} \cdot d\vec{l}$ around a closed path is not zero, and when a charge moves around a closed path, the field does a nonzero amount of work on it. It follows that for such a field the concept of *potential* has no meaning. We call such a field a **nonelectrostatic field.** In contrast, an electro*static* field is *always* conservative, as we discussed in Section 23.1, and always has an associated potential function. Despite this difference, the fundamental effect of *any* electric field is to exert a force $\vec{F} = q\vec{E}$ on a charge q. This relationship is valid whether $\vec{E}$ is a conservative field produced by a charge distribution or a nonconservative field caused by changing magnetic flux.

So a changing magnetic field acts as a source of electric field of a sort that we *cannot* produce with any static charge distribution. This may seem strange, but it's the way nature behaves. What's more, we'll see in Section 29.7 that a changing *electric* field acts as a source of *magnetic* field. We'll explore this symmetry between the two fields in greater detail in our study of electromagnetic waves in Chapter 32.

If any doubt remains in your mind about the reality of magnetically induced electric fields, consider a few of the many practical applications (Fig. 29.18). In the playback head of a tape deck, currents are induced in a stationary coil as the variously magnetized regions of the tape move past it. Computer disk drives operate on the same principle. Pickups in electric guitars use currents induced in stationary pickup coils by the vibration of nearby ferromagnetic strings. Alternators in most cars use rotating magnets to induce currents in stationary coils. The list goes on and on; whether we realize it or not, magnetically induced electric fields play an important role in everyday life.

29.18 Applications of induced electric fields. (a) Data are stored on a computer hard disk in a pattern of magnetized areas on the surface of the disk. To read these data, a coil on a movable arm is placed next to the spinning disk. The coil experiences a changing magnetic flux, inducing a current whose characteristics depend on the pattern coded on the disk. (b) This hybrid automobile has both a gasoline engine and an electric motor. As the car comes to a halt, the spinning wheels run the motor backward so that it acts as a generator. The resulting induced current is used to recharge the car's batteries. (c) The rotating crankshaft of a piston-engine airplane spins a magnet, inducing an emf in an adjacent coil and generating the spark that ignites fuel in the engine cylinders. This keeps the engine running even if the airplane's other electrical systems fail.



Example 29.12 Induced electric fields

Suppose the long solenoid in Fig. 29.17a is wound with 500 turns per meter and the current in its windings is increasing at the rate of 100 A/s. The cross-sectional area of the solenoid is 4.0 cm^2 = 4.0×10^{-4} m^2. (a) Find the magnitude of the induced emf in the wire loop outside the solenoid. (b) Find the magnitude of the induced electric field within the loop if its radius is 2.0 cm.

SOLUTION

IDENTIFY: As in Fig. 29.17b, the increasing magnetic field inside the solenoid causes a change in the magnetic flux through the wire loop and hence induces an electric field $\vec{E}$ around the loop. Our target variables are the induced emf $\mathcal{E}$ and the magnitude of $\vec{E}$.

SET UP: We use Eq. (29.8) to determine the emf. Determining the field magnitude E is simplified because the loop and the solenoid share the same central axis. Hence, by symmetry, the electric field is tangent to the loop and has the same magnitude all the way around its circumference. This makes it easy to find E from the emf $\mathcal{E}$ using Eq. (29.9).

EXECUTE: (a) From Eq. (29.8), the induced emf is

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\mu_0 nA\frac{dI}{dt} \\ &= -(4\pi \times 10^{-7}\ \text{Wb/A}\cdot\text{m})(500\ \text{turns/m}) \\ &\quad \times (4.0 \times 10^{-4}\ \text{m}^2)(100\ \text{A/s}) \\ &= -25 \times 10^{-6}\ \text{Wb/s} = -25 \times 10^{-6}\ \text{V} = -25\ \mu\text{V}\end{aligned}$$

(b) By symmetry the line integral $\oint \vec{E} \cdot d\vec{l}$ has absolute value $2\pi rE$ (disregarding the direction in which we integrate around the loop). This is equal to the absolute value of the emf, so

$$E = \frac{|\mathcal{E}|}{2\pi r} = \frac{25 \times 10^{-6}\ \text{V}}{2\pi(2.0 \times 10^{-2}\ \text{m})} = 2.0 \times 10^{-4}\ \text{V/m}$$

EVALUATE: In Fig. 29.17b the magnetic flux *into* the plane of the figure is increasing. According to the right-hand rule for induced emf (illustrated in Fig. 29.6), a positive emf would be clockwise around the loop; the negative sign of $\mathcal{E}$ shows that the emf is in the counterclockwise direction. Can you also show this using Lenz's law?

Test Your Understanding of Section 29.5 If you wiggle a magnet back and forth in your hand, are you generating an electric field? If so, is this electric field conservative?

*29.6 Eddy Currents

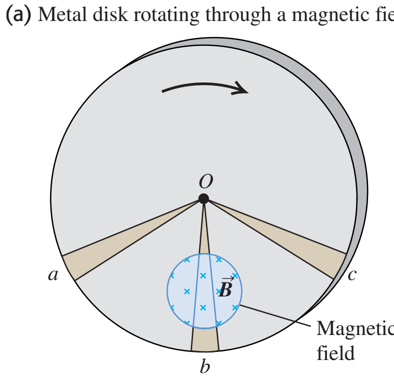
In the examples of induction effects that we have studied, the induced currents have been confined to well-defined paths in conductors and other components forming a circuit. However, many pieces of electrical equipment contain masses of metal moving in magnetic fields or located in changing magnetic fields. In situations like these we can have induced currents that circulate throughout the volume of a material. Because their flow patterns resemble swirling eddies in a river, we call these **eddy currents.**

As an example, consider a metallic disk rotating in a magnetic field perpendicular to the plane of the disk but confined to a limited portion of the disk's area, as shown in Fig. 29.19a. Sector Ob is moving across the field and has an emf induced in it. Sectors Oa and Oc are not in the field, but they provide return conducting paths for charges displaced along Ob to return from b to O. The result is a circulation of eddy currents in the disk, somewhat as sketched in Fig. 29.19b.

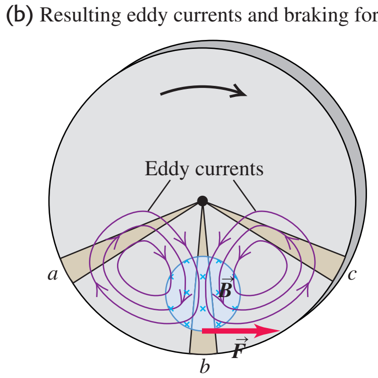
We can use Lenz's law to decide on the direction of the induced current in the neighborhood of sector Ob. This current must experience a magnetic force $\vec{F} = I\vec{L} \times \vec{B}$ that *opposes* the rotation of the disk, and so this force must be to the right in Fig. 29.19b. Since $\vec{B}$ is directed into the plane of the disk, the current and hence $\vec{L}$ have downward components. The return currents lie outside the field, so they do not experience magnetic forces. The interaction between the eddy currents

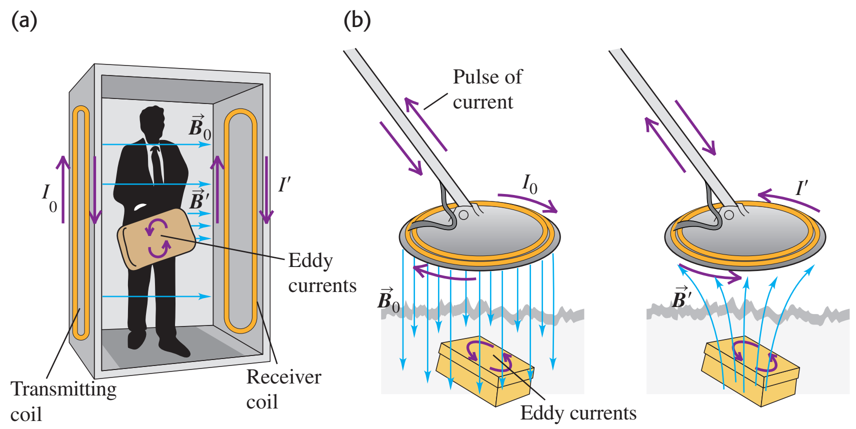
29.19 Eddy currents induced in a rotating metal disk.

(a) Metal disk rotating through a magnetic field



(b) Resulting eddy currents and braking force





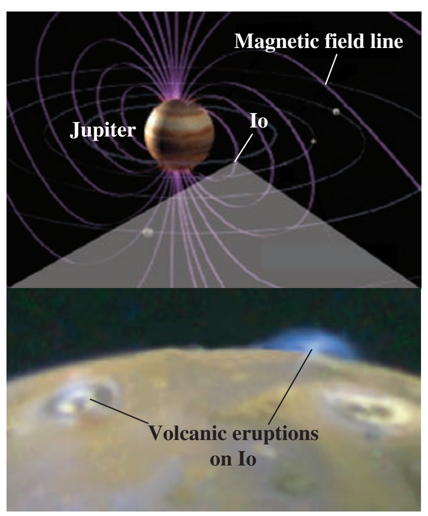
29.20 (a) A metal detector at an airport security checkpoint generates an alternating magnetic field $\vec{B}_0$. This induces eddy currents in a conducting object carried through the detector. The eddy currents in turn produce an alternating magnetic field $\vec{B}'$, and this field induces a current in the detector's receiver coil. (b) Portable metal detectors work on the same principle.

and the field causes a braking action on the disk. Such effects can be used to stop the rotation of a circular saw quickly when the power is turned off. Some sensitive balances use this effect to damp out vibrations. Eddy current braking is used on some electrically powered rapid-transit vehicles. Electromagnets mounted in the cars induce eddy currents in the rails; the resulting magnetic fields cause braking forces on the electromagnets and thus on the cars.

Eddy currents have many other practical uses. The shiny metal disk in the electric power company's meter outside your house rotates as a result of eddy currents. These currents are induced in the disk by magnetic fields caused by sinusoidally varying currents in a coil. In induction furnaces, eddy currents are used to heat materials in completely sealed containers for processes in which it is essential to avoid the slightest contamination of the materials. The metal detectors used at airport security checkpoints (Fig. 29.20a) operate by detecting eddy currents induced in metallic objects. Similar devices (Fig. 29.20b) are used to find buried treasure such as bottlecaps and lost pennies.

A particularly dramatic example of eddy currents in action is Jupiter's moon Io, which is slightly larger than the earth's moon (Fig. 29.21a). Io moves rapidly through Jupiter's intense magnetic field, and this sets up strong eddy currents within Io's interior. These currents dissipate energy at a rate of 10^{12} W, equivalent to setting off a one-kiloton nuclear weapon inside Io every four seconds! This dissipated energy helps to keep Io's interior hot and so helps to cause volcanic eruptions on its surface, like those in Fig. 29.21b. (Gravitational effects from Jupiter cause even more heating.)

29.21 As Jupiter's moon Io moves around its orbit, the planet's powerful magnetic field induces eddy currents within Io. The lower closeup image shows two simultaneous volcanic eruptions on Io, triggered in part by eddy current heating.



Eddy currents also have undesirable effects. In an alternating-current transformer, coils wrapped around an iron core carry a sinusoidally varying current. The resulting eddy currents in the core waste energy through I^2R heating and themselves set up an unwanted opposing emf in the coils. To minimize these effects, the core is designed so that the paths for eddy currents are as narrow as possible. We'll describe how this is done when we discuss transformers in detail in Section 31.6.

Test Your Understanding of Section 29.6 Suppose that the magnetic field in Fig. 29.19 were directed out of the plane of the figure and the disk were rotating counterclockwise. Compared to the directions of the force $\vec{F}$ and the eddy currents shown in Fig. 29.19b, what would the new directions be? (i) The force $\vec{F}$ and the eddy currents would both be in the same direction; (ii) the force $\vec{F}$ would be in the same direction, but the eddy currents would be in the opposite direction; (iii) the force $\vec{F}$ would be in the opposite direction, but the eddy currents would be in the same direction; (iv) the force $\vec{F}$ and the eddy currents would be in the opposite directions.

29.7 Displacement Current and Maxwell's Equations

We have seen that a varying magnetic field gives rise to an induced electric field. In one of the more remarkable examples of the symmetry of nature, it turns out that a varying *electric* field gives rise to a *magnetic* field. This effect is of tremendous importance, for it turns out to explain the existence of radio waves, gamma rays, and visible light, as well as all other forms of electromagnetic waves.

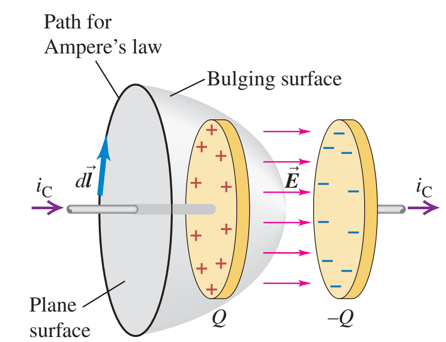
Generalizing Ampere's Law

To see the origin of the relationship between varying electric fields and magnetic fields, let's return to Ampere's law as given in Section 28.6, Eq. (28.20):

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

The problem with Ampere's law in this form is that it is *incomplete.* To see why, let's consider the process of charging a capacitor (Fig. 29.22). Conducting wires lead current i_C into one plate and out of the other; the charge Q increases, and the electric field $\vec{E}$ between the plates increases. The notation i_C indicates *conduction* current to distinguish it from another kind of current we are about to encounter, called *displacement* current i_D. We use lowercase i's and v's to denote instantaneous values of currents and potential differences, respectively, that may vary with time.

29.22 Parallel-plate capacitor being charged. The conduction current through the plane surface is i_C, but there is no conduction current through the surface that bulges out to pass between the plates. The two surfaces have a common boundary, so this difference in I_{encl} leads to an apparent contradiction in applying Ampere's law.



Let's apply Ampere's law to the circular path shown. The integral $\oint \vec{B} \cdot d\vec{l}$ around this path equals $\mu_0 I_{\text{encl}}$. For the plane circular area bounded by the circle, I_{encl} is just the current i_C in the left conductor. But the surface that bulges out to the right is bounded by the same circle, and the current through that surface is zero. So $\oint \vec{B} \cdot d\vec{l}$ is equal to $\mu_0 i_C$, and at the same time it is equal to zero! This is a clear contradiction.

But something else is happening on the bulged-out surface. As the capacitor charges, the electric field $\vec{E}$ and the electric *flux* Φ_E through the surface are increasing. We can determine their rates of change in terms of the charge and current. The instantaneous charge is $q = Cv$, where C is the capacitance and v is the instantaneous potential difference. For a parallel-plate capacitor, $C = \epsilon_0 A/d$, where A is the plate area and d is the spacing. The potential difference v between plates is $v = Ed$, where E is the electric field magnitude between plates. (We neglect fringing and assume that $\vec{E}$ is uniform in the region between the plates.) If this region is filled with a material with permittivity ϵ, we replace ϵ_0 by ϵ everywhere; we'll use ϵ in the following discussion.

Substituting these expressions for C and v into $q = Cv$, we can express the capacitor charge q as

$$q = Cv = \frac{\epsilon A}{d}(Ed) = \epsilon EA = \epsilon \Phi_E \tag{29.12}$$

where $\Phi_E = EA$ is the electric flux through the surface.

As the capacitor charges, the rate of change of q is the conduction current, $i_C = dq/dt$. Taking the derivative of Eq. (29.12) with respect to time, we get

$$i_C = \frac{dq}{dt} = \epsilon \frac{d\Phi_E}{dt} \tag{29.13}$$

Now, stretching our imagination a little, we invent a fictitious **displacement current** i_D in the region between the plates, defined as

$$i_D = \epsilon \frac{d\Phi_E}{dt} \quad \text{(displacement current)} \tag{29.14}$$

That is, we imagine that the changing flux through the curved surface in Fig. 29.22 is somehow equivalent, in Ampere's law, to a conduction current through that surface. We include this fictitious current, along with the real conduction current i_C, in Ampere's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_C + i_D)_{encl} \qquad \text{(generalized Ampere's law)} \tag{29.15}$$

Ampere's law in this form is obeyed no matter which surface we use in Fig. 29.22. For the flat surface, i_D is zero; for the curved surface, i_C is zero; and i_C for the flat surface equals i_D for the curved surface. Equation (29.15) remains valid in a magnetic material, provided that the magnetization is proportional to the external field and we replace μ_0 by μ.

The fictitious current i_D was invented in 1865 by the Scottish physicist James Clerk Maxwell (1831–1879), who called it displacement current. There is a corresponding *displacement current density* $j_D = i_D/A$; using $\Phi_E = EA$ and dividing Eq. (29.14) by A, we find

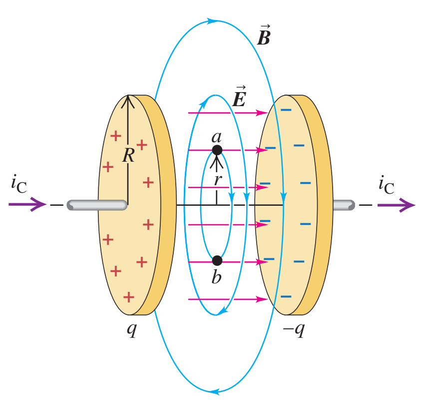
$$j_D = \epsilon \frac{dE}{dt} \tag{29.16}$$

We have pulled the concept out of thin air, as Maxwell did, but we see that it enables us to save Ampere's law in situations such as that in Fig. 29.22.

Another benefit of displacement current is that it lets us generalize Kirchhoff's junction rule, discussed in Section 26.2. Considering the left plate of the capacitor plate, we have conduction current into it but none out of it. But when we include the displacement current, we have conduction current coming in one side and an equal displacement current coming out the other side. With this generalized meaning of the term "current," we can speak of current going *through* the capacitor.

The Reality of Displacement Current

29.23 A capacitor being charged by a current i_C has a displacement current equal to i_C between the plates, with displacement-current density $j_D = \epsilon\, dE/dt$. This can be regarded as the source of the magnetic field between the plates.



You might well ask at this point whether displacement current has any real physical significance or whether it is just a ruse to satisfy Ampere's law and Kirchhoff's junction rule. Here's a fundamental experiment that helps to answer that question. We take a plane circular area between the capacitor plates, as shown in Fig. 29.23. If displacement current really plays the role in Ampere's law that we have claimed, then there ought to be a magnetic field in the region between the plates while the capacitor is charging. We can use our generalized Ampere's law, including displacement current, to predict what this field should be.

To be specific, let's picture round capacitor plates with radius R. To find the magnetic field at a point in the region between the plates at a distance r from the axis, we apply Ampere's law to a circle of radius r passing through the point, with $r < R$. This circle passes through points a and b in Fig. 29.23. The total current enclosed by the circle is j_D times its area, or $(i_D/\pi R^2)(\pi r^2)$. The integral $\oint \vec{B} \cdot d\vec{l}$ in Ampere's law is just B times the circumference $2\pi r$ of the circle, and because $i_D = i_C$ for the charging capacitor, Ampere's law becomes

$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 \frac{r^2}{R^2} i_C \qquad \text{or}$$

$$B = \frac{\mu_0}{2\pi} \frac{r}{R^2} i_C \tag{29.17}$$

This result predicts that in the region between the plates $\vec{B}$ is zero at the axis and increases linearly with distance from the axis. A similar calculation shows that *outside* the region between the plates (that is, for $r > R$), $\vec{B}$ is the same as though the wire were continuous and the plates not present at all.

When we *measure* the magnetic field in this region, we find that it really is there and that it behaves just as Eq. (29.17) predicts. This confirms directly the role of displacement current as a source of magnetic field. It is now established beyond reasonable doubt that displacement current, far from being just an artifice, is a fundamental fact of nature. Maxwell's discovery was the bold step of an extraordinary genius.

Maxwell's Equations of Electromagnetism

We are now in a position to wrap up in a single package *all* of the relationships between electric and magnetic fields and their sources. This package consists of four equations, called **Maxwell's equations.** Maxwell did not discover all of these equations single-handedly (though he did develop the concept of displacement current). But he did put them together and recognized their significance, particularly in predicting the existence of electromagnetic waves.

For now we'll state Maxwell's equations in their simplest form, for the case in which we have charges and currents in otherwise empty space. In Chapter 32 we'll discuss how to modify these equations if a dielectric or a magnetic material is present.

Two of Maxwell's equations involve an integral of $\vec{E}$ or $\vec{B}$ over a closed surface. The first is simply Gauss's law for electric fields, Eq. (22.8), which states that the surface integral of $E_\perp$ over any closed surface equals $1/\epsilon_0$ times the total charge Q_{encl} enclosed within the surface:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \qquad \text{(Gauss's law for } \vec{E}\text{)} \tag{29.18}$$

The second is the analogous relationship for *magnetic* fields, Eq. (27.8), which states that the surface integral of $B_\perp$ over any closed surface is always zero:

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(Gauss's law for } \vec{B}\text{)} \tag{29.19}$$

This statement means, among other things, that there are no magnetic monopoles (single magnetic charges) to act as sources of magnetic field.

The third equation is Ampere's law including displacement current. This states that both conduction current i_C and displacement current $\epsilon_0 d\Phi_E/dt$, where Φ_E is electric flux, act as sources of magnetic field:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{encl} \qquad \text{(Ampere's law)} \tag{29.20}$$

The fourth and final equation is Faraday's law. It states that a changing magnetic field or magnetic flux induces an electric field:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)} \tag{29.21}$$

If there is a changing magnetic flux, the line integral in Eq. (29.21) is not zero, which shows that the $\vec{E}$ field produced by a changing magnetic flux is not conservative. Recall that this line integral must be carried out over a *stationary* closed path.

It's worthwhile to look more carefully at the electric field $\vec{E}$ and its role in Maxwell's equations. In general, the total $\vec{E}$ field at a point in space can be the superposition of an electrostatic field $\vec{E}_c$ caused by a distribution of charges at rest and a magnetically induced, nonelectrostatic field $\vec{E}_n$. (The subscript c stands for Coulomb or conservative; the subscript n stands for non-Coulomb, nonelectrostatic, or nonconservative.) That is,

$$\vec{E} = \vec{E}_c + \vec{E}_n$$

The electrostatic part $\vec{E}_c$ is *always* conservative, so $\oint \vec{E}_c \cdot d\vec{l} = 0$. This conservative part of the field does not contribute to the integral in Faraday's law, so we can take $\vec{E}$ in Eq. (29.21) to be the total electric field $\vec{E}$, including both the part $\vec{E}_c$ due to charges and the magnetically induced part $\vec{E}_n$. Similarly, the nonconservative part $\vec{E}_n$ of the $\vec{E}$ field does not contribute to the integral in Gauss's law, because this part of the field is not caused by static charges. Hence $\oint \vec{E}_n \cdot d\vec{A}$ is always zero. We conclude that in all the Maxwell equations, $\vec{E}$ is the total electric field; these equations don't distinguish between conservative and nonconservative fields.

Symmetry in Maxwell's Equations

There is a remarkable symmetry in Maxwell's four equations. In empty space where there is no charge, the first two equations (Eqs. (29.18) and (29.19)) are identical in form, one containing $\vec{E}$ and the other containing $\vec{B}$. When we compare the second two equations, Eq. (29.20) says that a changing electric flux creates a magnetic field, and Eq. (29.21) says that a changing magnetic flux creates an electric field. In empty space, where there is no conduction current, $i_C = 0$ and the two equations have the same form, apart from a numerical constant and a negative sign, with the roles of $\vec{E}$ and $\vec{B}$ exchanged in the two equations.

We can rewrite Eqs. (29.20) and (29.21) in a different but equivalent form by introducing the definitions of electric and magnetic flux, $\Phi_E = \int \vec{E} \cdot d\vec{A}$ and $\Phi_B = \int \vec{B} \cdot d\vec{A}$, respectively. In empty space, where there is no charge or conduction current, $i_C = 0$ and $Q_{encl} = 0$, and we have

$$\oint \vec{B} \cdot d\vec{l} = \epsilon_0 \mu_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A} \qquad (29.22)$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} \qquad (29.23)$$

Again we notice the symmetry between the roles of $\vec{E}$ and $\vec{B}$ in these expressions.

The most remarkable feature of these equations is that a time-varying field of *either* kind induces a field of the other kind in neighboring regions of space. Maxwell recognized that these relationships predict the existence of electromagnetic disturbances consisting of time-varying electric and magnetic fields that travel or *propagate* from one region of space to another, even if no matter is present in the intervening space. Such disturbances, called *electromagnetic waves,* provide the physical basis for light, radio and television waves, infrared, ultraviolet, x rays, and the rest of the electromagnetic spectrum. We will return to this vitally important topic in Chapter 32.

Although it may not be obvious, *all* the basic relationships between fields and their sources are contained in Maxwell's equations. We can derive Coulomb's law from Gauss's law, we can derive the law of Biot and Savart from Ampere's law, and so on. When we add the equation that defines the $\vec{E}$ and $\vec{B}$ fields in terms of the forces that they exert on a charge q, namely,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad (29.24)$$

we have *all* the fundamental relationships of electromagnetism!

Finally, we note that Maxwell's equations would have even greater symmetry between the $\vec{E}$ and $\vec{B}$ fields if single magnetic charges (magnetic monopoles) existed. The right side of Eq. (29.19) would contain the total *magnetic* charge enclosed by the surface, and the right side of Eq. (29.21) would include a magnetic monopole current term. Perhaps you can begin to see why some physicists wish that magnetic monopoles existed; they would help to perfect the mathematical poetry of Maxwell's equations.

The discovery that electromagnetism can be wrapped up so neatly and elegantly is a very satisfying one. In conciseness and generality, Maxwell's equations are in the same league with Newton's laws of motion and the laws of thermodynamics. Indeed, a major goal of science is learning how to express very broad and general relationships in a concise and compact form. Maxwell's synthesis of electromagnetism stands as a towering intellectual achievement, comparable to the Newtonian synthesis we described at the end of Section 12.5 and to the development of relativity and quantum mechanics in the 20th century.

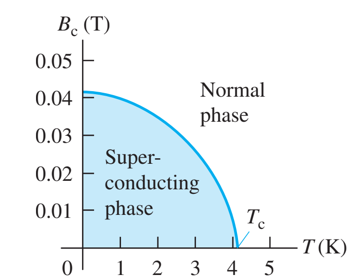
Test Your Understanding of Section 29.7 (a) Which of Maxwell's equations explains how a credit card reader works? (b) Which one describes how a wire carrying a steady current generates a magnetic field?

*29.8 Superconductivity

The most familiar property of a superconductor is the sudden disappearance of all electrical resistance when the material is cooled below a temperature called the *critical temperature,* denoted by T_c. We discussed this behavior and the circumstances of its discovery in Section 25.2. But superconductivity is far more than just the absence of measurable resistance. Superconductors also have extraordinary *magnetic* properties. We'll explore some of these properties in this section.

The first hint of unusual magnetic properties was the discovery that for any superconducting material the critical temperature T_c changes when the material is placed in an externally produced magnetic field $\vec{B}_0$. Figure 29.24 shows this dependence for mercury, the first element in which superconductivity was observed. As the external field magnitude B_0 increases, the superconducting transition occurs at lower and lower temperature. When B_0 is greater than 0.0412 T, *no* superconducting transition occurs. The minimum magnitude of magnetic field that is needed to eliminate superconductivity at a temperature below T_c is called the *critical field,* denoted by B_c.

29.24 Phase diagram for pure mercury, showing the critical magnetic field B_c and its dependence on temperature. Superconductivity is impossible above the critical temperature T_c. The curves for other superconducting materials are similar but with different numerical values.



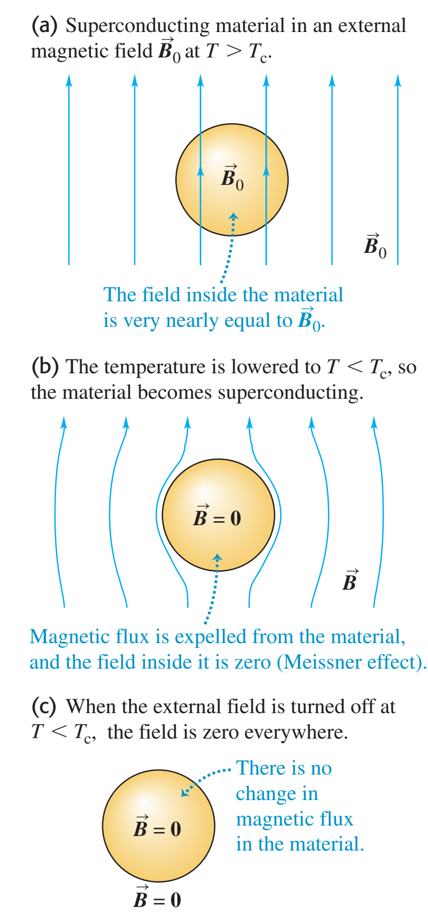
The Meissner Effect

Another aspect of the magnetic behavior of superconductors appears if we place a homogeneous sphere of a superconducting material in a uniform applied magnetic field $\vec{B}_0$ at a temperature T greater than T_c. The material is then in the normal phase, not the superconducting phase. The field is as shown in Figure 29.25a. Now we lower the temperature until the superconducting transition occurs. (We assume that the magnitude of $\vec{B}_0$ is not large enough to prevent the phase transition.) What happens to the field?

Measurements of the field outside the sphere show that the field lines become distorted as in Fig. 29.25b. There is no longer any field inside the material, except possibly in a very thin surface layer a hundred or so atoms thick. If a coil is wrapped around the sphere, the emf induced in the coil shows that during the superconducting transition the magnetic flux through the coil decreases from its initial value to zero; this is consistent with the absence of field inside the material. Finally, if the field is now turned off while the material is still in its superconducting phase, no emf is induced in the coil, and measurements show no field outside the sphere (Fig. 29.25c).

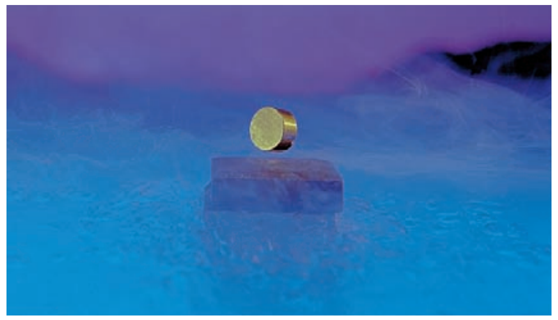
We conclude that during a superconducting transition in the presence of the field $\vec{B}_0$, all of the magnetic flux is expelled from the bulk of the sphere, and the

29.25 A superconducting material (a) above the critical temperature and (b), (c) below the critical temperature.



magnetic flux Φ_B through the coil becomes zero. This expulsion of magnetic flux is called the *Meissner effect.* As shown in Fig. 29.25b, this expulsion crowds the magnetic field lines closer together to the side of the sphere, increasing $\vec{B}$ there.

Superconductor Levitation and Other Applications



29.26 A superconductor (the black slab) exerts a repulsive force on a magnet (the metallic cylinder), supporting the magnet in midair.

The diamagnetic nature of a superconductor has some interesting *mechanical* consequences. A paramagnetic or ferromagnetic material is attracted by a permanent magnet because the magnetic dipoles in the material align with the nonuniform magnetic field of the permanent magnet. (We discussed this in Section 27.7.) For a diamagnetic material the magnetization is in the opposite sense, and a diamagnetic material is *repelled* by a permanent magnet. By Newton's third law the magnet is also repelled by the diamagnetic material. Figure 29.26 shows the repulsion between a specimen of a high-temperature superconductor and a magnet; the magnet is supported ("levitated") by this repulsive magnetic force.

The behavior we have described is characteristic of what are called *type-I superconductors.* There is another class of superconducting materials called *type-II superconductors.* When such a material in the superconducting phase is placed in a magnetic field, the bulk of the material remains superconducting, but thin filaments of material, running parallel to the field, may return to the normal phase. Currents circulate around the boundaries of these filaments, and there *is* magnetic flux inside them. Type-II superconductors are used for electromagnets because they usually have much larger values of B_c than do type-I materials, permitting much larger magnetic fields without destroying the superconducting state. Type-II superconductors have *two* critical magnetic fields: the first, B_{c1}, is the field at which magnetic flux begins to enter the material, forming the filaments just described, and the second, B_{c2}, is the field at which the material becomes normal.

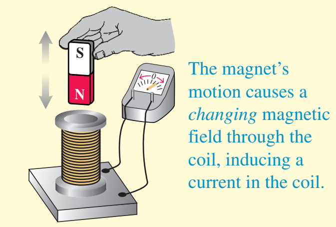
Many important and exciting applications of superconductors are under development. Superconducting electromagnets have been used in research laboratories for several years. Their advantages compared to conventional electromagnets include greater efficiency, compactness, and greater field magnitudes. Once a current is established in the coil of a superconducting electromagnet, no additional power input is required because there is no resistive energy loss. The coils can also be made more compact because there is no need to provide channels for the circulation of cooling fluids. Superconducting magnets routinely attain steady fields of the order of 10 T, much larger than the maximum fields that are available with ordinary electromagnets.

Superconductors are attractive for long-distance electric power transmission and for energy-conversion devices, including generators, motors, and transformers. Very sensitive measurements of magnetic fields can be made with superconducting quantum interference devices (SQUIDs), which can detect changes in magnetic flux of less than 10^{-14} Wb; these devices have applications in medicine, geology, and other fields. The number of potential uses for superconductors has increased greatly since the discovery in 1987 of high-temperature superconductors. These materials have critical temperatures that are above the temperature of liquid nitrogen (about 77 K) and so are comparatively easy to attain. Development of practical applications of superconductor science promises to be an exciting chapter in contemporary technology.

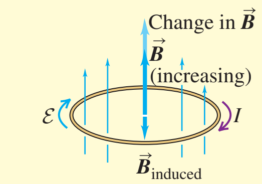
CHAPTER 29 SUMMARY

Faraday's law: Faraday's law states that the induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop. This relationship is valid whether the flux change is caused by a changing magnetic field, motion of the loop, or both. (See Examples 29.1–29.7.)

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad (29.3)$$



Lenz's law: Lenz's law states that an induced current or emf always tends to oppose or cancel out the change that caused it. Lenz's law can be derived from Faraday's law and is often easier to use. (See Examples 29.8 and 29.9.)



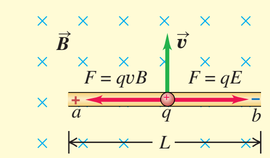
Motional emf: If a conductor moves in a magnetic field, a motional emf is induced. (See Examples 29.10 and 29.11.)

$$\mathcal{E} = vBL \qquad (29.6)$$

(conductor with length L moves in uniform $\vec{B}$ field, $\vec{L}$ and $\vec{v}$ both perpendicular to $\vec{B}$ and to each other)

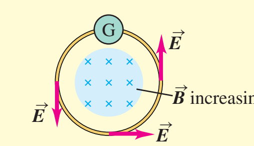
$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} \qquad (29.7)$$

(all or part of a closed loop moves in a $\vec{B}$ field)



Induced electric fields: When an emf is induced by a changing magnetic flux through a stationary conductor, there is an induced electric field $\vec{E}$ of nonelectrostatic origin. This field is nonconservative and cannot be associated with a potential. (See Example 29.12.)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad (29.10)$$



Displacement current and Maxwell's equations: A time-varying electric field generates a displacement current i_D, which acts as a source of magnetic field in exactly the same way as conduction current. The relationships between electric and magnetic fields and their sources can be stated compactly in four equations, called Maxwell's equations. Together they form a complete basis for the relationship of $\vec{E}$ and $\vec{B}$ fields to their sources.

$$i_D = \epsilon \frac{d\Phi_E}{dt} \qquad (29.14)$$

(displacement current)

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \qquad (29.18)$$

(Gauss's law for $\vec{E}$ fields)

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad (29.19)$$

(Gauss's law for $\vec{B}$ fields)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{encl} \qquad (29.20)$$

(Ampere's law including displacement current)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad (29.21)$$

(Faraday's law)

Key Terms

induced current, *994*
induced emf, *994*
Faraday's law of induction, *996*
Lenz's law, *1004*
motional electromotive force, *1006*
induced electric field, *1009*
nonelectrostatic field, *1010*
eddy currents, *1011*
displacement current, *1013*
Maxwell's equations, *1015*

Answer to Chapter Opening Question ?

As the magnetic stripe moves through the card reader, the coded pattern of magnetization in the stripe causes a varying magnetic flux and hence an induced current in the reader's circuits. If the card does not move, there is no induced emf or current and none of the credit card's information is read.

Answers to Test Your Understanding Questions

29.2 Answers: (a) (i), (b) (iii) (a) Initially there is magnetic flux into the plane of the page, which we call positive. While the loop is being squeezed, the flux is becoming less positive $(d\Phi_B/dt < 0)$ and so the induced emf is positive as in Fig. 29.6b $(\mathcal{E} = -d\Phi_B/dt > 0)$. If you point the thumb of your right hand into the page, your fingers curl clockwise, so this is the direction of positive induced emf. (b) Since the coil's shape is no longer changing, the magnetic flux is not changing and there is no induced emf.

29.3 Answers: (a) (i), (b) (iii) In (a), as in the original situation, the magnet and loop are approaching each other and the downward flux through the loop is increasing. Hence the induced emf and induced current are the same. In (b), since the magnet and loop are moving together, the flux through the loop is not changing and no emf is induced.

29.4 Answers: (a) (iii); (b) (i) or (ii); (c) (ii) or (iii) You will get the maximum motional emf if you hold the rod vertically, so that its length is perpendicular to both the magnetic field and the direction of motion. With this orientation, $\vec{L}$ is parallel to $\vec{v} \times \vec{B}$. If you hold the rod in any horizontal orientation, $\vec{L}$ will be perpendicular to $\vec{v} \times \vec{B}$ and no emf will be induced. If you walk due north or south, $\vec{v} \times \vec{B} = \mathbf{0}$ and no emf will be induced for any orientation of the rod.

29.5 Answers: yes, no The magnetic field at a fixed position changes as you move the magnet. Such induced electric fields are *not* conservative.

29.6 Answer: (iii) By Lenz's law, the force must oppose the motion of the disk through the magnetic field. Since the disk material is now moving to the right through the field region, the force $\vec{F}$ is to the left—that is, in the opposite direction to that shown in Fig. 29.19b. To produce a leftward magnetic force $\vec{F} = I\vec{L} \times \vec{B}$ on currents moving through a magnetic field $\vec{B}$ directed out of the plane of the figure, the eddy currents must be moving downward in the figure—that is, in the same direction shown in Fig. 29.19b.

29.7 Answers: (a) Faraday's law, (b) Ampere's law A credit card reader works by inducing currents in the reader's coils as the card's magnetized stripe is swiped (see the answer to the chapter opening question). Ampere's law describes how currents of all kinds (both conduction currents and displacement currents) give rise to magnetic fields.

PROBLEMS

For instructor-assigned homework, go to **www.masteringphysics.com**

Discussion Questions

Q29.1. A sheet of copper is placed between the poles of an electromagnet with the magnetic field perpendicular to the sheet. When the sheet is pulled out, a considerable force is required, and the force required increases with speed. Explain.

Q29.2. In Fig. 29.8, if the angular speed ω of the loop is doubled, then the frequency with which the induced current changes direction doubles, and the maximum emf also doubles. Why? Does the torque required to turn the loop change? Explain.

Q29.3. Two circular loops lie side by side in the same plane. One is connected to a source that supplies an increasing current; the other is a simple closed ring. Is the induced current in the ring in the same direction as the current in the loop connected to the source, or opposite? What if the current in the first loop is decreasing? Explain.

Q29.4. A farmer claimed that the high-voltage transmission lines running parallel to his fence induced dangerously high voltages on the fence. Is this within the realm of possibility? Explain. (The lines carry alternating current that changes direction 120 times each second.)

Q29.5. A long, straight conductor passes through the center of a metal ring, perpendicular to its plane. If the current in the conductor increases, is a current induced in the ring? Explain.

Q29.6. A student asserted that if a permanent magnet is dropped down a vertical copper pipe, it eventually reaches a terminal velocity even if there is no air resistance. Why should this be? Or should it?

Q29.7. An airplane is in level flight over Antarctica, where the magnetic field of the earth is mostly directed upward away from the ground. As viewed by a passenger facing toward the front of the plane, is the left or the right wingtip at higher potential? Does your answer depend on the direction the plane is flying?

Q29.8. Consider the situation in Exercise 29.19. In part (a), find the direction of the force that the large circuit exerts on the small one. Explain how this result is consistent with Lenz's law.

Q29.9. A metal rectangle is close to a long, straight, current-carrying wire, with two of its sides parallel to the wire. If the current in the long wire is decreasing, is the rectangle repelled by or attracted to the wire? Explain why this result is consistent with Lenz's law.

Q29.10. A square conducting loop is in a region of uniform, constant magnetic field. Can the loop be rotated about an axis along one side and no emf be induced in the loop? Discuss, in terms of the orientation of the rotation axis relative to the magnetic-field direction.

Q29.11. Example 29.7 discusses the external force that must be applied to the slidewire to move it at constant speed. If there were a break in the left-hand end of the U-shaped conductor, how much force would be needed to move the slidewire at constant speed? As in the example, you can ignore friction.

Q29.12. In the situation shown in Fig. 29.16, would it be appropriate to ask how much *energy* an electron gains during a complete trip around the wire loop with current I'? Would it be appropriate to ask what *potential difference* the electron moves through during such a complete trip? Explain your answers.

Q29.13. A metal ring is oriented with the plane of its area perpendicular to a spatially uniform magnetic field that increases at a steady rate. If the radius of the ring is doubled, by what factor do (a) the emf induced in the ring and (b) the electric field induced in the ring change?

Q29.14. For Eq. (29.6), show that if v is in meters per second, B in teslas, and L in meters, then the units of the right-hand side of the equation are joules per coulomb or volts (the correct SI units for $\mathcal{E}$).

Q29.15. Can one have a displacement current as well as a conduction current within a conductor? Explain.

Q29.16. Your physics study partner asks you to consider a parallel-plate capacitor that has a dielectric completely filling the volume between the plates. He then claims that Eqs. (29.13) and (29.14) show that the conduction current in the dielectric equals the displacement current in the dielectric. Do you agree? Explain.

Q29.17. Match the mathematical statements of Maxwell's equations as given in Section 29.7 to these verbal statements. (a) Closed electric field lines are evidently produced only by changing magnetic flux. (b) Closed magnetic field lines are produced both by the motion of electric charge and by changing electric flux. (c) Electric field lines can start on positive charges and end on negative charges. (d) Evidently there are no magnetic monopoles on which to start and end magnetic field lines.

Q29.18. If magnetic monopoles existed, the right-hand side of Eq. (29.21) would include a term proportional to the current of magnetic monopoles. Suppose a steady monopole current is moving in a long straight wire. Sketch the *electric* field lines that such a current would produce.

Q29.19. If magnetic monopoles existed, the right-hand side of Eq. (29.19) would be proportional to the total enclosed *magnetic* charge. Suppose an infinite line of magnetic monopoles were on the x-axis. Sketch the magnetic field lines that this line of monopoles would produce.

Exercises

Section 29.2 Faraday's Law

29.1. A flat, rectangular coil consisting of 50 turns measures 25.0 cm by 30.0 cm. It is in a uniform, 1.20-T, magnetic field, with the plane of the coil parallel to the field. In 0.222 s, it is rotated so that the plane of the coil is perpendicular to the field. (a) What is the change in the magnetic flux through the coil due to this rotation? (b) Find the magnitude of the average emf induced in the coil during this rotation.

29.2. In a physics laboratory experiment, a coil with 200 turns enclosing an area of 12 cm^2 is rotated in 0.040 s from a position where its plane is perpendicular to the earth's magnetic field to a position where its plane is parallel to the field. The earth's magnetic field at the lab location is 6.0×10^{-5} T. (a) What is the total magnetic flux through the coil before it is rotated? After it is rotated? b) What is the average emf induced in the coil?

29.3. Search Coils and Credit Cards. (a) Derive the equation relating the total charge Q that flows through a search coil (Conceptual Example 29.3) to the magnetic-field magnitude B. The search coil has N turns, each with area A, and the flux through the coil is decreased from its initial maximum value to zero in a time Δt. The resistance of the coil is R, and the total charge is $Q = I\Delta t$, where I is the average current induced by the change in flux. (b) In a credit card reader, the magnetic strip on the back of a credit card is rapidly "swiped" past a coil within the reader. Explain, using the same ideas that underlie the operation of a search coil, how the reader can decode the information stored in the pattern of magnetization on the strip. (c) Is it necessary that the credit card be "swiped" through the reader at exactly the right speed? Why or why not?

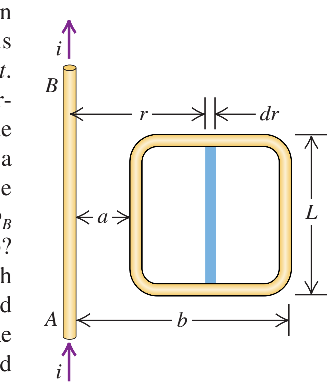
29.4. A closely wound search coil (Exercise 29.3) has an area of 3.20 cm^2, 120 turns, and a resistance of 60.0 Ω. It is connected to a charge-measuring instrument whose resistance is 45.0 Ω. When the coil is rotated quickly from a position parallel to a uniform magnetic field to a position perpendicular to the field, the instrument indicates a charge of 3.56×10^{-5} C. What is the magnitude of the field?

29.5. A circular loop of wire with a radius of 12.0 cm and oriented in the horizontal xy-plane is located in a region of uniform magnetic field. A field of 1.5 T is directed along the positive z-direction, which is upward. (a) If the loop is removed from the field region in a time interval of 2.0 ms, find the average emf that will be induced in the wire loop during the extraction process. (b) If the coil is viewed looking down on it from above, is the induced current in the loop clockwise or counterclockwise?

29.6. A coil 4.00 cm in radius, containing 500 turns, is placed in a uniform magnetic field that varies with time according to $B = (0.0120\ \text{T/s})t + (3.00 \times 10^{-5}\ \text{T/s}^4)t^4$. The coil is connected to a 600-Ω resistor, and its plane is perpendicular to the magnetic field. You can ignore the resistance of the coil. (a) Find the magnitude of the induced emf in the coil as a function of time. (b) What is the current in the resistor at time $t = 5.00$ s?

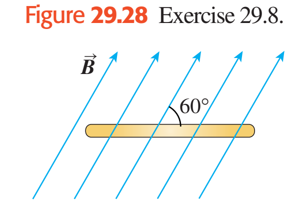
29.7. The current in the long, straight wire AB shown in Fig. 29.27 is upward and is increasing steadily at a rate di/dt. (a) At an instant when the current is i, what are the magnitude and direction of the field $\vec{B}$ at a distance r to the right of the wire? (b) What is the flux $d\Phi_B$ through the narrow, shaded strip? (c) What is the total flux through the loop? (d) What is the induced emf in the loop? (e) Evaluate the numerical value of the induced emf if $a = 12.0$ cm, $b = 36.0$ cm, $L = 24.0$ cm, and $di/dt = 9.60$ A/s.

Figure 29.27 Exercise 29.7.



29.8. A flat, circular, steel loop of radius 75 cm is at rest in a uniform magnetic field, as shown in an edge-on view in Fig. 29.28. The field is changing with time, according to $B(t) = (1.4\ \text{T})e^{-(0.057\ \text{s}^{-1})t}$. (a) Find the emf induced in the loop as a function of time. (b) When is the induced emf equal to $\frac{1}{10}$ of its initial value? (c) Find the direction of the current induced in the loop, as viewed from above the loop.

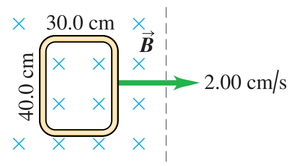
Figure 29.28 Exercise 29.8.



29.9. Shrinking Loop. A circular loop of flexible iron wire has an initial circumference of 165.0 cm, but its circumference is decreasing at a constant rate of 12.0 cm/s due to a tangential pull on the wire. The loop is in a constant, uniform magnetic field oriented perpendicular to the plane of the loop and with magnitude 0.500 T. (a) Find the emf induced in the loop at the instant when 9.0 s have passed. (b) Find the direction of the induced current in the loop as viewed looking along the direction of the magnetic field.

29.10. A rectangle measuring 30.0 cm by 40.0 cm is located inside a region of a spatially uniform magnetic field of 1.25 T, with the field perpendicular to the plane of the coil (Fig. 29.29). The coil is pulled out at a steady rate of 2.00 cm/s traveling perpendicular to the field lines. The region of the field ends abruptly as shown. Find the emf induced in this coil when it is (a) all inside the field; (b) partly inside the field; (c) all outside the field.

Figure 29.29 Exercise 29.10.



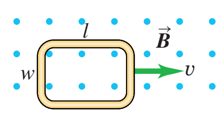
29.11. In a region of space, a magnetic field points in the $+x$-direction (toward the right). Its magnitude varies with position according to the formula $B_x = B_0 + bx$, where B_0 and b are positive constants, for $x \geq 0$. A flat coil of area A moves with uniform speed v from right to left with the plane of its area always perpendicular to this field. (a) What is the emf induced in this coil while it is to the right of the origin? (b) As viewed from the origin, what is the direction (clockwise or counterclockwise) of the current induced in the coil? (c) If instead the coil moved from left to right, what would be the answers to parts (a) and (b)?

29.12. Back emf. A motor with a brush-and-commutator arrangement, as described in Example 29.5, has a circular coil with radius 2.5 cm and 150 turns of wire. The magnetic field has magnitude 0.060 T, and the coil rotates at 440 rev/min. (a) What is the maximum emf induced in the coil? (b) What is the average back emf?

29.13. The armature of a small generator consists of a flat, square coil with 120 turns and sides with a length of 1.60 cm. The coil rotates in a magnetic field of 0.0750 T. What is the angular speed of the coil if the maximum emf produced is 24.0 mV?

29.14. A flat, rectangular coil of dimensions l and w is pulled with uniform speed v through a uniform magnetic field B with the plane of its area perpendicular to the field (Fig. 29.30). (a) Find the emf induced in this coil. (b) If the speed and magnetic field are both tripled, what is the induced emf?

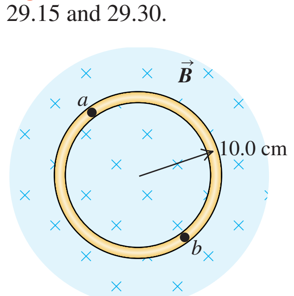
Figure 29.30 Exercise 29.14.



Section 29.3 Lenz's Law

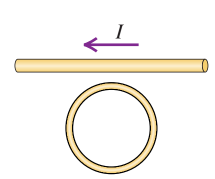
29.15. A circular loop of wire is in a region of spatially uniform magnetic field, as shown in Fig. 29.31. The magnetic field is directed into the plane of the figure. Determine the direction (clockwise or counterclockwise) of the induced current in the loop when (a) B is increasing; (b) B is decreasing; (c) B is constant with value B_0. Explain your reasoning.

Figure 29.31 Exercise 29.15 and 29.30.



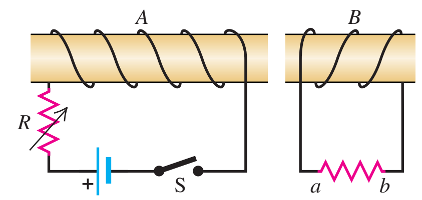
29.16. The current in Fig. 29.32 obeys the equation $I(t) = I_0 e^{-bt}$, where $b > 0$. Find the direction (clockwise or counterclockwise) of the current induced in the round coil for $t > 0$.

Figure 29.32 Exercise 29.16.



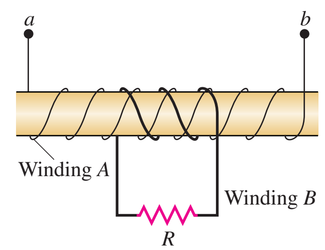
29.17. Using Lenz's law, determine the direction of the current in resistor ab of Fig. 29.33 when (a) switch S is opened after having been closed for several minutes; (b) coil B is brought closer to coil A with the switch closed; (c) the resistance of R is decreased while the switch remains closed.

Figure 29.33 Exercise 29.17.



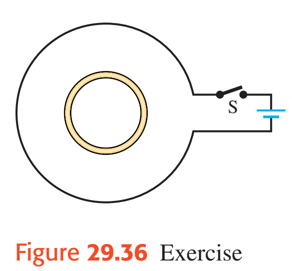
29.18. A cardboard tube is wrapped with two windings of insulated wire wound in opposite directions, as shown in Fig. 29.34. Terminals a and b of winding A may be connected to a battery through a reversing switch. State whether the induced current in the resistor R is from left to right or from right to left in the following circumstances: (a) the current in winding A is from a to b and is increasing; (b) the current in winding A is from b to a and is decreasing; (c) the current in winding A is from b to a and is increasing.

Figure 29.34 Exercise 29.18.



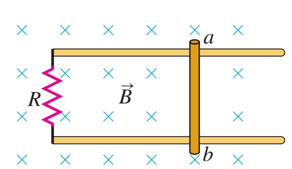
29.19. A small, circular ring is inside a larger loop that is connected to a battery and a switch, as shown in Fig. 29.35. Use Lenz's law to find the direction of the current induced in the small ring (a) just after switch S is closed; (b) after S has been closed a long time; (c) just after S has been reopened after being closed a long time.

Figure 29.35 Exercise 29.19.



29.20. A 1.50-m-long metal bar is pulled to the right at a steady 5.0 m/s perpendicular to a uniform, 0.750-T magnetic field. The bar rides on parallel metal rails connected through a 25.0-Ω resistor, as shown in Fig. 29.36, so the apparatus makes a complete circuit. You can ignore the resistance of the bar and the rails. (a) Calculate the magnitude of the emf induced in the circuit. (b) Find the direction of the current induced in the circuit (i) using the magnetic force on the charges in the moving bar; (ii) using Faraday's law; (iii) using Lenz's law. (c) Calculate the current through the resistor.

Figure 29.36 Exercise 29.20 and Problem 29.64.

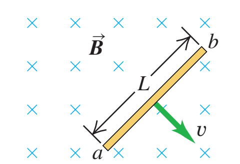


Section 29.4 Motional Electromotive Force

29.21. In Fig. 29.37 a conducting rod of length $L = 30.0$ cm moves in a magnetic field $\vec{B}$ of magnitude 0.450 T directed into the plane of the figure. The rod moves with speed $v = 5.00$ m/s in the direction

shown. (a) what is the potential difference between the ends of the rod? (b) Which point, a to b, is at higher potential? (c) When the charges in the rod are in equilibrium, what are the magnitude and direction of the electric field within the rod? (d) When the charges in the rod are in equilibrium, which point, a or b, has an excess of positive charge? (e) What is the potential difference across the rod if it moves (i) parallel to ab and (ii) directly out of the page?

Figure 29.37 Exercise 29.21.



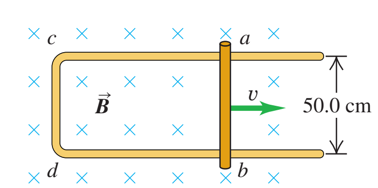
29.22. For the situation in Exercise 29.20, find (a) the motional emf in the bar and (b) the current through the resistor.

29.23. Are Motional emfs a Practical Source of Electricity? How fast (in m/s and mph) would a 5.00-cm copper bar have to move at right angles to a 0.650-T magnetic field to generate 1.50 V (the same as a AA battery) across its ends? Does this seem like a practical way to generate electricity?

29.24. Motional emfs in Transportation. Airplanes and trains move through the earth's magnetic field at rather high speeds, so it is reasonable to wonder whether this field can have a substantial effect on them. We shall use a typical value of 0.50 G for the earth's field (a) The French TGV train and the Japanese "bullet train" reach speeds of up to 180 mph moving on tracks about 1.5 m apart. At top speed moving perpendicular to the earth's magnetic field, what potential difference is induced across the tracks as the wheels roll? Does this seem large enough to produce noticeable effects? (b) The Boeing 747-400 aircraft has a wingspan of 64.4 m and a cruising speed of 565 mph. If there is no wind blowing (so that this is also their speed relative to the ground), what is the maximum potential difference that could be induced between the opposite tips of the wings? Does this seem large enough to cause problems with the plane?

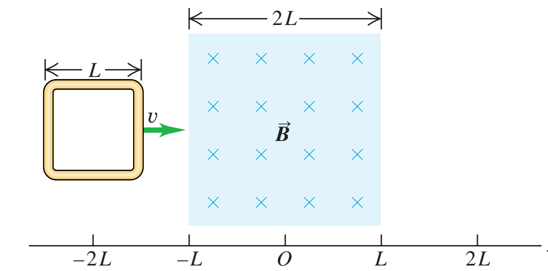
29.25. The conducting rod ab shown in Fig. 29.38 makes contact with metal rails ca and db. The apparatus is in a uniform magnetic field of 0.800 T, perpendicular to the plane of the figure (a) Find the magnitude of the emf induced in the rod when it is moving toward the right with a speed 7.50 m/s. (b) In what direction does the current flow in the rod? (c) If the resistance of the circuit $abdc$ is 1.50 Ω (assumed to be constant), find the force (magnitude and direction) required to keep the rod moving to the right with a constant speed of 7.50 m/s. You can ignore friction. (d) Compare the rate at which mechanical work is done by the force (Fv) with the rate at which thermal energy is developed in the circuit (I^2R).

Figure 29.38 Exercise 29.25.



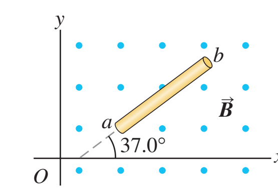
29.26. A square loop of wire with side length L and resistance R is moved at constant speed v across a uniform magnetic field confined to a square region whose sides are twice the length of those of the square loop (Fig. 29.39). (a) Graph the external force F needed to move the loop at constant speed as a function of the coordinate x from $x = -2L$ to $x = +2L$. (The coordinate x is measured from the center of the magnetic-field region to the center of the loop. It is negative when the center of the loop is to the left of the center of the magnetic-field region. Take positive force to be to the right.) (b) Graph the induced current in the loop as a function of x. Take counterclockwise currents to be positive.

Figure 29.39 Exercise 29.26.



29.27. A 1.41-m bar moves through a uniform, 1.20-T magnetic field with a speed of 2.50 m/s (Fig. 29.40). In each case, find the emf induced between the ends of this bar and identify which, if any, end (a or b) is at the higher potential. The bar moves in the direction of (a) the $+x$-axis; (b) the $-y$-axis; (c) the $+z$-axis. (d) How should this bar move so that the emf across its ends has the greatest possible value with b at a higher potential than a, and what is this maximum emf?

Figure 29.40 Exercise 29.27.



Section 29.5 Induced Electric Fields

29.28. A long, thin solenoid has 900 turns per meter and radius 2.50 cm. The current in the solenoid is increasing at a uniform rate of 60.0 A/s. What is the magnitude of the induced electric field at a point near the center of the solenoid and (a) 0.500 cm from the axis of the solenoid; (b) 1.00 cm from the axis of the solenoid?

29.29. The magnetic field within a long, straight solenoid with a circular cross section and radius R is increasing at a rate of dB/dt. (a) What is the rate of change of flux through a circle with radius r_1 inside the solenoid, normal to the axis of the solenoid, and with center on the solenoid axis? (b) Find the magnitude of the induced electric field inside the solenoid, at a distance r_1 from its axis. Show the direction of this field in a diagram. (c) What is the magnitude of the induced electric field *outside* the solenoid, at a distance r_2 from the axis? (d) Graph the magnitude of the induced electric field as a function of the distance r from the axis from $r = 0$ to $r = 2R$. (e) What is the magnitude of the induced emf in a circular turn of radius $R/2$ that has its center on the solenoid axis? (f) What is the magnitude of the induced emf if the radius in part (e) is R? (g) What is the induced emf if the radius in part (e) is $2R$?

29.30. The magnetic field $\vec{B}$ at all points within the colored circle shown in Fig. 29.31 has an initial magnitude of 0.750 T. (The circle could represent approximately the space inside a long, thin solenoid.) The magnetic field is directed into the plane of the diagram and is decreasing at the rate of −0.0350 T/s. (a) What is the shape of the field lines of the induced electric field shown in Fig. 29.31, within the colored circle? (b) What are the magnitude and direction of this field at any point on the circular conducting ring with radius 0.100 m? (c) What is the current in the ring if its resistance is 4.00 Ω? (d) What is the emf between points a and b on the ring? (e) If the ring is cut at some point and the ends are separated slightly, what will be the emf between the ends?

29.31. A long, thin solenoid has 400 turns per meter and radius 1.10 cm. The current in the solenoid is increasing at a uniform rate

di/dt. The induced electric field at a point near the center of the solenoid and 3.50 cm from its axis is 8.00×10^{-6} V/m. Calculate di/dt.

29.32. A metal ring 4.50 cm in diameter is placed between the north and south poles of large magnets with the plane of its area perpendicular to the magnetic field. These magnets produce an initial uniform field of 1.12 T between them but are gradually pulled apart, causing this field to remain uniform but decrease steadily at 0.250 T/s. (a) What is the magnitude of the electric field induced in the ring? (b) In which direction (clockwise or counterclockwise) does the current flow as viewed by someone on the south pole of the magnet?

29.33. A long, straight solenoid with a cross-sectional area of 8.00 cm^2 is wound with 90 turns of wire per centimeter, and the windings carry a current of 0.350 A. A second winding of 12 turns encircles the solenoid at its center. The current in the solenoid is turned off such that the magnetic field of the solenoid becomes zero in 0.0400 s. What is the average induced emf in the second winding?

Section 29.7 Displacement Current and Maxwell's Equations

29.34. A dielectric of permittivity 3.5×10^{-11} F/m completely fills the volume between two capacitor plates. For $t > 0$ the electric flux through the dielectric is $(8.0 \times 10^3\ \text{V}\cdot\text{m/s}^3)t^3$. The dielectric is ideal and nonmagnetic; the conduction current in the dielectric is zero. At what time does the displacement current in the dielectric equal 21 μA?

29.35. The electric flux through a certain area of a dielectric is $(8.76 \times 10^3\ \text{V}\cdot\text{m/s}^4)t^4$. The displacement current through that area is 12.9 pA at time $t = 26.1$ ms. Calculate the dielectric constant for the dielectric.

29.36. A parallel-plate, air-filled capacitor is being charged as in Fig. 29.23. The circular plates have radius 4.00 cm, and at a particular instant the conduction current in the wires is 0.280 A. (a) What is the displacement current density j_D in the air space between the plates? (b) What is the rate at which the electric field between the plates is changing? (c) What is the induced magnetic field between the plates at a distance of 2.00 cm from the axis? (d) At 1.00 cm from the axis?

29.37. Displacement Current in a Dielectric. Suppose that the parallel plates in Fig. 29.23 have an area of 3.00 cm^2 and are separated by a 2.50-mm-thick sheet of dielectric that completely fills the volume between the plates. The dielectric has dielectric constant 4.70. (You can ignore fringing effects.) At a certain instant, the potential difference between the plates is 120 V and the conduction current i_C equals 6.00 mA. At this instant, what are (a) the charge q on each plate; (b) the rate of change of charge on the plates; (c) the displacement current in the dielectric?

29.38. In Fig. 29.23 the capacitor plates have area 5.00 cm^2 and separation 2.00 mm. The plates are in vacuum. The charging current i_C has a *constant* value of 1.80 mA. At $t = 0$ the charge on the plates is zero. (a) Calculate the charge on the plates, the electric field between the plates, and the potential difference between the plates when $t = 0.500\ \mu$s. (b) Calculate dE/dt, the time rate of change of the electric field between the plates. Does dE/dt vary in time? (c) Calculate the displacement current density j_D between the plates, and from this the total displacement current i_D. How do i_C and i_D compare?

29.39. Displacement Current in a Wire. A long, straight, copper wire with a circular cross-sectional area of 2.1 mm^2 carries a current of 16 A. The resistivity of the material is $2.0 \times 10^{-8}\ \Omega\cdot$m. (a) What is the uniform electric field in the material? (b) If the current is changing at the rate of 4000 A/s, at what rate is the electric field in the material changing? (c) What is the displacement current density in the material in part (b)? (*Hint:* Since K for copper is very close to 1, use $\epsilon = \epsilon_0$.) (d) If the current is changing as in part (b), what is the magnitude of the magnetic field 6.0 cm from the center of the wire? Note that both the conduction current and the displacement current should be included in the calculation of B. Is the contribution from the displacement current significant?

*Section 29.8 Superconductivity

***29.40.** A long, straight wire made of a type-I superconductor carries a constant current I along its length. Show that the current cannot be uniformly spread over the wire's cross section but instead must all be at the surface.

***29.41.** A type-II superconductor in an external field between B_{c1} and B_{c2} has regions that contain magnetic flux and have resistance, and also has superconducting regions. What is the resistance of a long, thin cylinder of such material?

***29.42.** At temperatures near absolute zero, B_c approaches 0.142 T for vanadium, a type-I superconductor. The normal phase of vanadium has a magnetic susceptibility close to zero. Consider a long, thin vanadium cylinder with its axis parallel to an external magnetic field $\vec{B}_0$ in the $+x$-direction. At points far from the ends of the cylinder, by symmetry, all the magnetic vectors are parallel to the x-axis. At temperatures near absolute zero, what are the resultant magnetic field $\vec{B}$ and the magnetization $\vec{M}$ inside and outside the cylinder (far from the ends) for (a) $\vec{B}_0 = (0.130\ \text{T})\hat{\imath}$ and (b) $\vec{B}_0 = (0.260\ \text{T})\hat{\imath}$?

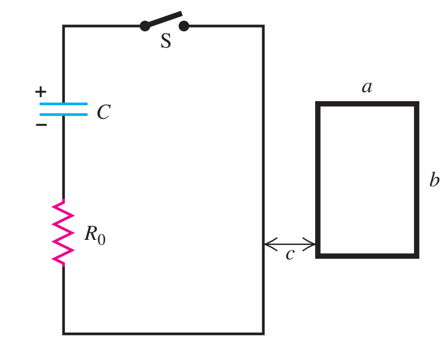
***29.43.** The compound SiV_3 is a type-II superconductor. At temperatures near absolute zero the two critical fields are $B_{c1} =$ 55.0 mT and $B_{c2} = 15.0$ T. The normal phase of SiV_3 has a magnetic susceptibility close to zero. A long, thin SiV_3 cylinder has its axis parallel to an external magnetic field $\vec{B}_0$ in the $+x$-direction. At points far from the ends of the cylinder, by symmetry, all the magnetic vectors are parallel to the x-axis. At a temperature near absolute zero the external magnetic field is slowly increased from zero. What are the resultant magnetic field $\vec{B}$ and the magnetization $\vec{M}$ inside the cylinder at points far from its ends (a) just before the magnetic flux begins to penetrate the material, and (b) just after the material becomes completely normal?

Problems

29.44. A Changing Magnetic Field. You are testing a new data-acquisition system. This system allows you to record a graph of the current in a circuit as a function of time. As part of the test, you are using a circuit made up of a 4.00-cm-radius, 500-turn coil of copper wire connected in series to a 600-Ω resistor. Copper has resistivity $1.72 \times 10^{-8}\Omega\cdot$m, and the wire used for the coil has diameter 0.0300 mm. You place the coil on a table that is tilted 30.0° from the horizontal and that lies between the poles of an electromagnet. The electromagnet generates a vertically upward magnetic field that is zero for $t < 0$, equal to $(0.120\ \text{T}) \times (1 - \cos \pi t)$ for $0 \le t \le 1.00$ s, and equal to 0.240 T for $t > 1.00$ s. (a) Draw the graph that should be produced by your data-acquisition system. (This is a full-featured system, so the graph will include labels and numerical values on its axes.) (b) If you were looking vertically downward at the coil, would the current be flowing clockwise or counterclockwise?

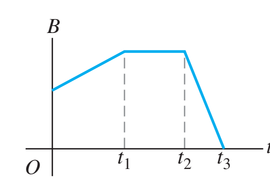
29.45. In the circuit shown in Fig. 29.41 the capacitor has capacitance $C = 20\ \mu$F and is initially charged to 100 V with the polarity shown. The resistor R_0 has resistance 10 Ω. At time $t = 0$ the switch is closed. The small circuit is not connected in any way to the large one. The wire of the small circuit has a resistance of 1.0 Ω/m and contains 25 loops. The large circuit is a rectangle 2.0 m by 4.0 m, while the small one has dimensions $a = 10.0$ cm and $b = 20.0$ cm. The distance c is 5.0 cm. (The figure is not drawn to scale.) Both circuits are held stationary. Assume that only the wire nearest the small circuit produces an appreciable magnetic field through it. (a) Find the current in the large circuit 200 μs after S is closed. (b) Find the current in the small circuit 200 μs after S is closed. (*Hint:* See Problem 29.7.) (c) Find the direction of the current in the small circuit. (d) Justify why we can ignore the magnetic field from all the wires of the large circuit except for the wire closest to the small circuit.

Figure 29.41 Problem 29.45.



29.46. A flat coil is oriented with the plane of its area at right angles to a spatially uniform magnetic field. The magnitude of this field varies with time according to the graph in Fig. 29.42. Sketch a qualitative (but accurate!) graph of the emf induced in the coil as a function of time. Be sure to identify the times t_1, t_2, and t_3 on your graph.

Figure 29.42 Problem 29.46.



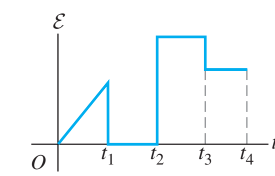
29.47. A circular wire loop of radius a and resistance R initially has a magnetic flux through it due to an external magnetic field. The external field then decreases to zero. A current is induced in the loop while the external field is changing; however, this current does not stop at the instant that the external field stops changing. The reason is that the current itself generates a magnetic field, which gives rise to a flux through the loop. If the current changes, the flux through the loop changes as well, and an induced emf appears in the loop to oppose the change. (a) The magnetic field at the center of the loop of radius a produced by a current i in the loop is given by $B = \mu_0 i/2a$. If we use the crude approximation that the field has this same value at all points within the loop, what is the flux of this field through the loop? (b) By using Faraday's law, Eq. (29.3), and the relationship $\mathcal{E} = iR$, show that after the external field has stopped changing, the current in the loop obeys the differential equation

$$\frac{di}{dt} = -\left(\frac{2R}{\pi\mu_0 a}\right)i$$

(c) If the current has the value i_0 at $t = 0$, the instant that the external field stops changing, solve the equation in part (b) to find i as a function of time for $t > 0$. (*Hint:* In Section 26.4 we encountered a similar differential equation, Eq. (26.15), for the quantity q. This equation for i may be solved in the same way.) (d) If the loop has radius $a = 50$ cm and resistance $R = 0.10\ \Omega$, how long after the external field stops changing will the current be equal to $0.010 i_0$ (that is, $\frac{1}{100}$ of its initial value)? (e) In solving the examples in this chapter, we ignored the effects described in this problem. Explain why this is a good approximation.

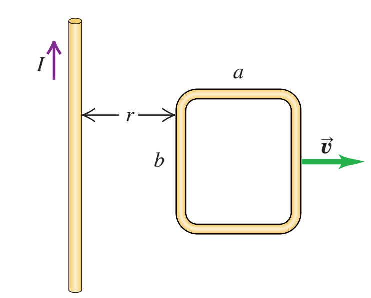
29.48. A coil is stationary in a spatially uniform, external, time-varying magnetic field. The emf induced in this coil as a function of time is shown if Fig. 29.43. Sketch a clear qualitative graph of the external magnetic field as a function of time, given that it started from zero. Include the points t_1, t_2, t_3, and t_4 on your graph.

Figure 29.43 Problem 29.48.



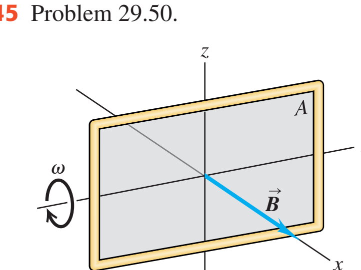
29.49. In Fig. 29.44 the loop is being pulled to the right at constant speed v. A constant current I flows in the long wire, in the direction shown. (a) Calculate the magnitude of the net emf $\mathcal{E}$ induced in the loop. Do this two ways: (i) by using Faraday's law of induction (*Hint:* See Problem 29.7) and (ii) by looking at the emf induced in each segment of the loop due to its motion. (b) Find the direction (clockwise or counterclockwise) of the current induced in the loop. Do this two ways: (i) using Lenz's law and (ii) using the magnetic force on charges in the loop. (c) Check your answer for the emf in part (a) in the following special cases to see whether it is physically reasonable: (i) The loop is stationary; (ii) the loop is very thin, so $a \to 0$; (iii) the loop gets very far from the wire.

Figure 29.44 Problem 29.49.



29.50. Suppose the loop in Fig. 29.45 is (a) rotated about the y-axis; (b) rotated about the x-axis; (c) rotated about an edge parallel to the z-axis. What is the maximum induced emf in each case if $A = 600\ \text{cm}^2$, $\omega = 35.0$ rad/s, and $B = 0.450$ T?

Figure 29.45 Problem 29.50.

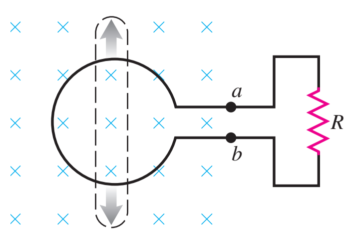


29.51. As a new electrical engineer for the local power company, you are assigned the project of designing a generator of sinusoidal ac voltage with a maximum voltage of 120 V. Besides plenty of wire, you have two strong magnets that can produce a constant uniform magnetic field of 1.5 T over a square area of 10.0 cm on a side when they are 12.0 cm apart. The basic design should consist of a square coil turning in the uniform magnetic field. To have an acceptable coil resistance, the coil can have at most 400 loops. What is the minimum rotation rate (in rpm) of the coil so it will produce the required voltage?

29.52. Make a Generator? You are shipwrecked on a deserted tropical island. You have some electrical devices that you could operate using a generator but you have no magnets. The earth's magnetic field at your location is horizontal and has magnitude 8.0×10^{-5} T, and you decide to try to use this field for a generator by rotating a large circular coil of wire at a high rate. You need to produce a peak emf of 9.0 V and estimate that you can rotate the coil at 30 rpm by turning a crank handle. You also decide that to have an acceptable coil resistance, the maximum number of turns the coil can have is 2000. (a) What area must the coil have? (b) If the coil is circular, what is the maximum translational speed of a point on the coil as it rotates? Do you think this device is feasible? Explain.

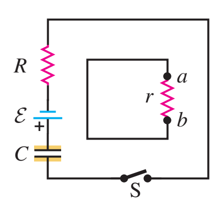
29.53. A flexible circular loop 6.50 cm in diameter lies in a magnetic field with magnitude 0.950 T, directed into the plane of the page as shown in Fig. 29.46. The loop is pulled at the points indicated by the arrows, forming a loop of zero area in 0.250 s. (a) Find the average induced emf in the circuit. (b) What is the direction of the current in R: from a to b or from b to a? Explain your reasoning.

Figure 29.46 Problem 29.53.



29.54. A Circuit Within a Circuit. Fig. 29.47 shows a small circuit within a larger one, both lying on the surface of a table. The switch is closed at $t = 0$ with the capacitor initially uncharged. Assume that the small circuit has no appreciable effect on the larger one. (a) What is the direction (a to b or to a) of the current in the resistor r (i) the instant after the switch is closed and (ii) one time constant after the switch is closed? (b) Sketch a graph of the current in the small circuit as a function of time, calling clockwise positive.

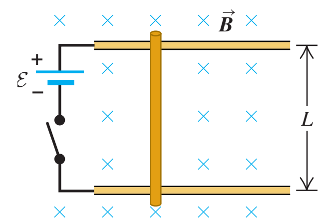
Figure 29.47 Problem 29.54.



29.55. Terminal Speed. A conducting rod with length L, mass m, and resistance R moves without friction on metal rails as shown in Fig. 29.11. A uniform magnetic field $\vec{B}$ is directed into the plane of the figure. The rod starts from rest and is acted on by a constant force $\vec{F}$ directed to the right. The rails are infinitely long and have negligible resistance. (a) Graph the speed of the rod as a function of time. (b) Find an expression for the terminal speed (the speed when the acceleration of the rod is zero).

29.56. Terminal Speed. A bar of length $L = 0.8$ m is free to slide without friction on horizontal rails, as shown in Fig. 29.48. There is a uniform magnetic field $B = 1.5$ T directed into the plane of the figure. At one end of the rails there is a battery with emf $\mathcal{E} = 12$ V and a switch. The bar has mass 0.90 kg and resistance 5.0 Ω, and all other resistance in the circuit can be ignored. The switch is closed at time $t = 0$. (a) Sketch the speed of the bar as a function of time. (b) Just after the switch is closed, what is the acceleration of the bar? (c) What is the acceleration of the bar when its speed is 2.0 m/s? (d) What is the terminal speed of the bar?

Figure 29.48 Problem 29.56.

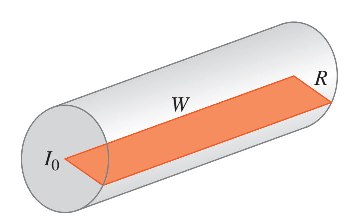


29.57. Antenna emf. A satellite, orbiting the earth at the equator at an altitude of 400 km, has an antenna that can be modeled as a 2.0-m-long rod. The antenna is oriented perpendicular to the earth's surface. At the equator, the earth's magnetic field is essentially horizontal and has a value of 8.0×10^{-5} T; ignore any changes in B with altitude. Assuming the orbit is circular, determine the induced emf between the tips of the antenna.

29.58. emf in a Bullet. At the equator, the earth's magnetic field is approximately horizontal, is directed toward the north, and has a value of 8×10^{-5} T. (a) Estimate the emf induced between the top and bottom of a bullet shot horizontally at a target on the equator if the bullet is shot toward the east. Assume the bullet has a length of 1 cm and a diameter of 0.4 cm and is traveling at 300 m/s. Which is at higher potential: the top or bottom of the bullet? (b) What is the emf if the bullet travels south? (c) What is the emf induced between the front and back of the bullet for any horizontal velocity?

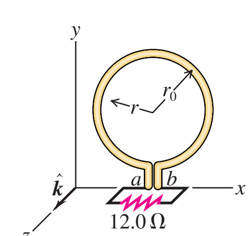
29.59. A very long, cylindrical wire of radius R carries a current I_0 uniformly distributed across the cross section of the wire. Calculate the magnetic flux through a rectangle that has one side of length W running down the center of the wire and another side of length R, as shown in Fig. 29.49 (see Problem 29.7).

Figure 29.49 Problem 29.59.



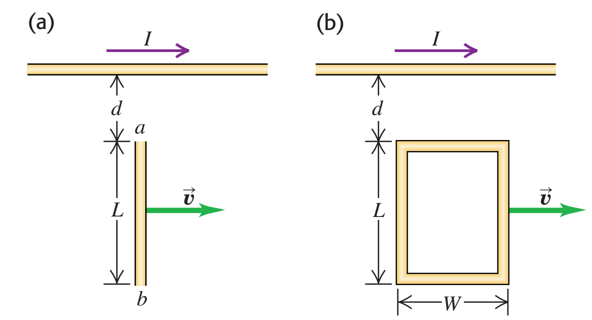
29.60. A circular conducting ring with radius $r_0 = 0.0420$ m lies in the xy-plane in a region of uniform magnetic field $\vec{B} = B_0[1 - 3(t/t_0)^2 + 2(t/t_0)^3]\hat{k}$. In this expression, $t_0 = 0.0100$ s and is constant, t is time, $\hat{k}$ is the unit vector in the $+z$-direction, and $B_0 = 0.0800$ T and is constant. At points a and b (Fig. 29.50) there is a small gap in the ring with wires leading to an external circuit of resistance $R = 12.0\ \Omega$. There is no magnetic field at the location of the external circuit. (a) Derive an expression, as a function of time, for the total magnetic flux Φ_B through the ring. (b) Determine the emf induced in the ring at time $t = 5.00 \times 10^{-3}$ s. What is the polarity of the emf? (c) Because of the internal resistance of the ring, the current through R at the time given in part (b) is only 3.00 mA. Determine the internal resistance of the ring. (d) Determine the emf in the ring at a time $t = 1.21 \times 10^{-2}$ s. What is the polarity of the emf? (e) Determine the time at which the current through R reverses its direction.

Figure 29.50 Problem 29.60.



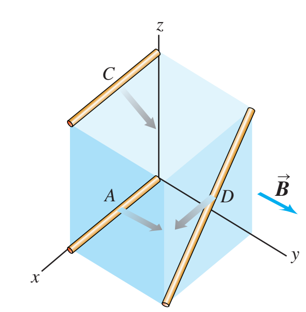
29.61. The long, straight wire shown in Fig. 29.51a carries constant current I. A metal bar with length L is moving at constant velocity $\vec{v}$, as shown in the figure. Point a is a distance d from the wire. (a) Calculate the emf induced in the bar. (b) Which point, a or b, is at higher potential? (c) If the bar is replaced by a rectangular wire loop of resistance R (Fig. 29.51b), what is the magnitude of the current induced in the loop?

Figure 29.51 Problem 29.61.



29.62. The cube shown in Fig. 29.52, 50.0 cm on a side, is in a uniform magnetic field of 0.120 T, directed along the positive y-axis. Wires A, C, and D move in the directions indicated, each with a speed of 0.350 m/s. (Wire A moves parallel to the xy-plane, C moves at an angle of 45.0° below the xy-plane, and D moves parallel to the xz-plane.) What is the potential difference between the ends of each wire?

Figure 29.52 Problem 29.62.

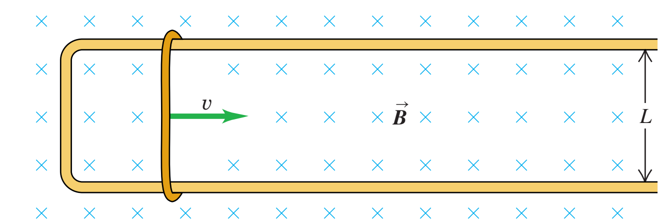


29.63. A slender rod, 0.240 m long, rotates with an angular speed of 8.80 rad/s about an axis through one end and perpendicular to the rod. The plane of rotation of the rod is perpendicular to a uniform magnetic field with a magnitude of 0.650 T. (a) What is the induced emf in the rod? (b) What is the potential difference between its ends? (c) Suppose instead the rod rotates at 8.80 rad/s about an axis through its center and perpendicular to the rod. In this case, what is the potential difference between the ends of the rod? Between the center of the rod and one end?

29.64. A Magnetic Exercise Machine. You have designed a new type of exercise machine with an extremely simple mechanism (Fig. 29.36). A vertical bar of silver (chosen for its low resistivity and because it makes the machine look cool) with length $L = 3.0$ m is free to move left or right without friction on silver rails. The entire apparatus is placed in a horizontal, uniform magnetic field of strength 0.25 T. When you push the bar to the left or right, the bar's motion sets up a current in the circuit that includes the bar. The resistance of the bar and the rails can be neglected. The magnetic field exerts a force on the current-carrying bar, and this force opposes the bar's motion. The health benefit is from the exercise that you do in working against this force. (a) Your design goal is that the person doing the exercise is to do work at the rate of 25 watts when moving the bar at a steady 2.0 m/s. What should be the resistance R? (b) You decide you want to be able to vary the power required from the person, to adapt the machine to the person's strength and fitness. If the power is to be increased to 50 W by altering R while leaving the other design parameters constant, should R be increased or decreased? Calculate the value of R for 50 W. (c) When you start to construct a prototype machine, you find it is difficult to produce a 0.25-T magnetic field over such a large area. If you decrease the length of the bar to 0.20 m while leaving B, v, and R the same as in part (a), what will be the power required of the person?

29.65. A rectangular loop with width L and a slide wire with mass m are as shown in Fig. 29.53. A uniform magnetic field $\vec{B}$ is directed perpendicular to the plane of the loop into the plane of the figure. The slide wire is given an initial speed of v_0 and then released. There is no friction between the slide wire and the loop, and the resistance of the loop is negligible in comparison to the resistance R of the slide wire. (a) Obtain an expression for F, the magnitude of the force exerted on the wire while it is moving at speed v. (b) Show that the distance x that the wire moves before coming to rest is $x = mv_0R/a^2B^2$.

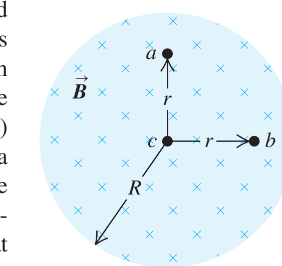
Figure 29.53 Problem 29.65.



29.66. A 25.0-cm-long metal rod lies in the xy-plane and makes an angle of 36.9° with the positive x-axis and an angle of 53.1° with the positive y-axis. The rod is moving in the $+x$-direction with a speed of 4.20 m/s. The rod is in a uniform magnetic field $\vec{B} = (0.120\ \text{T})\hat{\imath} - (0.220\ \text{T})\hat{\jmath} - (0.0900\ \text{T})\hat{k}$. (a) What is the magnitude of the emf induced in the rod? (b) Indicate in a sketch which end of the rod is at higher potential.

29.67. The magnetic field $\vec{B}$, at all points within a circular region of radius R, is uniform in space and directed into the plane of the page as shown in Fig. 29.54. (The region could be a cross section inside the windings of a long, straight solenoid.) If the magnetic field is increasing at a rate dB/dt, what are the magnitude and direction of the force on a stationary positive point charge q located at points a, b, and c? (Point a is a distance r above the center of the region, point b is a distance r to the right of the center, and point c is at the center of the region.)

Figure 29.54 Problem 29.67.



29.68. An airplane propeller of total length L rotates around its center with angular speed ω in a magnetic field that is perpendicular to the plane of rotation. Modeling the propeller as a thin, uniform bar, find the potential difference between (a) the center and either end of the propeller and (b) the two ends. (c) If the field is the earth's field of 0.50 G and the propeller turns at 220 rpm and is 2.0 m long, what is the potential difference between the middle and either end? It this large enough to be concerned about?

29.69. It is impossible to have a uniform electric field that abruptly drops to zero in a region of space in which the magnetic field is constant and in which there are no electric charges. To prove this statement, use the method of contradiction: Assume that such a case *is* possible and then show that your assumption contradicts a law of nature. (a) In the bottom half of a piece of paper, draw evenly spaced horizontal lines representing a uniform electric field to your right. Use dashed lines to draw a rectangle *abcda* with horizontal side *ab* in the electric-field region and horizontal side *cd* in the top half of your paper where $E = 0$. (b) Show that integration around your rectangle contradicts Faraday's law, Eq. (29.21).

29.70. Falling Square Loop. A vertically oriented, square loop of copper wire falls from a region where the field $\vec{B}$ is horizontal, uniform, and perpendicular to the plane of the loop, into a region where the field is zero. The loop is released from rest and initially is entirely within the magnetic-field region. Let the side length of the loop be s and let the diameter of the wire be d. The resistivity of copper is ρ_R and the density of copper is ρ_m. If the loop reaches its terminal speed while its upper segment is still in the magnetic-field region, find an expression for the terminal speed.

29.71. In a region of space where there are no conduction or displacement currents, it is impossible to have a uniform magnetic field that abruptly drops to zero. To prove this statement, use the method of contradiction: Assume that such a case *is* possible, and then show that your assumption contradicts a law of nature. (a) In the bottom half of a piece of paper, draw evenly spaced horizontal lines representing a uniform magnetic field to your right. Use dashed lines to draw a rectangle *abcda* with horizontal side *ab* in the magnetic-field region and horizontal side *cd* in the top half of your paper where $B = 0$. (b) Show that integration around your rectangle contradicts Ampere's law, Eq. (29.15).

29.72. A capacitor has two parallel plates with area A separated by a distance d. The space between plates is filled with a material having dielectric constant K. The material is not a perfect insulator but has resistivity ρ. The capacitor is initially charged with charge of magnitude Q_0 on each plate that gradually discharges by conduction through the dielectric. (a) Calculate the conduction current density $j_C(t)$ in the dielectric. (b) Show that at any instant the displacement current density in the dielectric is equal in magnitude to the conduction current density but opposite in direction, so the *total* current density is zero at every instant.

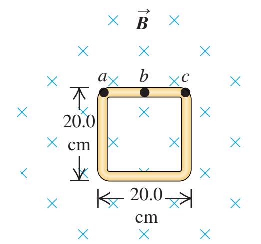
29.73. A rod of pure silicon (resistivity $\rho = 2300\ \Omega \cdot \text{m}$) is carrying a current. The electric field varies sinusoidally with time according to $E = E_0 \sin \omega t$, where $E_0 = 0.450\ \text{V/m}$, $\omega = 2\pi f$, and the frequency $f = 120$ Hz. (a) Find the magnitude of the maximum conduction current density in the wire. (b) Assuming $\epsilon = \epsilon_0$, find the maximum displacement current density in the wire, and compare with the result of part (a). (c) At what frequency f would the maximum conduction and displacement densities become equal if $\epsilon = \epsilon_0$ (which is not actually the case)? (d) At the frequency determined in part (c), what is the relative *phase* of the conduction and displacement currents?

Challenge Problems

29.74. A square, conducting, wire loop of side L, total mass m, and total resistance R initially lies in the horizontal xy-plane, with corners at $(x, y, z) = (0, 0, 0)$, $(0, L, 0)$, $(L, 0, 0)$, and $(L, L, 0)$. There is a uniform, upward magnetic field $\vec{B} = B\hat{k}$ in the space within and around the loop. The side of the loop that extends from $(0, 0, 0)$ to $(L, 0, 0)$ is held in place on the x-axis; the rest of the loop is free to pivot around this axis. When the loop is released, it begins to rotate due to the gravitational torque. (a) Find the *net* torque (magnitude and direction) that acts on the loop when it has rotated through an angle ϕ from its original orientation and is rotating downward at an angular speed ω. (b) Find the angular acceleration of the loop at the instant described in part (a). (c) Compared to the case with zero magnetic field, does it take the loop a longer or shorter time to rotate through 90°? Explain. (d) Is mechanical energy conserved as the loop rotates downward? Explain.

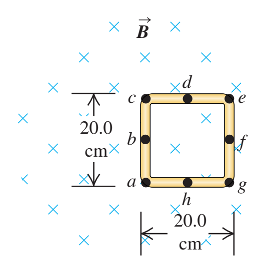
29.75. A square conducting loop, 20.0 cm on a side, is placed in the same magnetic field as shown in Exercise 29.30. (See Fig. 29.55; the center of the square loop is at the center of the magnetic-field region.) (a) Copy Fig. 29.55, and draw vectors to show the directions and relative magnitudes of the induced electric field $\vec{E}$ at points a, b, and c. (b) Prove that the component of $\vec{E}$ along the loop has the same value at every point of the loop and is equal to that of the ring shown in Fig. 29.31 (see Exercise 29.30). (c) What current is induced in the loop if its resistance is 1.90 Ω? (d) What is the potential difference between points a and b?

Figure 29.55 Challenge Problem 29.75.



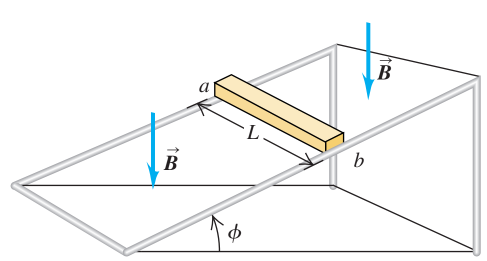
29.76. A uniform, square, conducting loop, 20.0 cm on a side, is placed in the same magnetic field as shown in Exercise 29.30, with side ac along a diameter and with point b at the center of the field (Fig. 29.56). (a) Copy Fig. 29.56, and draw vectors to show the direction and relative magnitude of the induced electric field $\vec{E}$ at the lettered points. (b) What is the induced emf in side ac? (c) What is the induced emf in the loop? (d) What is the current in the loop if its resistance is 1.90 Ω? (e) What is the potential difference between points a and c? Which is at higher potential?

Figure 29.56 Challenge Problem 29.76.



29.77. A metal bar with length L, mass m, and resistance R is placed on frictionless metal rails that are inclined at an angle ϕ above the horizontal. The rails have negligible resistance. A uniform magnetic field of magnitude B is directed downward as shown in Fig. 29.57. The bar is released from rest and slides down the rails. (a) Is the direction of the current induced in the bar from a to b or from b to a? (b) What is the terminal speed of the bar? (c) What is the induced current in the bar when the terminal speed has been reached? (d) After the terminal speed has been reached, at what rate is electrical energy being converted to thermal energy in the resistance of the bar? (e) After the terminal speed has been reached, at what rate is work being done on the bar by gravity? Compare your answer to that in part (d).

Figure 29.57 Challenge Problem 29.77.



29.78. Consider a uniform metal disk rotating through a perpendicular magnetic field $\vec{B}$, as shown in Fig. 29.19a. The disk has mass m, radius R, and thickness t, is made of a material with resistivity ρ, and is rotating clockwise in Fig. 29.19a with angular speed ω. The magnetic field is directed into the plane of the disk. Suppose that the region to which the magnetic field is confined is not circular, as shown in Fig. 29.19a, but is a small square with sides of length L $(L \ll R)$ centered a distance d from the point O (the center of the disk). The sides of this square are horizontal and vertical in Fig. 29.19a. (a) Show that the current induced within the square is approximately equal to $I = \omega dBLt/\rho$. In which direction does this current flow? (*Hint:* Assume that the resistance to the current is confined to the region of the square. The current also encounters resistance as it flows outside the region to which the magnetic field is confined, as shown in Fig. 29.19b; however, this resistance is relatively small, since the current can flow through such a wide area. Recall Eq. (25.10) for resistance, given in Section 25.3.) (b) Show that the induced current gives rise to a torque of approximate magnitude $\tau = \omega d^2B^2L^2t/\rho$ that opposes the rotation of the disk (that is, a counterclockwise torque). (c) What would be the magnitudes and directions of the induced current and torque if the direction of $\vec{B}$ were still into the plane of the disk but the disk rotated counterclockwise? What if the direction of $\vec{B}$ were out of the plane and the disk rotated counterclockwise?