ALTERNATING CURRENT

31

?Waves from a broadcasting station produce an alternating current in the circuits of a radio (like the one in this classic car). If a radio is tuned to a station at a frequency of 1000 kHz, does it also detect the transmissions from a station broadcasting at 600 kHz?

LEARNING GOALS

By studying this chapter, you will learn:

- How phasors make it easy to describe sinusoidally varying quantities.
- How to use reactance to describe the voltage across a circuit element that carries an alternating current.
- How to analyze an *L-R-C* series circuit with a sinusoidal emf.
- What determines the amount of power flowing into or out of an alternating-current circuit.
- How an *L-R-C* series circuit responds to sinusoidal emfs of different frequencies.
- Why transformers are useful, and how they work.

During the 1880s in the United States there was a heated and acrimonious debate between two inventors over the best method of electric-power distribution. Thomas Edison favored direct current (dc)—that is, steady current that does not vary with time. George Westinghouse favored **alternating current (ac),** with sinusoidally varying voltages and currents. He argued that transformers (which we will study in this chapter) can be used to step the voltage up and down with ac but not with dc; low voltages are safer for consumer use, but high voltages and correspondingly low currents are best for long-distance power transmission to minimize i^2R losses in the cables.

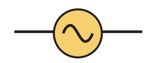
Eventually, Westinghouse prevailed, and most present-day household and industrial power-distribution systems operate with alternating current. Any appliance that you plug into a wall outlet uses ac, and many battery-powered devices such as radios and cordless telephones make use of the dc supplied by the battery to create or amplify alternating currents. Circuits in modern communication equipment, including pagers and television, also make extensive use of ac.

In this chapter we will learn how resistors, inductors, and capacitors behave in circuits with sinusoidally varying voltages and currents. Many of the principles that we found useful in Chapters 25, 28, and 30 are applicable, along with several new concepts related to the circuit behavior of inductors and capacitors. A key concept in this discussion is *resonance,* which we studied in Chapter 13 for mechanical systems.

31.1 Phasors and Alternating Currents

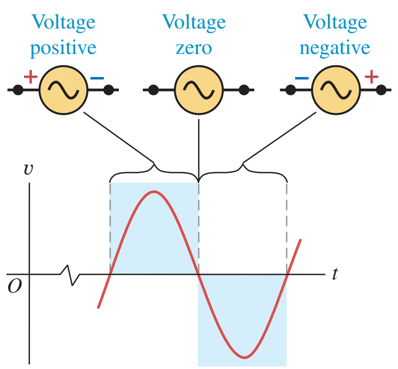
To supply an alternating current to a circuit, a source of alternating emf or voltage is required. An example of such a source is a coil of wire rotating with constant angular velocity in a magnetic field, which we discussed in Example 29.4 (Section 29.2). This develops a sinusoidal alternating emf and is the prototype of the commercial alternating-current generator or *alternator* (see Fig. 29.8).

We use the term **ac source** for any device that supplies a sinusoidally varying voltage (potential difference) v or current i. The usual circuit-diagram symbol for an ac source is



A sinusoidal voltage might be described by a function such as

$$v = V\cos\omega t \quad (31.1)$$



31.1 The voltage across a sinusoidal ac source.

In this expression, v (lowercase) is the *instantaneous* potential difference; V (uppercase) is the maximum potential difference, which we call the **voltage amplitude;** and ω is the *angular frequency,* equal to 2π times the frequency f (Fig. 31.1).

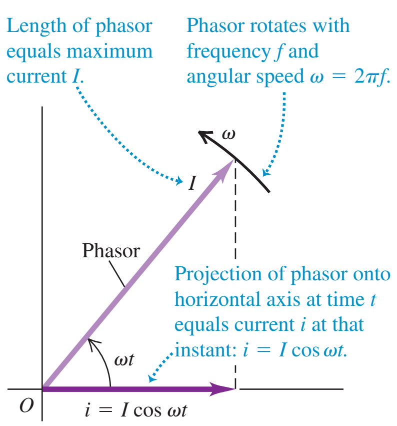
In the United States and Canada, commercial electric-power distribution systems always use a frequency of $f = 60$ Hz, corresponding to $\omega = (2\pi \text{ rad})(60 \text{ s}^{-1}) = 377$ rad/s; in much of the rest of the world, $f = 50$ Hz ($\omega = 314$ rad/s) is used. Similarly, a sinusoidal current might be described as

$$i = I\cos\omega t \quad (31.2)$$

where i (lowercase) is the instantaneous current and I (uppercase) is the maximum current or **current amplitude.**

Phasor Diagrams

To represent sinusoidally varying voltages and currents, we will use rotating vector diagrams similar to those we used in the study of simple harmonic motion in Section 13.2 (see Figs. 13.5b and 13.6). In these diagrams the instantaneous value of a quantity that varies sinusoidally with time is represented by the *projection* onto a horizontal axis of a vector with a length equal to the amplitude of the quantity. The vector rotates counterclockwise with constant angular speed ω. These rotating vectors are called **phasors,** and diagrams containing them are called **phasor diagrams.** Figure 31.2 shows a phasor diagram for the sinusoidal current described by Eq. (31.2). The projection of the phasor onto the horizontal axis at time t is $I\cos\omega t$; this is why we chose to use the cosine function rather than the sine in Eq. (31.2).



31.2 A phasor diagram.

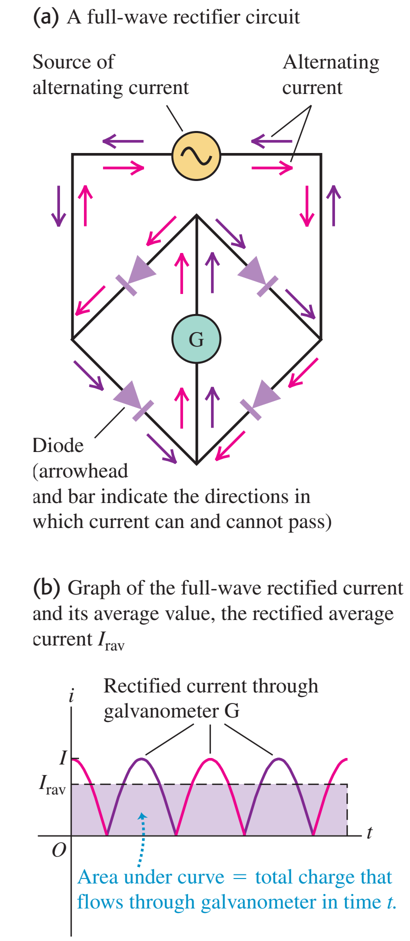
CAUTION **Just what is a phasor?** A phasor is not a real physical quantity with a direction in space, such as velocity, momentum, or electric field. Rather, it is a *geometric* entity that helps us to describe and analyze physical quantities that vary sinusoidally with time. In Section 13.2 we used a single phasor to represent the position of a point mass undergoing simple harmonic motion. In this chapter we will use phasors to *add* sinusoidal voltages and currents. Combining sinusoidal quantities with phase differences then becomes a matter of vector addition. We will find a similar use for phasors in Chapters 35 and 36 in our study of interference effects with light.

Rectified Alternating Current

How do we measure a sinusoidally varying current? In Section 26.3 we used a d'Arsonval galvanometer to measure steady currents. But if we pass a *sinusoidal* current through a d'Arsonval meter, the torque on the moving coil varies sinusoidally, with one direction half the time and the opposite direction the other half. The needle may wiggle a little if the frequency is low enough, but its average deflection is zero. Hence a d'Arsonval meter by itself isn't very useful for measuring alternating currents.

To get a measurable one-way current through the meter, we can use *diodes,* which we described in Section 25.3. A diode (or rectifier) is a device that conducts better in one direction than in the other; an ideal diode has zero resistance for one direction of current and infinite resistance for the other. One possible arrangement is shown in Fig. 31.3a. The current through the galvanometer G is always upward, regardless of the direction of the current from the ac source (i.e., which part of the cycle the source is in). The current through G is as shown by the graph in Fig. 31.2b. It pulsates but always has the same direction, and the average meter deflection is *not* zero. This arrangement of diodes is called a *full-wave rectifier circuit.*

31.3 (a) A full-wave rectifier circuit. (b) Graph of the resulting current through the galvanometer G.



The **rectified average current** I_{rav} is defined so that during any whole number of cycles, the total charge that flows is the same as though the current were constant with a value equal to I_{rav}. The notation I_{rav} and the name *rectified average* current emphasize that this is *not* the average of the original sinusoidal current. In Fig. 31.3b the total charge that flows in time t corresponds to the area under the curve of i versus t (recall that $i = dq/dt$, so q is the integral of t); this area must equal the rectangular area with height I_{rav}. We see that I_{rav} is less than the maximum current I; the two are related by

$$I_{rav} = \frac{2}{\pi}I = 0.637I \quad \text{(rectified average value of a sinusoidal current)} \quad (31.3)$$

(The factor of $2/\pi$ is the average value of $|\cos\omega t|$ or of $|\sin\omega t|$; see Example 29.5 in Section 29.2.) The galvanometer deflection is proportional to I_{rav}. The galvanometer scale can be calibrated to read I, I_{rav}, or, most commonly, I_{rms} (discussed below).

Root-Mean-Square (rms) Values

A more useful way to describe a quantity that can be either positive or negative is the *root-mean-square (rms) value.* We used rms values in Section 18.3 in connection with the speeds of molecules in a gas. We *square* the instantaneous current i, take the *average* (mean) value of i^2, and finally take the *square root* of that average. This procedure defines the **root-mean-square current,** denoted as I_{rms} (Fig. 31.4). Even when i is negative, i^2 is always positive, so I_{rms} is never zero (unless i is zero at every instant).

Here's how we obtain I_{rms} for a sinusoidal current, like that shown in Fig. 31.4. If the instantaneous current is given by $i = I\cos\omega t$, then

$$i^2 = I^2\cos^2\omega t$$

Using a double-angle formula from trigonometry,

$$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$$

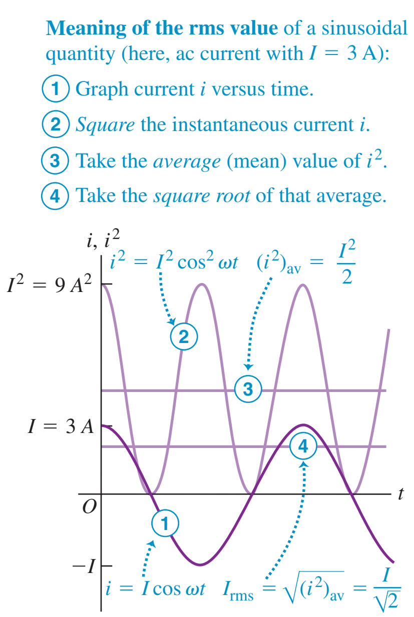
we find

$$i^2 = I^2\frac{1}{2}(1 + \cos 2\omega t) = \frac{1}{2}I^2 + \frac{1}{2}I^2\cos 2\omega t$$

The average of $\cos 2\omega t$ is zero because it is positive half the time and negative half the time. Thus the average of i^2 is simply $I^2/2$. The square root of this is I_{rms}:

$$I_{rms} = \frac{I}{\sqrt{2}} \quad \text{(root-mean-square value of a sinusoidal current)} \quad (31.4)$$

31.4 Calculating the root-mean-square (rms) value of an alternating current.



In the same way, the root-mean-square value of a sinusoidal voltage with amplitude (maximum value) V is

$$V_{rms} = \frac{V}{\sqrt{2}} \quad \text{(root-mean-square value of a sinusoidal voltage)} \quad (31.5)$$

We can convert a rectifying ammeter into a voltmeter by adding a series resistor, just as for the dc case discussed in Section 26.3. Meters used for ac voltage and current measurements are nearly always calibrated to read rms values, not maximum or rectified average. Voltages and currents in power distribution systems are always described in terms of their rms values. The usual household power supply, "120-volt ac," has an rms voltage of 120 V (Fig. 31.5). The voltage amplitude is

$$V = \sqrt{2}V_{rms} = \sqrt{2}(120\text{ V}) = 170\text{ V}$$

31.5 This wall socket delivers a root-mean-square voltage of 120 V. Sixty times per second, the instantaneous voltage across its terminals varies from $(\sqrt{2})(120\text{ V}) = 170$ V to -170 V and back again.

Example 31.1 Current in a personal computer

The plate on the back of a personal computer says that it draws 2.7 A from a 120-V, 60-Hz line. For this computer, what are (a) the average current, (b) the average of the square of the current, and (c) the current amplitude?

SOLUTION

IDENTIFY: This example is about alternating current.

SET UP: In parts (b) and (c) we use the idea that the root-mean-square current, given by Eq. (31.4), is the *square root* of the *mean* (average) of the *square* of the current.

EXECUTE: (a) The average of *any* sinusoidal alternating current, over any whole number of cycles, is zero.

(b) The current given is the rms value: $I_{rms} = 2.7$ A. The target variable $(i^2)_{av}$ is the *mean* of the *square* of the current. The rms current is the square root of this target variable, so

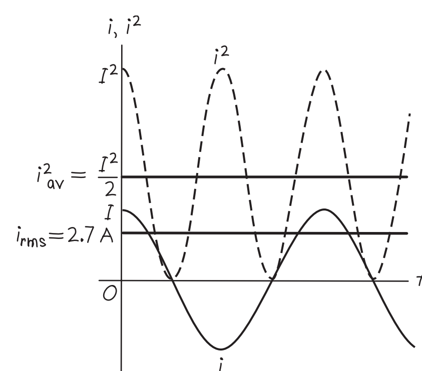
$$I_{rms} = \sqrt{(i^2)_{av}} \quad \text{or} \quad (i^2)_{av} = (I_{rms})^2 = (2.7\text{ A})^2 = 7.3\text{ A}^2$$

(c) From Eq. (31.4) the current amplitude I is

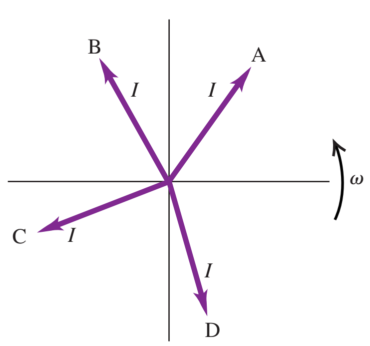
$$I = \sqrt{2}I_{rms} = \sqrt{2}(2.7\text{ A}) = 3.8\text{ A}$$

Figure 31.6 shows our graphs of i and i^2.

EVALUATE: Why would we be interested in the average of the square of the current? Recall that the rate at which energy is dissipated in a resistor R equals i^2R. This rate varies if the current is alternating, so it is best described by its average value $(i^2)_{av}R = I_{rms}{}^2R$. We make use of this idea in Section 31.4.



31.6 Our graphs of the current i and the square of the current i^2 versus time t.



Test Your Understanding of Section 31.1 The figure at left shows four different current phasors with the same angular frequency ω. At the time shown, which phasor corresponds to (a) a positive current that is becoming more positive; (b) a positive current that is decreasing toward zero; (c) a negative current that is becoming more negative; (d) a negative current that is decreasing in magnitude toward zero?

31.2 Resistance and Reactance

In this section we will derive voltage–current relationships for individual circuit elements carrying a sinusoidal current. We'll consider resistors, inductors, and capacitors.

Resistor in an ac Circuit

First let's consider a resistor with resistance R through which there is a sinusoidal current given by Eq. (31.2): $i = I\cos\omega t$. The positive direction of current is counterclockwise around the circuit, as in Fig. 31.7a. The current amplitude (maximum current) is I. From Ohm's law the instantaneous potential v_R of point a with respect to point b (that is, the instantaneous voltage across the resistor) is

$$v_R = iR = (IR)\cos\omega t \quad (31.6)$$

The maximum voltage V_R, the *voltage amplitude,* is the coefficient of the cosine function:

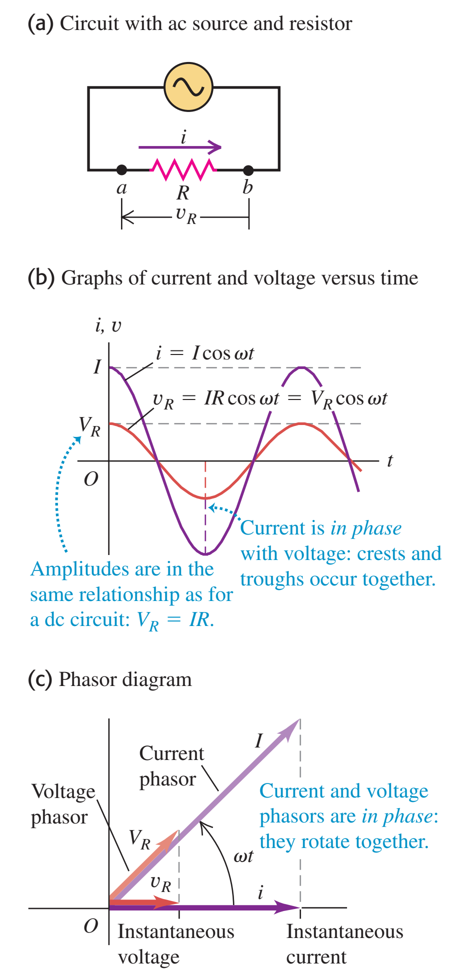
$$V_R = IR \quad \text{(amplitude of voltage across a resistor, ac circuit)} \quad (31.7)$$

Hence we can also write

$$v_R = V_R\cos\omega t \quad (31.8)$$

The current i and voltage v_R are both proportional to $\cos\omega t$, so the current is *in phase* with the voltage. Equation (31.7) shows that the current and voltage amplitudes are related in the same way as in a dc circuit.

Figure 31.7b shows graphs of i and v_R as functions of time. The vertical scales for current and voltage are different, so the relative heights of the two curves are not significant. The corresponding phasor diagram is given in Fig. 31.7c. Because i and v_R are *in phase* and have the same frequency, the current and voltage phasors rotate together; they are parallel at each instant. Their projections on the horizontal axis represent the instantaneous current and voltage, respectively.

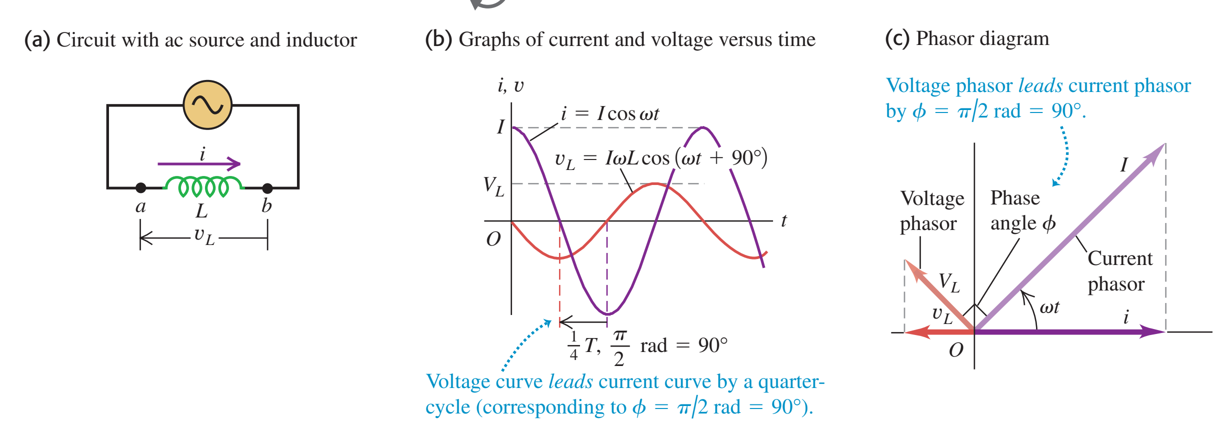


31.7 Resistance R connected across an ac source.

Inductor in an ac Circuit

Next, we replace the resistor in Fig. 31.7 with a pure inductor with self-inductance L and zero resistance (Fig. 31.8a). Again we assume that the current is $i = I\cos\omega t$, with the positive direction of current taken as counterclockwise around the circuit.

Although there is no resistance, there is a potential difference v_L between the inductor terminals a and b because the current varies with time, giving rise to a self-induced emf. The induced emf in the direction of i is given by Eq. (30.7), $\mathcal{E} = -L\,di/dt$; however, the voltage v_L is *not* simply equal to $\mathcal{E}$. To see why, notice that if the current in the inductor is in the positive (counterclockwise) direction from a to b and is increasing, then di/dt is positive and the induced emf is directed to the left to oppose the increase in current; hence point a is at higher potential than is point b. Thus the potential of point a with respect to point b is positive and is given by $v_L = +L\,di/dt$, the *negative* of the induced emf. (You should convince



31.8 Inductance L connected across an ac source.

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yourself that this expression gives the correct sign of v_L in *all* cases, including i counterclockwise and decreasing, i clockwise and increasing, and i clockwise and decreasing; you should also review Section 30.2.) So we have

$$v_L = L\frac{di}{dt} = L\frac{d}{dt}(I\cos\omega t) = -I\omega L\sin\omega t \quad (31.9)$$

The voltage v_L across the inductor at any instant is proportional to the *rate of change* of the current. The points of maximum voltage on the graph correspond to maximum steepness of the current curve, and the points of zero voltage are the points where the current curve instantaneously levels off at its maximum and minimum values (Fig. 31.8b). The voltage and current are "out of step" or *out of phase* by a quarter-cycle. Since the voltage peaks occur a quarter-cycle earlier than the current peaks, we say that the voltage *leads* the current by 90°. The phasor diagram in Fig. 31.8c also shows this relationship; the voltage phasor is ahead of the current phasor by 90°.

We can also obtain this phase relationship by rewriting Eq. (31.9) using the identity $\cos(A + 90°) = -\sin A$:

$$v_L = I\omega L\cos(\omega t + 90°) \quad (31.10)$$

This result shows that the voltage can be viewed as a cosine function with a "head start" of 90° relative to the current.

As we have done in Eq. (31.10), we will usually describe the phase of the *voltage* relative to the *current,* not the reverse. Thus if the current i in a circuit is

$$i = I\cos\omega t$$

and the voltage v of one point with respect to another is

$$v = V\cos(\omega t + \phi)$$

we call ϕ the **phase angle;** it gives the phase of the *voltage* relative to the *current*. For a pure resistor, $\phi = 0$, and for a pure inductor, $\phi = 90°$.

From Eq. (31.9) or (31.10) the amplitude V_L of the inductor voltage is

$$V_L = I\omega L \quad (31.11)$$

We define the **inductive reactance** X_L of an inductor as

$$X_L = \omega L \qquad \text{(inductive reactance)} \quad (31.12)$$

Using X_L, we can write Eq. (31.11) in a form similar to Eq. (31.7) for a resistor $(V_R = IR)$:

$$V_L = IX_L \qquad \text{(amplitude of voltage across an inductor, ac circuit)} \quad (31.13)$$

Because X_L is the ratio of a voltage and a current, its SI unit is the ohm, the same as for resistance.

CAUTION **Inductor voltage and current are not in phase** Keep in mind that Eq. (31.13) is a relationship between the *amplitudes* of the oscillating voltage and current for the inductor in Fig. 31.8a. It does *not* say that the voltage at any instant is equal to the current at that instant multiplied by X_L. As Fig. 31.8b shows, the voltage and current are 90° out of phase. Voltage and current are in phase only for resistors, as in Eq. (31.6).

The Meaning of Inductive Reactance

The inductive reactance X_L is really a description of the self-induced emf that opposes any change in the current through the inductor. From Eq. (31.13), for a given current amplitude I the voltage $v_L = +L\,di/dt$ across the inductor and the self-induced emf $\mathcal{E} = -L\,di/dt$ both have an amplitude V_L that is directly proportional to X_L. According to Eq. (31.12), the inductive reactance and self-induced emf increase with more rapid variation in current (that is, increasing angular frequency ω) and increasing inductance L.

If an oscillating voltage of a given amplitude V_L is applied across the inductor terminals, the resulting current will have a smaller amplitude I for larger values of X_L. Since X_L is proportional to frequency, a high-frequency voltage applied to the inductor gives only a small current, while a lower-frequency voltage of the same amplitude gives rise to a larger current. Inductors are used in some circuit applications, such as power supplies and radio-interference filters, to block high frequencies while permitting lower frequencies or dc to pass through. A circuit device that uses an inductor for this purpose is called a *low-pass filter* (see Problem 31.50).

Example 31.2 An inductor in an ac circuit

Suppose you want the current amplitude in a pure inductor in a radio receiver to be 250 μA when the voltage amplitude is 3.60 V at a frequency of 1.60 MHz (corresponding to the upper end of the AM broadcast band). (a) What inductive reactance is needed? What inductance? (b) If the voltage amplitude is kept constant, what will be the current amplitude through this inductor at 16.0 MHz? At 160 kHz?

SOLUTION

IDENTIFY: We are not told about any other elements of the circuit of which the inductor is part. Nor should we care about those other elements, since from the perspective of this example, all they do is provide the inductor with an oscillating voltage. Hence all of those other circuit elements are lumped into the ac source shown in Fig. 31.8a.

SET UP: We are given the current amplitude I and the voltage amplitude V. Our target variables in part (a) are the inductive reactance X_L at 1.60 MHz and the inductance L, which we find using Eqs. (31.13) and (31.12). Once we know L, we use these same two equations to find the inductive reactance and current amplitude at any other frequency.

EXECUTE: (a) From Eq. (31.13),

$$X_L = \frac{V_L}{I} = \frac{3.60\ \text{V}}{250 \times 10^{-6}\ \text{A}} = 1.44 \times 10^4\ \Omega = 14.4\ \text{k}\Omega$$

From Eq. (31.12), with $\omega = 2\pi f$, we find

$$L = \frac{X_L}{2\pi f} = \frac{1.44 \times 10^4\ \Omega}{2\pi(1.60 \times 10^6\ \text{Hz})} = 1.43 \times 10^{-3}\ \text{H} = 1.43\ \text{mH}$$

(b) Combining Eqs. (31.12) and (31.13), we find that the current amplitude is $I = V_L/X_L = V_L/\omega L = V_L/2\pi f L$. Thus the current amplitude is inversely proportional to the frequency f. Since $I = 250\ \mu\text{A}$ at $f = 1.60$ MHz, the current amplitude at 16.0 MHz (ten times the original frequency) will be one-tenth as great, or 25.0 μA; at 160 kHz = 0.160 MHz (one-tenth of the original frequency) the current amplitude is ten times as great, or 2500 μA = 2.50 mA.

EVALUATE: In general, the lower the frequency of an oscillating voltage applied across an inductor, the greater the amplitude of the oscillating current that results.

Capacitor in an ac Circuit

Finally, we connect a capacitor with capacitance C to the source, as in Fig. 31.9a, producing a current $i = I\cos\omega t$ through the capacitor. Again, the positive direction of current is counterclockwise around the circuit.

CAUTION **Alternating current through a capacitor** You may object that charge can't really move through the capacitor because its two plates are insulated from each other. True enough, but as the capacitor charges and discharges, there is at each instant a current i into one plate, an equal current out of the other plate, and an equal *displacement* current between the plates just as though the charge were being conducted through the capacitor. (You may want to review the discussion of displacement current in Section 29.7.) Thus we often speak about alternating current *through* a capacitor.

To find the instantaneous voltage v_C across the capacitor—that is, the potential of point a with respect to point b—we first let q denote the charge on the left-hand plate of the capacitor in Fig. 31.9a (so $-q$ is the charge on the right-hand plate). The current i is related to q by $i = dq/dt$; with this definition, positive current corresponds to an increasing charge on the left-hand capacitor plate. Then

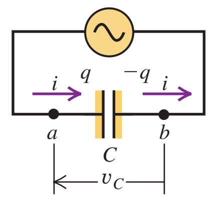
$$i = \frac{dq}{dt} = I\cos\omega t$$

Integrating this, we get

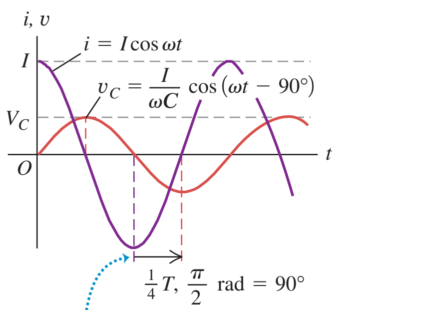
$$q = \frac{I}{\omega}\sin\omega t \quad (31.14)$$

31.9 Capacitor C connected across an ac source.

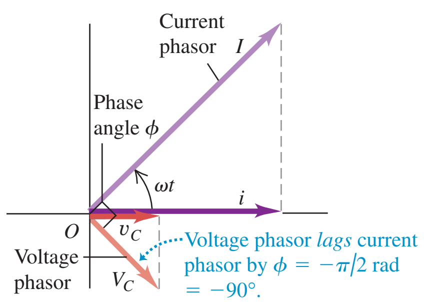
(a) Circuit with ac source and capacitor



(b) Graphs of current and voltage versus time



(c) Phasor diagram



Also, from Eq. (24.1) the charge q equals the voltage v_C multiplied by the capacitance, $q = Cv_C$. Using this in Eq. (31.14), we find

$$v_C = \frac{I}{\omega C}\sin\omega t \tag{31.15}$$

The instantaneous current i is equal to the rate of change dq/dt of the capacitor charge q; since $q = Cv_C$, i is also proportional to the rate of change of voltage. (Compare to an inductor, for which the situation is reversed and v_L is proportional to the rate of change of i.) Figure 31.9b shows v_C and i as functions of t. Because $i = dq/dt = C\,dv_C/dt$, the current has its greatest magnitude when the v_C curve is rising or falling most steeply and is zero when the v_C curve instantaneously levels off at its maximum and minimum values.

The capacitor voltage and current are out of phase by a quarter-cycle. The peaks of voltage occur a quarter-cycle *after* the corresponding current peaks, and we say that the voltage *lags* the current by 90°. The phasor diagram in Fig. 31.9c shows this relationship; the voltage phasor is behind the current phasor by a quarter-cycle, or 90°.

We can also derive this phase difference by rewriting Eq. (31.15), using the identity $\cos(A - 90°) = \sin A$:

$$v_C = \frac{I}{\omega C}\cos(\omega t - 90°) \tag{31.16}$$

This corresponds to a phase angle $\phi = -90°$. This cosine function has a "late start" of 90° compared with the current $i = I\cos\omega t$.

Equations (31.15) and (31.16) show that the *maximum* voltage V_C (the voltage amplitude) is

$$V_C = \frac{I}{\omega C} \tag{31.17}$$

To put this expression in a form similar to Eq. (31.7) for a resistor, $V_R = IR$, we define a quantity X_C, called the **capacitive reactance** of the capacitor, as

$$X_C = \frac{1}{\omega C} \quad \text{(capacitive reactance)} \tag{31.18}$$

Then

$$V_C = IX_C \quad \text{(amplitude of voltage across a capacitor, ac circuit)} \tag{31.19}$$

The SI unit of X_C is the ohm, the same as for resistance and inductive reactance, because X_C is the ratio of a voltage and a current.

CAUTION Capacitor voltage and current are not in phase Remember that Eq. (31.19) for a capacitor, like Eq. (31.13) for an inductor, is *not* a statement about the instantaneous values of voltage and current. The instantaneous values are actually 90° out of phase, as Fig. 31.9b shows. Rather, Eq. (31.19) relates the *amplitudes* of the voltage and current.

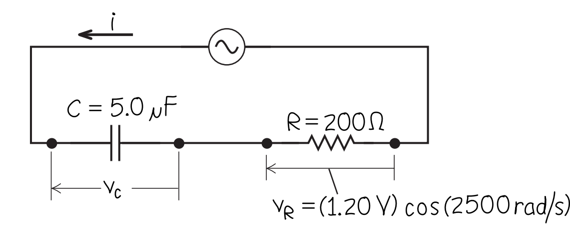
The Meaning of Capacitive Reactance

The capacitive reactance of a capacitor is inversely proportional both to the capacitance C and to the angular frequency ω; the greater the capacitance and the higher the frequency, the *smaller* the capacitive reactance X_C. Capacitors tend to pass high-frequency current and to block low-frequency currents and dc, just the opposite of inductors. A device that preferentially passes signals of high frequency is called a *high-pass filter* (see Problem 31.49).

Example 31.3 A resistor and a capacitor in an ac circuit

A 200-Ω resistor is connected in series with a 5.0-μF capacitor. The voltage across the resistor is $v_R = (1.20\text{ V})\cos(2500\text{ rad/s})t$. (a) Derive an expression for the circuit current. (b) Determine the capacitive reactance of the capacitor. (c) Derive an expression for the voltage across the capacitor.

31.10 Our sketch for this problem.



SOLUTION

IDENTIFY: Since this is a series circuit, the current is the same through the capacitor as through the resistor. Our target variables are the current i, capacitive reactance X_C, and capacitor voltage v_C.

SET UP: Figure 31.10 shows the circuit. We find the current through the resistor, and hence through the circuit as a whole, using Eq. (31.6). We use Eq. (31.18) to find the capacitive reactance X_C, Eq. (31.19) to find the voltage amplitude, and Eq. (31.16) to write an expression for the instantaneous voltage across the capacitor.

EXECUTE: (a) Using $v_R = iR$, we find that the current i in the resistor and through the circuit as a whole is

$$i = \frac{v_R}{R} = \frac{(1.20\text{ V})\cos(2500\text{ rad/s})t}{200\ \Omega} = (6.0 \times 10^{-3}\text{ A})\cos(2500\text{ rad/s})t$$

(b) From Eq. (31.18), the capacitive reactance at ω = 2500 rad/s is

$$X_C = \frac{1}{\omega C} = \frac{1}{(2500\text{ rad/s})(5.0 \times 10^{-6}\text{ F})} = 80\ \Omega$$

(c) From Eq. (31.19), the amplitude V_C of the voltage across the capacitor is

$$V_C = IX_C = (6.0 \times 10^{-3}\text{ A})(80\ \Omega) = 0.48\text{ V}$$

The 80-Ω reactance of the capacitor is 40% of the resistor's 200-Ω resistance, so the value of V_C is 40% of V_R. The instantaneous capacitor voltage v_C is given by Eq. (31.16):

$$v_C = V_C\cos(\omega t - 90°) = (0.48\text{ V})\cos[(2500\text{ rad/s})t - \pi/2\text{ rad}]$$

EVALUATE: Although the *current* through the capacitor is the same as through the resistor, the *voltages* across these two devices are different in both amplitude and phase. Note that in the expression for v_C we converted the 90° to $\pi/2$ rad so that all the angular quantities have the same units. In ac circuit analysis, phase angles are often given in degrees, so be careful to convert to radians when necessary.

Comparing ac Circuit Elements

Table 31.1 summarizes the relationships of voltage and current amplitudes for the three circuit elements we have discussed. Note again that *instantaneous* voltage and current are proportional in a resistor, where there is zero phase difference between v_R and i (see Fig. 31.7b). The instantaneous voltage and current are *not* proportional in an inductor or capacitor, because there is a 90° phase difference in both cases (see Figs. 31.8b and 31.9b).

Figure 31.11 shows how the resistance of a resistor and the reactances of an inductor and a capacitor vary with angular frequency ω. Resistance R is independent of frequency, while the reactances X_L and X_C are not. If $\omega = 0$, corresponding to a dc circuit, there is *no* current through a capacitor because $X_C \to \infty$, and there is no inductive effect because $X_L = 0$. In the limit $\omega \to \infty$, X_L also approaches infinity, and the current through an inductor becomes vanishingly small; recall that the self-induced emf opposes rapid changes in current. In this same limit, X_C and the voltage across a capacitor both approach zero; the current changes direction so rapidly that no charge can build up on either plate.

Figure 31.12 shows an application of the above discussion to a loudspeaker system. Low-frequency sounds are produced by the *woofer,* which is a speaker

31.11 Graphs of R, X_L, and X_C as functions of angular frequency ω.

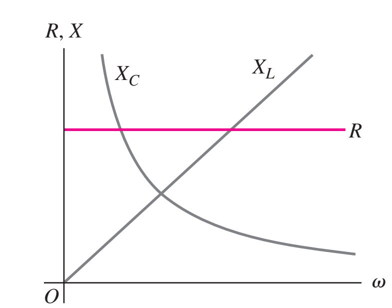
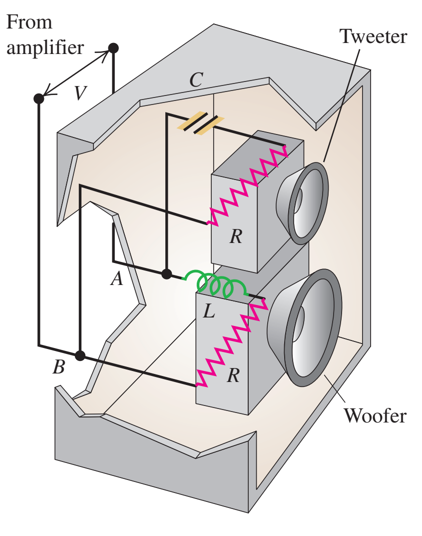


Table 31.1 Circuit Elements with Alternating Current

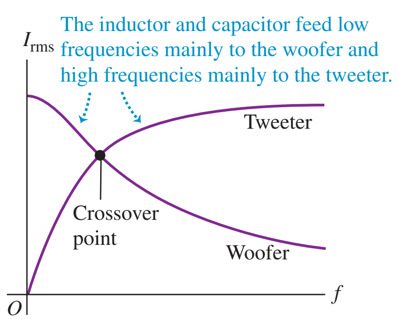
Circuit Element	Amplitude Relationship	Circuit Quantity	Phase of v
Resistor	$V_R = IR$	R	In phase with i
Inductor	$V_L = IX_L$	$X_L = \omega L$	Leads i by 90°
Capacitor	$V_C = IX_C$	$X_C = 1/\omega C$	Lags i by 90°

31.12 (a) The two speakers in this loud speaker system are connected in parallel to the amplifier. (b) Graphs of current amplitude in the tweeter and woofer as functions of frequency for a given amplifier voltage amplitude.

(a) A crossover network in a loudspeaker system



(b) Graphs of rms current as functions of frequency for a given amplifier voltage



with large diameter; the *tweeter,* a speaker with smaller diameter, produces high-frequency sounds. In order to route signals of different frequency to the appropriate speaker, the woofer and tweeter are connected in parallel across the amplifier output. The capacitor in the tweeter branch blocks the low-frequency components of sound but passes the higher frequencies; the inductor in the woofer branch does the opposite.

Test Your Understanding of Section 31.2 An oscillating voltage of fixed amplitude is applied across a circuit element. If the frequency of this voltage is increased, will the amplitude of the current through the element (i) increase, (ii) decrease, or (iii) remain the same if it is (a) a resistor, (b) an inductor, or (c) a capacitor?

31.3 The *L-R-C* Series Circuit

Many ac circuits used in practical electronic systems involve resistance, inductive reactance, and capacitive reactance. A simple example is a series circuit containing a resistor, an inductor, a capacitor, and an ac source, as shown in Fig. 31.13a. (In Section 30.6 we considered the behavior of the current in an *L-R-C* series circuit *without* a source.)

To analyze this and similar circuits, we will use a phasor diagram that includes the voltage and current phasors for each of the components. In this circuit, because of Kirchhoff's loop rule, the instantaneous *total* voltage v_{ad} across all three components is equal to the source voltage at that instant. We will show that the phasor representing this total voltage is the *vector sum* of the phasors for the individual voltages. Complete phasor diagrams for this circuit are shown in Figs. 31.13b and 31.13c. These may appear complex, but we'll explain them one step at a time.

Let's assume that the source supplies a current i given by $i = I\cos\omega t$. Because the circuit elements are connected in series, the current at any instant is the same at every point in the circuit. Thus a *single phasor I*, with length proportional to the current amplitude, represents the current in *all* circuit elements.

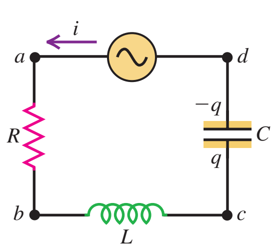
As in Section 31.2, we use the symbols v_R, v_L, and v_C for the instantaneous voltages across R, L, and C, and the symbols V_R, V_L, and V_C for the maximum voltages. We denote the instantaneous and maximum *source* voltages by v and V. Then, in Fig. 31.13a, $v = v_{ad}$, $v_R = v_{ab}$, $v_L = v_{bc}$, and $v_C = v_{cd}$.

We have shown that the potential difference between the terminals of a resistor is *in phase* with the current in the resistor and that its maximum value V_R is given by Eq. (31.7):

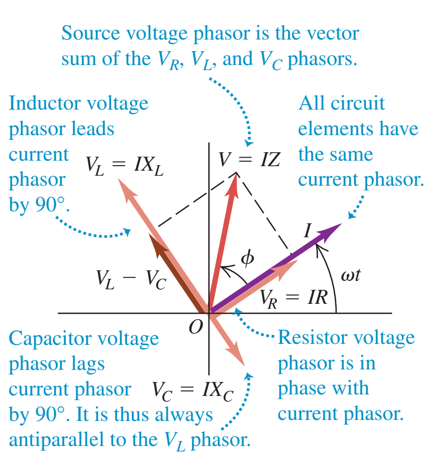
$$V_R = IR$$

31.13 An *L-R-C* series circuit with an ac source.

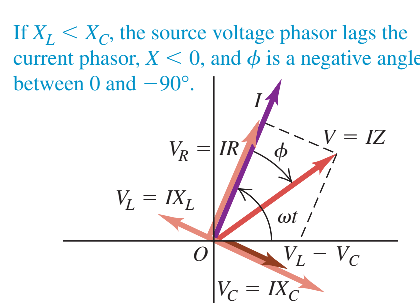
(a) Series *R-L-C* circuit



(b) Phasor diagram for the case $X_L > X_C$



(c) Phasor diagram for the case $X_L < X_C$



The phasor V_R in Fig. 31.13b, in phase with the current phasor I, represents the voltage across the resistor. Its projection onto the horizontal axis at any instant gives the instantaneous potential difference v_R.

The voltage across an inductor *leads* the current by 90°. Its voltage amplitude is given by Eq. (31.13):

$$V_L = IX_L$$

The phasor V_L in Fig. 31.13b represents the voltage across the inductor, and its projection onto the horizontal axis at any instant equals v_L.

The voltage across a capacitor *lags* the current by 90°. Its voltage amplitude is given by Eq. (31.19):

$$V_C = IX_C$$

The phasor V_C in Fig. 31.13b represents the voltage across the capacitor, and its projection onto the horizontal axis at any instant equals v_C.

The instantaneous potential difference v between terminals a and d is equal at every instant to the (algebraic) sum of the potential differences v_R, v_L, and v_C. That is, it equals the sum of the *projections* of the phasors V_R, V_L, and V_C. But the sum of the projections of these phasors is equal to the *projection* of their *vector sum.* So the vector sum V must be the phasor that represents the source voltage v and the instantaneous total voltage v_{ad} across the series of elements.

To form this vector sum, we first subtract the phasor V_C from the phasor V_L. (These two phasors always lie along the same line, with opposite directions.) This gives the phasor $V_L - V_C$. This is always at right angles to the phasor V_R, so from the Pythagorean theorem the magnitude of the phasor V is

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \qquad (31.20)$$

We define the **impedance** Z of an ac circuit as the ratio of the voltage amplitude across the circuit to the current amplitude in the circuit. From Eq. (31.20) the impedance of the *L-R-C* series circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad (31.21)$$

so we can rewrite Eq. (31.20) as

$$V = IZ \quad \text{(amplitude of voltage across an ac circuit)} \qquad (31.22)$$

While Eq. (31.21) is valid only for an *L-R-C* series circuit, we can use Eq. (31.22) to define the impedance of *any* network of resistors, inductors, and capacitors as the ratio of the amplitude of the voltage across the network to the current amplitude. The SI unit of impedance is the ohm.

The Meaning of Impedance and Phase Angle

Equation (31.22) has a form similar to $V = IR$, with impedance Z in an ac circuit playing the role of resistance R in a dc circuit. Just as direct current tends to follow the path of least resistance, so alternating current tends to follow the path of lowest impedance (Fig. 31.14). Note, however, that impedance is actually a function of R, L, and C, as well as of the angular frequency ω. We can see this by substituting Eq. (31.12) for X_L and Eq. (31.18) for X_C into Eq. (31.21), giving the following complete expression for Z for a series circuit:

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{R^2 + [\omega L - (1/\omega C)]^2} \end{aligned} \quad \text{(impedance of an } L\text{-}R\text{-}C \text{ series circuit)} \qquad (31.23)$$

ActivPhysics Online
14.3 AC Circuits: The Driven Oscillator (Questions 6, 7, and 10)

31.14 This gas-filled glass sphere has an alternating voltage between its surface and the electrode at its center. The glowing streamers show the resulting alternating current that passes through the gas. When you touch the outside of the sphere, your fingertips and the inner surface of the sphere act as the plates of a capacitor, and the sphere and your body together form an *L-R-C* series circuit. The current (which is low enough to be harmless) is drawn to your fingers because the path through your body has a low impedance.

Hence for a given amplitude V of the source voltage applied to the circuit, the amplitude $I = V/Z$ of the resulting current will be different at different frequencies. We'll explore this frequency dependence in detail in Section 31.5.

In the phasor diagram shown in Fig. 31.13b, the angle ϕ between the voltage and current phasors is the phase angle of the source voltage v with respect to the current i; that is, it is the angle by which the source voltage leads the current. From the diagram,

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I(X_L - X_C)}{IR} = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{\omega L - 1/\omega C}{R} \quad \text{(phase angle of an } L\text{-}R\text{-}C \text{ series circuit)} \qquad (31.24)$$

If the current is $i = I\cos\omega t$, then the source voltage v is

$$v = V\cos(\omega t + \phi) \qquad (31.25)$$

Figure 31.13b shows the behavior of a circuit in which $X_L > X_C$. Figure 31.13c shows the behavior when $X_L < X_C$; the voltage phasor V lies on the opposite side of the current phasor I and the voltage *lags* the current. In this case, $X_L - X_C$ is *negative*, $\tan\phi$ is negative, and ϕ is a negative angle between 0 and $-90°$. Since X_L and X_C depend on frequency, the phase angle ϕ depends on frequency as well. We'll examine the consequences of this in Section 31.5.

All of the expressions that we've developed for an *L-R-C* series circuit are still valid if one of the circuit elements is missing. If the resistor is missing, we set $R = 0$; if the inductor is missing, we set $L = 0$. But if the capacitor is missing, we set $C = \infty$, corresponding to the absence of any potential difference $(v_C = q/C = 0)$ or any capacitive reactance $(X_C = 1/\omega C = 0)$.

In this entire discussion we have described magnitudes of voltages and currents in terms of their *maximum* values, the voltage and current *amplitudes.* But we remarked at the end of Section 31.1 that these quantities are usually described in terms of rms values, not amplitudes. For any sinusoidally varying quantity the rms value is always $1/\sqrt{2}$ times the amplitude. All the relationships between voltage and current that we have derived in this and the preceding sections are still valid if we use rms quantities throughout instead of amplitudes. For example, if we divide Eq. (31.22) by $\sqrt{2}$, we get

$$\frac{V}{\sqrt{2}} = \frac{I}{\sqrt{2}}Z$$

which we can rewrite as

$$V_{\text{rms}} = I_{\text{rms}}Z \qquad (31.26)$$

We can translate Eqs. (31.7), (31.13), and (31.19) in exactly the same way.

We have considered only ac circuits in which an inductor, a resistor, and a capacitor are in series. You can do a similar analysis for a *parallel L-R-C* circuit; see Problem 31.54.

Finally, we remark that in this section we have been describing the *steady-state* condition of a circuit, the state that exists after the circuit has been connected to the source for a long time. When the source is first connected, there may be additional voltages and currents, called *transients,* whose nature depends on the time in the cycle when the circuit is initially completed. A detailed analysis of transients is beyond our scope. They always die out after a sufficiently long time, and they do not affect the steady-state behavior of the circuit. But they can cause dangerous and damaging surges in power lines, which is why delicate electronic systems such as computers are often provided with power-line surge protectors.

Problem-Solving Strategy 31.1 Alternating-Current Circuits

MP

IDENTIFY *the relevant concepts:* All of the concepts that we used to analyze direct-current circuits also apply to alternating-current circuits. However, we must be careful to distinguish between the amplitudes of alternating currents and voltages and their instantaneous values. We must also keep in mind the distinctions between resistance (for resistors), reactance (for inductors or capacitors), and impedance (for composite circuits).

SET UP *the problem* using the following steps:

1. Draw a diagram of the circuit and label all known and unknown quantities.
2. Determine the target variables.

EXECUTE *the solution* as follows:

1. Use the relationships derived in Sections 31.2 and 31.3 to solve for the target variables, using the following hints.
2. In ac circuit problems it is nearly always easiest to work with angular frequency ω. If you are given the ordinary frequency f, expressed in Hz, convert it using the relationship $\omega = 2\pi f$.
3. Keep in mind a few basic facts about phase relationships. For a resistor, voltage and current are always *in phase,* and the two corresponding phasors in a phasor diagram always have the same direction. For an inductor, the voltage always *leads* the current by 90° (i.e., $\phi = +90°$), and the voltage phasor is always turned 90° counterclockwise from the current phasor. For a capacitor, the voltage always *lags* the current by 90° (i.e., $\phi = -90°$), and the voltage phasor is always turned 90° clockwise from the current phasor.
4. Remember that with ac circuits, all voltages and currents are sinusoidal functions of time instead of being constant, but Kirchhoff's rules hold nonetheless at each instant. Thus, in a series circuit, the instantaneous current is the same in all circuit elements; in a parallel circuit, the instantaneous potential difference is the same across all circuit elements.
5. Inductive reactance, capacitive reactance, and impedance are analogous to resistance; each represents the ratio of voltage amplitude V to current amplitude I in a circuit element or combination of elements. Keep in mind, however, that phase relationships play an essential role. The effects of resistance and reactance have to be combined by *vector* addition of the corresponding voltage phasors, as in Figs. 31.13b and 31.13c. When you have several circuit elements in series, for example, you can't just *add* all the numerical values of resistance and reactance to get the impedance; that would ignore the phase relationships.

EVALUATE *your answer:* When working with a series *L-R-C* circuit, you can check your results by comparing the values of the inductive reactance X_L and the capacitive reactance X_C. If $X_L > X_C$, then the voltage amplitude across the inductor is greater than that across the capacitor and the phase angle ϕ is positive (between 0 and 90°). If $X_L < X_C$, then the voltage amplitude across the inductor is less than that across the capacitor and the phase angle ϕ is negative (between 0 and $-90°$).

Example 31.4 An *L-R-C* series circuit I

In the series circuit of Fig. 31.13a, suppose $R = 300\ \Omega$, $L = 60$ mH, $C = 0.50\ \mu$F, $V = 50$ V, and $\omega = 10{,}000$ rad/s. Find the reactances X_L and X_C, the impedance Z, the current amplitude I, the phase angle ϕ, and the voltage amplitude across each circuit element.

SOLUTION

IDENTIFY: This problem uses the ideas developed in Section 31.2 and this section about the behavior of circuit elements in an ac circuit.

SET UP: We use Eqs. (31.12) and (31.18) to determine the reactances and Eq. (31.23) to find the impedance. We then use Eq. (31.22) to find the current amplitude and Eq. (31.24) to calculate the phase angle. Given this information, the relationships in Table 31.1 tell us the voltage amplitudes.

EXECUTE: The inductive and capacitive reactances are

$$X_L = \omega L = (10{,}000\ \text{rad/s})(60\ \text{mH}) = 600\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(10{,}000\ \text{rad/s})(0.50 \times 10^{-6}\ \text{F})} = 200\ \Omega$$

The impedance Z of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(300\ \Omega)^2 + (600\ \Omega - 200\ \Omega)^2} = 500\ \Omega$$

With source voltage amplitude $V = 50$ V the current amplitude is

$$I = \frac{V}{Z} = \frac{50\ \text{V}}{500\ \Omega} = 0.10\ \text{A}$$

The phase angle ϕ is

$$\phi = \arctan\frac{X_L - X_C}{R} = \arctan\frac{400\ \Omega}{300\ \Omega} = 53°$$

From Table 31.1, the voltage amplitudes V_R, V_L, and V_C across the resistor, inductor, and capacitor, respectively, are

$$V_R = IR = (0.10\ \text{A})(300\ \Omega) = 30\ \text{V}$$
$$V_L = IX_L = (0.10\ \text{A})(600\ \Omega) = 60\ \text{V}$$
$$V_C = IX_C = (0.10\ \text{A})(200\ \Omega) = 20\ \text{V}$$

EVALUATE: Note that $X_L > X_C$ and hence the voltage amplitude across the inductor is greater than across the capacitor and ϕ is negative. The value $\phi = -53°$ means that the voltage *leads* the current by 53°; this is like the situation shown in Fig. 31.13b.

Note that the source voltage amplitude $V = 50$ V is *not* equal to the sum of the voltage amplitudes across the separate circuit elements. (That is, $50\ \text{V} \neq 30\ \text{V} + 60\ \text{V} + 20\ \text{V}$.) Make sure you understand why not!

Example 31.5 An *L-R-C* series circuit II

For the *L-R-C* series circuit described in Example 31.4, describe the time dependence of the instantaneous current and each instantaneous voltage.

SOLUTION

IDENTIFY: In Example 31.4 we found the *amplitudes* of the current and voltages. Now our task is to find expressions for the *instantaneous values* of the current and voltages. As we learned in Section 31.2, the voltage across a resistor is in phase with the current but the voltages across an inductor or capacitor are not. We also learned in this section that ϕ is the phase angle between the source voltage and the current.

SET UP: If we describe the current using Eq. (31.2), the voltages are given by Eq. (31.8) for the resistor, Eq. (31.10) for the inductor, Eq. (31.16) for the capacitor, and Eq. (31.25) for the source.

EXECUTE: The current and all of the voltages oscillate with the same angular frequency, $\omega = 10{,}000$ rad/s, and hence with the same period, $2\pi/\omega = 2\pi/(10{,}000\ \text{rad/s}) = 6.3 \times 10^{-4}\ \text{s} = 0.63\ \text{ms}$. Using Eq. (31.2), the current is

$$i = I\cos\omega t = (0.10\ \text{A})\cos(10{,}000\ \text{rad/s})t$$

This choice simply means that we choose $t = 0$ to be an instant when the current is maximum. The resistor voltage is *in phase* with the current, so

$$v_R = V_R\cos\omega t = (30\ \text{V})\cos(10{,}000\ \text{rad/s})t$$

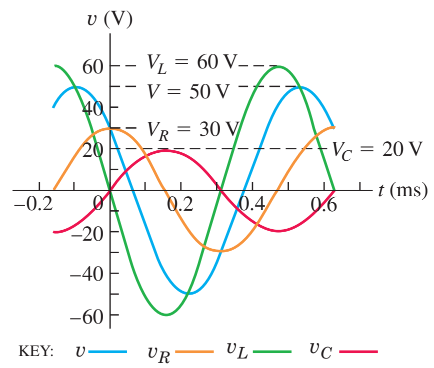
The inductor voltage *leads* the current by 90°, so

$$v_L = V_L\cos(\omega t + 90°) = -V_L\sin\omega t$$
$$= -(60\ \text{V})\sin(10{,}000\ \text{rad/s})t$$

The capacitor voltage *lags* the current by 90°, so

$$v_C = V_C\cos(\omega t - 90°) = V_C\sin\omega t$$
$$= (20\ \text{V})\sin(10{,}000\ \text{rad/s})t$$

31.15 Graphs of the source voltage v, resistor voltage v_R, inductor voltage v_L, and capacitor voltage v_C as functions of time for the situation of Example 31.4. The current, which is not shown, is in phase with the resistor voltage.



Finally, the source voltage (equal to the voltage across the entire combination of resistor, inductor, and capacitor) *leads* the current by $\phi = 53°$, so

$$v = V\cos(\omega t + \phi)$$
$$= (50\ \text{V})\cos\left[(10{,}000\ \text{rad/s})t + \left(\frac{2\pi\ \text{rad}}{360°}\right)(53°)\right]$$
$$= (50\ \text{V})\cos[(10{,}000\ \text{rad/s})t + 0.93\ \text{rad}]$$

EVALUATE: Figure 31.15 graphs the various voltages versus time. The inductor voltage has a larger amplitude than the capacitor voltage because $X_L > X_C$. While the source voltage amplitude V is not equal to the sum of the individual voltage amplitudes V_R, V_L, and V_C, the *instantaneous* source voltage v is always equal to the sum of the instantaneous voltages v_R, v_L, and v_C. You should verify this by measuring the values of the voltages shown in the graph at different values of the time t.

Test Your Understanding of Section 31.3 Rank the following ac circuits in order of their current amplitude, from highest to lowest value. (i) the circuit in Example 31.4; (ii) the circuit in Example 31.4 with the capacitor and inductor both removed; (iii) the circuit in Example 31.4 with the resistor and capacitor both removed; (iv) the circuit in Example 31.4 with the resistor and inductor both removed.

31.4 Power in Alternating-Current Circuits

Alternating currents play a central role in systems for distributing, converting, and using electrical energy, so it's important to look at power relationships in ac circuits. For an ac circuit with instantaneous current i and current amplitude I, we'll consider an element of that circuit across which the instantaneous potential difference is v with voltage amplitude V. The instantaneous power p delivered to this circuit element is

$$p = vi$$

Let's first see what this means for individual circuit elements. We'll assume in each case that $i = I\cos\omega t$.

Power in a Resistor

Suppose first that the circuit element is a *pure resistor R*, as in Fig. 31.7a; then $v = v_R$ and i are *in phase.* We obtain the graph representing p by multiplying the heights of the graphs of v and i in Fig. 31.7b at each instant. This graph is shown by the black curve in Fig. 31.16a. The product vi is always positive because v and i are always either both positive or both negative. Hence energy is supplied *to* the resistor at every instant for both directions of i, although the power is not constant.

The power curve for a pure resistor is symmetrical about a value equal to one-half its maximum value VI, so the *average power* P_{av} is

$$P_{\text{av}} = \frac{1}{2}VI \qquad \text{(for a pure resistor)} \qquad (31.27)$$

An equivalent expression is

$$P_{\text{av}} = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}} = V_{\text{rms}}I_{\text{rms}} \qquad \text{(for a pure resistor)} \qquad (31.28)$$

Also, $V_{\text{rms}} = I_{\text{rms}}R$, so we can express P_{av} by any of the equivalent forms

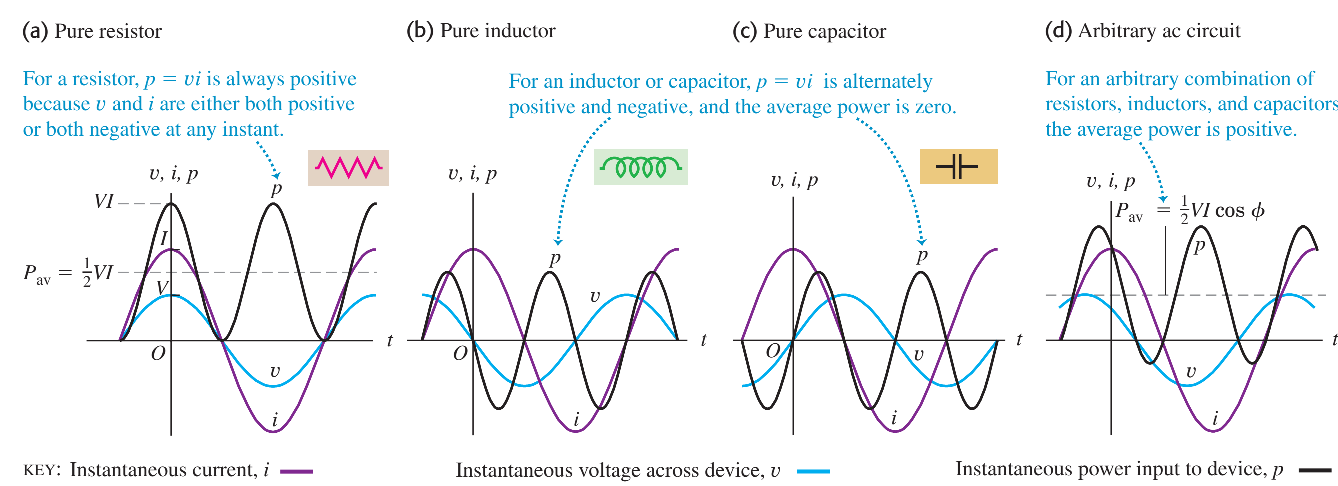
$$P_{\text{av}} = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R} = V_{\text{rms}}I_{\text{rms}} \qquad \text{(for a pure resistor)} \qquad (31.29)$$

Note that the expressions in Eq. (31.29) have the same form as the corresponding relationships for a dc circuit, Eq. (25.18). Also note that they are valid only for pure resistors, not for more complicated combinations of circuit elements.

Power in an Inductor

Next we connect the source to a pure inductor L, as in Fig. 31.8a. The voltage $v = v_L$ leads the current i by 90°. When we multiply the curves of v and i, the product vi is *negative* during the half of the cycle when v and i have *opposite* signs. The power curve, shown in Fig. 31.16b, is symmetrical about the horizontal axis; it is positive half the time and negative the other half, and the average power is zero. When p is positive, energy is being supplied to set up the magnetic field in the inductor; when p is negative, the field is collapsing and the inductor is returning energy to the source. The net energy transfer over one cycle is zero.

31.16 Graphs of current, voltage, and power as functions of time for (a) a pure resistor, (b) a pure inductor, (c) a pure capacitor, and (d) an arbitrary ac circuit that can have resistance, inductance, and capacitance.



Power in a Capacitor

Finally, we connect the source to a pure capacitor C, as in Fig. 31.9a. The voltage $v = v_C$ lags the current i by 90°. Figure 31.16c shows the power curve; the average power is again zero. Energy is supplied to charge the capacitor and is returned to the source when the capacitor discharges. The net energy transfer over one cycle is again zero.

Power in a General ac Circuit

In *any* ac circuit, with any combination of resistors, capacitors, and inductors, the voltage v across the entire circuit has some phase angle ϕ with respect to the current i. Then the instantaneous power p is given by

$$p = vi = [V\cos(\omega t + \phi)][I\cos\omega t] \tag{31.30}$$

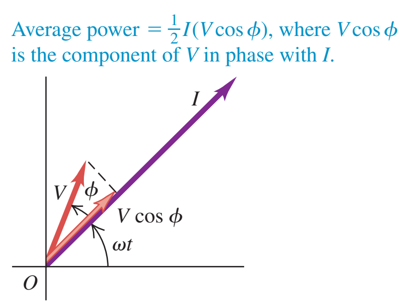
The instantaneous power curve has the form shown in Fig. 31.16d. The area between the positive loops and the horizontal axis is greater than the area between the negative loops and the horizontal axis, and the average power is positive.

We can derive from Eq. (31.30) an expression for the *average* power P_{av} by using the identity for the cosine of the sum of two angles:

$$\begin{aligned} p &= [V(\cos\omega t\cos\phi - \sin\omega t\sin\phi)][I\cos\omega t] \\ &= VI\cos\phi\cos^2\omega t - VI\sin\phi\cos\omega t\sin\omega t \end{aligned}$$

From the discussion in Section 31.1 that led to Eq. (31.4), we see that the average value of $\cos^2\omega t$ (over one cycle) is $\frac{1}{2}$. The average value of $\cos\omega t\sin\omega t$ is zero because this product is equal to $\frac{1}{2}\sin 2\omega t$, whose average over a cycle is zero. So the average power P_{av} is

$$P_{av} = \frac{1}{2}VI\cos\phi = V_{rms}I_{rms}\cos\phi \quad \text{(average power into a general ac circuit)} \tag{31.31}$$



31.17 Using phasors to calculate the average power for an arbitrary ac circuit.

When v and i are in phase, so $\phi = 0$, the average power equals $\frac{1}{2}VI = V_{rms}I_{rms}$; when v and i are 90° out of phase, the average power is zero. In the general case, when v has a phase angle ϕ with respect to i, the average power equals $\frac{1}{2}I$ multiplied by $V\cos\phi$, the component of the voltage phasor that is *in phase* with the current phasor. Figure 31.17 shows the general relationship of the current and voltage phasors. For the L-R-C series circuit, Figs. 31.13b and 31.13c show that $V\cos\phi$ equals the voltage amplitude V_R for the resistor; hence Eq. (31.31) is the average power dissipated in the resistor. On average there is no energy flow into or out of the inductor or capacitor, so none of P_{av} goes into either of these circuit elements.

The factor $\cos\phi$ is called the **power factor** of the circuit. For a pure resistance, $\phi = 0$, $\cos\phi = 1$, and $P_{av} = V_{rms}I_{rms}$. For a pure inductor or capacitor, $\phi = \pm 90°$, $\cos\phi = 0$, and $P_{av} = 0$. For an L-R-C series circuit the power factor is equal to R/Z; we leave the proof of this statement to you (see Exercise 31.27).

A low power factor (large angle ϕ of lag or lead) is usually undesirable in power circuits. The reason is that for a given potential difference, a large current is needed to supply a given amount of power. This results in large i^2R losses in the transmission lines. Your electric power company may charge a higher rate to a client with a low power factor. Many types of ac machinery draw a *lagging* current; that is, the current drawn by the machinery lags the applied voltage. Hence the voltage leads the current, so $\phi > 0$ and $\cos\phi < 1$. The power factor can be corrected toward the ideal value of 1 by connecting a capacitor in parallel with the load. The current drawn by the capacitor *leads* the voltage (that is, the voltage across the capacitor lags the current), which compensates for the lagging current in the other branch of the circuit. The capacitor itself absorbs no net power from the line.

Example 31.6 Power in a hair dryer

An electric hair dryer is rated at 1500 W at 120 V. The rated power of this hair dryer, or of any other ac device, is the *average* power drawn by the device, and the rated voltage is the *rms* voltage. Calculate (a) the resistance, (b) the rms current, and (c) the maximum instantaneous power. Assume that the hair dryer is a pure resistor. (The hair dryer's heating element acts as a resistor.)

SOLUTION

IDENTIFY: We assume that the hair dryer is a pure resistor. We are given the average power $P_{av} = 1500$ W and the rms voltage $V_{rms} = 120$ V. Our target variables are the resistance R, the rms current I_{rms}, and the maximum value of the instantaneous power p.

SET UP: We solve Eq. (31.29) to determine the resistance R. We find the rms current from V_{rms} and P_{av} using Eq. (31.28), and we find the maximum instantaneous power from Eq. (31.30).

EXECUTE: (a) From Eq. (31.29), the resistance is

$$R = \frac{V_{rms}^{\ 2}}{P_{av}} = \frac{(120\text{ V})^2}{1500\text{ W}} = 9.6\ \Omega$$

(b) From Eq. (31.28),

$$I_{rms} = \frac{P_{av}}{V_{rms}} = \frac{1500\text{ W}}{120\text{ V}} = 12.5\text{ A}$$

(c) For a pure resistor, the voltage and current are in phase and the phase angle ϕ is zero. Hence from Eq. (31.30), the instantaneous power is $p = VI\cos^2\omega t$ and the maximum instantaneous power is $p_{max} = VI$. From Eq. (31.27), this is twice the average power P_{av}, so

$$p_{max} = VI = 2P_{av} = 2(1500\text{ W}) = 3000\text{ W}$$

EVALUATE: We can confirm our result in part (b) by using Eq. (31.7): $I_{rms} = V_{rms}/R = (120\text{ V})/(9.6\ \Omega) = 12.5$ A. Note that some manufacturers of stereo amplifiers state power outputs in terms of the peak value rather than the lower average value, to mislead the unwary consumer.

Example 31.7 Power in an *L-R-C* series circuit

For the L-R-C series circuit of Example 31.4, (a) calculate the power factor; and (b) calculate the average power delivered to the entire circuit and to each circuit element.

SOLUTION

IDENTIFY: We can use all of the results found in Example 31.4.

SET UP: The power factor is simply the cosine of the phase angle ϕ, and Eq. (31.31) allows us to find the average power delivered in terms of ϕ and the amplitudes of voltage and current.

EXECUTE: (a) The power factor is $\cos\phi = \cos 53° = 0.60$.

(b) From Eq. (31.31) the average power delivered to the circuit is

$$P_{av} = \frac{1}{2}VI\cos\phi = \frac{1}{2}(50\text{ V})(0.10\text{ A})(0.60) = 1.5\text{ W}$$

EVALUATE: While P_{av} is the average power delivered to the L-R-C combination, all of this power is dissipated in the resistor. The average power delivered to a pure inductor or pure capacitor is always zero (see Figs. 31.16b and 31.16c).

Test Your Understanding of Section 31.4 Figure 31.16d shows that during part of a cycle of oscillation, the instantaneous power delivered to the circuit is negative. This means that energy is being extracted from the circuit. (a) Where is the energy extracted from? (i) the resistor; (ii) the inductor; (iii) the capacitor; (iv) the ac source; (v) more than one of these. (b) Where does the energy go? (i) the resistor; (ii) the inductor; (iii) the capacitor; (iv) the ac source; (v) more than one of these.

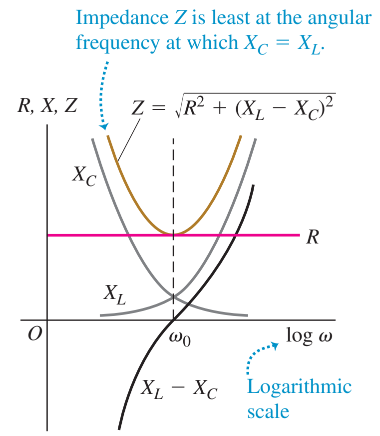
31.5 Resonance in Alternating-Current Circuits

Much of the practical importance of L-R-C series circuits arises from the way in which such circuits respond to sources of different angular frequency ω. For example, one type of tuning circuit used in radio receivers is simply an L-R-C series circuit. A radio signal of any given frequency produces a current of the same frequency in the receiver circuit, but the amplitude of the current is *greatest* if the signal frequency equals the particular frequency to which the receiver circuit is "tuned." This effect is called *resonance*. The circuit is designed so that signals at other than the tuned frequency produce currents that are too small to make an audible sound come out of the radio's speakers.

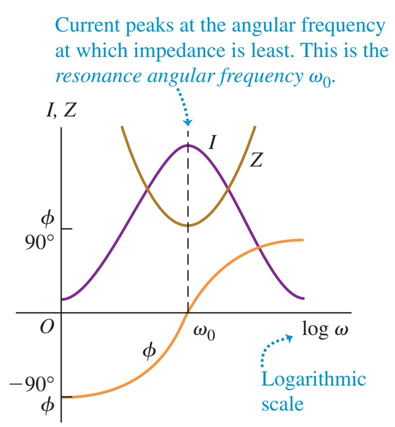
ActivPhysics Online
14.3 AC Circuits: The Driven Oscillator (Questions 8, 9, and 11)

31.18 How variations in the angular frequency of an ac circuit affect (a) reactances, resistance, and impedance, and (b) impedance, current amplitude, and phase angle.

(a) Reactance, resistance, and impedance as functions of angular frequency



(b) Impedance, current, and phase angle as functions of angular frequency



To see how an *L-R-C* series circuit can be used in this way, suppose we connect an ac source with constant voltage amplitude V but adjustable angular frequency ω across an *L-R-C* series circuit. The current that appears in the circuit has the same angular frequency as the source and a current amplitude $I = V/Z$, where Z is the impedance of the *L-R-C* series circuit. This impedance depends on the frequency, as Eq. (31.23) shows. Figure 31.18a shows graphs of R, X_L, X_C, and Z as functions of ω. We have used a logarithmic angular frequency scale so that we can cover a wide range of frequencies. As the frequency increases, X_L increases and X_C decreases; hence there is always one frequency at which X_L and X_C are equal and $X_L - X_C$ is zero. At this frequency the impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$ has its *smallest* value, equal simply to the resistance R.

Circuit Behavior at Resonance

As we vary the angular frequency ω of the source, the current amplitude $I = V/Z$ varies as shown in Fig. 31.18b; the *maximum* value of I occurs at the frequency at which the impedance Z is *minimum.* This peaking of the current amplitude at a certain frequency is called **resonance.** The angular frequency ω_0 at which the resonance peak occurs is called the **resonance angular frequency.** This is the angular frequency at which the inductive and capacitive reactances are equal, so at resonance,

$$X_L = X_C \qquad \omega_0 L = \frac{1}{\omega_0 C} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad \text{(}L\text{-}R\text{-}C\text{ series circuit at resonance)} \tag{31.32}$$

Note that this is equal to the natural angular frequency of oscillation of an *L-C* circuit, which we derived in Section 30.5, Eq. (30.22). The **resonance frequency** f_0 is $\omega_0/2\pi$. This is the frequency at which the greatest current appears in the circuit for a given source voltage amplitude; in other words, f_0 is the frequency to which the circuit is "tuned."

It's instructive to look at what happens to the *voltages* in an *L-R-C* series circuit at resonance. The current at any instant is the same in L and C. The voltage across an inductor always *leads* the current by 90°, or $\frac{1}{4}$ cycle, and the voltage across a capacitor always *lags* the current by 90°. Therefore the instantaneous voltages across L and C always differ in phase by 180°, or $\frac{1}{2}$ cycle; they have opposite signs at each instant. At the resonance frequency, and *only* at the resonance frequency, $X_L = X_C$ and the voltage amplitudes $V_L = IX_L$ and $V_C = IX_C$ are *equal;* then the instantaneous voltages across L and C add to zero at each instant, and the *total* voltage v_{bd} across the *L-C* combination in Fig. 31.13a is exactly zero. The voltage across the resistor is then equal to the source voltage. So at the resonance frequency the circuit behaves as if the inductor and capacitor weren't there at all!

The *phase* of the voltage relative to the current is given by Eq. (31.24). At frequencies below resonance, X_C is greater than X_L; the capacitive reactance dominates, the voltage *lags* the current, and the phase angle ϕ is between zero and −90°. Above resonance, the inductive reactance dominates; the voltage *leads* the current, and the phase angle is between zero and +90°. This variation of ϕ with angular frequency is shown in Fig. 31.18b.

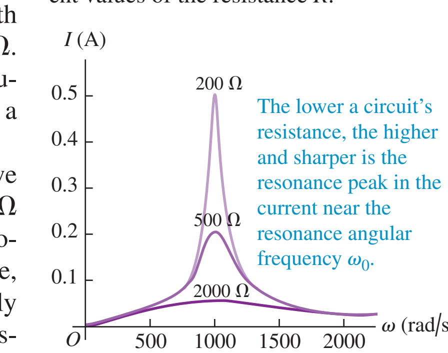
Tailoring an ac Circuit

If we can vary the inductance L or the capacitance C of a circuit, we can also vary the resonance frequency. This is exactly how a radio or television receiving set is "tuned" to receive a particular station. In the early days of radio this was accomplished by use of capacitors with movable metal plates whose overlap could be varied to change C. (This is what is being done with the radio tuning knob shown in the photograph that opens this chapter.) A more modern approach is to vary L by using a coil with a ferrite core that slides in or out.

In a series *L-R-C* circuit the impedance reaches its minimum value and the current its maximum value at the resonance frequency. The middle curve in Fig. 31.19 is a graph of current as a function of frequency for such a circuit, with source voltage amplitude $V = 100$ V, $L = 2.0$ H, $C = 0.50\ \mu$F, and $R = 500\ \Omega$. This curve is called a *response curve* or a *resonance curve.* The resonance angular frequency is $\omega_0 = (LC)^{-1/2} = 1000$ rad/s. As we expect, the curve has a peak at this angular frequency.

The resonance frequency is determined by L and C; what happens when we change R? Figure 31.19 also shows graphs of I as a function of ω for $R = 200\ \Omega$ and for $R = 2000\ \Omega$. The curves are similar for frequencies far away from resonance, where the impedance is dominated by X_L or X_C. But near resonance, where X_L and X_C nearly cancel each other, the curve is higher and more sharply peaked for small values of R and broader and flatter for large values of R. At resonance, $Z = R$ and $I = V/R$, so the maximum height of the curve is inversely proportional to R.

31.19 Graph of current amplitude I as a function of angular frequency ω for an *L-R-C* series circuit with $V = 100$ V, $L = 2.0$ H, $C = 0.50\ \mu$F, and three different values of the resistance R.



The shape of the response curve is important in the design of radio and television receiving circuits. The sharply peaked curve is what makes it possible to discriminate between two stations broadcasting on adjacent frequency bands. But if the peak is *too* sharp, some of the information in the received signal is lost, such as the high-frequency sounds in music. The shape of the resonance curve is also related to the overdamped and underdamped oscillations that we described in Section 30.6. A sharply peaked resonance curve corresponds to a small value of R and a lightly damped oscillating system; a broad, flat curve goes with a large value of R and a heavily damped system.

In this section we have discussed resonance in an *L-R-C series* circuit. Resonance can also occur in an ac circuit in which the inductor, resistor, and capacitor are connected in *parallel.* We leave the details to you (see Problem 31.55).

Resonance phenomena occur not just in ac circuits, but in all areas of physics. We discussed examples of resonance in *mechanical* systems in Sections 13.8 and 16.5. The amplitude of a mechanical oscillation peaks when the driving-force frequency is close to a natural frequency of the system; this is analogous to the peaking of the current in an *L-R-C* series circuit. We suggest that you review the sections on mechanical resonance now, looking for the analogies. Other important examples of resonance occur in atomic and nuclear physics and in the study of fundamental particles (high-energy physics).

Example 31.8 Tuning a radio

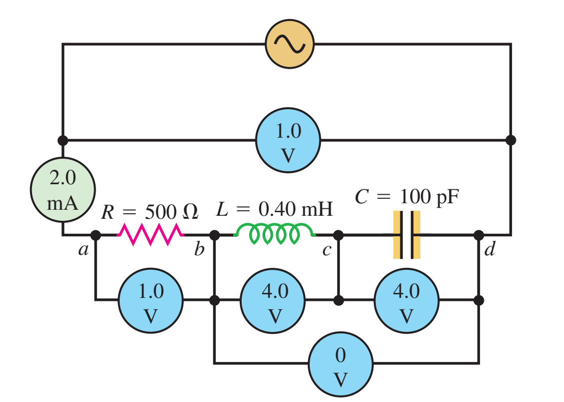
The series circuit in Fig. 31.20 is similar to arrangements that are sometimes used in radio tuning circuits. This circuit is connected to the terminals of an ac source with a constant rms terminal voltage of 1.0 V and a variable frequency. Find (a) the resonance frequency; (b) the inductive reactance, the capacitive reactance, and the impedance at the resonance frequency; (c) the rms current at resonance; and (d) the rms voltage across each circuit element at resonance.

SOLUTION

IDENTIFY: The circuit in Fig. 31.20 is a series *L-R-C* circuit, but with meters added to measure the rms current and voltages (which are our target variables).

SET UP: Equation (31.32) includes the formula for the resonance angular frequency ω_0, from which we find the resonance frequency f_0. We find the remaining target variables using the results of Sections 31.2 and 31.3.

31.20 A radio tuning circuit at resonance. The circles denote rms current and voltages.



Continued

EXECUTE: (a) The resonance angular frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.40 \times 10^{-3}\ \text{H})(100 \times 10^{-12}\ \text{F})}}$$
$$= 5.0 \times 10^6\ \text{rad/s}$$

The corresponding frequency $f_0 = \omega_0/2\pi$ is

$$f_0 = 8.0 \times 10^5\ \text{Hz} = 800\ \text{kHz}$$

This corresponds to the lower part of the AM radio band.

(b) At this frequency,

$$X_L = \omega L = (5.0 \times 10^6\ \text{rad/s})(0.40 \times 10^{-3}\ \text{H}) = 2000\ \Omega$$
$$X_C = \frac{1}{\omega C} = \frac{1}{(5.0 \times 10^6\ \text{rad/s})(100 \times 10^{-12}\ \text{F})} = 2000\ \Omega$$

Since $X_L = X_C$ and $X_L - X_C = 0$, Eq. (31.23) shows that the impedance Z at resonance is equal to the resistance: $Z = R = 500\ \Omega$.

(c) From Eq. (31.26) the rms current at resonance is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R} = \frac{1.0\ \text{V}}{500\ \Omega} = 0.0020\ \text{A} = 2.0\ \text{mA}$$

(d) The rms potential difference across the resistor is

$$V_{R\text{-rms}} = I_{\text{rms}}R = (0.0020\ \text{A})(500\ \Omega) = 1.0\ \text{V}$$

The rms potential differences across the inductor and capacitor are, respectively:

$$V_{L\text{-rms}} = I_{\text{rms}}X_L = (0.0020\ \text{A})(2000\ \Omega) = 4.0\ \text{V}$$
$$V_{C\text{-rms}} = I_{\text{rms}}X_C = (0.0020\ \text{A})(2000\ \Omega) = 4.0\ \text{V}$$

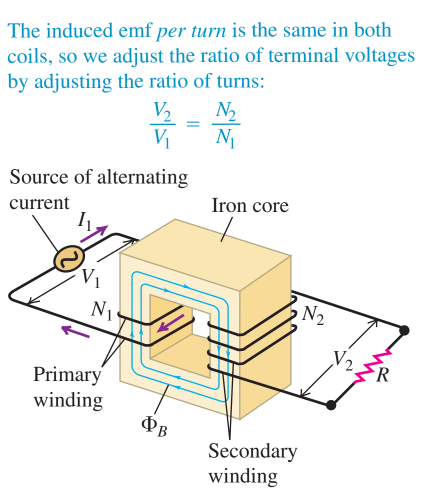
EVALUATE: The potential differences across the inductor and the capacitor have equal rms values and amplitudes, but are 180° out of phase and so add to zero at each instant. Note also that at resonance, $V_{R\text{-rms}}$ is equal to the source voltage V_{rms}, while in this example, $V_{L\text{-rms}}$ and $V_{C\text{-rms}}$ are both considerably *larger* than V_{rms}.

Test Your Understanding of Section 31.5 How does the resonance frequency of an *L-R-C* series circuit change if the plates of the capacitor are brought closer together? (i) It increases; (ii) it decreases; (iii) it is unaffected.

31.6 Transformers

One of the great advantages of ac over dc for electric-power distribution is that it is much easier to step voltage levels up and down with ac than with dc. For long-distance power transmission it is desirable to use as high a voltage and as small a current as possible; this reduces i^2R losses in the transmission lines, and smaller wires can be used, saving on material costs. Present-day transmission lines routinely operate at rms voltages of the order of 500 kV. On the other hand, safety considerations and insulation requirements dictate relatively low voltages in generating equipment and in household and industrial power distribution. The standard voltage for household wiring is 120 V in the United States and Canada and 240 V in many other countries. The necessary voltage conversion is accomplished by the use of **transformers.**

How Transformers Work



31.21 Schematic diagram of an idealized step-up transformer. The primary is connected to an ac source; the secondary is connected to a device with resistance R.

Figure 31.21 shows an idealized transformer. The key components of the transformer are two coils or *windings,* electrically insulated from each other but wound on the same core. The core is typically made of a material, such as iron, with a very large relative permeability K_m. This keeps the magnetic field lines due to a current in one winding almost completely within the core. Hence almost all of these field lines pass through the other winding, maximizing the *mutual inductance* of the two windings (see Section 30.1). The winding to which power is supplied is called the **primary;** the winding from which power is delivered is called the **secondary.** The circuit symbol for a transformer with an iron core, such as those used in power distribution systems, is

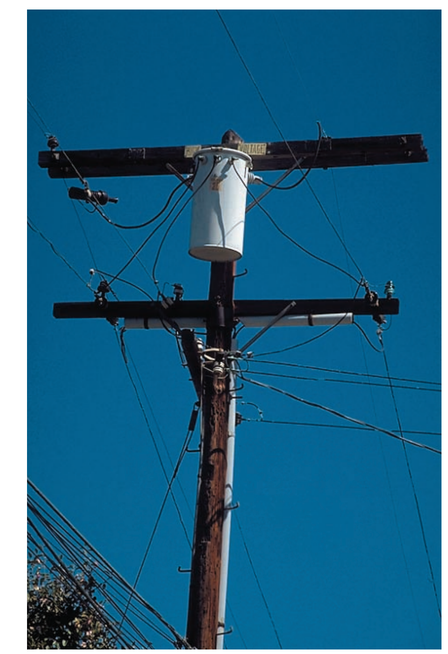
Here's how a transformer works. The ac source causes an alternating current in the primary, which sets up an alternating flux in the core; this induces an emf in each winding, in accordance with Faraday's law. The induced emf in the secondary gives rise to an alternating current in the secondary, and this delivers energy to the device to which the secondary is connected. All currents and emfs have the same frequency as the ac source.

Let's see how the voltage across the secondary can be made larger or smaller in amplitude than the voltage across the primary. We neglect the resistance of the windings and assume that all the magnetic field lines are confined to the iron core, so at any instant the magnetic flux Φ_B is the same in each turn of the primary and secondary windings. The primary winding has N_1 turns and the secondary winding has N_2 turns. When the magnetic flux changes because of changing currents in the two coils, the resulting induced emfs are

$$\mathcal{E}_1 = -N_1\frac{d\Phi_B}{dt} \quad \text{and} \quad \mathcal{E}_2 = -N_2\frac{d\Phi_B}{dt} \qquad (31.33)$$

The flux *per turn* Φ_B is the same in both the primary and the secondary, so Eqs. (31.33) show that the induced emf *per turn* is the same in each. The ratio of the secondary emf $\mathcal{E}_2$ to the primary emf $\mathcal{E}_1$ is therefore equal at any instant to the ratio of secondary to primary turns:

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1} \qquad (31.34)$$



31.22 The cylindrical can near the top of this power pole is a step-down transformer. It converts the high-voltage ac in the power lines to low-voltage (120 V) ac, which is then distributed to the surrounding homes and businesses.

Since $\mathcal{E}_1$ and $\mathcal{E}_2$ both oscillate with the same frequency as the ac source, Eq. (31.34) also gives the ratio of the amplitudes or of the rms values of the induced emfs. If the windings have zero resistance, the induced emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ are equal to the terminal voltages across the primary and the secondary, respectively; hence

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \quad \text{(terminal voltages of transformer primary and secondary)} \qquad (31.35)$$

where V_1 and V_2 are either the amplitudes or the rms values of the terminal voltages. By choosing the appropriate turns ratio N_2/N_1, we may obtain any desired secondary voltage from a given primary voltage. If $N_2 > N_1$, as in Fig. 31.21, then $V_2 > V_1$ and we have a *step-up* transformer; if $N_2 < N_1$, then $V_2 < V_1$ and we have a *step-down* transformer. At a power generating station, step-up transformers are used; the primary is connected to the power source and the secondary is connected to the transmission lines, giving the desired high voltage for transmission. Near the consumer, step-down transformers lower the voltage to a value suitable for use in home or industry (Fig. 31.22).

Even the relatively low voltage provided by a household wall socket is too high for many electronic devices, so a further step-down transformer is necessary. This is the role of an "ac adapter" (also called a "power cube" or "power adapter"), such as those used to recharge a mobile phone or laptop computer from line voltage. Such adapters contain a step-down transformer that converts line voltage to a lower value, typically 3 to 12 volts, as well as diodes to convert alternating current to the direct current that small electronic devices require (Fig. 31.23).

31.23 An ac adapter like this one converts household ac into low-voltage dc for use in electronic devices. It contains a step-down transformer to lower the voltage and diodes to rectify the output current (see Fig. 31.3).

Energy Considerations for Transformers

If the secondary circuit is completed by a resistance R, then the amplitude or rms value of the current in the secondary circuit is $I_2 = V_2/R$. From energy considerations, the power delivered to the primary equals that taken out of the secondary (since there is no resistance in the windings), so

$$V_1I_1 = V_2I_2 \quad \text{(currents in transformer primary and secondary)} \qquad (31.36)$$

We can combine Eqs. (31.35) and (31.36) and the relationship $I_2 = V_2/R$ to eliminate V_2 and I_2; we obtain

$$\frac{V_1}{I_1} = \frac{R}{(N_2/N_1)^2} \qquad (31.37)$$

This shows that when the secondary circuit is completed through a resistance R, the result is the same as if the *source* had been connected directly to a resistance equal to R divided by the square of the turns ratio, $(N_2/N_1)^2$. In other words, the transformer "transforms" not only voltages and currents, but resistances as well. More generally, we can regard a transformer as "transforming" the *impedance* of the network to which the secondary circuit is completed.

Equation (31.37) has many practical consequences. The power supplied by a source to a resistor depends on the resistances of both the resistor and the source. It can be shown that the power transfer is greatest when the two resistances are *equal.* The same principle applies in both dc and ac circuits. When a high-impedance ac source must be connected to a low-impedance circuit, such as an audio amplifier connected to a loudspeaker, the source impedance can be *matched* to that of the circuit by use of a transformer with an appropriate turns ratio N_2/N_1.

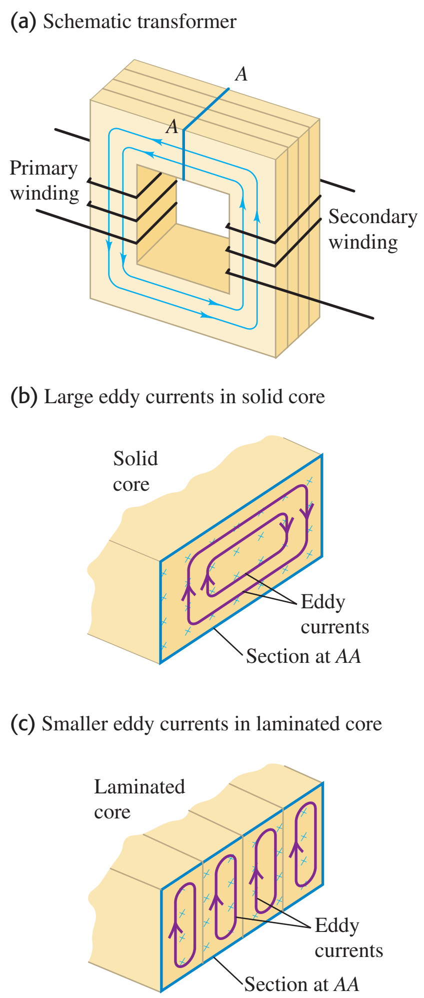
Real transformers always have some energy losses. (That's why an ac adapter like the one shown in Fig. 31.23 feels warm to the touch after it's been in use for a while; the transformer is heated by the dissipated energy.) The windings have some resistance, leading to i^2R losses. There are also energy losses through hysteresis in the core (see Section 28.8). Hysteresis losses are minimized by the use of soft iron with a narrow hysteresis loop.

Another important mechanism for energy loss in a transformer core involves eddy currents (see Section 29.6). Consider a section AA through an iron transformer core (Fig. 31.24a). Since iron is a conductor, any such section can be pictured as several conducting circuits, one within the other (Fig. 31.24b). The flux through each of these circuits is continually changing, so eddy currents circulate in the entire volume of the core, with lines of flow that form planes perpendicular to the flux. These eddy currents are very undesirable; they waste energy through i^2R heating and themselves set up an opposing flux.

The effects of eddy currents can be minimized by the use of a *laminated* core, that is, one built up of thin sheets or laminae. The large electrical surface resistance of each lamina, due either to a natural coating of oxide or to an insulating varnish, effectively confines the eddy currents to individual laminae (Fig. 31.24c). The possible eddy-current paths are narrower, the induced emf in each path is smaller, and the eddy currents are greatly reduced. The alternating magnetic field exerts forces on the current-carrying laminae that cause them to vibrate back and forth; this vibration causes the characteristic "hum" of an operating transformer. You can hear this same "hum" from the magnetic ballast of a fluorescent light fixture (see Section 30.2).

Thanks to the use of soft iron cores and lamination, transformer efficiencies are usually well over 90%; in large installations they may reach 99%.

31.24 (a) Primary and secondary windings in a transformer. (b) Eddy currents in the iron core, shown in the cross section at AA. (c) Using a laminated core reduces the eddy currents.



Example 31.9 "Wake up and smell the (transformer)!"

A friend brings back from Europe a device that she claims to be the world's greatest coffeemaker. Unfortunately, it was designed to operate from a 240-V line to obtain the 960 W of power that it needs. (a) What can she do to operate it at 120 V? (b) What current will the coffeemaker draw from the 120-V line? (c) What is the resistance of the coffeemaker? (The voltages are rms values.)

SOLUTION

IDENTIFY: Our friend needs a step-up transformer to convert the 120-V ac available in the home to the 240-V ac that the coffeemaker requires. This problem is about the properties of this transformer.

SET UP: We use Eq. (31.35) to determine the transformer turns ratio N_2/N_1, the relationship $P_{av} = V_{rms}I_{rms}$ for a resistor to find the current draw, and Eq. (31.37) to calculate the resistance.

EXECUTE: (a) To get $V_2 = 240$ V from $V_1 = 120$ V, the required turns ratio is $N_2/N_1 = V_2/V_1 = (240\text{ V})/(120\text{ V}) = 2$. That is, the secondary coil (connected to the coffeemaker) should have twice as many turns as the primary coil (connected to the 120-V line).

(b) The rms current I_1 in the 120-V primary is found by using $P_{av} = V_1I_1$, where P_{av} is the average power drawn by the coffeemaker and hence the power supplied by the 120-V line. (We're assuming that there are no energy losses in the transformer.) Hence $I_1 = P_{av}/V_1 = (960\text{ W})/(120\text{ V}) = 8.0$ A. The secondary current is then $I_2 = P_{av}/V_2 = (960\text{ W})/(240\text{ V}) = 4.0$ A.

(c) We have $V_1 = 120$ V, $I_1 = 8.0$ A, and $N_2/N_1 = 2$, so

$$\frac{V_1}{I_1} = \frac{120\text{ V}}{8.0\text{ A}} = 15\ \Omega$$

From Eq. (31.37),

$$R = 2^2(15\ \Omega) = 60\ \Omega$$

EVALUATE: As a check, $V_2/R = (240\text{ V})/(60\ \omega) = 4.0\ A = I_2$, the same value obtained previously. you can also check this result for r by using the expression $P_{av} = V_2^2/R$ for the power drawn by the coffeemaker.

Test Your Understanding of Section 31.6 Each of the following four transformers has 1000 turns in its primary coil. Rank the transformers from largest to smallest number of turns in the secondary coil. (i) converts 120-V ac into 6.0-V ac; (ii) converts 120-V ac into 240-V ac; (iii) converts 240-V ac into 6.0-V ac; (iv) converts 240-V ac into 120-V ac.

CHAPTER 31 SUMMARY

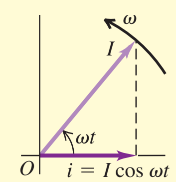
Phasors and alternating current: An alternator or ac source produces an emf that varies sinusoidally with time. A sinusoidal voltage or current can be represented by a phasor, a vector that rotates counterclockwise with constant angular velocity ω equal to the angular frequency of the sinusoidal quantity. Its projection on the horizontal axis at any instant represents the instantaneous value of the quantity.

For a sinusoidal current, the rectified average and rms (root-mean-square) currents are proportional to the current amplitude I. Similarly, the rms value of a sinusoidal voltage is proportional to the voltage amplitude V. (See Example 31.1.)

$$I_{rav} = \frac{2}{\pi}I = 0.637I \quad (31.3)$$

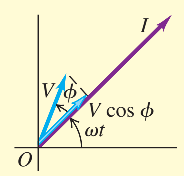
$$I_{rms} = \frac{I}{\sqrt{2}} \quad (31.4)$$

$$V_{rms} = \frac{V}{\sqrt{2}} \quad (31.5)$$



Voltage, current, and phase angle: In general, the instantaneous voltage between two points in an ac circuit is not in phase with the instantaneous current passing through those points. The quantity ϕ is called the phase angle of the voltage relative to the current.

$$i = I\cos\omega t$$
$$v = V\cos(\omega t + \phi) \quad (31.2)$$

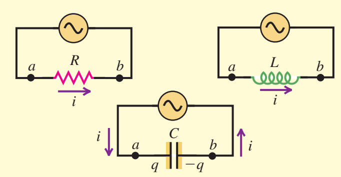


Resistance and reactance: The voltage across a resistor R is in phase with the current. The voltage across an inductor L leads the current by 90° ($\phi = +90°$), while the voltage across a capacitor C lags the current by 90° ($\phi = -90°$). The voltage amplitude across each type of device is proportional to the current amplitude I. An inductor has inductive reactance $X_L = \omega L$, and a capacitor has capacitive reactance $X_C = 1/\omega C$. (See Examples 31.2 and 31.3.)

$$V_R = IR \quad (31.7)$$

$$V_L = IX_L \quad (31.13)$$

$$V_C = IX_C \quad (31.19)$$

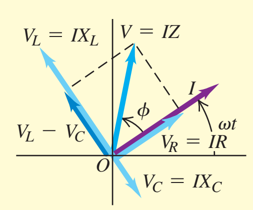


Impedance and the *L-R-C* series circuit: In a general ac circuit, the voltage and current amplitudes are related by the circuit impedance Z. In an L-R-C series circuit, the values of L, R, C, and the angular frequency ω determine the impedance and the phase angle ϕ of the voltage relative to the current. (See Examples 31.4 and 31.5.)

$$V = IZ \quad (31.22)$$

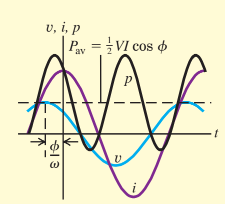
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + [\omega L - (1/\omega C)]^2} \quad (31.23)$$

$$\tan\phi = \frac{\omega L - 1/\omega C}{R} \quad (31.24)$$



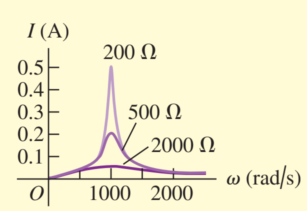
Power in ac circuits: The average power input P_{av} to an ac circuit depends on the voltage and current amplitudes (or, equivalently, their rms values) and the phase angle ϕ of the voltage relative to the current. The quantity $\cos\phi$ is called the power factor. (See Examples 31.6 and 31.7.)

$$P_{av} = \frac{1}{2}VI\cos\phi = V_{rms}I_{rms}\cos\phi \quad (31.31)$$



Resonance in ac circuits: In an L-R-C series circuit, the current becomes maximum and the impedance becomes minimum at an angular frequency called the resonance angular frequency. This phenomenon is called resonance. At resonance the voltage and current are in phase, and the impedance Z is equal to the resistance R. (See Example 31.8.)

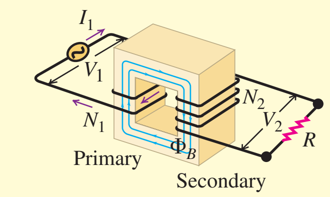
$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (31.32)$$



Transformers: A transformer is used to transform the voltage and current levels in an ac circuit. In an ideal transformer with no energy losses, if the primary winding has N_1 turns and the secondary winding has N_2 turns, the amplitudes (or rms values) of the two voltages are related by Eq. (31.35). The amplitudes (or rms values) of the primary and secondary voltages and currents are related by Eq. (31.36). (See Example 31.9.)

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \quad (31.35)$$

$$V_1I_1 = V_2I_2 \quad (31.36)$$



Key Terms

alternating current (ac), *1061*
ac source, *1062*
voltage amplitude, *1062*
current amplitude, *1062*
phasor, *1062*
phasor diagram, *1062*
rectified average current, *1063*
root-mean-square (rms) current, *1063*
phase angle, *1066*
inductive reactance, *1066*
capacitive reactance, *1068*
impedance, *1071*
power factor, *1076*
resonance, *1078*
resonance angular frequency, *1078*
resonance frequency, *1078*
transformer, *1080*
primary, *1080*
secondary, *1080*

Answer to Chapter Opening Question ?

Yes. In fact, the radio simultaneously detects transmissions at *all* frequencies. However, a radio is an L-R-C series circuit, and at any given time it is tuned to have a resonance at just one frequency. Hence the response of the radio to that frequency is much greater than its response to any other frequency, which is why you hear only one broadcasting station through the radio's speaker. (You can sometimes hear a second station if its frequency is sufficiently close to the tuned frequency.)

Answers to Test Your Understanding Questions

31.1 Answers: (a) D; (b) A; (c) B; (d) C For each phasor, the actual current is represented by the projection of that phasor onto the horizontal axis. The phasors all rotate counterclockwise around the origin with angular speed ω, so at the instant shown the projection of phasor A is positive but trending toward zero; the projection of phasor B is negative and becoming more negative; the projection of phasor C is negative but trending toward zero; and the projection of phasor D is positive and becoming more positive.

31.2 Answers: (a) (iii); (b) (ii); (c) (i) For a resistor, $V_R = IR$, so $I = V_R/R$. The voltage amplitude V_R and resistance R do not change with frequency, so the current amplitude I remains constant. For an inductor, $V_L = IX_L = I\omega L$, so $I = V_L/\omega L$. The voltage amplitude V_L and inductance L are constant, so the current amplitude I decreases as the frequency increases. For a capacitor, $V_C = IX_C = I/\omega C$, so $I = V_C\omega C$. The voltage amplitude V_C and capacitance C are constant, so the current amplitude I increases as the frequency increases.

31.3 Answer: (iv), (ii), (i), (iii) For the circuit in Example 31.4, $I = V/Z = (50\text{ V})/(500\ \Omega) = 0.10$ A. If the capacitor and inductor are removed so that only the ac source and resistor remain, the circuit is like that shown in Fig. 31.7a; then $I = V/R = (50\text{ V})/(300\ \Omega) = 0.17$ A. If the resistor and capacitor are removed so that only the ac source and inductor remain, the circuit is like that shown in Fig. 31.8a; then $I = V/X_L = (50\text{ V})/(600\ \Omega) = 0.083$ A. Finally, if the resistor and inductor are removed so that only the ac source and capacitor remain, the circuit is like that shown in Fig. 31.9a; then $I = V/X_C = (50\text{ V})/(200\ \Omega) = 0.25$ A.

31.4 Answers: (a) (v); (b) (iv) The energy cannot be extracted from the resistor, since energy is dissipated in a resistor and cannot be recovered. Instead, the energy must be extracted from either the inductor (which stores magnetic-field energy) or the capacitor (which stores electric-field energy). Positive power means that energy is being transferred from the ac source to the circuit, so *negative* power implies that energy is being transferred back into the source.

31.5 Answer: (ii) The capacitance C increases if the plate spacing is decreased (see Section 24.1). Hence the resonance frequency $f_0 = \omega_0/2\pi = 1/2\pi\sqrt{LC}$ decreases.

31.6 Answer: (ii), (iv), (i), (iii) From Eq. (31.35) the turns ratio is $N_2/N_1 = V_2/V_1$, so the number of turns in the secondary is $N_2 = N_1V_2/V_1$. Hence for the four cases we have (i) $N_2 = (1000)(6.0\text{ V})/(120\text{ V}) = 50$ turns; (ii) $N_2 = (1000)(240\text{ V})/(120\text{ V}) = 2000$ turns; (iii) $N_2 = (1000)(6.0\text{ V})/(240\text{ V}) = 25$ turns; and (iv) $N_2 = (1000)(120\text{ V})/(240\text{ V}) = 500$ turns. Note that (i), (iii), and (iv) are step-down transformers with fewer turns in the secondary than in the primary, while (ii) is a step-up transformer with more turns in the secondary than in the primary.

PROBLEMS

Discussion Questions

Q31.1. Household electric power in most of western Europe is supplied at 240 V, rather than the 120 V that is standard in the United States and Canada. What are the advantages and disadvantages of each system?

Q31.2. The current in an ac power line changes direction 120 times per second, and its average value is zero. Explain how it is possible for power to be transmitted in such a system.

Q31.3. In an ac circuit, why is the average power for an inductor and a capacitor zero, but not for a resistor?

Q31.4. Equation (31.14) was derived by using the relationship $i = dq/dt$ between the current and the charge on the capacitor. In Fig. 31.9a the positive counterclockwise current increases the charge on the capacitor. When the charge on the left plate is positive but decreasing in time, is $i = dq/dt$ still correct or should it be $i = -dq/dt$? Is $i = dq/dt$ still correct when the right-hand plate has positive charge that is increasing or decreasing in magnitude? Explain.

Q31.5. Fluorescent lights often use an inductor, called a ballast, to limit the current through the tubes. Why is it better to use an inductor rather than a resistor for this purpose?

Q31.6. Equation (31.9) says that $v_{ab} = L\,di/dt$ (see Fig. 31.8a). Using Faraday's law, explain why point a is at higher potential than point b when i is in the direction shown in Fig. 31.8a and is increasing in magnitude. When i is counterclockwise and decreasing in magnitude, is $v_{ab} = L\,di/dt$ still correct, or should it be $v_{ab} = -L\,di/dt$? Is $v_{ab} = L\,di/dt$ still correct when i is clockwise and increasing or decreasing in magnitude? Explain.

Q31.7. Is it possible for the power factor of an *L-R-C* series ac circuit to be zero? Justify your answer on *physical* grounds.

Q31.8. In a series *L-R-C* circuit, can the instantaneous voltage across the capacitor exceed the source voltage at that same instant? Can this be true for the instantaneous voltage across the inductor? Across the resistor? Explain.

Q31.9. In a series *L-R-C* circuit, what are the phase angle ϕ and power factor $\cos\phi$ when the resistance is much smaller than the inductive or capacitive reactance and the circuit is operated far from resonance? Explain.

Q31.10. When a series *L-R-C* circuit is connected across a 120-V ac line, the voltage rating of the capacitor may be exceeded even if it is rated at 200 or 400 V. How can this be?

Q31.11. In Example 31.6 (Section 31.4), a hair dryer was treated as a pure resistor. But because there are coils in the heating element and in the motor that drives the blower fan, a hair dryer also has inductance. Qualitatively, does including an inductance increase or decrease the values of R, I_{rms}, and P?

Q31.12. A light bulb and a parallel-plate capacitor with air between the plates are connected in series to an ac source. What happens to the brightness of the bulb when a dielectric is inserted between the plates of the capacitor? Explain.

Q31.13. A coil of wire wrapped on a hollow tube and a light bulb are connected in series to an ac source. What happens to the brightness of the bulb when an iron rod is inserted in the tube?

Q31.14. A circuit consists of a light bulb, a capacitor, and an inductor connected in series to an ac source. What happens to the brightness of the bulb when the inductor is removed? When the inductor is left in the circuit but the capacitor is removed? Explain.

Q31.15. A circuit consists of a light bulb, a capacitor, and an inductor connected in series to an ac source. Is it possible for both the capacitor and the inductor to be removed and the brightness of the bulb to remain the same? Explain.

Q31.16. Can a transformer be used with dc? Explain. What happens if a transformer designed for 120-V ac is connected to a 120-V dc line?

Q31.17. An ideal transformer has N_1 windings in the primary and N_2 windings in its secondary. If you double only the number of secondary windings, by what factor does (a) the voltage amplitude in the secondary change, and (b) the effective resistance of the secondary circuit change?

Q31.18. Some electrical appliances operate equally well on ac or dc, and others work only on ac or only on dc. Give examples of each, and explain the differences.

Exercises

Section 31.1 Phasors and Alternating Currents

31.1. The plate on the back of a certain computer scanner says that the unit draws 0.34 A of current from a 120-V, 60-Hz line. Find (a) the root-mean-square current, (b) the current amplitude, (c) the average current; (d) the average square of the current.

31.2. A sinusoidal current $i = I\cos\omega t$ has an rms value $I_{rms} = 2.10$ A. (a) What is the current amplitude? (b) The current is passed through a full-wave rectifier circuit. What is the rectified average current? (c) Which is larger: I_{rms} or I_{rav}? Explain, using graphs of i^2 and of the rectified current.

31.3. The voltage across the terminals of an ac power supply varies with time according to Eq. (31.1). The voltage amplitude is $V = 45.0$ V. What are (a) the root-mean-square potential difference V_{rms}? and (b) the average potential difference V_{av} between the two terminals of the power supply?

Section 31.2 Resistance and Reactance

31.4. A 2.20-μF capacitor is connected across an ac source whose voltage amplitude is kept constant at 60.0 V but whose frequency can be varied. Find the current amplitude when the angular frequency is (a) 100 rad/s; (b) 1000 rad/s; (c) 10,000 rad/s. (d) Show the results of parts (a) through (c) in a plot of log I versus log ω.

31.5. A 5.00-H inductor with negligible resistance is connected across the ac source of Exercise 31.4. Find the current amplitude when the angular frequency is (a) 100 rad/s; (b) 1000 rad/s; (c) 10,000 rad/s. (d) Show the results of parts (a) through (c) in a plot of log I versus log ω.

31.6. A capacitance C and an inductance L are operated at the same angular frequency. (a) At what angular frequency will they have the same reactance? (b) If $L = 5.00$ mH and $C = 3.50\ \mu$F, what is the numerical value of the angular frequency in part (a), and what is the reactance of each element?

31.7. In each circuit described next, an ac voltage source producing a current $i = I\cos\omega t$ is connected to an additional circuit element. (a) The ac source is connected across a resistor R. Sketch graphs of the current in the circuit and the potential difference across the resistor as functions of time, covering two cycles of oscillation. Put both graphs on the *same* set of axes so you can compare them. (b) Do the same as in part (a), except suppose the resistor is replaced by an inductor L. Sketch the same graphs as in part (a), but this time across the inductor instead of the resistor. (c) Do the same as in part (a), except suppose the resistor is replaced by a capacitor C. Sketch the same graphs as in part (a), except now across the capacitor instead of the resistor. (d) Sketch phasor diagrams for each of the preceding cases.

31.8. (a) Compute the reactance of a 0.450-H inductor at frequencies of 60.0 Hz and 600 Hz. (b) Compute the reactance of a 2.50-μF capacitor at the same frequencies. (c) At what frequency is the reactance of a 0.450-H inductor equal to that of a 2.50-μF capacitor?

31.9. (a) What is the reactance of a 3.00-H inductor at a frequency of 80.0 Hz? (b) What is the inductance of an inductor whose reactance is 120 Ω at 80.0 Hz? (c) What is the reactance of a 4.00-μF capacitor at a frequency of 80.0 Hz? (d) What is the capacitance of a capacitor whose reactance is 120 Ω at 80.0 Hz?

31.10. A Radio Inductor. You want the current amplitude through a 0.450-mH inductor (part of the circuitry for a radio receiver) to be 2.60 mA when a sinusoidal voltage with amplitude 12.0 V is applied across the inductor. What frequency is required?

31.11. Kitchen Capacitance. The wiring for a refrigerator contains a starter capacitor. A voltage of amplitude 170 V and frequency 60.0 Hz applied across the capacitor is to produce a current amplitude of 0.850 A through the capacitor. What capacitance C is required?

31.12. A 250-Ω resistor is connected in series with a 4.80-μF capacitor. The voltage across the capacitor is $v_C = (7.60\text{ V})\sin[(120\text{ rad/s})t]$. (a) Determine the capacitive reactance of the capacitor. (b) Derive an expression for the voltage v_R across the resistor.

31.13. A 150-Ω resistor is connected in series with a 0.250-H inductor. The voltage across the resistor is $v_R = (3.80\text{ V})\cos[(720\text{ rad/s})t]$. (a) Derive an expression for the circuit current. (b) Determine the inductive reactance of the inductor. (c) Derive an expression for the voltage v_L across the inductor.

Section 31.3 The *L-R-C* Series Circuit

31.14. You have a 200-Ω resistor, a 0.400-H inductor, and a 6.00-μF capacitor. Suppose you take the resistor and inductor and make a series circuit with a voltage source that has voltage amplitude 30.0 V and an angular frequency of 250 rad/s. (a) What is the impedance of the circuit? (b) What is the current amplitude? (c) What are the voltage amplitudes across the resistor and across the inductor? (d) What is the phase angle ϕ of the source voltage with respect to the current? Does the source voltage lag or lead the current? (e) Construct the phasor diagram.

31.15. (a) For the *R-L* circuit of Exercise 31.14, graph v, v_R, and v_L versus t for $t = 0$ to $t = 50.0$ ms. The current is given by $i = I\cos\omega t$, so $v = V\cos(\omega t + \phi)$. (b) What are v, v_R, and v_L at $t = 20.0$ ms? Compare $v_R + v_L$ to v at this instant. (c) Repeat part (b) for $t = 40.0$ ms.

31.16. Repeat Exercise 31.14 with the circuit consisting of only the capacitor and the inductor in series. For part (c), calculate the voltage amplitudes across the capacitor and across the inductor.

31.17. Repeat Exercise 31.14 with the circuit consisting of only the resistor and the capacitor in series. For part (c), calculate the voltage amplitudes across the resistor and across the capacitor.

31.18. (a) For the *R-C* circuit of Exercise 31.17, graph v, v_R, and v_C versus t for $t = 0$ to $t = 50.0$ ms. The current is given by $i = I\cos\omega t$, so $v = V\cos(\omega t + \phi)$. (b) What are v, v_R, and v_C at $t = 20.0$ ms? Compare $v_R + v_C$ to v at this instant. (c) Repeat part (b) for $t = 40.0$ ms.

31.19. The resistor, inductor, capacitor, and voltage source described in Exercise 31.14 are connected to form an *L-R-C* series circuit. (a) What is the impedance of the circuit? (b) What is the current amplitude? (c) What is the phase angle of the source voltage with respect to the current? Does the source voltage lag or lead the current? (d) What are the voltage amplitudes across the resistor, inductor, and capacitor? (e) Explain how it is possible for the voltage amplitude across the capacitor to be greater than the voltage amplitude across the source.

31.20. (a) For the *L-R-C* circuit of Exercise 31.19, graph v, v_R, v_L, and v_C versus t for $t = 0$ to $t = 50.0$ ms. The current is given by $i = I\cos\omega t$, so $v = V\cos(\omega t + \phi)$. (b) What are v, v_R, v_L, and v_C at $t = 20.0$ ms? Compare $v_R + v_L + v_C$ to v at this instant. (c) Repeat part (b) for $t = 40.0$ ms.

31.21. Analyzing an *L-R-C* Circuit. You have a 200-Ω resistor, a 0.400-H inductor, a 5.00-μF capacitor, and a variable-frequency ac source with an amplitude of 3.00 V. You connect all four elements together to form a series circuit. (a) At what frequency will the current in the circuit be greatest? What will be the current amplitude at this frequency? (b) What will be the current amplitude at an angular frequency of 400 rad/s? At this frequency, will the source voltage lead or lag the current?

31.22. An *L-R-C* series circuit is constructed using a 175-Ω resistor, a 12.5-μF capacitor, and an 8.00-mH inductor, all connected across an ac source having a variable frequency and a voltage amplitude of 25.0 V. (a) At what angular frequency will the impedance be smallest, and what is the impedance at this frequency? (b) At the angular frequency in part (a), what is the maximum current through the inductor? (c) At the angular frequency in part (a), find the potential difference across the ac source, the resistor, the capacitor, and the inductor at the instant that the current is equal to one-half its greatest positive value. (d) In part (c), how are the potential differences across the resistor, inductor, and capacitor related to the potential difference across the ac source?

31.23. In an *L-R-C* series circuit, the rms voltage across the resistor is 30.0 V, across the capacitor it is 90.0 V, and across the inductor it is 50.0 V. What is the rms voltage of the source?

31.24. Define the reactance X of an *L-R-C* circuit to be $X = X_L - X_C$. (a) Show that $X = 0$ when the angular frequency ω of the current is equal to the resonance angular frequency ω_0. (b) What is the sign of X when $\omega > \omega_0$? (c) What is the sign of X when $\omega < \omega_0$? (d) Graph X versus ω.

Section 31.4 Power in Alternating-Current Circuits

31.25. The power of a certain CD player operating at 120 V rms is 20.0 W. Assuming that the CD player behaves like a pure resistance, find (a) the maximum instantaneous power; (b) the rms current; (c) the resistance of this player.

31.26. In a series *L-R-C* circuit, the components have the following values: $L = 20.0$ mH, $C = 140$ nF, and $R = 350$ Ω. The generator has an rms voltage of 120 V and a frequency of 1.25 kHz. Determine (a) the power supplied by the generator; and (b) the power dissipated in the resistor.

31.27. (a) Show that for an *L-R-C* series circuit the power factor is equal to R/Z. (*Hint:* Use the phasor diagram; see Fig. 31.13b.) (b) Show that for any ac circuit, not just one containing pure resistance only, the average power delivered by the voltage source is given by $P_{av} = I_{rms}^{\ 2}R$.

31.28. An *L-R-C* series circuit is connected to a 120-Hz ac source that has $V_{rms} = 80.0$ V. The circuit has a resistance of 75.0 Ω and an impedance at this frequency of 105 Ω. What average power is delivered to the circuit by the source?

31.29. An *L-R-C* series circuit with $L = 0.120$ H, $R = 240$ Ω, and $C = 7.30\ \mu$F carries an rms current of 0.450 A with a frequency of 400 Hz. (a) What are the phase angle and power factor for this circuit? (b) What is the impedance of the circuit? (c) What is the rms voltage of the source? (d) What average power is delivered by the source? (e) What is the average rate at which electrical energy is converted to thermal energy in the resistor? (f) What is the average rate at which electrical energy is dissipated (converted to other forms) in the capacitor? (g) In the inductor?

31.30. A series ac circuit contains a 250-Ω resistor, a 15-mH inductor, a 3.5-μF capacitor, and an ac power source of voltage amplitude 45 V operating at an angular frequency of 360 rad/s. (a) What is the power factor of this circuit? (b) Find the average power delivered to the entire circuit. (c) What is the average power delivered to the resistor, to the capacitor, and to the inductor?

Section 31.5 Resonance in Alternating-Current Circuits

31.31. In an L-R-C series circuit, $R = 300\ \Omega$, $L = 0.400$ H, and $C = 6.00 \times 10^{-8}$ F. When the ac source operates at the resonance frequency of the circuit, the current amplitude is 0.500 A. (a) What is the voltage amplitude of the source? (b) What is the amplitude of the voltage across the resistor, across the inductor, and across the capacitor? (c) What is the average power supplied by the source?

31.32. An L-R-C series circuit consists of a source with voltage amplitude 120 V and angular frequency 50.0 rad/s, a resistor with $R = 400\ \Omega$ an inductor with $L = 9.00$ H, and a capacitor with capacitance C. (a) For what value of C will the current amplitude in the circuit be a maximum? (b) When C has the value calculated in part (a), what is the amplitude of the voltage across the inductor?

31.33. In an L-R-C series circuit, $R = 150\ \Omega$, $L = 0.750$ H, and $C = 0.0180\ \mu$F. The source has voltage amplitude $V = 150$ V and a frequency equal to the resonance frequency of the circuit. (a) What is the power factor? (b) What is the average power delivered by the source? (c) The capacitor is replaced by one with $C = 0.0360\ \mu$F and the source frequency is adjusted to the new resonance value. Then what is the average power delivered by the source?

31.34. In an L-R-C series circuit, $R = 400\ \Omega$, $L = 0.350$ H, and $C = 0.0120\ \mu$F. (a) What is the resonance angular frequency of the circuit? (b) The capacitor can withstand a peak voltage of 550 V. If the voltage source operates at the resonance frequency, what maximum voltage amplitude can it have if the maximum capacitor voltage is not exceeded?

31.35. A series circuit consists of an ac source of variable frequency, a 115-Ω resistor, a 1.25-μF capacitor, and a 4.50-mH inductor. Find the impedance of this circuit when the angular frequency of the ac source is adjusted to (a) the resonance angular frequency; (b) twice the resonance angular frequency; (c) half the resonance angular frequency.

31.36. In an L-R-C series circuit, $L = 0.280$ H and $C = 4.00\ \mu$F. The voltage amplitude of the source is 120 V. (a) What is the resonance angular frequency of the circuit? (b) When the source operates at the resonance angular frequency, the current amplitude in the circuit is 1.70 A. What is the resistance R of the resistor? (c) At the resonance angular frequency, what are the peak voltages across the inductor, the capacitor, and the resistor?

Section 31.6 Transformers

31.37. A Step-Down Transformer. A transformer connected to a 120-V (rms) ac line is to supply 12.0 V (rms) to a portable electronic device. The load resistance in the secondary is 5.00 Ω. (a) What should the ratio of primary to secondary turns of the transformer be? (b) What rms current must the secondary supply? (c) What average power is delivered to the load? (d) What resistance connected directly across the 120-V line would draw the same power as the transformer? Show that this is equal to 5.00 Ω times the square of the ratio of primary to secondary turns.

31.38. A Step-Up Transformer. A transformer connected to a 120-V (rms) ac line is to supply 13,000 V (rms) for a neon sign. To reduce shock hazard, a fuse is to be inserted in the primary circuit; the fuse is to blow when the rms current in the secondary circuit exceeds 8.50 mA. (a) What is the ratio of secondary to primary turns of the transformer? (b) What power must be supplied to the transformer when the rms secondary current is 8.50 mA? (c) What current rating should the fuse in the primary circuit have?

31.39. Off to Europe! You plan to take your hair blower to Europe, where the electrical outlets put out 240 V instead of the 120 V seen in the United States. The blower puts out 1600 W at 120 V. (a) What could you do to operate your blower via the 240-V line in Europe? (b) What current will your blower draw from a European outlet? (c) What resistance will your blower appear to have when operated at 240 V?

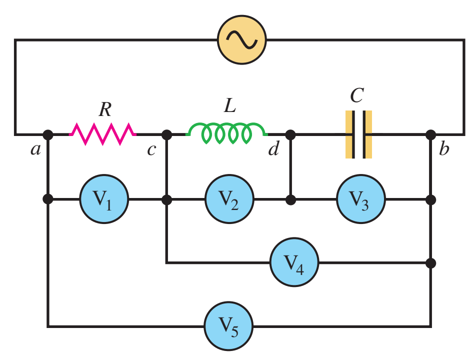
Problems

31.40. Figure 31.12a shows the crossover network in a loudspeaker system. One branch consists of a capacitor C and a resistor R in series (the tweeter). This branch is in parallel with a second branch (the woofer) that consists of an inductor L and a resistor R in series. The same source voltage with angular frequency ω is applied across each parallel branch. (a) What is the impedance of the tweeter branch? (b) What is the impedance of the woofer branch? (c) Explain why the currents in the two branches are equal when the impedances of the branches are equal. (d) Derive an expression for the frequency f that corresponds to the crossover point in Fig. 31.12b.

31.41. A coil has a resistance of 48.0 Ω. At a frequency of 80.0 Hz the voltage across the coil leads the current in it by 52.3°. Determine the inductance of the coil.

31.42. Five infinite-impedance voltmeters, calibrated to read rms values, are connected as shown in Fig 31.25. Let $R = 200\ \Omega$, $L = 0.400$ H, $C = 6.00\ \mu$F, and $V = 30.0$ V. What is the reading of each voltmeter if (a) $\omega = 200$ rad/s; and (b) $\omega = 1000$ rad/s?

Figure 31.25 Problem 31.42.



31.43. A sinusoidal current is given by $i = I\cos\omega t$. The full-wave rectified current is shown in Fig. 31.3b. (a) Let t_1 and t_2 be the two smallest positive times at which the rectified current is zero. Express t_1 and t_2 in terms of ω. (b) Find the area under the rectified i versus t curve between t_1 and t_2 by computing the integral $\int_{t_1}^{t_2} i\,dt$. Since $dq = i\,dt$, this area equals the charge that flows during the t_1 to t_2 time interval. (c) Set the result in part (b) equal to $I_{rav}(t_2 - t_1)$ and calculate I_{rav} in terms of the current amplitude I. Compare your answer to Eq. (31.3).

31.44. A large electromagnetic coil is connected to a 120-Hz ac source. The coil has resistance 400 Ω, and at this source frequency the coil has inductive reactance 250 Ω. (a) What is the inductance of the coil? (b) What must the rms voltage of the source be if the coil is to consume an average electrical power of 800 W?

31.45. A series circuit has an impedance of 60.0 Ω and a power factor of 0.720 at 50.0 Hz. The source voltage lags the current. (a) What circuit element, an inductor or a capacitor, should be placed in series with the circuit to raise its power factor? (b) What size element will raise the power factor to unity?

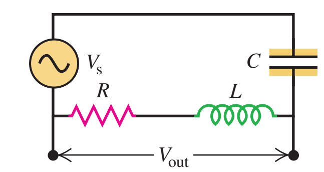
31.46. A circuit consists of a resistor and a capacitor in series with an ac source that supplies an rms voltage of 240 V. At the frequency of the source the reactance of the capacitor is 50.0 Ω. The rms current in the circuit is 3.00 A. What is the average power supplied by the source?

31.47. An L-R-C series circuit consists of a 50.0-Ω resistor, a 10.0-μF capacitor, a 3.50-mH inductor, and an ac voltage source of voltage amplitude 60.0 V operating at 1250 Hz. (a) Find the current amplitude and the voltage amplitudes across the inductor, the resistor, and the capacitor. Why can the voltage amplitudes add up to *more* than 60.0 V? (b) If the frequency is now doubled, but nothing else is changed, which of the quantities in part (a) will change? Find the new values for those that do change.

31.48. At a frequency ω_1 the reactance of a certain capacitor equals that of a certain inductor. (a) If the frequency is changed to $\omega_2 = 2\omega_1$, what is the ratio of the reactance of the inductor to that of the capacitor? Which reactance is larger? (b) If the frequency is changed to $\omega_3 = \omega_1/3$, what is the ratio of the reactance of the inductor to that of the capacitor? Which reactance is larger? (c) If the capacitor and inductor are placed in series with a resistor of resistance R to form a series L-R-C circuit, what will be the resonance angular frequency of the circuit?

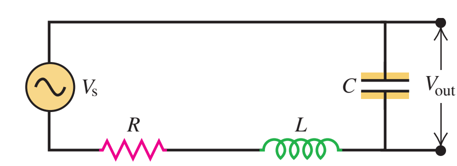
31.49. A High-Pass Filter. One application of L-R-C series circuits is to high-pass or low-pass filters, which filter out either the low- or high-frequency components of a signal. A high-pass filter is shown in Fig. 31.26, where the output voltage is taken across the L-R combination. (The L-R combination represents an inductive coil that also has resistance due to the large length of wire in the coil.) Derive an expression for V_{out}/V_s, the ratio of the output and source voltage amplitudes, as a function of the angular frequency ω of the source. Show that when ω is small, this ratio is proportional to ω and thus is small, and show that the ratio approaches unity in the limit of large frequency.

Figure 31.26 Problem 31.49.



31.50. A Low-Pass Filter. Figure 31.27 shows a low-pass filter (see Problem 31.49); the output voltage is taken across the capacitor in an L-R-C series circuit. Derive an expression for V_{out}/V_s, the ratio of the output and source voltage amplitudes, as a function of the angular frequency ω of the source. Show that when ω is large, this ratio is proportional to ω^{-2} and thus is very small, and show that the ratio approaches unity in the limit of small frequency.

Figure 31.27 Problem 31.50.



31.51. An L-R-C series circuit is connected to an ac source of constant voltage amplitude V and variable angular frequency ω. (a) Show that the current amplitude, as a function of ω, is

$$I = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

(b) Show that the average power dissipated in the resistor is

$$P = \frac{V^2 R/2}{R^2 + (\omega L - 1/\omega C)^2}$$

(c) Show that I and P are *both* maximum when $\omega = 1/\sqrt{LC}$; that is, when the source frequency equals the resonance frequency of the circuit. (d) Graph P as a function of ω for $V = 100$ V, $R = 200\ \Omega$, $L = 2.0$ H, and $C = 0.50\ \mu$F. Compare to the light purple curve in Fig. 31.19. Discuss the behavior of I and P in the limits $\omega = 0$ and $\omega \to \infty$.

31.52. An L-R-C series circuit is connected to an ac source of constant voltage amplitude V and variable angular frequency ω. Using the results of Problem 31.51, find an expression for (a) the amplitude V_L of the voltage across the inductor as a function of ω; and (b) the amplitude V_C of the voltage across the capacitor as a function of ω. (c) Graph V_L and V_C as functions of ω for $V = 100$ V, $R = 200\ \Omega$, $L = 2.0$ H, and $C = 0.50\ \mu$F. (d) Discuss the behavior of V_L and V_C in the limits $\omega = 0$ and $\omega \to \infty$. For what value of ω is $V_L = V_C$? What is the significance of this value of ω?

31.53. An L-R-C series circuit is connected to an ac source of constant voltage amplitude V and variable angular frequency ω. (a) Show that the time-averaged energy stored in the inductor is $U_B = \frac{1}{4}LI^2$ and the time-averaged energy stored in the capacitor is $U_E = \frac{1}{4}CV^2$. (b) Use the results of Problems 31.51 and 31.52 to find expressions for U_B and U_E as functions of ω. (c) Graph U_B and U_E as functions of ω for $V = 100$ V, $R = 200\ \Omega$, $L = 2.0$ H, and $C = 0.50\ \mu$F. (d) Discuss the behavior of U_B and U_E in the limits $\omega = 0$ and $\omega \to \infty$. For what value of ω is $U_B = U_E$? What is the significance of this value of ω?

31.54. The L-R-C Parallel Circuit. A resistor, inductor, and capacitor are connected in parallel to an ac source with voltage amplitude V and angular frequency ω. Let the source voltage be given by $v = V\cos\omega t$. (a) Show that the instantaneous voltages v_R, v_L, and v_C at any instant are each equal to v and that $i = i_R + i_L + i_C$, where i is the current through the source and i_R, i_L, and i_C are the currents through the resistor, the inductor, and the capacitor, respectively. (b) What are the phases of i_R, i_L, and i_C with respect to v? Use current phasors to represent i, i_R, i_L, and i_C. In a phasor diagram, show the phases of these four currents with respect to v. (c) Use the phasor diagram of part (b) to show that the current amplitude I for the current i through the source is given by $I = \sqrt{I_R^2 + (I_C - I_L)^2}$. (d) Show that the result of part (c) can be written as $I = V/Z$, with $1/Z = \sqrt{1/R^2 + (\omega C - 1/\omega L)^2}$.

31.55. Parallel Resonance. The impedance of an L-R-C parallel circuit was derived in Problem 31.54. (a) Show that at the resonance angular frequency $\omega_0 = 1/\sqrt{LC}$, $I_C = I_L$, and I is a *minimum*. (b) Since I is a minimum at resonance, is it correct to say that the power delivered to the resistor is also a minimum at $\omega = \omega_0$? Explain. (c) At resonance, what is the phase angle of the source current with respect to the source voltage? How does this compare to the phase angle for an L-R-C *series* circuit at resonance? (d) Draw the circuit diagram for an L-R-C parallel circuit. Arrange the circuit elements in your diagram so that the resistor is closest to the ac source. Justify the following statement: When the angular frequency of the source is $\omega = \omega_0$, there is *no* current flowing between (i) the part of the circuit that includes the source and the resistor and (ii) the part that includes the inductor and capacitor, so you could cut the wires connecting these two parts of the circuit without affecting the currents. (e) Is the statement in part (d) still valid if we consider that any real inductor or capacitor also has some resistance of its own? Explain.

31.56. A 400-Ω resistor and a 6.00-μF capacitor are connected in parallel to an ac generator that supplies an rms voltage of 220 V at an angular frequency of 360 rad/s. Use the results of Problem 31.54. Note that since there is no inductor in the circuit, the $1/\omega L$ term is not present in the expression for Z. Find (a) the current amplitude in the resistor; (b) the current amplitude in the capacitor; (c) the phase angle of the source current with respect to the source voltage; (d) the amplitude of the current through the generator. (e) Does the source current lag or lead the source voltage?

31.57. An L-R-C parallel circuit is connected to an ac source of constant voltage amplitude V and variable angular frequency ω. (a) Using the results of Problem 31.54, find expressions for the amplitudes I_R, I_L, and I_C of the currents through the resistor, inductor, and capacitor as functions of ω. (b) Graph I_R, I_L, and I_C as functions of ω for $V = 100$ V, $R = 200\ \Omega$, $L = 2.0$ H, and $C = 0.50\ \mu$F. (c) Discuss the behavior of I_L and I_C in the limits $\omega = 0$ and $\omega \rightarrow \infty$. Explain why I_L and I_C behave as they do in these limits. (d) Calculate the resonance frequency (in Hz) of the circuit, and sketch the phasor diagram at the resonance frequency. (e) At the resonance frequency, what is the current amplitude through the source? (f) At the resonance frequency, what is the current amplitude through the resistor, through the inductor, and through the capacitor?

31.58. An L-R-C series circuit consists of a 2.50-μF capacitor, a 5.00-mH inductor, and a 75.0-Ω resistor connected across an ac source of voltage amplitude 15.0 V having variable frequency. (a) Under what circumstances is the average power delivered to the circuit equal to $\frac{1}{2}V_{\text{rms}}I_{\text{rms}}$? (b) Under the conditions of part (a), what is the average power delivered to each circuit element and what is the maximum current through the capacitor?

31.59. In an L-R-C series circuit the magnitude of the phase angle is 54.0°, with the source voltage lagging the current. The reactance of the capacitor is 350 Ω, and the resistor resistance is 180 Ω. The average power delivered by the source is 140 W. Find (a) the reactance of the inductor; (b) the rms current; (c) the rms voltage of the source.

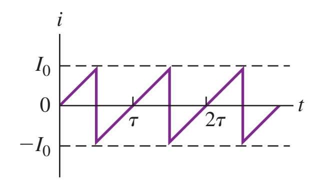
31.60. An L-R-C series circuit has $R = 500\ \Omega$, $L = 2.00$ H, $C = 0.500\ \mu$F, and $V = 100$ V. (a) For $\omega = 800$ rad/s, calculate V_R, V_L, V_C, and ϕ. Using a single set of axes, graph v, v_R, v_L, and v_C as functions of time. Include two cycles of v on your graph. (b) Repeat part (a) for $\omega = 1000$ rad/s. (c) Repeat part (a) for $\omega = 1250$ rad/s.

31.61. In an L-R-C series circuit, the source has a voltage amplitude of 120 V, $R = 80.0\ \Omega$, and the reactance of the capacitor is 480 Ω. The voltage amplitude across the capacitor is 360 V. (a) What is the current amplitude in the circuit? (b) What is the impedance? (c) What two values can the reactance of the inductor have? (d) For which of the two values found in part (c) is the angular frequency less than the resonance angular frequency? Explain.

31.62. A series circuit consists of a 1.50-mH inductor, a 125-Ω resistor, and a 25.0-nF capacitor connected across an ac source having an rms voltage of 35.0 V and variable frequency. (a) At what angular frequency will the current amplitude be equal to $\frac{1}{3}$ of its maximum possible value? (b) At the frequency in part (a) what are the current amplitude and the voltage amplitude across each of the circuit elements (including the ac source)?

31.63. The current in a certain circuit varies with time as shown in Fig. 31.28. Find the average current and the rms current in terms of I_0.

Figure 31.28 Problem 31.63.



31.64. The Resonance Width. Consider an L-R-C series circuit with a 1.80-H inductor, a 0.900-μF capacitor, and a 300-Ω resistor. The source has terminal rms voltage $V_{\text{rms}} = 60.0$ V and variable angular frequency ω. (a) What is the resonance angular frequency ω_0 of the circuit? (b) What is the rms current through the circuit at resonance, $I_{\text{rms-0}}$? (c) For what two values of the angular frequency, ω_1 and ω_2, is the rms current half the resonance value? (d) The quantity $|\omega_1 - \omega_2|$ defines the *resonance width.* Calculate $I_{\text{rms-0}}$ and the resonance width for $R = 300\ \Omega$, $30.0\ \Omega$, and $3.00\ \Omega$. Describe how your results compare to the discussion in Section 31.5.

31.65. An inductor, a capacitor, and a resistor are all connected in series across an ac source. If the resistance, inductance, and capacitance are all doubled, by what factor does each of the following quantities change? Indicate whether they increase or decrease: (a) the resonance angular frequency; (b) the inductive reactance; (c) the capacitive reactance. (d) Does the impedance double?

31.66. A transformer consists of 275 primary windings and 834 secondary windings. If the potential difference across the primary coil is 25.0 V, (a) what is the voltage across the secondary coil, and (b) what is the effective load resistance of the secondary coil if it is connected across a 125-Ω resistor?

31.67. You want to double the resonance angular frequency of a series R-L-C circuit by changing only the *pertinent* circuit elements all by the same factor. (a) Which ones should you change? (b) By what factor should you change them?

31.68. A resistance R, capacitance C, and inductance L are connected in series to a voltage source with amplitude V and variable angular frequency ω. If $\omega = \omega_0$, the resonance angular frequency, find (a) the maximum current in the resistor; (b) the maximum voltage across the capacitor; (c) the maximum voltage across the inductor; (d) the maximum energy stored in the capacitor; (e) the maximum energy stored in the inductor. Give your answers in terms of R, C, L, and V.

31.69. Repeat Problem 31.68 for the case $\omega = \omega_0/2$.

31.70. Repeat Problem 31.68 for the case $\omega = 2\omega_0$.

31.71. Finding an Unknown Inductance. Your boss gives you an inductor and asks you to measure its inductance. You have available a resistor, an ac voltmeter of high impedance, a capacitor, and an ac source. Explain how you might use these to determine the inductance, and cite any other piece of equipment you may need. Be sure to explain clearly how to use the equipment and what you need to measure to find the unknown inductance.

31.72. An L-R-C series circuit draws 220 W from a 120-V (rms), 50.0-Hz ac line. The power factor is 0.560, and the source voltage leads the current. (a) What is the net resistance R of the circuit? (b) Find the capacitance of the series capacitor that will result in a power factor of unity when it is added to the original circuit. (c) What power will then be drawn from the supply line?

31.73. In an L-R-C series circuit the current is given by $i = I\cos\omega t$. The voltage amplitudes for the resistor, inductor, and capacitor are V_R, V_L, and V_C. (a) Show that the instantaneous power into the resistor is $p_R = V_R I\cos^2\omega t = \frac{1}{2}V_R I(1 + \cos 2\omega t)$. What does this expression give for the average power into the resistor? (b) Show that the instantaneous power into the inductor is $p_L = -V_L I\sin\omega t\cos\omega t = -\frac{1}{2}V_L I\sin 2\omega t$. What does this expression give for the average power into the inductor? (c) Show that the instantaneous power into the capacitor is $p_C = V_C I\sin\omega t\cos\omega t = \frac{1}{2}V_C I\sin 2\omega t$. What does this expression give for the average power into the capacitor? (d) The instantaneous power delivered by the source is shown in Section 31.4 to be $p = VI\cos\omega t\,(\cos\phi\cos\omega t - \sin\phi\sin\omega t)$. Show that $p_R + p_L + p_C$ equals p at each instant of time.

Challenge Problems

31.74. (a) At what angular frequency is the voltage amplitude across the *resistor* in an L-R-C series circuit at maximum value? (b) At what angular frequency is the voltage amplitude across the *inductor* at maximum value? (c) At what angular frequency is the voltage amplitude across the *capacitor* at maximum value? (You may want to refer to the results of Problem 31.52.)

31.75. Complex Numbers in a Circuit. The voltage across a circuit element in an ac circuit is not necessarily in phase with the current through that circuit element. Therefore the voltage amplitudes across the circuit elements in a branch in an ac circuit do not add algebraically. A method that is commonly employed to simplify the analysis of an ac circuit driven by a sinusoidal source is to represent the impedance Z as a *complex* number. The resistance R is taken to be the real part of the impedance, and the reactance $X = X_L - X_C$ is taken to be the imaginary part. Thus, for a branch containing a resistor, inductor, and capacitor in series, the complex impedance is $Z_{\text{cpx}} = R + iX$, where $i^2 = -1$. If the voltage amplitude across the branch is V_{cpx}, we define a *complex* current amplitude by $I_{\text{cpx}} = V_{\text{cpx}}/Z_{\text{cpx}}$. The *actual* current amplitude is the absolute value of the complex current amplitude, that is, $I = (I_{\text{cpx}}{}^{*}I_{\text{cpx}})^{1/2}$. The phase angle ϕ of the current with respect to the source voltage is given by $\tan\phi = \text{Im}(I_{\text{cpx}})/\text{Re}(I_{\text{cpx}})$. The voltage amplitudes $V_{R\text{-cpx}}$, $V_{L\text{-cpx}}$, and $V_{C\text{-cpx}}$ across the resistance, inductance, and capacitance, respectively, are found by multiplying I_{cpx} by R, iX_L, or $-iX_C$, respectively. From the complex representation for the voltage amplitudes, the voltage across a branch is just the algebraic sum of the voltages across each circuit element; $V_{\text{cpx}} = V_{R\text{-cpx}} + V_{L\text{-cpx}} + V_{C\text{-cpx}}$. The actual value of any current amplitude or voltage amplitude is the absolute value of the corresponding complex quantity. Consider the series L-R-C circuit shown in Fig. 31.29. The values of the circuit elements, the source voltage amplitude, and the source angular frequency are as shown. Use the phasor diagram techniques presented in Section 31.1 to solve for (a) the current amplitude; and (b) the phase angle ϕ of the current with respect to the source voltage. (Note that this angle is the negative of the phase angle defined in Fig. 31.13.) Now analyze the same circuit using the complex-number approach. (c) Determine the complex impedance of the circuit, Z_{cpx}. Take the absolute value to obtain Z, the actual impedance of the circuit. (d) Take the voltage amplitude of the source, V_{cpx}, to be real, and find the complex current amplitude I_{cpx}. Find the actual current amplitude by taking the absolute value of I_{cpx}. (e) Find the phase angle ϕ of the current with respect to the source voltage by using the real and imaginary parts of I_{cpx}, as explained above. (f) Find the complex representations of the voltages across the resistance, the inductance, and the capacitance. (g) Adding the answers found in part (f), verify that the sum of these complex numbers is real and equal to 200 V, the voltage of the source.

Figure 31.29 Challenge Problem 31.75.