GEOMETRIC OPTICS 34

?How do magnifying lenses work? At what distance from the object being examined do they provide the sharpest view?

LEARNING GOALS

By studying this chapter, you will learn:

- How a plane mirror forms an image.
- Why concave and convex mirrors form different kinds of image.
- How images can be formed by a curved interface between two transparent materials.
- What aspects of a lens determine the type of image that it produces.
- What determines the field of view of a camera lens.
- What causes various defects in human vision, and how they can be corrected.
- The principle of the simple magnifier.
- How microscopes and telescopes work.

Your reflection in the bathroom mirror, the view of the moon through a telescope, the patterns seen in a kaleidoscope—all of these are examples of *images.* In each case the object that you're looking at appears to be in a different place than its actual position: Your reflection is on the other side of the mirror, the moon appears to be much closer when seen through a telescope, and objects seen in a kaleidoscope seem to be in many places at the same time. In each case, light rays that come from a point on an object are deflected by reflection or refraction (or a combination of the two), so they converge toward or appear to diverge from a point called an *image point.* Our goal in this chapter is to see how this is done and to explore the different kinds of images that can be made with simple optical devices.

To understand images and image formation, all we need are the ray model of light, the laws of reflection and refraction, and some simple geometry and trigonometry. The key role played by geometry in our analysis is the reason for the name *geometric optics* that is given to the study of how light rays form images. We'll begin our analysis with one of the simplest image-forming optical devices, a plane mirror. We'll go on to study how images are formed by curved mirrors, by refracting surfaces, and by thin lenses. Our results will lay the foundation for understanding many familiar optical instruments, including camera lenses, magnifiers, the human eye, microscopes, and telescopes.

34.1 Reflection and Refraction at a Plane Surface

Before discussing what is meant by an image, we first need the concept of **object** as it is used in optics. By an *object* we mean anything from which light rays radiate. This light could be emitted by the object itself if it is *self-luminous,* like the glowing filament of a light bulb. Alternatively, the light could be emitted by

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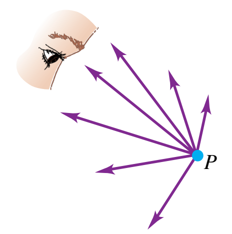
15.4 Geometric Optics: Plane Mirrors

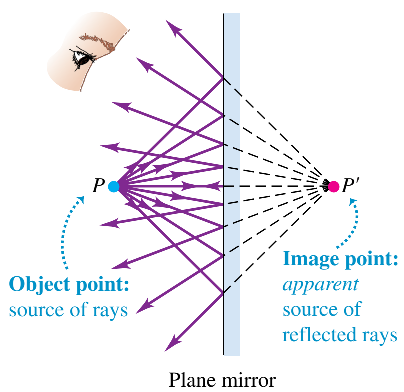
34.1 Light rays radiate from a point object P in all directions.



another source (such as a lamp or the sun) and then reflected from the object; an example is the light you see coming from the pages of this book. Figure 34.1 shows light rays radiating in all directions from an object at a point P. For an observer to see this object directly, there must be no obstruction between the object and the observer's eyes. Note that light rays from the object reach the observer's left and right eyes at different angles; these differences are processed by the observer's brain to infer the *distance* from the observer to the object.

The object in Fig. 34.1 is a **point object** that has no physical extent. Real objects with length, width, and height are called **extended objects.** To start with, we'll consider only an idealized point object, since we can always think of an extended object as being made up of a very large number of point objects.

34.2 Light rays from the object at point P are reflected from a plane mirror. The reflected rays entering the eye look as though they had come from image point P'.



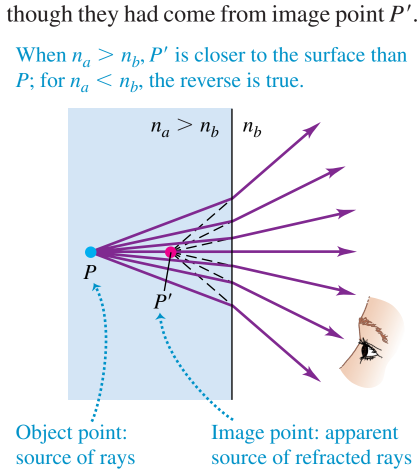
Suppose some of the rays from the object strike a smooth, plane reflecting surface (Fig. 34.2). This could be the surface of a material with a different index of refraction, which reflects part of the incident light, or a polished metal surface that reflects almost 100% of the light that strikes it. We will always draw the reflecting surface as a black line with a shaded area behind it, as in Fig. 34.2. Bathroom mirrors have a thin sheet of glass that lies in front of and protects the reflecting surface; we'll ignore the effects of this thin sheet.

According to the law of reflection, all rays striking the surface are reflected at an angle from the normal equal to the angle of incidence. Since the surface is plane, the normal is in the same direction at all points on the surface, and we have *specular* reflection. After the rays are reflected, their directions are the same as though they had come from point P'. We call point P an *object point* and point P' the corresponding *image point,* and we say that the reflecting surface forms an **image** of point P. An observer who can see only the rays reflected from the surface, and who doesn't know that he's seeing a reflection, *thinks* that the rays originate from the image point P'. The image point is therefore a convenient way to describe the directions of the various reflected rays, just as the object point P describes the directions of the rays arriving at the surface *before* reflection.

If the surface in Fig. 34.2 were *not* smooth, the reflection would be *diffuse,* and rays reflected from different parts of the surface would go in uncorrelated directions (see Fig. 33.6b). In this case there would not be a definite image point P' from which all reflected rays seem to emanate. You can't see your reflection in the surface of a tarnished piece of metal because its surface is rough; polishing the metal smoothes the surface so that specular reflection occurs and a reflected image becomes visible.

An image is also formed by a plane *refracting* surface, as shown in Fig. 34.3. Rays coming from point P are refracted at the interface between two optical materials. When the angles of incidence are small, the final directions of the rays after refraction are the same as though they had come from point P', as shown, and again we call P' an *image point.* In Section 33.2 we described how this effect makes underwater objects appear closer to the surface than they really are (see Fig. 33.9).

34.3 Light rays from the object at point P are refracted at the plane interface. The refracted rays entering the eye look as though they had come from image point P'.

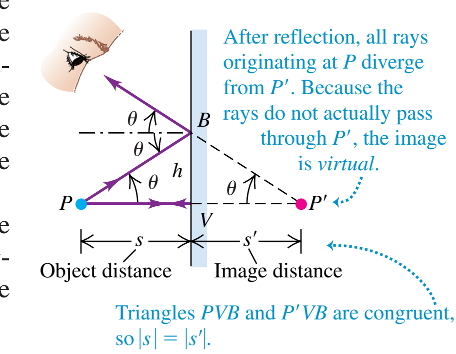


In both Figs. 34.2 and 34.3 the rays do not actually pass through the image point P'. Indeed, if the mirror in Fig. 34.2 is opaque, there is no light at all on its right side. If the outgoing rays don't actually pass through the image point, we call the image a **virtual image.** Later we will see cases in which the outgoing rays really *do* pass through an image point, and we will call the resulting image a **real image.** The images that are formed on a projection screen, on the photographic film in a camera, and on the retina of your eye are real images.

Image Formation by a Plane Mirror

Let's concentrate for now on images produced by *reflection;* we'll return to refraction later in the chapter. To find the precise location of the virtual image P' that a plane mirror forms of an object at P, we use the construction shown in Fig. 34.4. The figure shows two rays diverging from an object point P at a distance s to the left of a plane mirror. We call s the **object distance.** The ray PV is incident normally on the mirror (that is, it is perpendicular to the mirror surface), and it returns along its original path.

34.4 Construction for determining the location of the image formed by a plane mirror. The image point P' is as far behind the mirror as the object point P is in front of it.



The ray PB makes an angle θ with PV. It strikes the mirror at an angle of incidence θ and is reflected at an equal angle with the normal. When we extend the two reflected rays backward, they intersect at point P', at a distance s' behind the mirror. We call s' the **image distance.** The line between P and P' is perpendicular to the mirror. The two triangles PVB and $P'VB$ are congruent, so P and P' are at equal distances from the mirror, and s and s' have equal magnitudes. The image point P' is located exactly opposite the object point P as far *behind* the mirror as the object point is from the front of the mirror.

We can repeat the construction of Fig. 34.4 for each ray diverging from P. The directions of *all* the outgoing reflected rays are the same as though they had originated at point P', confirming that P' is the *image* of P. No matter where the observer is located, she will always see the image at the point P'.

Sign Rules

Before we go further, let's introduce some general sign rules. These may seem unnecessarily complicated for the simple case of an image formed by a plane mirror, but we want to state the rules in a form that will be applicable to *all* the situations we will encounter later. These will include image formation by a plane or spherical reflecting or refracting surface, or by a pair of refracting surfaces forming a lens. Here are the rules:

1. **Sign rule for the object distance:** When the object is on the same side of the reflecting or refracting surface as the incoming light, the object distance s is positive; otherwise, it is negative.
2. **Sign rule for the image distance:** When the image is on the same side of the reflecting or refracting surface as the outgoing light, the image distance s' is positive; otherwise, it is negative.
3. **Sign rule for the radius of curvature of a spherical surface:** When the center of curvature C is on the same side as the outgoing light, the radius of curvature is positive; otherwise, it is negative.

34.5 For both of these situations, the object distance s is positive (rule 1) and the image distance s' is negative (rule 2).

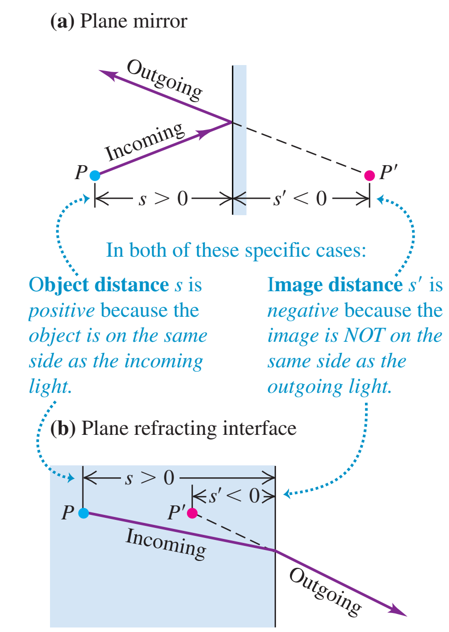


Figure 34.5 illustrates rules 1 and 2 for two different situations. For a mirror the incoming and outgoing sides are always the same; for example, in Figs. 34.2, 34.4, and 34.5a they are both the left side. For the refracting surfaces in Figs. 34.3 and 34.5b the incoming and outgoing sides are on the left and right sides, respectively, of the interface between the two materials. (Note that other textbooks may use different rules.)

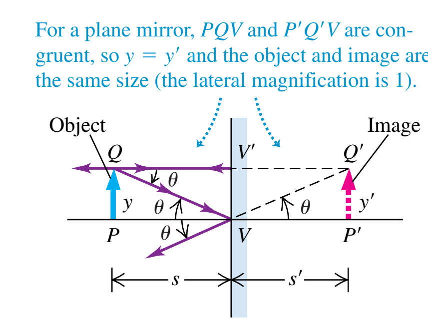
In Figs. 34.4 and 34.5a the object distance s is *positive* because the object point P is on the incoming side (the left side) of the reflecting surface. The image distance s' is *negative* because the image point P' is *not* on the outgoing side (the left side) of the surface. The object and image distances s and s' are related simply by

$$s = -s' \qquad \text{(plane mirror)} \tag{34.1}$$

For a plane reflecting or refracting surface, the radius of curvature is infinite and not a particularly interesting or useful quantity; in these cases we really don't need sign rule 3. But this rule will be of great importance when we study image formation by *curved* reflecting and refracting surfaces later in the chapter.

Image of an Extended Object: Plane Mirror

34.6 Construction for determining the height of an image formed by reflection at a plane reflecting surface.



Next we consider an *extended* object with finite size. For simplicity we often consider an object that has only one dimension, like a slender arrow, oriented parallel to the reflecting surface; an example is the arrow PQ in Fig. 34.6. The distance from the head to the tail of an arrow oriented in this way is called its *height;* in Fig. 34.6 the height is y. The image formed by such an extended object is an extended image; to each point on the object, there corresponds a point on the image. Two of the rays from Q are shown; *all* the rays from Q appear to diverge from its image point Q' after reflection. The image of the arrow is the line $P'Q'$, with height y'. Other points of the object PQ have image points between P' and Q'. The triangles PQV and $P'Q'V$ are congruent, so the object PQ and image $P'Q'$ have the same size and orientation, and $y = y'$.

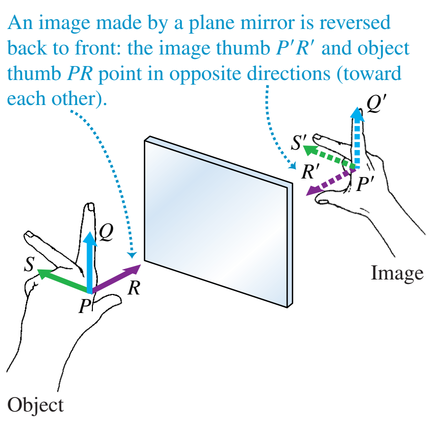
The ratio of image height to object height, y'/y, in *any* image-forming situation is called the **lateral magnification** m; that is,

$$m = \frac{y'}{y} \quad \text{(lateral magnification)} \qquad (34.2)$$

Thus for a plane mirror the lateral magnification m is unity. When you look at yourself in a plane mirror, your image is the same size as the real you.

In Fig. 34.6 the image arrow points in the *same* direction as the object arrow; we say that the image is **erect.** In this case, y and y' have the same sign, and the lateral magnification m is positive. The image formed by a plane mirror is always erect, so y and y' have both the same magnitude and the same sign; from Eq. (34.2) the lateral magnification of a plane mirror is always $m = +1$. Later we will encounter situations in which the image is **inverted;** that is, the image arrow points in the direction *opposite* to that of the object arrow. For an inverted image, y and y' have *opposite* signs, and the lateral magnification m is *negative.*

The object in Fig. 34.6 has only one dimension. Figure 34.7 shows a *three-dimensional* object and its three-dimensional virtual image formed by a plane mirror. The object and image are related in the same way as a left hand and a right hand.



34.7 The image formed by a plane mirror is virtual, erect, and reversed. It is the same size as the object.

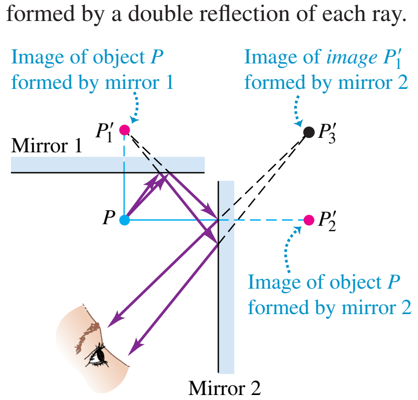
34.8 The image formed by a plane mirror is reversed; the image of a right hand is a left hand, and so on. (The hand is resting on a horizontal mirror.) Are images of the letters H and A reversed?

CAUTION Reflections in a plane mirror At this point, you may be asking, "Why does a plane mirror reverse images left and right but not top and bottom?" This question is quite misleading! As Fig. 34.7 shows, the up-down image $P'Q'$ and the left-right image $P'S'$ are parallel to their objects and are not reversed at all. Only the front-back image $P'R'$ is reversed relative to PR. Hence it's most correct to say that a plane mirror reverses *back to front.* To verify this object-image relationship, point your thumbs along PR and $P'R'$, your forefingers along PQ and $P'Q'$, and your middle fingers along PS and $P'S'$. When an object and its image are related in this way, the image is said to be **reversed;** this means that *only* the front-back dimension is reversed.

The reversed image of a three-dimensional object formed by a plane mirror is the same *size* as the object in all its dimensions. When the transverse dimensions of object and image are in the same direction, the image is erect. Thus a plane mirror always forms an erect but reversed image. Figure 34.8 illustrates this point.

An important property of all images formed by reflecting or refracting surfaces is that an *image* formed by one surface or optical device can serve as the *object* for a second surface or device. Figure 34.9 shows a simple example. Mirror 1 forms an image P_1' of the object point P, and mirror 2 forms another image P_2', each in the way we have just discussed. But in addition, the image P_1' formed by mirror 1 serves an object for mirror 2, which then forms an image of this object at point P_3' as shown. Similarly, mirror 1 uses the image P_2' formed by mirror 2 as an object and forms an image of it. We leave it to you to show that this image point is also at P_3'. The idea that an image formed by one device can act as the object for a second device is of great importance in geometric optics. We will use it later in this chapter to locate the image formed by two successive curved-surface refractions in a lens. This idea will help us to understand image formation by combinations of lenses, as in a microscope or a refracting telescope.

34.9 Images P_1' and P_2' are formed by a single reflection of each ray from the object at P. Image P_3', located by treating either of the other images as an object, is formed by a double reflection of each ray.



Test Your Understanding of Section 34.1 If you walk directly toward a plane mirror at a speed v, at what speed does your image approach you? (i) slower than v; (ii) v; (iii) faster than v but slower than $2v$; (iv) $2v$; (v) faster than $2v$.

34.2 Reflection at a Spherical Surface

A plane mirror produces an image that is the same size as the object. But there are many applications for mirrors in which the image and object must be of different sizes. A magnifying mirror used when applying makeup gives an image that is *larger* than the object, and surveillance mirrors (used in stores to help spot shoplifters) give an image that is *smaller* than the object. There are also applications of mirrors in which a *real* image is desired, so light rays do indeed pass through the image point P'. A plane mirror by itself cannot perform any of these tasks. Instead, *curved* mirrors are used.

Image of a Point Object: Spherical Mirror

We'll consider the special (and easily analyzed) case of image formation by a *spherical* mirror. Figure 34.10a shows a spherical mirror with radius of curvature R, with its concave side facing the incident light. The **center of curvature** of the surface (the center of the sphere of which the surface is a part) is at C, and the **vertex** of the mirror (the center of the mirror surface) is at V. The line CV is called the **optic axis.** Point P is an object point that lies on the optic axis; for the moment, we assume that the distance from P to V is greater than R.

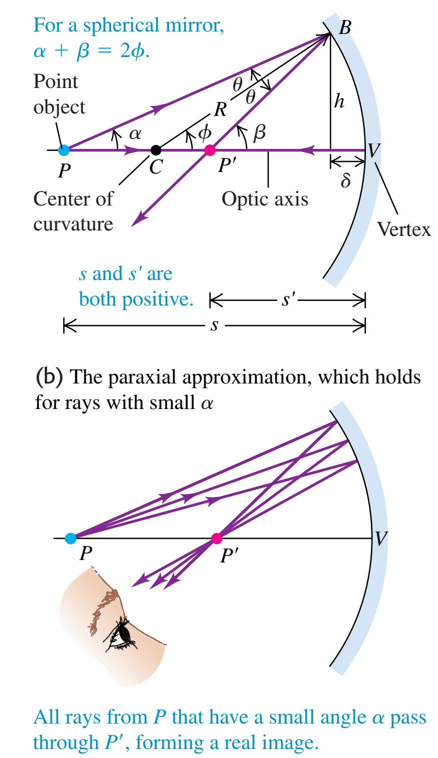
Ray PV, passing through C, strikes the mirror normally and is reflected back on itself. Ray PB, at an angle α with the axis, strikes the mirror at B, where the angles of incidence and reflection are θ. The reflected ray intersects the axis at point P'. We will show shortly that *all* rays from P intersect the axis at the *same* point P', as in Fig. 34.10b, provided that the angle α is small. Point P' is therefore the *image* of object point P. Unlike the reflected rays in Fig. 34.1, the reflected rays in Fig. 34.10b actually do intersect at point P', then diverge from P' *as if* they had originated at this point. Thus P' is a *real* image.

To see the usefulness of having a real image, suppose that the mirror is in a darkened room in which the only source of light is a self-luminous object at P. If you place a small piece of photographic film at P', all the rays of light coming from point P that reflect off the mirror will strike the same point P' on the film; when developed, the film will show a single bright spot, representing a sharply focused image of the object at point P. This principle is at the heart of most astronomical telescopes, which use large concave mirrors to make photographs of celestial objects. With a *plane* mirror like that in Fig. 34.2, placing a piece of film at the image point P' would be a waste of time; the light rays never actually pass through the image point, and the image can't be recorded on film. Real images are *essential* for photography.

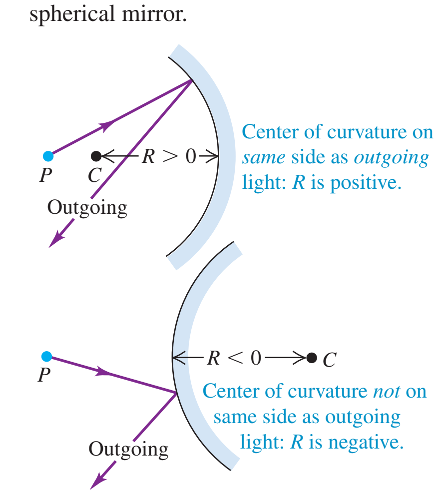
Let's now find the location of the real image point P' in Fig. 34.10a and prove the assertion that all rays from P intersect at P' (provided that their angle with the optic axis is small). The object distance, measured from the vertex V, is s; the image distance, also measured from V, is s'. The signs of s, s', and the radius of curvature R are determined by the sign rules given in Section 34.1. The object point P is on the same side as the incident light, so according to sign rule 1, s is positive. The image point P' is on the same side as the reflected light, so according to sign rule 2, the image distance s' is also positive. The center of curvature C is on the same side as the reflected light, so according to sign rule 3, R, too, is positive; R is always positive when reflection occurs at the *concave* side of a surface Fig. 34.11).

34.10 (a) A concave spherical mirror forms a real image of a point object P on the mirror's optic axis. (b) The eye sees some of the outgoing rays and perceives them as having come from P'.

(a) Construction for finding the position P' of an image formed by a concave spherical mirror



34.11 The sign rule for the radius of a spherical mirror.



We now use the following theorem from plane geometry: An exterior angle of a triangle equals the sum of the two opposite interior angles. Applying this theorem to triangles *PBC* and *P′BC* in Fig. 34.10a, we have

$$\phi = \alpha + \theta \qquad \beta = \phi + \theta$$

Eliminating θ between these equations gives

$$\alpha + \beta = 2\phi \tag{34.3}$$

We may now compute the image distance s'. Let h represent the height of point B above the optic axis, and let δ represent the short distance from V to the foot of this vertical line. We now write expressions for the tangents of α, β, and ϕ, remembering that s, s', and R are all positive quantities:

$$\tan\alpha = \frac{h}{s - \delta} \qquad \tan\beta = \frac{h}{s' - \delta} \qquad \tan\phi = \frac{h}{R - \delta}$$

These trigonometric equations cannot be solved as simply as the corresponding algebraic equations for a plane mirror. However, *if the angle α is small,* the angles β and ϕ are also small. The tangent of an angle that is much less than one radian is nearly equal to the angle itself (measured in radians), so we can replace $\tan\alpha$ by α, and so on, in the equations above. Also, if α is small, we can neglect the distance δ compared with s', s, and R. So for small angles we have the following approximate relationships:

$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$

Substituting these into Eq. (34.3) and dividing by h, we obtain a general relationship among s, s', and R:

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \qquad \text{(object–image relationship, spherical mirror)} \tag{34.4}$$

This equation does not contain the angle α. Hence *all* rays from P that make sufficiently small angles with the axis intersect at P' after they are reflected; this verifies our earlier assertion. Such rays, nearly parallel to the axis and close to it, are called **paraxial rays.** (The term **paraxial approximation** is often used for the approximations we have just described.) Since all such reflected light rays converge on the image point, a concave mirror is also called a *converging mirror.*

Be sure you understand that Eq. (34.4), as well as many similar relationships that we will derive later in this chapter and the next, is only *approximately* correct. It results from a calculation containing approximations, and it is valid only for paraxial rays. If we increase the angle α that a ray makes with the optic axis, the point P' where the ray intersects the optic axis moves somewhat closer to the vertex than for a paraxial ray. As a result, a spherical mirror, unlike a plane mirror, does not form a precise point image of a point object; the image is "smeared out." This property of a spherical mirror is called **spherical aberration.** When the primary mirror of the Hubble Space Telescope (Fig. 34.12a) was manufactured, tiny errors were made in its shape that led to an unacceptable amount of spherical aberration (Fig. 34.12b). The performance of the telescope improved dramatically after the installation of corrective optics (Fig. 34.12c).

If the radius of curvature becomes infinite $(R = \infty)$, the mirror becomes *plane,* and Eq. (34.4) reduces to Eq. (34.1) for a plane reflecting surface.

Focal Point and Focal Length

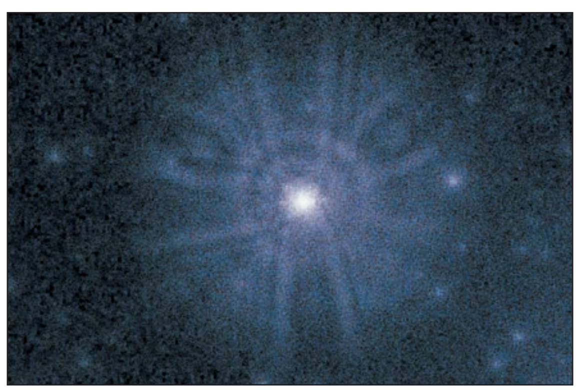
When the object point P is very far from the spherical mirror $(s = \infty)$, the incoming rays are parallel. (The star shown in Fig. 34.12c is an example of such a distant object.) From Eq. (34.4) the image distance s' in this case is given by

$$\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \qquad s' = \frac{R}{2}$$

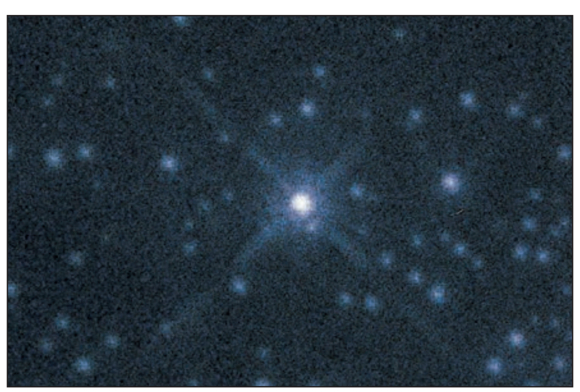
34.12 (a), (b) Soon after the Hubble Space Telescope (HST) was placed in orbit in 1990, it was discovered that the concave primary mirror (also called the *objective mirror*) was too shallow by about 1/50 the width of a human hair, leading to spherical aberration of the star's image. (c) After corrective optics were installed in 1993, the effects of spherical aberration were almost completely eliminated.

(a) The 2.4-m-diameter primary mirror of the Hubble Space Telescope

(b) A star seen with the original mirror



(c) The same star with corrective optics



The situation is shown in Fig. 34.13a. The beam of incident parallel rays converges, after reflection from the mirror, to a point F at a distance $R/2$ from the vertex of the mirror. The point F at which the incident parallel rays converge is called the **focal point;** we say that these rays are brought to a focus. The distance from the vertex to the focal point, denoted by f, is called the **focal length.** We see that f is related to the radius of curvature R by

$$f = \frac{R}{2} \qquad \text{(focal length of a spherical mirror)} \tag{34.5}$$

The opposite situation is shown in Fig. 34.13b. Now the *object* is placed at the focal point F, so the object distance is $s = f = R/2$. The image distance s' is again given by Eq. (34.4):

$$\frac{2}{R} + \frac{1}{s'} = \frac{2}{R} \qquad \frac{1}{s'} = 0 \qquad s' = \infty$$

With the object at the focal point, the reflected rays in Fig. 34.13b are parallel to the optic axis; they meet only at a point infinitely far from the mirror, so the image is at infinity.

Thus the focal point F of a spherical mirror has the properties that (1) any incoming ray parallel to the optic axis is reflected through the focal point and (2) any incoming ray that passes through the focal point is reflected parallel to the optic axis. For spherical mirrors these statements are true only for paraxial rays. For parabolic mirrors these statements are *exactly* true; this is why parabolic mirrors are preferred for astronomical telescopes. Spherical or parabolic mirrors are used in flashlights and headlights to form the light from the bulb into a parallel beam. Some solar-power plants use an array of plane mirrors to simulate an approximately spherical concave mirror; light from the sun is collected by the mirrors and directed to the focal point, where a steam boiler is placed. (The concepts of focal point and focal length also apply to lenses, as we'll see in Section 34.4.)

We will usually express the relationship between object and image distances for a mirror, Eq. (34.4), in terms of the focal length f:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(object–image relationship, spherical mirror)} \tag{34.6}$$

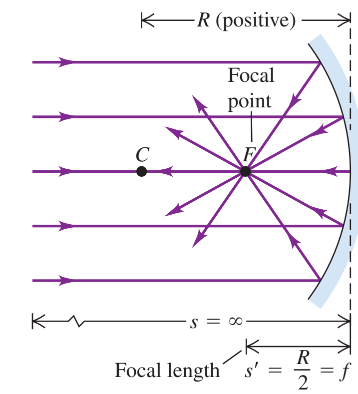
Image of an Extended Object: Spherical Mirror

Now suppose we have an object with *finite* size, represented by the arrow *PQ* in Fig. 34.14, perpendicular to the optic axis *CV*. The image of P formed by paraxial rays is at P'. The object distance for point Q is very nearly equal to that for point P, so the image $P'Q'$ is nearly straight and perpendicular to the axis. Note that the object and image arrows have different sizes, y and y', respectively, and that they have opposite orientation. In Eq. (34.2) we defined the *lateral magnification m* as the ratio of image size y' to object size y:

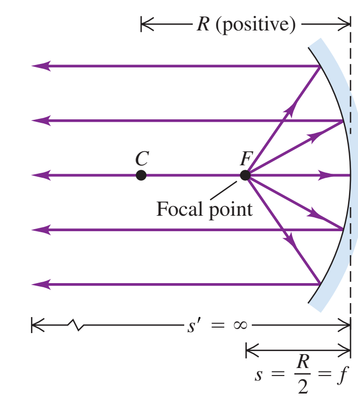
$$m = \frac{y'}{y}$$

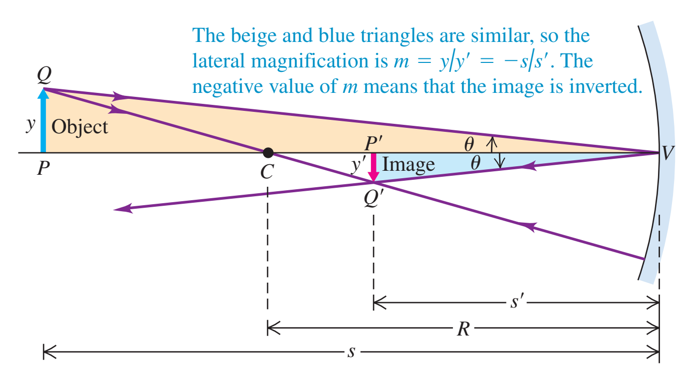
34.13 The focal point and focal length of a concave mirror.

(a) All parallel rays incident on a spherical mirror reflect through the focal point.



(b) Rays diverging from the focal point reflect to form parallel outgoing rays.





34.14 Construction for determining the position, orientation, and height of an image formed by a concave spherical mirror.

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15.5 Spherical Mirrors: Ray Diagrams
15.6 Spherical Mirrors: The Mirror Equation
15.7 Spherical Mirrors: Linear Magnification m
15.8 Spherical Mirrors: Problems

Because triangles PVQ and $P'VQ'$ in Fig. 34.14 are *similar,* we also have the relationship $y/s = -y'/s'$. The negative sign is needed because object and image are on opposite sides of the optic axis; if y is positive, y' is negative. Therefore

$$m = \frac{y'}{y} = -\frac{s'}{s} \qquad \text{(lateral magnification, spherical mirror)} \qquad (34.7)$$

If m is positive, the image is erect in comparison to the object; if m is negative, the image is *inverted* relative to the object, as in Fig. 34.14. For a *plane* mirror, $s = -s'$, so $y' = y$ and $m = +1$; since m is positive, the image is erect, and since $|m| = 1$, the image is the same size as the object.

CAUTION **Lateral magnification can be less than 1** Although the ratio of image size to object size is called the *lateral magnification,* the image formed by a mirror or lens may be larger than, smaller than, or the same size as the object. If it is smaller, then the lateral magnification is less than unity in absolute value: $|m| < 1$. The image formed by an astronomical telescope mirror or a camera lens is usually *much* smaller than the object. For example, the image of the bright star shown in Fig. 34.12c is just a few millimeters across, while the star itself is hundreds of thousands of kilometers in diameter. ▌

In our discussion of concave mirrors we have so far considered only objects that lie *outside* or at the focal point, so that the object distance s is greater than or equal to the (positive) focal length f. In this case the image point is on the same side of the mirror as the outgoing rays, and the image is real and inverted. If an object is placed *inside* the focal point of a concave mirror, so that $s < f$, the resulting image is *virtual* (that is, the image point is on the opposite side of the mirror from the object), *erect,* and *larger* than the object. Mirrors used when applying makeup (referred to at the beginning of this section) are concave mirrors; in use, the distance from the face to the mirror is less than the focal length, and an enlarged, erect image is seen. You can prove these statements about concave mirrors by applying Eqs. (34.6) and (34.7) (see Exercise 34.11). We'll also be able to verify these results later in this section, after we've learned some graphical methods for relating the positions and sizes of the object and the image.

Example 34.1 Image formation by a concave mirror I

A concave mirror forms an image, on a wall 3.00 m from the mirror, of the filament of a headlight lamp 10.0 cm in front of the mirror. (a) What are the radius of curvature and focal length of the mirror? (b) What is the height of the image if the height of the object is 5.00 mm?

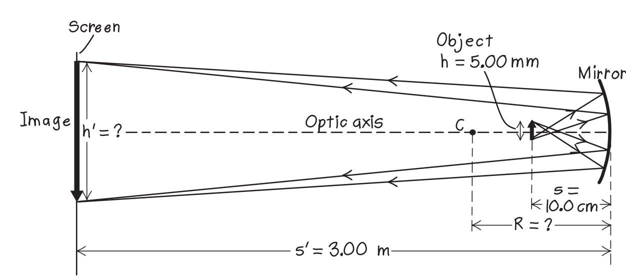
SOLUTION

IDENTIFY: This problem uses the ideas developed in this section. Our target variables are the radius of curvature R, focal length f, and image height y'.

SET UP: Figure 34.15 shows the situation. We are given the distances from the mirror to the object (s) and from the mirror to the image (s'). We use the object–image relationship given by Eq. (34.6) to determine the focal length f, and then find the radius of curvature R using Eq. (34.5). Equation (34.7) lets us calculate the image height y' from the distances s and s' and the object height y.

EXECUTE: (a) Both the object and the image are on the concave side of the mirror (the reflective side), so both object distance and image distance are positive; we have $s = 10.0$ cm and $s' = 300$ cm. From Eq. (34.4),

34.15 Our sketch for this problem.



$$\frac{1}{10.0 \text{ cm}} + \frac{1}{300 \text{ cm}} = \frac{2}{R}$$

$$R = \frac{2}{0.100 \text{ cm}^{-1} + 3.33 \times 10^{-3} \text{ cm}^{-1}} = 19.4 \text{ cm}$$

The focal length of the mirror is $f = R/2 = 9.7$ cm.

(b) From Eq. (34.7) the lateral magnification is

$$m = \frac{y'}{y} = -\frac{s'}{s} = -\frac{300 \text{ cm}}{10.0 \text{ cm}} = -30.0$$

Because m is negative, the image is inverted. The height of the image is 30.0 times the height of the object, or $(30.0)(5.00 \text{ mm}) = 150$ mm.

EVALUATE: Note that the object is placed just outside the focal point ($s = 10.0$ cm compared to $f = 9.7$ cm). This is very similar to what is done in automobile headlights. With the filament close to the focal point, the concave mirror produces a beam of nearly parallel rays.

Conceptual Example 34.2 Image formation by a concave mirror II

In Example 34.1, suppose that the left half of the mirror's reflecting surface is covered with nonreflective soot. What effect will this have on the image of the filament?

SOLUTION

It would be natural to guess that the image would now show only half of the filament. But in fact the image will still show the *entire* filament. The explanation can be seen by examining Fig. 34.10b. Light rays coming from any object point P are reflected from *all* parts of the mirror and converge on the corresponding image point P'. If part of the mirror surface is made nonreflective or is removed altogether, the light rays from the remaining reflective surface still form an image of every part of the object.

The only effect of reducing the reflecting area is that the image becomes dimmer because less light energy reaches the image point. In our example the reflective area of the mirror is reduced by one-half, and the image will be one-half as bright. *Increasing* the reflective area makes the image brighter. To make reasonably bright images of distant stars, astronomical telescopes use mirrors that are up to several meters in diameter. Figure 34.12a shows an example.

Convex Mirrors

In Fig. 34.16a the *convex* side of a spherical mirror faces the incident light. The center of curvature is on the side opposite to the outgoing rays; according to sign rule 3 in Section 34.1, R is negative (see Fig. 34.11). Ray PB is reflected, with the angles of incidence and reflection both equal to θ. The reflected ray, projected backward, intersects the axis at P'. As with a concave mirror, *all* rays from P that are reflected by the mirror diverge from the same point P', provided that the angle α is small. Therefore P' is the image of P. The object distance s is positive, the image distance s' is negative, and the radius of curvature R is *negative* for a *convex* mirror.

Figure 34.16b shows two rays diverging from the head of the arrow PQ and the virtual image $P'Q'$ of this arrow. The same procedure that we used for a concave mirror can be used to show that for a convex mirror,

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

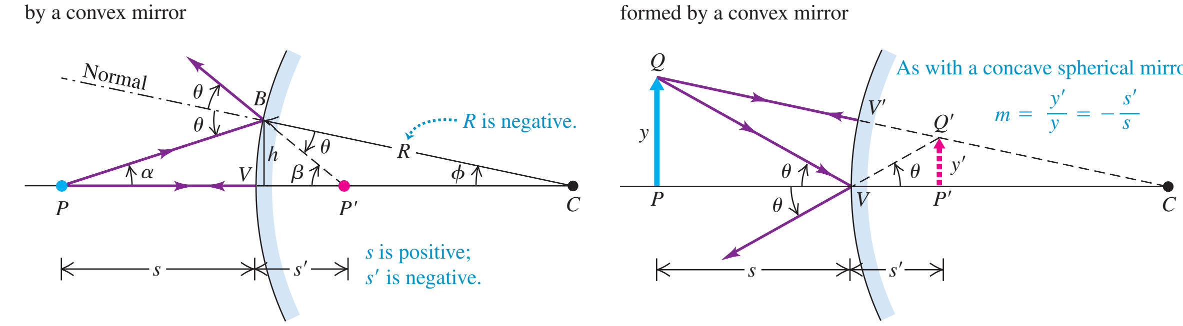
and the lateral magnification is

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

34.16 Image formation by a convex mirror.

(a) Construction for finding the position of an image formed by a convex mirror

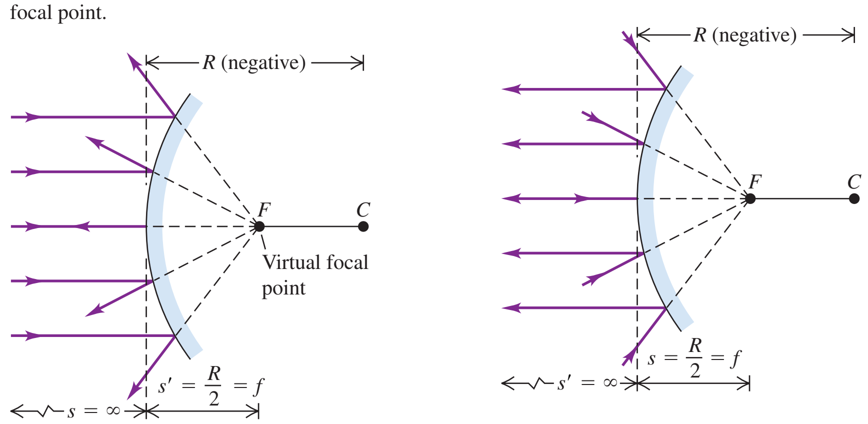
(b) Construction for finding the magnification of an image formed by a convex mirror



34.17 The focal point and focal length of a convex mirror.

(a) Paraxial rays incident on a convex spherical mirror diverge from a virtual focal point.

(b) Rays aimed at the virtual focal point are parallel to the axis after reflection.



These expressions are exactly the same as Eqs. (34.4) and (34.7) for a concave mirror. Thus when we use our sign rules consistently, Eqs. (34.4) and (34.7) are valid for both concave and convex mirrors.

When R is negative (convex mirror), incoming rays that are parallel to the optic axis are not reflected through the focal point F. Instead, they diverge as though they had come from the point F at a distance f *behind* the mirror, as shown in Fig. 34.17a. In this case, f is the focal length, and F is called a *virtual focal point*. The corresponding image distance s' is negative, so both f and R are negative, and Eq. (34.5), $f = R/2$, holds for convex as well as concave mirrors. In Fig. 34.17b the incoming rays are converging as though they would meet at the virtual focal point F, and they are reflected parallel to the optic axis.

In summary, Eqs. (34.4) through (34.7), the basic relationships for image formation by a spherical mirror, are valid for both concave and convex mirrors, provided that we use the sign rules consistently.

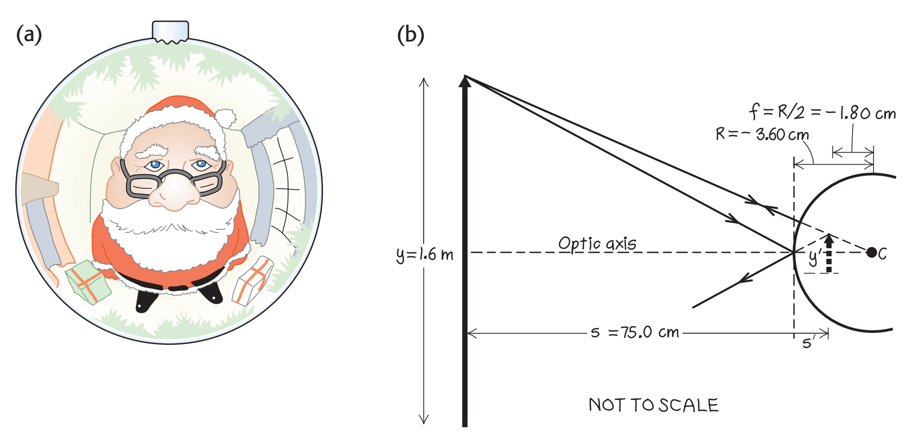
Example 34.3 Santa's image problem

Santa checks himself for soot, using his reflection in a shiny silvered Christmas tree ornament 0.750 m away (Fig. 34.18a). The diameter of the ornament is 7.20 cm. Standard reference works state that he is a "right jolly old elf," so we estimate his height to be 1.6 m. Where and how tall is the image of Santa formed by the ornament? Is it erect or inverted?

SOLUTION

IDENTIFY: Santa is the object, and the surface of the ornament closest to Santa acts as a convex mirror. The relationships among object distance, image distance, focal length, and magnification are the same as for concave mirrors, provided we use the sign rules consistently.

34.18 (a) The ornament forms a virtual, reduced, erect image of Santa. (b) Our sketch of two of the rays forming the image.



SET UP: Figure 34.18b shows the situation. Since the mirror is convex, its radius of curvature and focal length are negative. The object distance is $s = 0.750 \text{ m} = 75.0 \text{ cm}$ and Santa's height is $y = 1.6 \text{ m}$. We use Eq. (34.6) to determine the image distance s', and then use Eq. (34.7) to find the lateral magnification m and hence the image height y'. The sign of m tells us whether the image is erect or inverted.

EXECUTE: The radius of the convex mirror (half the diameter) is $R = -(7.20 \text{ cm})/2 = -3.60 \text{ cm}$, and the focal length is $f = R/2 = -1.80 \text{ cm}$. From Eq. (34.6),

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-1.80 \text{ cm}} - \frac{1}{75.0 \text{ cm}}$$

$$s' = -1.76 \text{ cm}$$

Because s' is negative, the image is behind the mirror—that is, on the side opposite to the outgoing light (Fig. 34.18b)—and it is virtual. The image is about halfway between the front surface of the ornament and its center.

The lateral magnification m is given by Eq. (34.7):

$$m = \frac{y'}{y} = -\frac{s'}{s} = -\frac{-1.76 \text{ cm}}{75.0 \text{ cm}} = 0.0234$$

Because m is positive, the image is erect. It is only about 0.0234 as tall as Santa himself:

$$y' = my = (0.0234)(1.6 \text{ m}) = 3.8 \times 10^{-2} \text{ m} = 3.8 \text{ cm}$$

EVALUATE: When the object distance s is positive, a convex mirror *always* forms an erect, virtual, reduced, reversed image. For this reason, convex mirrors are used for shoplifting surveillance in stores, at blind intersections, and as "wide-angle" rear-view mirrors for cars and trucks (including those that bear the legend "Objects in mirror are closer than they appear").

Graphical Methods for Mirrors

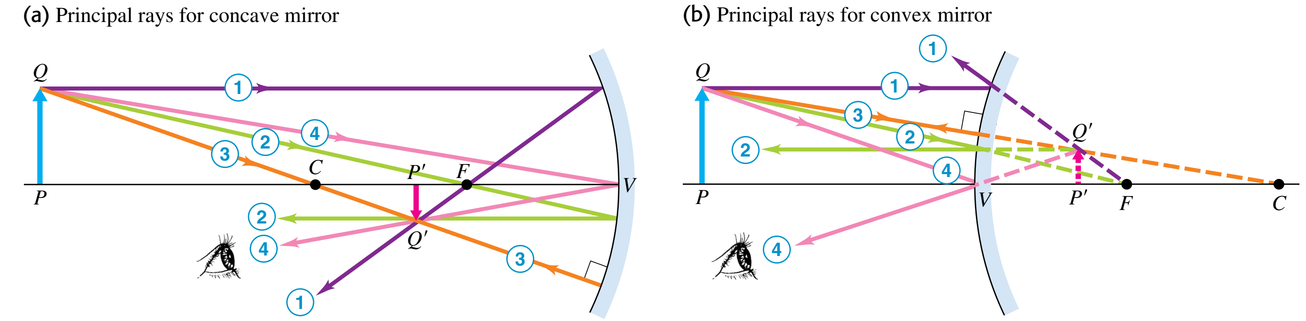
In Examples 34.1 and 34.3, we used Eqs. (34.6) and (34.7) to find the position and size of the image formed by a mirror. We can also determine the properties of the image by a simple *graphical* method. This method consists of finding the point of intersection of a few particular rays that diverge from a point of the object (such as point Q in Fig. 34.19) and are reflected by the mirror. Then (neglecting aberrations) *all* rays from this object point that strike the mirror will intersect at the same point. For this construction we always choose an object point that is *not* on the optic axis. Four rays that we can usually draw easily are shown in Fig. 34.19. These are called **principal rays.**

1. *A ray parallel to the axis,* after reflection, passes through the focal point F of a concave mirror or appears to come from the (virtual) focal point of a convex mirror.
2. *A ray through (or proceeding toward) the focal point F* is reflected parallel to the axis.
3. *A ray along the radius* through or away from the center of curvature C intersects the surface normally and is reflected back along its original path.
4. *A ray to the vertex V* is reflected forming equal angles with the optic axis.

34.19 The graphical method of locating an image formed by spherical mirror. The colors of the rays are for identification only; they do not refer to specific colors of light.

(a) Principal rays for concave mirror

(b) Principal rays for convex mirror



(a)
1. Ray parallel to axis reflects through focal point.
2. Ray through focal point reflects parallel to axis.
3. Ray through center of curvature intersects the surface normally and reflects along its original path.
4. Ray to vertex reflects symmetrically around optic axis.

(b)
1. Reflected parallel ray appears to come from focal point.
2. Ray toward focal point reflects parallel to axis.
3. As with concave mirror: Ray radial to center of curvature intersects the surface normally and reflects along its original path.
4. As with concave mirror: Ray to vertex reflects symmetrically around optic axis.

Once we have found the position of the image point by means of the intersection of any two of these principal rays (1, 2, 3, 4), we can draw the path of any other ray from the object point to the same image point.

CAUTION **Principal rays are not the only rays** Although we've emphasized the principal rays, in fact *any* ray from the object that strikes the mirror will pass through the image point (for a real image) or appear to originate from the image point (for a virtual image). Usually, you only need to draw the principal rays, because these are all you need to locate the image.

Problem-Solving Strategy 34.1 Image Formation by Mirrors

IDENTIFY *the relevant concepts:* There are two different and complementary ways to solve problems involving image formation by mirrors. One approach uses equations, while the other involves drawing a principal-ray diagram. A successful problem solution uses *both* approaches.

SET UP *the problem:* Determine the target variables. The three key quantities are the focal length, object distance, and image distance; typically you'll be given two of these and will have to determine the third.

EXECUTE *the solution* as follows:

1. The principal-ray diagram is to geometric optics what the free-body diagram is to mechanics. In any problem involving image formation by a mirror, *always* draw a principal-ray diagram first if you have enough information. (The same advice should be followed when dealing with lenses in the following sections.)
2. It is usually best to orient your diagrams consistently with the incoming rays traveling from left to right. Don't draw a lot of other rays at random; stick with the principal rays, the ones you know something about. Use a ruler and measure distances carefully! A freehand sketch will *not* give good results.
3. If your principal rays don't converge at a real image point, you may have to extend them straight backward to locate a virtual image point, as in Fig. 34.19b. We recommend drawing the extensions with broken lines. Another useful aid is to color-code the different principal rays, as is done in Fig. 34.19.
4. Check your results using Eq. (34.6), $1/s + 1/s' = 1/f$, and the lateral magnification equation, Eq. (34.7). The results you find using this equation must be consistent with your principal-ray diagram; if not, double check both your calculations and your diagram.
5. Pay careful attention to signs on object and image distances, radii of curvature, and object and image heights. A negative sign on any of these quantities *always* has significance. Use the equations and the sign rules carefully and consistently, and they will tell you the truth! Note that the *same* sign rules (given in Section 34.1) work for all four cases in this chapter: reflection and refraction from plane and spherical surfaces.

EVALUATE *your answer:* You've already checked your results by using both diagrams and equations. But it always helps to take a look back and ask yourself, "Do these results make sense?"

Example 34.4 Concave mirror, different object distances

A concave mirror has a radius of curvature with absolute value 20 cm. Find graphically the image of an object in the form of an arrow perpendicular to the axis of the mirror at each of the following object distances: (a) 30 cm, (b) 20 cm, (c) 10 cm, and (d) 5 cm. Check the construction by *computing* the size and lateral magnification of each image.

SOLUTION

IDENTIFY: This problem asks us to use *both* graphical methods and calculations to find the image made by a mirror. This is a good practice to follow in all problems that involve image formation.

SET UP: We are given the radius of curvature $R = 20$ cm (positive since the mirror is concave) and hence the focal length $f = R/2 = 10$ cm. In each case we are told the object distance s and are asked to find the image distance s' and the lateral magnification $m = -s'/s$.

EXECUTE: Figure 34.20 shows the principal-ray diagrams for the four cases. Study each of these diagrams carefully, comparing each numbered ray with the description above. Several points are worth noting. First, in (b) the object and image distances are equal. Ray 3 cannot be drawn in this case because a ray from Q through the center of curvature C does not strike the mirror. Ray 2 cannot be drawn in (c) because a ray from Q toward F also does not strike the mirror. In this case the outgoing rays are parallel, corresponding to an infinite image distance. In (d) the outgoing rays have no real intersection point; they must be extended backward to find the point from which they appear to diverge—that is, from the *virtual image point* Q'. The case shown in (d) illustrates the general observation that an object placed inside the focal point of a concave mirror produces a virtual image.

Measurements of the figures, with appropriate scaling, give the following approximate image distances: (a) 15 cm; (b) 20 cm; (c) ∞ or $-\infty$ (because the outgoing rays are parallel and do not converge at any finite distance); (d) -10 cm. To *compute* these distances, we use Eq. (34.6) with $f = 10$ cm:

$$\text{(a)}\quad \frac{1}{30\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} \qquad s' = 15\text{ cm}$$

$$\text{(b)}\quad \frac{1}{20\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} \qquad s' = 20\text{ cm}$$

$$\text{(c)}\quad \frac{1}{10\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} \qquad s' = \infty \ (\text{or } -\infty)$$

$$\text{(d)}\quad \frac{1}{5\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} \qquad s' = -10\text{ cm}$$

In (a) and (b) the image is real; in (d) it is virtual. In (c) the image is formed at infinity.

The lateral magnifications measured from the figures are approximately (a) $-\frac{1}{2}$; (b) -1; (c) ∞ or $-\infty$ (because the image distance is infinite); (d) $+2$. *Computing* the magnifications from Eq. (34.7), we find:

$$\text{(a)}\quad m = -\frac{15\text{ cm}}{30\text{ cm}} = -\frac{1}{2}$$

$$\text{(b)}\quad m = -\frac{20\text{ cm}}{20\text{ cm}} = -1$$

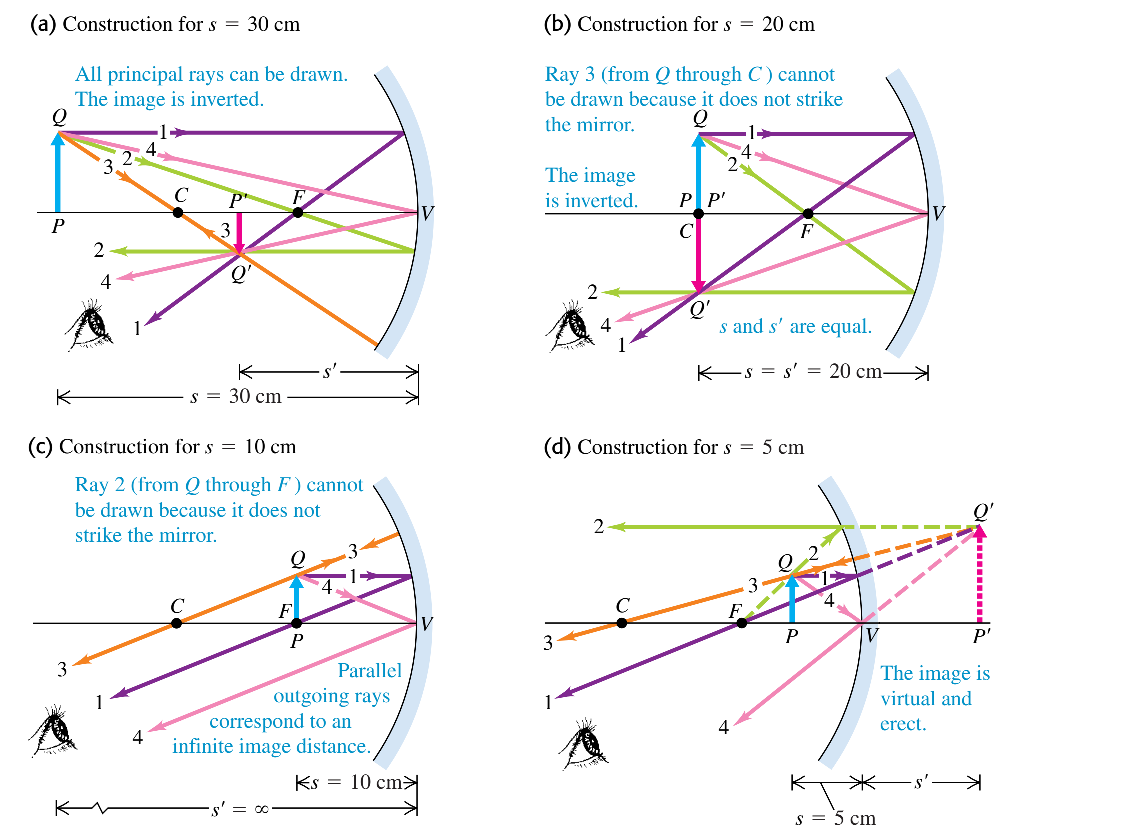
$$\text{(c)}\quad m = -\frac{\infty\text{ cm}}{10\text{ cm}} = -\infty \ (\text{or } +\infty)$$

$$\text{(d)}\quad m = -\frac{-10\text{ cm}}{5\text{ cm}} = +2$$

In (a) and (b) the image is inverted; in (d) it is erect.

EVALUATE: Notice the trend as the object is moved closer to the mirror. When the object is far from the mirror, as in Fig. 34.20a, the image is smaller than the object, inverted, and real. As the object distance decreases, the image moves farther from the mirror and increases in size (Fig. 34.20b). When the object is at the focal point, the image is at infinity (Fig. 34.20c). If the object is moved inside the focal point, the image becomes larger than the object, erect, and virtual (Fig. 34.20d). You can test these conclusions by looking at objects reflected in the concave bowl of a metal spoon.

34.20 Using principal-ray diagrams to locate the image $P'Q'$ made by a concave mirror.



Test Your Understanding of Section 34.2 A cosmetics mirror is designed so that your reflection appears right-side up and enlarged. (a) Is the mirror concave or convex? (b) To see an enlarged image, what should be the distance from the mirror (of focal length f) to your face? (i) $|f|$; (ii) less than $|f|$; (iii) greater than $|f|$.

34.3 Refraction at a Spherical Surface

As we mentioned in Section 34.1, images can be formed by refraction as well as by reflection. To begin with, let's consider refraction at a spherical surface—that is, at a spherical interface between two optical materials with different indexes of refraction. This analysis is directly applicable to some real optical systems, such as the human eye. It also provides a stepping-stone for the analysis of lenses, which usually have *two* spherical (or nearly spherical) surfaces.

Image of a Point Object: Spherical Refracting Surface

In Fig. 34.21 a spherical surface with radius R forms an interface between two materials with different indexes of refraction n_a and n_b. The surface forms an image P' of an object point P; we want to find how the object and image distances (s and s') are related. We will use the same sign rules that we used for spherical mirrors. The center of curvature C is on the outgoing side of the surface, so R is positive. Ray PV strikes the vertex V and is perpendicular to the surface (that is, to the plane that is tangent to the surface at the point of incidence V). It passes into the second material without deviation. Ray PB, making an angle α with the axis, is incident at an angle θ_a with the normal and is refracted at an angle θ_b. These rays intersect at P', a distance s' to the right of the vertex. The figure is drawn for the case $n_a < n_b$. The object and image distances are both positive.

We are going to prove that if the angle α is small, *all* rays from P intersect at the same point P', so P' is the *real image* of P. We use much the same approach as we did for spherical mirrors in Section 34.2. We again use the theorem that an exterior angle of a triangle equals the sum of the two opposite interior angles; applying this to the triangles PBC and $P'BC$ gives

$$\theta_a = \alpha + \phi \qquad \phi = \beta + \theta_b \tag{34.8}$$

From the law of refraction,

$$n_a \sin\theta_a = n_b \sin\theta_b$$

Also, the tangents of α, β, and ϕ are

$$\tan\alpha = \frac{h}{s+\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta} \tag{34.9}$$

For paraxial rays, θ_a and θ_b are both small in comparison to a radian, and we may approximate both the sine and tangent of either of these angles by the angle itself (measured in radians). The law of refraction then gives

$$n_a\theta_a = n_b\theta_b$$

Combining this with the first of Eqs. (34.8), we obtain

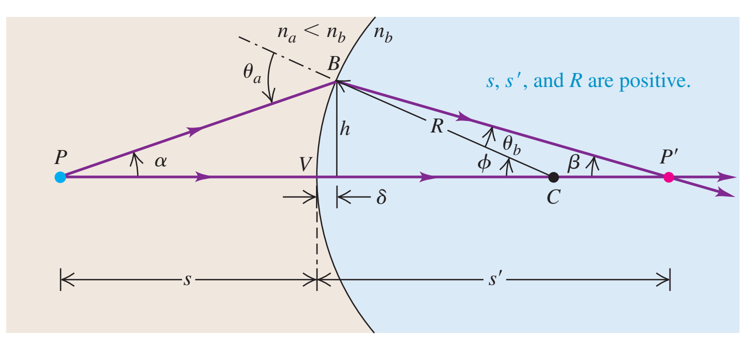
$$\theta_b = \frac{n_a}{n_b}(\alpha + \phi)$$

When we substitute this into the second of Eqs. (34.8), we get

$$n_a\alpha + n_b\beta = (n_b - n_a)\phi \tag{34.10}$$

Now we use the approximations $\tan\alpha = \alpha$, and so on, in Eqs. (34.9) and also neglect the small distance δ; those equations then become

$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$



34.21 Construction for finding the position of the image point P' of a point object P formed by refraction at a spherical surface. The materials to the left and right of the interface have refractive indexes n_a and n_b, respectively. In the case shown here, $n_a < n_b$.

Finally, we substitute these into Eq. (34.10) and divide out the common factor h. We obtain

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \quad \text{(object–image relationship, spherical refracting surface)} \tag{34.11}$$

This equation does not contain the angle α, so the image distance is the same for *all* paraxial rays emanating from P; this proves our assertion that P' is the image of P.

To obtain the lateral magnification m for this situation, we use the construction in Fig. 34.22. We draw two rays from point Q, one through the center of curvature C and the other incident at the vertex V. From the triangles PQV and $P'Q'V$,

$$\tan\theta_a = \frac{y}{s} \qquad \tan\theta_b = \frac{-y'}{s'}$$

and from the law of refraction,

$$n_a \sin\theta_a = n_b \sin\theta_b$$

For small angles,

$$\tan\theta_a = \sin\theta_a \qquad \tan\theta_b = \sin\theta_b$$

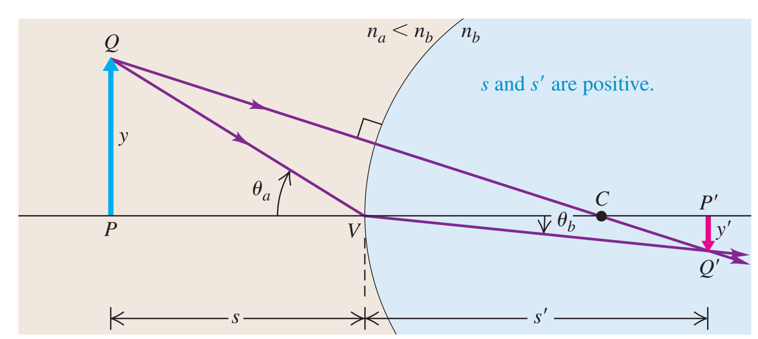
so finally

$$\frac{n_a y}{s} = -\frac{n_b y'}{s'} \qquad \text{or}$$

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s} \quad \text{(lateral magnification, spherical refracting surface)} \tag{34.12}$$

Equations (34.11) and (34.12) can be applied to both convex and concave refracting surfaces, provided that you use the sign rules consistently. It doesn't matter whether n_b is greater or less than n_a. To verify these statements, you should construct diagrams like Figs. 34.21 and 34.22 for the following three cases: (i) $R > 0$ and $n_a > n_b$, (ii) $R < 0$ and $n_a < n_b$, and (iii) $R < 0$ and $n_a > n_b$. Then in each case, use your diagram to again derive Eqs. (34.11) and (34.12).

Here's a final note on the sign rule for the radius of curvature R of a surface. For the convex reflecting surface in Fig. 34.16, we considered R negative, but the convex *refracting* surface in Fig. 34.21 has a *positive* value of R. This may seem inconsistent, but it isn't. The rule is that R is positive if the center of curvature C is on the outgoing side of the surface and negative if C is on the other side. For the convex reflecting surface in Fig. 34.16, R is negative because point C is to the right of the surface but outgoing rays are to the left. For the convex refracting surface in Fig. 34.21, R is positive because both C and the outgoing rays are to the right of the surface.



34.22 Construction for determining the height of an image formed by refraction at a spherical surface. In the case shown here, $n_a < n_b$.

34.23 Light rays refract as they pass through the curved surfaces of these water droplets.

Refraction at a curved surface is one reason gardeners avoid watering plants at midday. As sunlight enters a water drop resting on a leaf (Fig. 34.23), the light rays are refracted toward each other as in Figs. 34.21 and 34.22. The sunlight that strikes the leaf is therefore more concentrated and able to cause damage.

An important special case of a spherical refracting surface is a *plane* surface between two optical materials. This corresponds to setting $R = \infty$ in Eq. (34.11). In this case,

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0 \qquad \text{(plane refracting surface)} \tag{34.13}$$

To find the lateral magnification m for this case, we combine this equation with the general relationship, Eq. (34.12), obtaining the simple result

$$m = 1$$

That is, the image formed by a *plane* refracting surface always has the same lateral size as the object, and it is always erect.

An example of image formation by a plane refracting surface is the appearance of a partly submerged drinking straw or canoe paddle. When viewed from some angles, the object appears to have a sharp bend at the water surface because the submerged part appears to be only about three-quarters of its actual distance below the surface. (We commented on the appearance of a submerged object in Section 33.2; see Fig. 33.9.)

Example 34.5 Image formation by refraction I

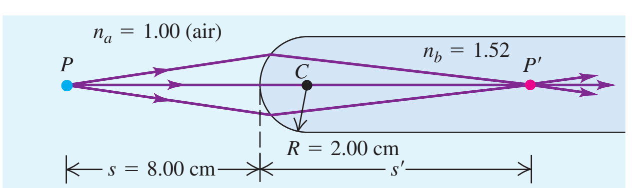
A cylindrical glass rod in air (Fig. 34.24) has index of refraction 1.52. One end is ground to a hemispherical surface with radius $R = 2.00$ cm. (a) Find the image distance of a small object on the axis of the rod, 8.00 cm to the left of the vertex. (b) Find the lateral magnification.

SOLUTION

IDENTIFY: This problem uses the ideas of refraction at a curved surface. Our target variables are the image distance s' and the lateral magnification m.

SET UP: Here material a is air $(n_a = 1.00)$ and material b is the glass of which the rod is made $(n_b = 1.52)$. We are given $s = 8.00$ cm; the radius of the spherical surface is positive $(R = +2.00 \text{ cm})$ because the center of curvature is on the outgoing side of the surface. We use Eq. (34.11) to determine the image distance and Eq. (34.12) to find the lateral magnification.

34.24 The glass rod in air forms a real image.



EXECUTE: (a) From Eq. (34.11),

$$\frac{1.00}{8.00 \text{ cm}} + \frac{1.52}{s'} = \frac{1.52 - 1.00}{+2.00 \text{ cm}}$$

$$s' = +11.3 \text{ cm}$$

(b) From Eq. (34.12),

$$m = -\frac{n_a s'}{n_b s} = -\frac{(1.00)(11.3 \text{ cm})}{(1.52)(8.00 \text{ cm})} = -0.929$$

EVALUATE: Because the image distance s' is positive, the image is formed 11.3 cm to the *right* of the vertex (on the outgoing side), as shown in Fig. 34.24. The value of m tells us that the image is somewhat smaller than the object, and it is inverted. If the object is an arrow 1.000 mm high, pointing upward, the image is an arrow 0.929 mm high, pointing downward.

Example 34.6 Image formation by refraction II

The glass rod in Example 34.5 is immersed in water (index of refraction $n = 1.33$), as shown in Fig. 34.25. The other quantities have the same values as before. Find the image distance and lateral magnification.

SOLUTION

IDENTIFY: The situation is the same as in Example 34.5 except that now $n_a = 1.33$.

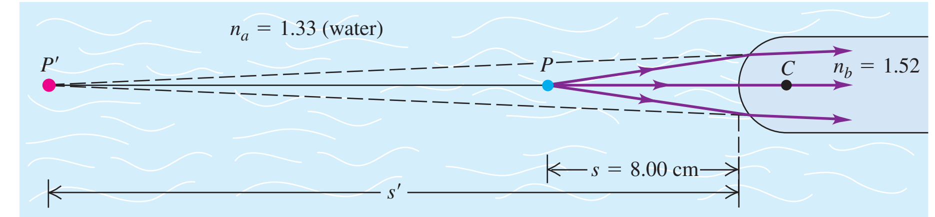
SET UP: As in Example 34.5, we use Eqs. (34.11) and (34.12) to determine s' and m, respectively.

EXECUTE: From Eq. (34.11),

$$\frac{1.33}{8.00 \text{ cm}} + \frac{1.52}{s'} = \frac{1.52 - 1.33}{+2.00 \text{ cm}}$$

$$s' = -21.3 \text{ cm}$$

34.25 When immersed in water, the glass rod forms a virtual image.



The lateral magnification in this case is

$$m = -\frac{(1.33)(-21.3 \text{ cm})}{(1.52)(8.00 \text{ cm})} = +2.33$$

EVALUATE: The negative value of s' means that after the rays are refracted by the surface, they are not converging but *appear* to diverge from a point 21.3 cm to the *left* of the vertex. We saw a similar case in the reflection of light from a convex mirror; we call the point a *virtual image.* In this example the surface forms a virtual image 21.3 cm to the left of the vertex. The vertical image is erect (because m is positive) and 2.33 times as large as the object.

Example 34.7 Apparent depth of a swimming pool

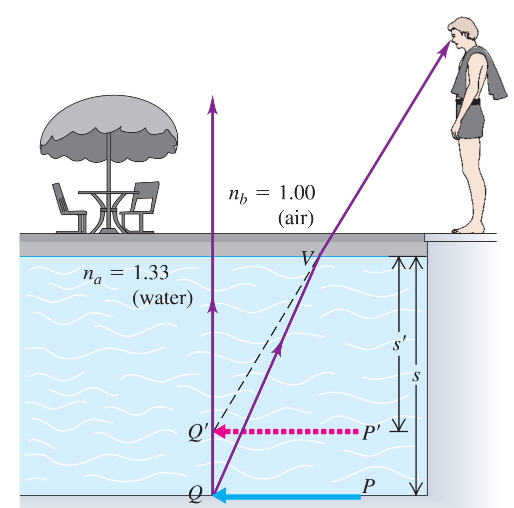
Swimming pool owners know that the pool always looks shallower than it really is and that it is important to identify the deep parts conspicuously so that people who can't swim won't jump into water that's over their heads. If a nonswimmer looks straight down into water that is actually 2.00 m (about 6 ft, 7 in.) deep, how deep does it appear to be?

SOLUTION

IDENTIFY: The surface of the water acts as a plane refracting surface.

SET UP: Figure 34.26 shows the situation. To determine the apparent depth of the pool, we imagine that there is an arrow PQ painted on the bottom of the pool. The refracting surface of the pool forms a virtual image $P'Q'$ of this arrow. We use Eq. (34.13) to find the depth of this arrow; it tells us the apparent depth of the pool.

34.26 Arrow $P'Q'$ is the virtual image of the underwater arrow PQ. The angles of the ray with the vertical are exaggerated for clarity.



EXECUTE: The object distance is the actual depth of the pool, $s = 2.00$ m. Material a is the water $(n_a = 1.33)$ and material b is air $(n_b = 1.00)$. The position of the image is given by Eq. (34.13):

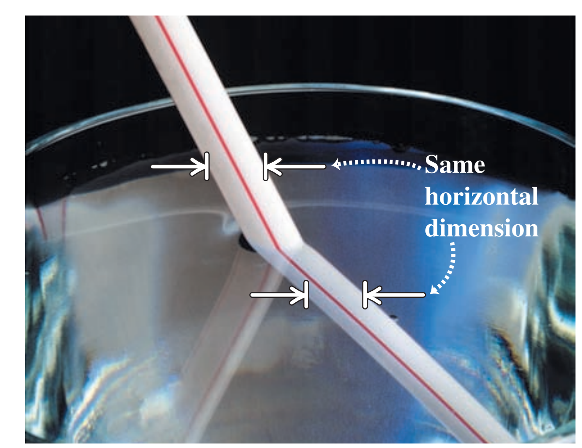
$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{1.33}{2.00 \text{ m}} + \frac{1.00}{s'} = 0$$

$$s' = -1.50 \text{ m}$$

The image distance is negative. From the sign rules in Section 34.1, this means that the image is virtual and on the incoming side of the refracting surface—that is, on the same side as the object. The apparent depth is 1.50 m (about 4 ft, 11 in.), or only three-quarters of the actual depth. A 6-ft nonswimmer who didn't allow for this effect would be in trouble.

EVALUATE: Recall that the lateral magnification for a plane refracting surface is $m = 1$. Hence the image $P'Q'$ of the arrow is the same *horizontal length* as the actual arrow PQ. Only its depth is different. You can see this effect in Fig. 34.27.

34.27 The submerged portion of this straw appears to be at a shallower depth (closer to the surface) than it actually is.



Test Your Understanding of Section 34.3 The water droplets in Fig. 34.23 have radius of curvature R and index of refraction $n = 1.33$. Can they form an image of the sun on the leaf?

34.4 Thin Lenses

ActivPhysics Online
15.9 Thin Lens Ray Diagram
15.10 Converging Thin Lenses
15.11 Diverging Thin Lenses

The most familiar and widely used optical device (after the plane mirror) is the *lens.* A lens is an optical system with two refracting surfaces. The simplest lens has two *spherical* surfaces close enough together that we can neglect the distance between them (the thickness of the lens); we call this a **thin lens.** If you wear eyeglasses or contact lenses while reading, you are viewing these words through a pair of thin lenses. We can analyze thin lenses in detail using the results of Section 34.3 for refraction by a single spherical surface. However, we postpone this analysis until later in the section so that we can first discuss the properties of thin lenses.

Properties of a Lens

A lens of the shape shown in Fig. 34.28 has the property that when a beam of rays parallel to the axis passes through the lens, the rays converge to a point F_2 (Fig. 34.28a) and form a real image at that point. Such a lens is called a **converging lens.** Similarly, rays passing through point F_1 emerge from the lens as a beam of parallel rays (Fig. 34.28b). The points F_1 and F_2 are called the first and second *focal points,* and the distance f (measured from the center of the lens) is called the *focal length.* Note the similarities between the two focal points of a converging lens and the single focal point of a concave mirror (Fig. 34.13). As for a concave mirror, the focal length of a converging lens is defined to be a *positive* quantity, and such a lens is also called a *positive lens.*

The central horizontal line in Fig. 34.28 is called the *optic axis,* as with spherical mirrors. The centers of curvature of the two spherical surfaces lie on and define the optic axis. The two focal lengths in Fig. 34.28, both labeled f, *are always equal* for a thin lens, even when the two sides have different curvatures. We will derive this somewhat surprising result later in the section, when we derive the relationship of f to the index of refraction of the lens and the radii of curvature of its surfaces.

34.28 F_1 and F_2 are the first and second focal points of a converging thin lens. The numerical value of f is positive.

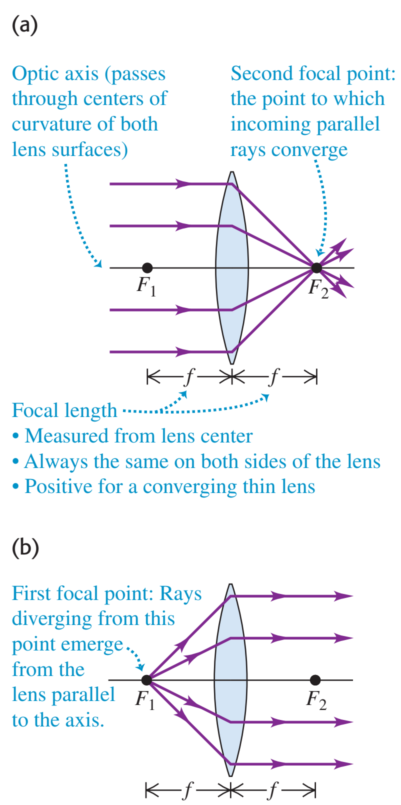
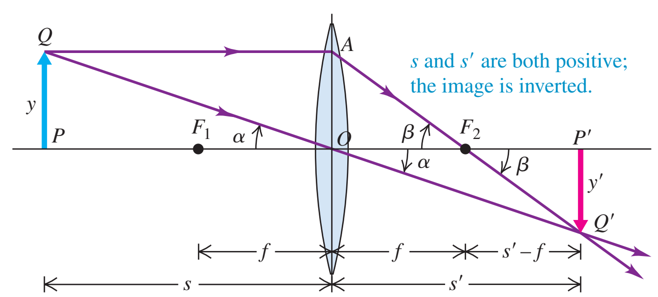


Image of an Extended Object: Converging Lens

Like a concave mirror, a converging lens can form an image of an extended object. Figure 34.29 shows how to find the position and lateral magnification of an image made by a thin converging lens. Using the same notation and sign rules as before, we let s and s' be the object and image distances, respectively, and let y and y' be the object and image heights. Ray QA, parallel to the optic axis before refraction, passes through the second focal point F_2 after refraction. Ray QOQ' passes undeflected straight through the center of the lens because at the center the two surfaces are parallel and (we have assumed) very close together.

34.29 Construction used to find image position for a thin lens. To emphasize that the lens is assumed to be very thin, the ray QAQ' is shown as bent at the midplane of the lens rather than at the two surfaces and ray QOQ' is shown as a straight line.



There is refraction where the ray enters and leaves the material but no net change in direction.

The two angles labeled α in Fig. 34.29 are equal. Therefore the two right triangles PQO and $P'Q'O$ are *similar,* and ratios of corresponding sides are equal. Thus

$$\frac{y}{s} = -\frac{y'}{s'} \quad \text{or} \quad \frac{y'}{y} = -\frac{s'}{s} \qquad (34.14)$$

(The reason for the negative sign is that the image is below the optic axis and y' is negative.) Also, the two angles labeled β are equal, and the two right triangles OAF_2 and $P'Q'F_2$ are similar, so

$$\frac{y}{f} = -\frac{y'}{s' - f} \quad \text{or}$$

$$\frac{y'}{y} = -\frac{s' - f}{f} \qquad (34.15)$$

We now equate Eqs. (34.14) and (34.15), divide by s', and rearrange to obtain

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{(object–image relationship, thin lens)} \qquad (34.16)$$

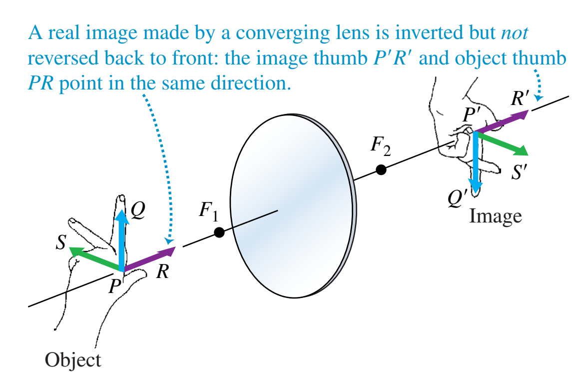
This analysis also gives the lateral magnification $m = y'/y$ for the lens; from Eq. (34.14),

$$m = -\frac{s'}{s} \quad \text{(lateral magnification, thin lens)} \qquad (34.17)$$

The negative sign tells us that when s and s' are both positive, as in Fig. 34.29, the image is *inverted,* and y and y' have opposite signs.

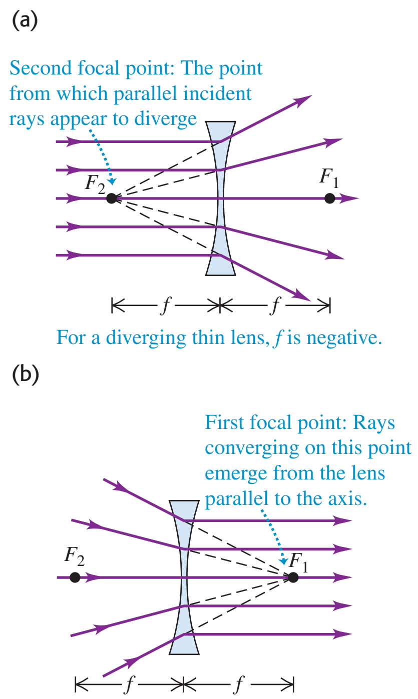
Equations (34.16) and (34.17) are the basic equations for thin lenses. They are *exactly* the same as the corresponding equations for spherical mirrors, Eqs. (34.6) and (34.7). As we will see, the same sign rules that we used for spherical mirrors are also applicable to lenses. In particular, consider a lens with a positive focal length (a converging lens). When an object is outside the first focal point F_1 of this lens (that is, when $s > f$), the image distance s' is positive (that is, the image is on the same side as the outgoing rays); this image is real and inverted, as in Fig. 34.29. An object placed inside the first focal point of a converging lens, so that $s < f$, produces an image with a negative value of s'; this image is located on the same side of the lens as the object and is virtual, erect, and larger than the object. You can verify these statements algebraically using Eqs. (34.16) and (34.17); we'll also verify them in the next section, using graphical methods analogous to those introduced for mirrors in Section 34.2.

Figure 34.30 shows how a lens forms a three-dimensional image of a three-dimensional object. Point R is nearer the lens than point P. From Eq. (34.16),



34.30 The image $S'P'Q'R'$ of a three-dimensional object $SPQR$ is not reversed by a lens.

34.31 F_2 and F_1 are the second and first focal points of a diverging thin lens, respectively. The numerical value of f is negative.



34.32 Various types of lenses.

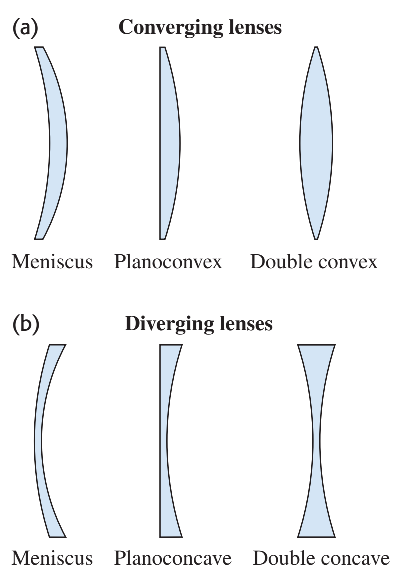


image point R' is farther from the lens than is image point P', and the image $P'R'$ points in the same direction as the object PR. Arrows $P'S'$ and $P'Q'$ are reversed relative to PS and PQ.

Let's compare Fig. 34.30 with Fig. 34.7, which shows the image formed by a plane *mirror.* We note that the image formed by the lens is inverted, but it is *not* reversed front to back along the optic axis. That is, if the object is a left hand, its image is also a left hand. You can verify this by pointing your left thumb along PR, your left forefinger along PQ, and your left middle finger along PS. Then rotate your hand 180°, using your thumb as an axis; this brings the fingers into coincidence with $P'Q'$ and $P'S'$. In other words, an *inverted* image is equivalent to an image that has been rotated by 180° about the lens axis.

Diverging Lenses

So far we have been discussing *converging* lenses. Figure 34.31 shows a **diverging lens;** the beam of parallel rays incident on this lens *diverges* after refraction. The focal length of a diverging lens is a negative quantity, and the lens is also called a *negative lens.* The focal points of a negative lens are reversed, relative to those of a positive lens. The second focal point, F_2, of a negative lens is the point from which rays that are originally parallel to the axis *appear to diverge* after refraction, as in Fig. 34.31a. Incident rays converging toward the first focal point F_1, as in Fig. 34.31b, emerge from the lens parallel to its axis. Comparing with Section 34.2, you can see that a diverging lens has the same relationship to a converging lens as a convex mirror has to a concave mirror.

Equations (34.16) and (34.17) apply to *both* positive and negative lenses. Figure 34.32 shows various types of lenses, both converging and diverging. Here's an important observation: *Any lens that is thicker at its center than at its edges is a converging lens with positive f; and any lens that is thicker at its edges than at its center is a diverging lens with negative f* (provided that the lens has a greater index of refraction than the surrounding material). We can prove this using the *lensmaker's equation,* which it is our next task to derive.

The Lensmaker's Equation

We'll now derive Eq. (34.16) in more detail and at the same time derive the *lensmaker's equation,* which is a relationship among the focal length f, the index of refraction n of the lens, and the radii of curvature R_1 and R_2 of the lens surfaces. We use the principle that an image formed by one reflecting or refracting surface can serve as the object for a second reflecting or refracting surface.

We begin with the somewhat more general problem of two spherical interfaces separating three materials with indexes of refraction n_a, n_b, and n_c, as shown in Fig. 34.33. The object and image distances for the first surface are s_1 and s_1', and those for the second surface are s_2 and s_2'. We assume that the lens is thin, so that the distance t between the two surfaces is small in comparison with the object and

34.33 The image formed by the first surface of a lens serves as the object for the second surface. The distances s_1' and s_2 are taken to be equal; this is a good approximation if the lens thickness t is small.

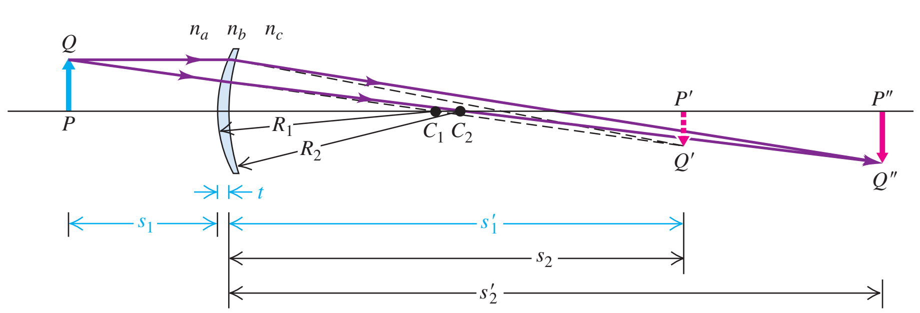


image distances and can therefore be neglected. This is usually the case with eyeglass lenses (Fig. 34.34). Then s_2 and s_1' have the same magnitude but opposite sign. For example, if the first image is on the outgoing side of the first surface, s_1' is positive. But when viewed as an object for the second surface, the first image is *not* on the incoming side of that surface. So we can say that $s_2 = -s_1'$.

We need to use the single-surface equation, Eq. (34.11), twice, once for each surface. The two resulting equations are

$$\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}$$

$$\frac{n_b}{s_2} + \frac{n_c}{s_2'} = \frac{n_c - n_b}{R_2}$$

Ordinarily, the first and third materials are air or vacuum, so we set $n_a = n_c = 1$. The second index n_b is that of the lens, which we can call simply n. Substituting these values and the relationship $s_2 = -s_1'$, we get

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1}$$

$$-\frac{n}{s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$

To get a relationship between the initial object position s_1 and the final image position s_2', we add these two equations. This eliminates the term n/s_1', and we obtain

$$\frac{1}{s_1} + \frac{1}{s_2'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Finally, thinking of the lens as a single unit, we call the object distance simply s instead of s_1, and we call the final image distance s' instead of s_2'. Making these substitutions, we have

$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (34.18)$$

Now we compare this with the other thin-lens equation, Eq. (34.16). We see that the object and image distances s and s' appear in exactly the same places in both equations and that the focal length f is given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{(lensmaker's equation for a thin lens)} \qquad (34.19)$$

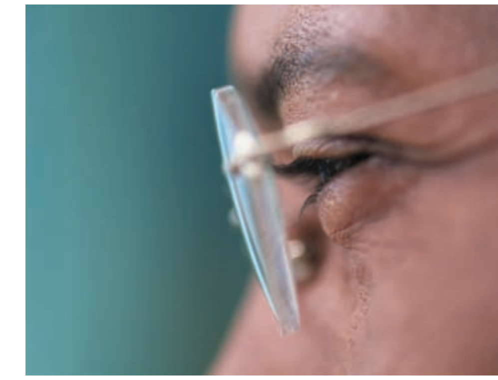
This is the **lensmaker's equation.** In the process of rederiving the relationship between object distance, image distance, and focal length for a thin lens, we have also derived an expression for the focal length f of a lens in terms of its index of refraction n and the radii of curvature R_1 and R_2 of its surfaces. This can be used to show that all the lenses in Fig. 34.32a are converging lenses with positive focal lengths and that all the lenses in Fig. 34.32b are diverging lenses with negative focal lengths (see Exercise 34.30).

We use all our sign rules from Section 34.1 with Eqs. (34.18) and (34.19). For example, in Fig. 34.35, s, s', and R_1 are positive, but R_2 is negative.

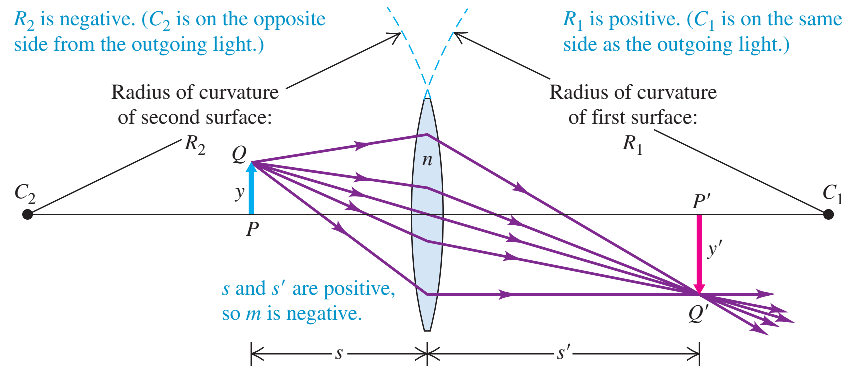
It is not hard to generalize Eq. (34.19) to the situation in which the lens is immersed in a material with an index of refraction greater than unity. We invite you to work out the lensmaker's equation for this more general situation.

We stress that the paraxial approximation is indeed an approximation! Rays that are at sufficiently large angles to the optic axis of a spherical lens will not be brought to the same focus as paraxial rays; this is the same problem of spherical

34.34 These eyeglass lenses satisfy the thin-lens approximation: Their thickness is small compared to the object and image distances.



34.35 A converging thin lens with a positive focal length f.



aberration that plagues spherical *mirrors* (see Section 34.2). To avoid this and other limitations of thin spherical lenses, lenses of more complicated shape are used in precision optical instruments.

Example 34.8 Determining the focal length of a lens

(a) Suppose the absolute values of the radii of curvature of the lens surfaces in Fig. 34.35 are both equal to 10 cm and the index of refraction is $n = 1.52$. What is the focal length f of the lens? (b) Suppose the lens in Fig. 34.31 also has $n = 1.52$, and the absolute values of the radii of curvature of its lens surfaces are also both equal to 10 cm. What is the focal length of this lens?

SOLUTION

IDENTIFY: We are asked to find the focal length of (a) a lens that is convex on both sides (Fig. 34.35) and (b) a lens that is concave on both sides (Fig. 34.31).

SET UP: We use the lensmaker's equation, Eq. (34.19), to determine the focal length in each situation. We take account of whether the surfaces are convex or concave by paying careful attention to the signs of the radii of curvature R_1 and R_2.

EXECUTE: (a) Figure 34.35 shows that the center of curvature of the first surface (C_1) is on the outgoing side of the lens, while the center of curvature of the second surface (C_2) is on the *incoming* side. Hence R_1 is positive but R_2 is negative: $R_1 = +10$ cm, $R_2 = -10$ cm. From Eq. (34.19),

$$\frac{1}{f} = (1.52 - 1)\left(\frac{1}{+10\text{ cm}} - \frac{1}{-10\text{ cm}}\right)$$

$$f = 9.6\text{ cm}$$

(b) For a double-concave lens the center of curvature of the first surface is on the *incoming* side, while the center of curvature of the second surface is on the outgoing side. Hence R_1 is negative and R_2 is positive: $R_1 = -10$ cm, $R_2 = +10$ cm. Again using Eq. (34.19),

$$\frac{1}{f} = (1.52 - 1)\left(\frac{1}{-10\text{ cm}} - \frac{1}{+10\text{ cm}}\right)$$

$$f = -9.6\text{ cm}$$

EVALUATE: In part (a) the focal length is positive, so this is a converging lens; this makes sense, since the lens is thicker at its center than at its edges. In part (b) the focal length is *negative,* so this is a *diverging* lens; this also makes sense, since the lens is thicker at its edges than at its center.

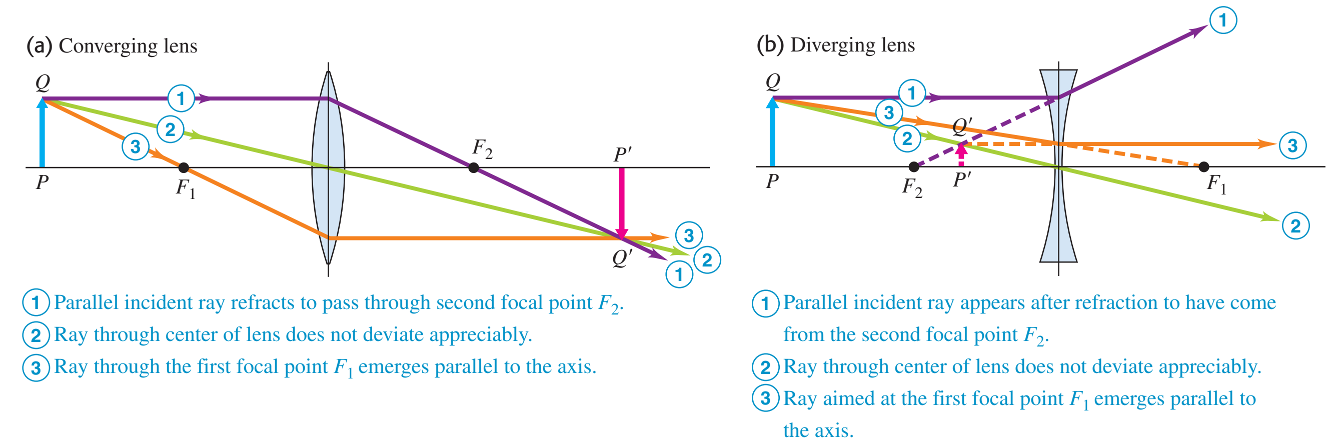
Graphical Methods for Lenses

We can determine the position and size of an image formed by a thin lens by using a graphical method very similar to the one we used in Section 34.2 for spherical mirrors. Again we draw a few special rays called *principal rays* that diverge from a point of the object that is *not* on the optic axis. The intersection of these rays, after they pass through the lens, determines the position and size of the image. In using this graphical method, we will consider the entire deviation of a ray as occurring at the midplane of the lens, as shown in Fig. 34.36. This is consistent with the assumption that the distance between the lens surfaces is negligible.

The three principal rays whose paths are usually easy to trace for lenses are shown in Fig. 34.36:

1. *A ray parallel to the axis* emerges from the lens in a direction that passes through the second focal point F_2 of a converging lens, or appears to come from the second focal point of a diverging lens.
2. *A ray through the center of the lens* is not appreciably deviated; at the center of the lens the two surfaces are parallel, so this ray emerges at essentially the same angle at which it enters and along essentially the same line.

34.36 The graphical method of locating an image formed by a thin lens. The colors of the rays are for identification only; they do not refer to specific colors of light. (Compare Fig. 34.19 for spherical mirrors.)



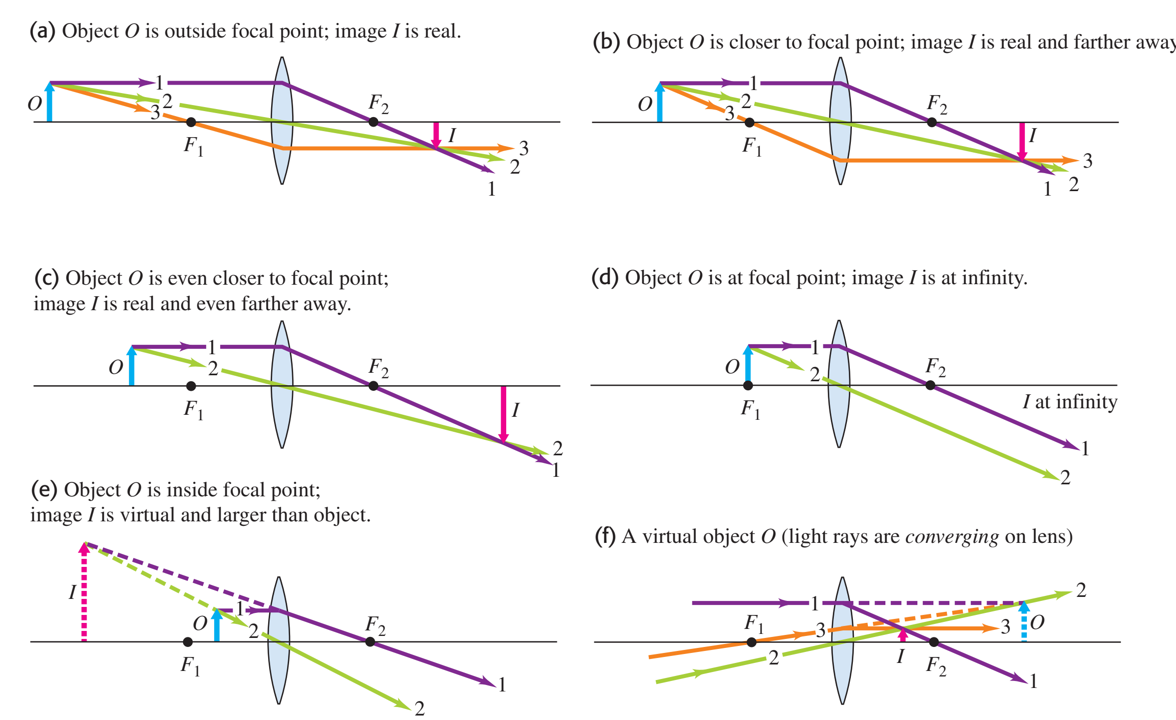
3. *A ray through (or proceeding toward) the first focal point* F_1 emerges parallel to the axis.

When the image is real, the position of the image point is determined by the intersection of any two rays 1, 2, and 3 (Fig. 34.36a). When the image is virtual, we extend the diverging outgoing rays backward to their intersection point to find the image point (Fig. 34.36b).

CAUTION Principal rays are not the only rays Keep in mind that *any* ray from the object that strikes the lens will pass through the image point (for a real image) or appear to originate from the image point (for a virtual image). (We made a similar comment about image formation by mirrors in Section 34.2.) We've emphasized the principal rays because they're the only ones you need to draw to locate the image.

Figure 34.37 shows principal-ray diagrams for a converging lens for several object distances. We suggest you study each of these diagrams very carefully, comparing each numbered ray with the above description.

34.37 Formation of images by a thin converging lens for various object distances. The principal rays are numbered. (Compare Fig. 34.20 for a concave spherical mirror.)



Parts (a), (b), and (c) of Fig. 34.37 help explain what happens in focusing a camera. For a photograph to be in sharp focus, the film must be at the position of the real image made by the camera's lens. The image distance increases as the object is brought closer, so the film is moved farther behind the lens (i.e., the lens is moved farther in front of the film). In Fig. 34.37d the object is at the focal point; ray 3 can't be drawn because it doesn't pass through the lens. In Fig. 34.37e the object distance is less than the focal length. The outgoing rays are divergent, and the image is *virtual;* its position is located by extending the outgoing rays backward, so the image distance s' is negative. Note also that the image is erect and larger than the object. (We'll see the usefulness of this in Section 34.6.) Figure 34.37f corresponds to a *virtual object.* The incoming rays do not diverge from a real object, but are *converging* as though they would meet at the tip of the virtual object O on the right side; the object distance s is negative in this case. The image is real and is located between the lens and the second focal point. This situation can arise if the rays that strike the lens in Fig. 34.37f emerge from another converging lens (not shown) to the left of the figure.

Problem-Solving Strategy 34.2 Image Formation by Thin Lenses

MP

IDENTIFY *the relevant concepts:* Problem-Solving Strategy 34.1 (Section 34.2) for mirrors is equally applicable here, and you should review it now. As for mirrors, you should solve problems involving image formation by lenses using *both* equations and a principal-ray diagram.

SET UP *the problem:* As always, determine the target variables.

EXECUTE *the solution* as follows:

1. Always begin with a principal-ray diagram if you have enough information. Orient your diagrams consistently so that light travels from left to right. Don't just sketch these diagrams; draw the rays with a ruler and measure the distances carefully.
2. Draw the principal rays so they bend at the midplane of the lens, as shown in Fig. 34.36. For a lens there are only three principal rays, compared to four for a mirror. Be sure to draw *all three* whenever possible. The intersection of any two determines the image, but if the third doesn't pass through the same intersection point, you know you have made a mistake. Redundancy can be useful in spotting errors.
3. If the outgoing principal rays don't converge at a real image point, the image is virtual. Then you have to extend the outgoing rays backward to find the virtual image point, which lies on the *incoming* side of the lens.
4. The same sign rules we have used for mirrors and single refracting surfaces (see Section 34.1) are also applicable for thin lenses. Be extremely careful to get your signs right and to interpret the signs of results correctly.
5. Use Eqs. (34.16) and (34.17) to confirm by calculation your graphical results for the image position and size. This gives an extremely useful consistency check.
6. The *image* from one lens or mirror may serve as the *object* for another. In that case, be careful in finding the object and image *distances* for this intermediate image; be sure you include the distance between the two elements (lenses and/or mirrors) correctly.

EVALUATE *your answer:* Cast a critical eye on your diagrams and calculations to make certain that your results are consistent.

Example 34.9 Image location and magnification with a converging lens

A converging lens has a focal length of 20 cm. Find graphically the image location for an object at each of the following distances from the lens: (a) 50 cm; (b) 20 cm; (c) 15 cm; (d) −40 cm. Determine the magnification in each case. Check your results by calculating the image position and lateral magnification from Eqs. (34.16) and (34.17), respectively.

SOLUTION

IDENTIFY: This problem illustrates the usefulness of both graphical and computational methods for problems with thin lenses, just as for problems with curved mirrors.

SET UP: In each case we are given the focal length $f = 20$ cm and the value of the object distance s. Our target variables are the image distance s' and the lateral magnification $m = -s'/s$.

EXECUTE: The appropriate principal-ray diagrams are shown in (a) Fig. 34.37a, (b) Fig. 34.37d, (c) Fig. 34.37e, and (d) Fig. 34.37f. The approximate image distances, from measurements of these diagrams, are 35 cm, $-\infty$, −40 cm, and 15 cm, and the approximate magnifications are $-\frac{2}{3}$, $+\infty$, and +3, and $+\frac{1}{3}$, respectively.

Calculating the image positions from Eq. (34.16), we find

(a) $\dfrac{1}{50\text{ cm}} + \dfrac{1}{s'} = \dfrac{1}{20\text{ cm}} \qquad s' = 33.3\text{ cm}$

(b) $\dfrac{1}{20\text{ cm}} + \dfrac{1}{s'} = \dfrac{1}{20\text{ cm}} \qquad s' = \pm\infty$

(c) $\dfrac{1}{15\text{ cm}} + \dfrac{1}{s'} = \dfrac{1}{20\text{ cm}} \qquad s' = -60\text{ cm}$

(d) $\dfrac{1}{-40\text{ cm}} + \dfrac{1}{s'} = \dfrac{1}{20\text{ cm}} \qquad s' = 13.3\text{ cm}$

The graphical results are fairly close to these except for part (c); the accuracy of the diagram in Fig. 34.37e is limited because the rays extended backward have nearly the same direction.

From Eq. (34.17) the lateral magnifications are

(a) $m = -\dfrac{33.3\text{ cm}}{50\text{ cm}} = -\dfrac{2}{3}$

(b) $m = -\dfrac{\pm\infty\text{ cm}}{20\text{ cm}} = \pm\infty$

(c) $m = -\dfrac{-60\text{ cm}}{15\text{ cm}} = +4$

(d) $m = -\dfrac{13.3\text{ cm}}{-40\text{ cm}} = +\dfrac{1}{3}$

EVALUATE: Note that s' is positive in parts (a) and (d) but negative in part (c). This makes sense: The image is real in parts (a) and (d) but virtual in part (c). The light rays that emerge from the lens in part (b) are parallel and never converge, so the image can be regarded as being at either $+\infty$ or $-\infty$.

The values of magnification tell us that the image is inverted in part (a) and erect in parts (c) and (d), in accordance with the principal-ray diagrams. The infinite value of magnification in part (b) is another way of saying that the image is formed infinitely far away.

Example 34.10 Image formation by a diverging lens

You are given a thin diverging lens. You find that a beam of parallel rays spreads out after passing through the lens, as though all the rays came from a point 20.0 cm from the center of the lens. You want to use this lens to form an erect virtual image that is $\frac{1}{3}$ the height of the object. (a) Where should the object be placed? (b) Draw a principal-ray diagram.

SOLUTION

IDENTIFY: The observation with parallel rays shows that the focal length is $f = -20$ cm. We want the lateral magnification to be $m = +\frac{1}{3}$ (positive because the image is to be erect).

SET UP: We use the given information to determine the ratio s'/s from Eq. (34.17) and then determine the object distance s with Eq. (34.16).

EXECUTE: (a) From Eq. (34.17), $m = +\frac{1}{3} = -s'/s$, so $s' = -s/3$. If we insert this result into Eq. (34.16), we find

$$\frac{1}{s} + \frac{1}{-s/3} = \frac{1}{-20.0\text{ cm}}$$

$$s = 40.0\text{ cm}$$

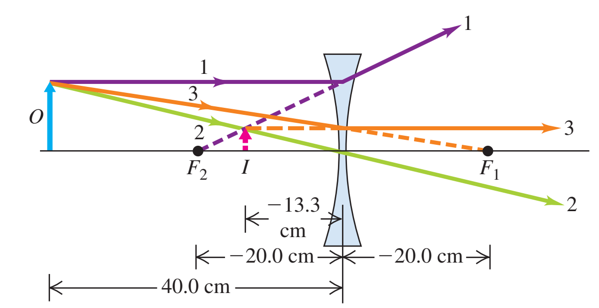
$$s' = -\frac{s}{3} = -\frac{40.0\text{ cm}}{3} = -13.3\text{ cm}$$

The image distance is negative, so the object and image are on the same side of the lens.

(b) Figure 34.38 is a principal-ray diagram for this problem, with the rays numbered in the same way as in Fig. 34.36b.

EVALUATE: A diverging lens is often mounted in the front door of a home. It provides the occupant of the home with an erect, reduced image of anyone standing outside the door. The occupant can see the outside person's entire height and decide whether to let him or her in.

34.38 Principal-ray diagram for an image formed by a thin diverging lens.



Example 34.11 An image of an image

An object 8.0 cm high is placed 12.0 cm to the left of a converging lens of focal length 8.0 cm. A second converging lens of focal length 6.0 cm is placed 36.0 cm to the right of the first lens. Both lenses have the same optic axis. Find the position, size, and orientation of the image produced by the two lenses in combination. (Combinations of converging lenses are used in telescopes and microscopes, to be discussed in Section 34.7.)

SOLUTION

IDENTIFY: The situation is shown in Fig. 34.39. The object O lies outside the first focal point F_1 of the first lens, so this lens produces a real image I. The light rays that strike the second lens diverge from this real image just as if I was a material object. Hence the *image* made by the *first* lens acts as an *object* for the *second* lens. Our goal is to determine the properties of the final image made by the second lens.

SET UP: We use both graphical and computational methods to determine the properties of the final image.

EXECUTE: In Fig. 34.39 (next page) we have drawn principal rays 1, 2, and 3 from the head of the object arrow O to find the position of the first image I and principal rays 1′, 2′, and 3′ from the head of the first image to find the position of the second image I' made by the second lens (even though rays 2′ and 3′ don't actually exist in this case). Note that the image is inverted *twice,* once by each lens, so the second image I' has the same orientation as the original object.

Continued

To *calculate* the position and size of the second image I', we must first find the position and size of the first image I. Applying Eq. (34.16), $1/s + 1/s' = 1/f$, to the first lens gives

$$\frac{1}{12.0\text{ cm}} + \frac{1}{s_1'} = \frac{1}{8.0\text{ cm}} \qquad s_1' = +24.0\text{ cm}$$

The first image I is 24.0 cm to the right of the first lens. The lateral magnification is $m_1 = -(24.0\text{ cm})/(12.0\text{ cm}) = -2.00$, so the height of the first image is $(-2.0)(8.0\text{ cm}) = -16.0\text{ cm}$.

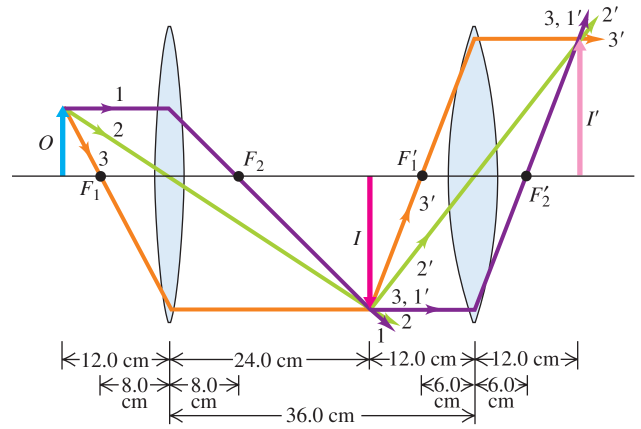
The first image is 36.0 cm − 24.0 cm = 12.0 cm to the left of the second lens, so the object distance for the second lens is +12.0 cm. Using Eq. (34.16) for the second lens gives the position of the second and final image:

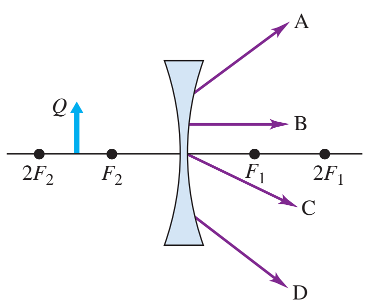
$$\frac{1}{12.0\text{ cm}} + \frac{1}{s_2'} = \frac{1}{6.0\text{ cm}} \qquad s_2' = +12.0\text{ cm}$$

The final image is 12.0 cm to the right of the second lens and 48.0 cm to the right of the first lens. The magnification produced by the second converging lens is $m_2 = -(12.0\text{ cm})/(12.0\text{ cm}) = -1.0$.

EVALUATE: The value of m_2 means that the final image is just as large as the first image but has the opposite orientation. This is also shown in the principal-ray diagram.

34.39 Principal-ray diagram for a combination of two converging lenses. The first lens makes a real image of the object. This real image acts as an object for the second lens.





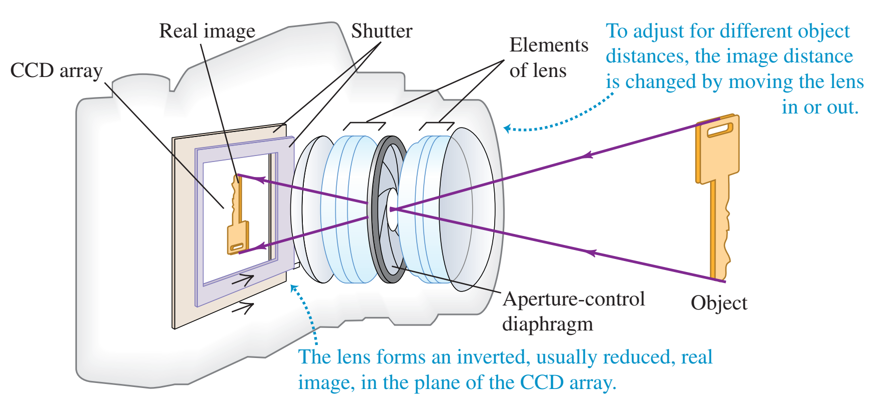
Test Your Understanding of Section 34.4 A diverging lens and an object are positioned as shown in the figure at left. Which of the rays A, B, C, and D could emanate from point Q at the top of the object?

34.5 Cameras

The concept of *image,* which is so central to understanding simple mirror and lens systems, plays an equally important role in the analysis of optical instruments (also called *optical devices*). Among the most common optical devices are cameras, which make an image of an object and record it either electronically or on film.

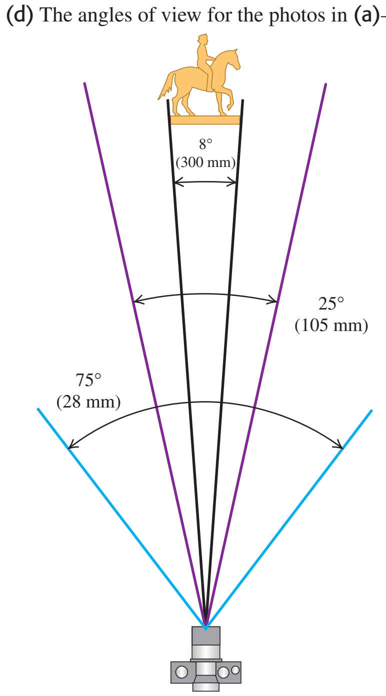
The basic elements of a **camera** are a light-tight box ("camera" is a Latin word meaning "a room or enclosure"), a converging lens, a shutter to open the lens for a prescribed length of time, and a light-sensitive recording medium (Fig. 34.40). In a digital camera this is an electronic detector called a charge-coupled device (CCD) array; in an older camera, this is photographic film. The lens forms an inverted real image on the recording medium of the object being photographed. High-quality camera lenses have several elements, permitting partial correction of various *aberrations,* including the dependence of index of refraction on wavelength and the limitations imposed by the paraxial approximation.

34.40 Key elements of a digital camera.



34.41 (a), (b), (c) Three photographs taken with the same camera from the same position in the Boston Public Garden using lenses with focal lengths f = 28 mm, 105 mm, and 300 mm. Increasing the focal length increases the image size proportionately. (d) The larger the value of f, the smaller the angle of view. The angles shown here are for a camera with image area 24 mm × 36 mm (corresponding to 35-mm film) and refer to the angle of view along the diagonal dimension of the film.

(d) The angles of view for the photos in (a)–(c)



When the camera is in proper *focus,* the position of the recording medium coincides with the position of the real image formed by the lens. The resulting photograph will then be as sharp as possible. With a converging lens, the image distance increases as the object distance decreases (see Figs. 34.41a, 34.41b, and 34.41c, and the discussion in Section 34.4). Hence in "focusing" the camera, we move the lens closer to the film for a distant object and farther from the film for a nearby object.

Camera Lenses: Focal Length

The choice of the focal length f for a camera lens depends on the film size and the desired angle of view. Figure 34.41 shows three photographs taken on 35-mm film with the same camera at the same position, but with lenses of different focal lengths. A lens of long focal length, called a *telephoto* lens, gives a small angle of view and a large image of a distant object (such as the statue in Fig. 34.41c); a lens of short focal length gives a small image and a wide angle of view (as in Fig. 34.41a) and is called a *wide-angle* lens. To understand this behavior, recall that the focal length is the distance from the lens to the image when the object is infinitely far away. In general, for *any* object distance, using a lens of longer focal length gives a greater image distance. This also increases the height of the image; as was discussed in Section 34.4, the ratio of the image height y' to the object height y (the *lateral magnification*) is equal in absolute value to the ratio of image distance s' to the object distance s [Eq. (34.17)]:

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

With a lens of short focal length, the ratio s'/s is small, and a distant object gives only a small image. When a lens with a long focal length is used, the image of this same object may entirely cover the area of the film. Hence the longer the focal length, the narrower the angle of view (Fig. 34.41d).

Camera Lenses: *f*-Number

For the film to record the image properly, the total light energy per unit area reaching the film (the "exposure") must fall within certain limits. This is controlled by the *shutter* and the *lens aperture.* The shutter controls the time interval during which light enters the lens. This is usually adjustable in steps corresponding to factors of about 2, often from 1 s to $\frac{1}{1000}$ s.

The intensity of light reaching the film is proportional to the area viewed by the camera lens and to the effective area of the lens. The size of the area that the lens "sees" is proportional to the square of the angle of view of the lens, and so is roughly proportional to $1/f^2$. The effective area of the lens is controlled by means of an adjustable lens aperture, or *diaphragm,* a nearly circular hole with variable diameter D; hence the effective area is proportional to D^2. Putting these factors together, we see that the intensity of light reaching the film with a particular lens is proportional to D^2/f^2. The light-gathering capability of a lens is commonly expressed by photographers in terms of the ratio f/D, called the ***f*-number** of the lens:

$$f\text{-number} = \frac{\text{Focal length}}{\text{Aperture diameter}} = \frac{f}{D} \qquad (34.20)$$

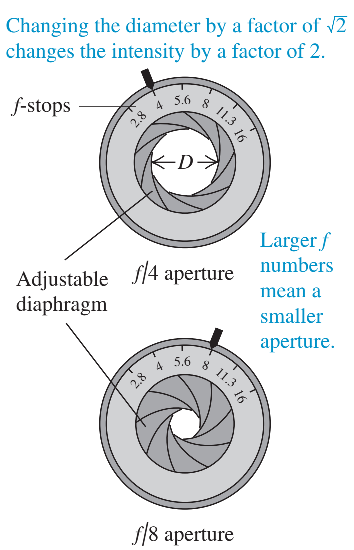
For example, a lens with a focal length $f = 50$ mm and an aperture diameter $D = 25$ mm is said to have an f-number of 2, or "an aperture of $f/2$." The light intensity reaching the film is *inversely* proportional to the square of the f-number.

For a lens with a variable-diameter aperture, increasing the diameter by a factor of $\sqrt{2}$ changes the f-number by $1/\sqrt{2}$ and increases the intensity at the film by a factor of 2. Adjustable apertures usually have scales labeled with successive numbers (often called *f-stops*) related by factors of $\sqrt{2}$, such as

$$f/2 \quad f/2.8 \quad f/4 \quad f/5.6 \quad f/8 \quad f/11 \quad f/16$$

and so on. The larger numbers represent smaller apertures and exposures, and each step corresponds to a factor of 2 in intensity (Fig. 34.42). The actual *exposure* (total amount of light reaching the film) is proportional to both the aperture area and the time of exposure. Thus $f/4$ and $\frac{1}{500}$ s, $f/5.6$ and $\frac{1}{250}$ s, and $f/8$ and $\frac{1}{125}$ s all correspond to the same exposure.

34.42 A camera lens with an adjustable diaphragm.

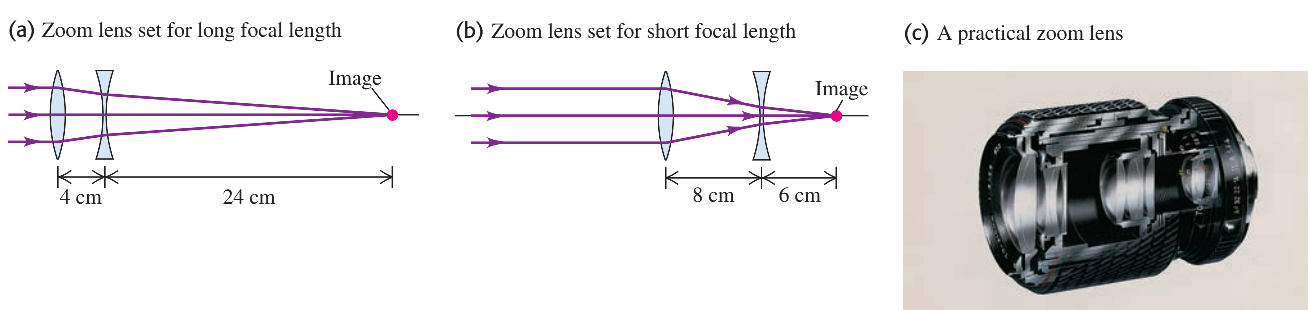


Zoom Lenses and Projectors

Many photographers use a *zoom lens,* which is not a single lens but a complex collection of several lens elements that give a continuously variable focal length, often over a range as great as 10 to 1. Figures 34.43a and 34.43b show a simple system with variable focal length, and Fig. 34.43c shows a typical zoom lens for a single-lens reflex camera. Zoom lenses give a range of image sizes of a given object. It is an enormously complex problem in optical design to keep the image in focus and maintain a constant f-number while the focal length changes. When you vary the focal length of a typical zoom lens, two groups of elements move within the lens and a diaphragm opens and closes.

A *projector* for viewing slides, digital images, or motion pictures operates very much like a camera in reverse. In a movie projector, a lamp shines on the film, which acts as an object for the projection lens. The lens forms a real, enlarged, inverted image of the film on the projection screen. Because the image is inverted, the film goes through the projector upside down so that the image on the screen appears right-side up.

34.43 A simple zoom lens uses a converging lens and a diverging lens in tandem. (a) When the two lenses are close together, the combination behaves like a single lens of long focal length. (b) If the two lenses are moved farther apart, the combination behaves like a short-focal-length lens. (c) A typical zoom lens for a single-lens reflex camera, containing twelve elements arranged in four groups.



Example 34.12 Photographic exposures

A common telephoto lens for a 35-mm camera has a focal length of 200 mm and a range of f-stops from $f/5.6$ to $f/45$. (a) What is the corresponding range of aperture diameters? (b) What is the corresponding range of intensity of the image on the film?

SOLUTION

IDENTIFY: Part (a) of this problem uses the relationship among focal length, aperture diameter, and f-number for a lens. Part (b) uses the relationship between intensity and aperture diameter.

SET UP: We use Eq. (34.20) to relate the diameter D (the target variable) to the f-number and the focal length $f = 200$ mm. The intensity of the light reaching the film is proportional to D^2/f^2; since f is the same in each case, we conclude that the intensity in this case is proportional to D^2, the square of the aperture diameter.

EXECUTE: (a) From Eq. (34.20) the range of diameters is from

$$D = \frac{f}{f\text{-number}} = \frac{200\text{ mm}}{5.6} = 36\text{ mm}$$

to

$$D = \frac{200\text{ mm}}{45} = 4.4\text{ mm}$$

(b) Because the intensity is proportional to the square of the diameter, the ratio of the intensity at $f/5.6$ to the intensity at $f/45$ is

$$\left(\frac{36\text{ mm}}{4.4\text{ mm}}\right)^2 = \left(\frac{45}{5.6}\right)^2 = 65 \qquad (\text{about } 2^6)$$

EVALUATE: If the correct exposure time at $f/5.6$ is $\frac{1}{1000}$ s, then at $f/45$ it is $(65)\left(\frac{1}{1000}\text{ s}\right) = \frac{1}{15}$ s to compensate for the lower intensity. This illustrates a general rule: The smaller the aperture and the larger the f-number, the longer the required exposure time. Nevertheless, many photographers prefer to use small apertures so that only the central part of the lens is used to make the image. This minimizes aberrations that occur near the edges of the lens and gives the sharpest possible images.

Test Your Understanding of Section 34.5 When used with 35-mm film (image area 24 mm × 36 mm), a lens with $f = 50$ mm gives a 45° angle of view and is called a "normal lens." When used with a CCD array that measures 5 mm × 5 mm, this same lens is (i) a wide-angle lens; (ii) a normal lens; (iii) a telephoto lens.

34.6 The Eye

The optical behavior of the eye is similar to that of a camera. The essential parts of the human eye, considered as an optical system, are shown in Fig. 34.44a. The eye is nearly spherical and about 2.5 cm in diameter. The front portion is somewhat more sharply curved and is covered by a tough, transparent membrane called the

34.44 (a) The eye. (b) There are two types of light-sensitive cells on the retina. The rods are more sensitive to light than the cones, but only the cones are sensitive to differences in color. A typical human eye contains about 1.3×10^8 rods and about 7×10^6 cones.

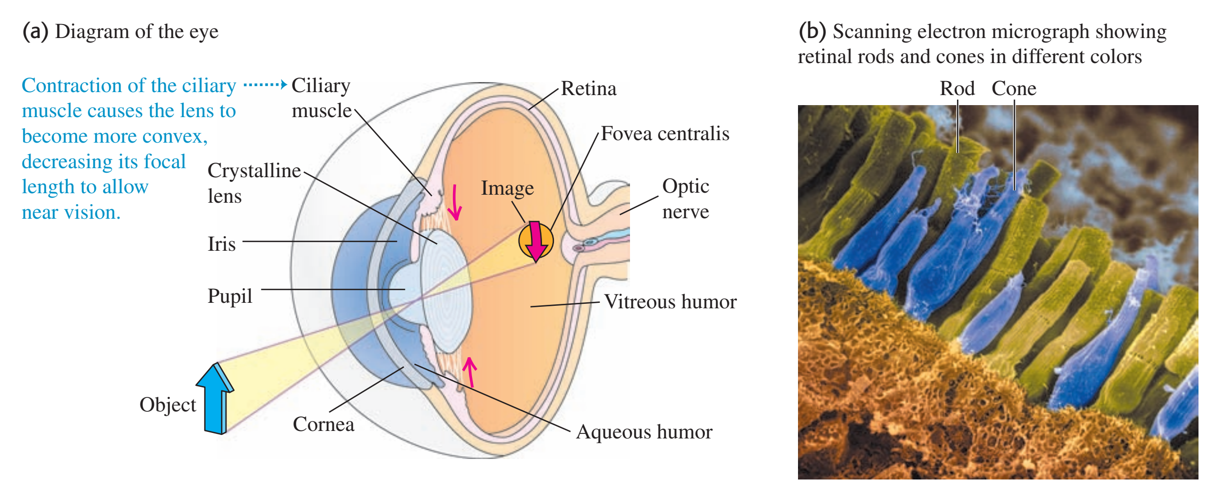
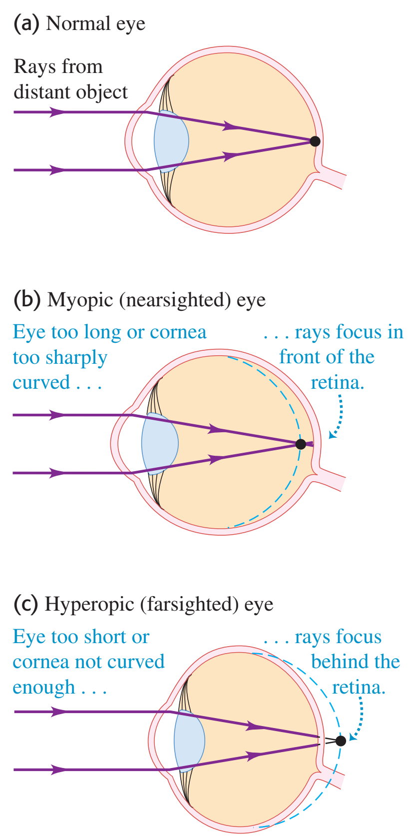


Table 34.1 Receding of Near Point with Age

Age (years)	Near Point (cm)
10	7
20	10
30	14
40	22
50	40
60	200

34.45 Refractive errors for (a) a normal eye, (b) a myopic (nearsighted) eye, and (c) a hyperopic (farsighted) eye viewing a very distant object. The dashed blue curve indicates the required position of the retina.



cornea. The region behind the cornea contains a liquid called the *aqueous humor.* Next comes the *crystalline lens,* a capsule containing a fibrous jelly, hard at the center and progressively softer at the outer portions. The crystalline lens is held in place by ligaments that attach it to the ciliary muscle, which encircles it. Behind the lens, the eye is filled with a thin watery jelly called the *vitreous humor.* The indexes of refraction of both the aqueous humor and the vitreous humor are about 1.336, nearly equal to that of water. The crystalline lens, while not homogeneous, has an average index of 1.437. This is not very different from the indexes of the aqueous and vitreous humors. As a result, most of the refraction of light entering the eye occurs at the outer surface of the cornea.

Refraction at the cornea and the surfaces of the lens produces a *real image* of the object being viewed. This image is formed on the light-sensitive *retina,* lining the rear inner surface of the eye. The retina plays the same role as the film in a camera. The *rods* and *cones* in the retina act like an array of miniature photocells (Fig. 34.44b); they sense the image and transmit it via the *optic nerve* to the brain. Vision is most acute in a small central region called the *fovea centralis,* about 0.25 mm in diameter.

In front of the lens is the *iris.* It contains an aperture with variable diameter called the *pupil,* which opens and closes to adapt to changing light intensity. The receptors of the retina also have intensity adaptation mechanisms.

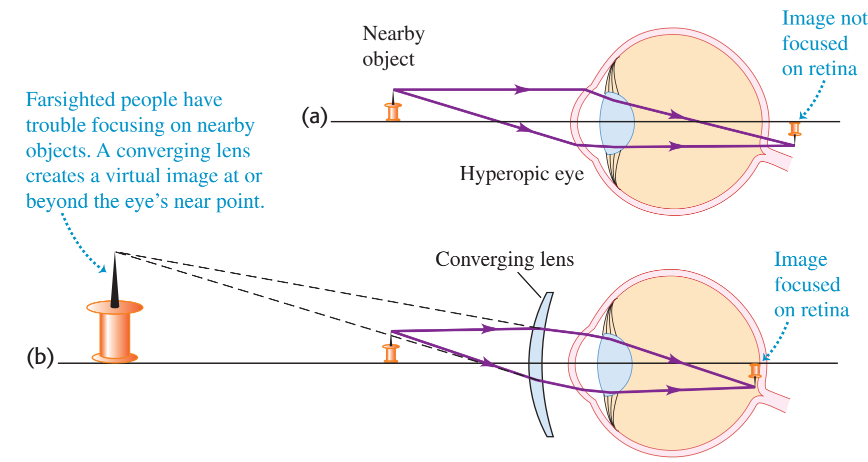
For an object to be seen sharply, the image must be formed exactly at the location of the retina. The eye adjusts to different object distances s by changing the focal length f of its lens; the lens-to-retina distance, corresponding to s', does not change. (Contrast this with focusing a camera, in which the focal length is fixed and the lens-to-film distance is changed.) For the normal eye, an object at infinity is sharply focused when the ciliary muscle is relaxed. To permit sharp imaging on the retina of closer objects, the tension in the ciliary muscle surrounding the lens increases, the ciliary muscle contracts, the lens bulges, and the radii of curvature of its surfaces decrease; this decreases the focal length. This process is called *accommodation.*

The extremes of the range over which distinct vision is possible are known as the *far point* and the *near point* of the eye. The far point of a normal eye is at infinity. The position of the near point depends on the amount by which the ciliary muscle can increase the curvature of the crystalline lens. The range of accommodation gradually diminishes with age because the crystalline lens grows throughout a person's life (it is about 50% larger at age 60 than at age 20) and the ciliary muscles are less able to distort a larger lens. For this reason, the near point gradually recedes as one grows older. This recession of the near point is called *presbyopia.* Table 34.1 shows the approximate position of the near point for an average person at various ages. For example, an average person 50 years of age cannot focus on an object that is closer than about 40 cm.

Defects of Vision

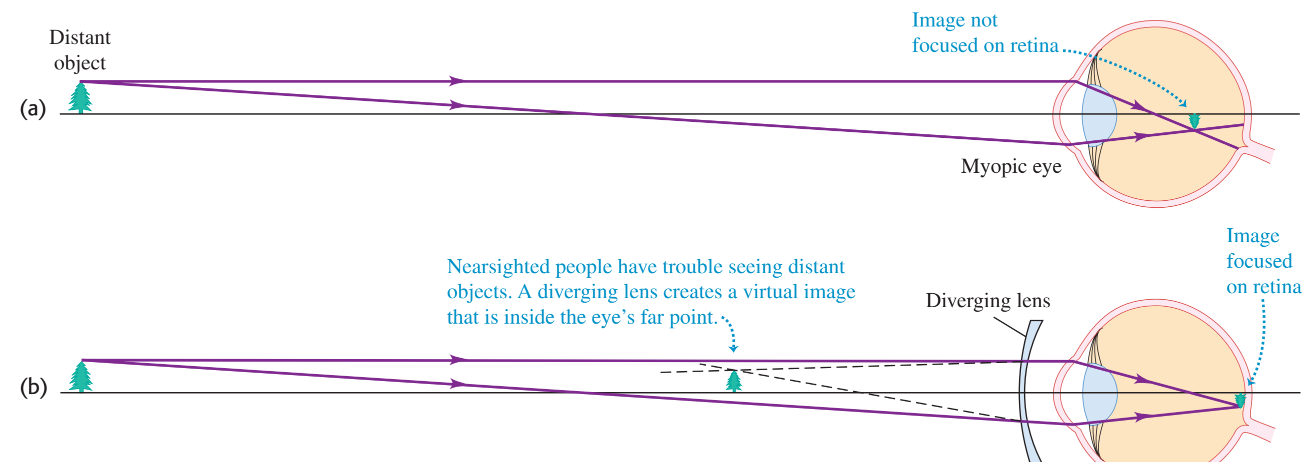
Several common defects of vision result from incorrect distance relationships in the eye. A normal eye forms an image on the retina of an object at infinity when the eye is relaxed (Fig. 34.45a). In the *myopic* (nearsighted) eye, the eyeball is too long from front to back in comparison with the radius of curvature of the cornea (or the cornea is too sharply curved), and rays from an object at infinity are focused in front of the retina (Fig. 34.45b). The most distant object for which an image can be formed on the retina is then nearer than infinity. In the *hyperopic* (farsighted) eye, the eyeball is too short or the cornea is not curved enough, and the image of an infinitely distant object is behind the retina (Fig. 34.45c). The myopic eye produces *too much* convergence in a parallel bundle of rays for an image to be formed on the retina; the hyperopic eye, *not enough* convergence.

All of these defects can be corrected by the use of corrective lenses (eyeglasses or contact lenses). The near point of either a presbyopic or a hyperopic



34.46 (a) An uncorrected hyperopic (farsighted) eye. (b) A positive (converging) lens gives the extra convergence needed for a hyperopic eye to focus the image on the retina.

34.47 (a) An uncorrected myopic (nearsighted) eye. (b) A negative (diverging) lens spreads the rays farther apart to compensate for the excessive convergence of the myopic eye.



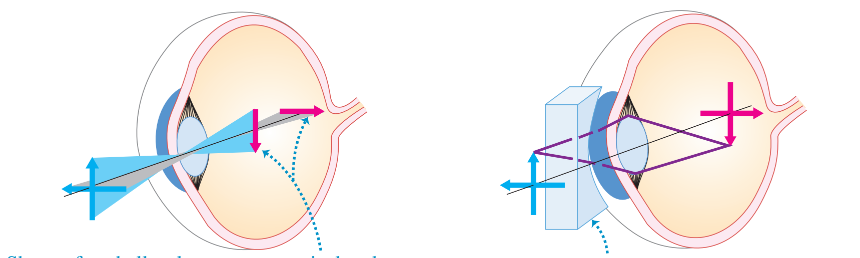
eye is *farther* from the eye than normal. To see clearly an object at normal reading distance (often assumed to be 25 cm), we need a lens that forms a virtual image of the object at or beyond the near point. This can be accomplished by a converging (positive) lens, as shown in Fig. 34.46. In effect the lens moves the object farther away from the eye to a point where a sharp retinal image can be formed. Similarly, correcting the myopic eye involves using a diverging (negative) lens to move the image closer to the eye than the actual object, as shown in Fig. 34.47.

Astigmatism is a different type of defect in which the surface of the cornea is not spherical but rather more sharply curved in one plane than in another. As a result, horizontal lines may be imaged in a different plane from vertical lines (Fig. 34.48a). Astigmatism may make it impossible, for example, to focus clearly on the horizontal and vertical bars of a window at the same time.

Astigmatism can be corrected by use of a lens with a *cylindrical* surface. For example, suppose the curvature of the cornea in a horizontal plane is correct to focus rays from infinity on the retina but the curvature in the vertical plane is too great to form a sharp retinal image. When a cylindrical lens with its axis horizontal is placed before the eye, the rays in a horizontal plane are unaffected, but the additional divergence of the rays in a vertical plane causes these to be sharply imaged on the retina (Fig. 34.48b).

Lenses for vision correction are usually described in terms of the **power,** defined as the reciprocal of the focal length expressed in meters. The unit of power is the **diopter.** Thus a lens with $f = 0.50$ m has a power of 2.0 diopters,

34.48 One type of astigmatism and how it is corrected.



$f = -0.25$ m corresponds to -4.0 diopters, and so on. The numbers on a prescription for glasses are usually powers expressed in diopters. When the correction involves both astigmatism and myopia or hyperopia, there are three numbers: one for the spherical power, one for the cylindrical power, and an angle to describe the orientation of the cylinder axis.

An alternative approach for correcting many defects of vision is to reshape the cornea. This is often done using a procedure called *laser-assisted in situ keratomileusis,* or LASIK. An incision is made into the cornea and a flap of outer corneal tissue is folded back. A pulsed ultraviolet laser with a beam only 50 μm wide (about $\frac{1}{200}$ the width of a human hair) is then used to vaporize away microscopic areas of the underlying tissue. The flap is then folded back into position, where it conforms to the new shape "carved" by the laser.

Example 34.13 Correcting for farsightedness

The near point of a certain hyperopic eye is 100 cm in front of the eye. To see clearly an object that is 25 cm in front of the eye, what contact lens is required?

SOLUTION

IDENTIFY: We want the lens to form a virtual image of the object at the near point of the eye, 100 cm from it. That is, when $s =$ 25 cm, we want s' to be 100 cm.

SET UP: Figure 34.49 shows the situation. We determine the required focal length of the contact lens using the object-image relationship for a thin lens, Eq. (34.16).

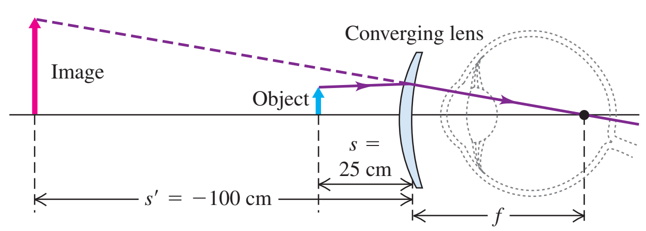
EXECUTE: From Eq. (34.16),

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{+25\text{ cm}} + \frac{1}{-100\text{ cm}}$$
$$f = +33\text{ cm}$$

We need a converging lens with focal length $f = 33$ cm. The corresponding power is $1/(0.33\text{ m})$, or $+3.0$ diopters.

EVALUATE: In this example we used a contact lens to correct hyperopia. Had we used eyeglasses, we would have had to account for the separation between the eye and the eyeglass lens, and a somewhat different power would have been required (see Example 34.14).

34.49 Using a contact lens to correct for farsightedness.



Example 34.14 Correcting for nearsightedness

The far point of a certain myopic eye is 50 cm in front of the eye. To see clearly an object at infinity, what eyeglass lens is required? Assume that the lens is worn 2 cm in front of the eye.

SOLUTION

IDENTIFY: The far point of a myopic eye is nearer than infinity. To see clearly objects beyond the far point, we need a lens that forms a virtual image of such objects no farther from the eye than the far point.

SET UP: Figure 34.50 shows the situation. Assume that the virtual image of the object at infinity is formed at the far point, 50 cm in front of the eye and 48 cm in front of the eyeglass lens. Then when $s = \infty$, s' -48 cm.

we want to be As in Example 34.13, we use the values of s and s' to calculate the required focal length.

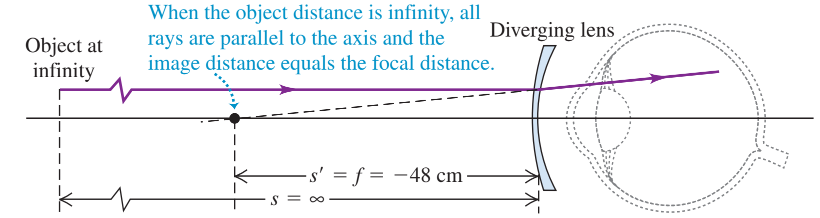
EXECUTE: From Eq. (34.16),

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{-48\text{ cm}}$$
$$f = -48\text{ cm}$$

We need a *diverging* lens with focal length $-48\text{ cm} = -0.48$ m. The power is -2.1 diopter.

EVALUATE: If a *contact* lens were used instead, we would need $f = -50$ cm and a power of -2.0 diopters. Can you see why?

34.50 Using a contact lens to correct for nearsightedness.



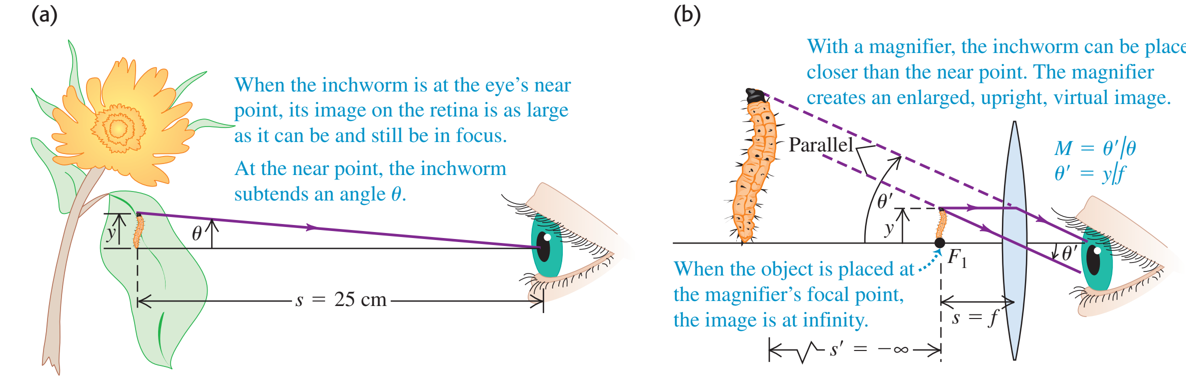
Test Your Understanding of Section 34.6 A certain eyeglass lens is thin at its center, even thinner at its top and bottom edges, and relatively thick at its left and right edges. What defects of vision is this lens intended to correct? (i) hyperopia for objects oriented both vertically and horizontally; (ii) myopia for objects oriented both vertically and horizontally; (iii) hyperopia for objects oriented vertically and myopia for objects oriented horizontally; (iv) hyperopia for objects oriented horizontally and myopia for objects oriented vertically.

34.7 The Magnifier

The apparent size of an object is determined by the size of its image on the retina. If the eye is unaided, this size depends on the *angle* θ subtended by the object at the eye, called its **angular size** (Fig. 34.51a).

To look closely at a small object, such as an insect or a crystal, you bring it close to your eye, making the subtended angle and the retinal image as large as possible. But your eye cannot focus sharply on objects that are closer than the near point, so the angular size of an object is greatest (that is, it subtends the largest possible viewing angle) when it is placed at the near point. In the following discussion we will assume an average viewer for whom the near point is 25 cm from the eye.

34.51 (a) The angular size θ is largest when the object is at the near point. (b) The magnifier gives a virtual image at infinity. This virtual image appears to the eye to be a real object subtending a larger angle θ' at the eye.



A converging lens can be used to form a virtual image that is larger and farther from the eye than the object itself, as shown in Fig. 34.51b. Then the object can be moved closer to the eye, and the angular size of the image may be substantially larger than the angular size of the object at 25 cm without the lens. A lens used in this way is called a **magnifier,** otherwise known as a *magnifying glass* or a *simple magnifier.* The virtual image is most comfortable to view when it is placed at infinity, so that the ciliary muscle of the eye is relaxed; this means that the object is placed at the focal point F_1 of the magnifier. In the following discussion we assume that this is done.

In Fig. 34.51a the object is at the near point, where it subtends an angle θ at the eye. In Fig. 34.51b a magnifier in front of the eye forms an image at infinity, and the angle subtended at the magnifier is θ'. The usefulness of the magnifier is given by the ratio of the angle θ' (with the magnifier) to the angle θ (without the magnifier). This ratio is called the **angular magnification** M:

$$M = \frac{\theta'}{\theta} \quad \text{(angular magnification)} \tag{34.21}$$

CAUTION **Angular magnification vs. lateral magnification** Don't confuse the *angular* magnification M with the *lateral* magnification m. Angular magnification is the ratio of the *angular* size of an image to the angular size of the corresponding object; lateral magnification refers to the ratio of the *height* of an image to the height of the corresponding object. For the situation shown in Fig. 34.51b, the angular magnification is about 3×, since the inchworm subtends an angle about three times larger than that in Fig. 34.51a; hence the inchworm will look about three times larger to the eye. The *lateral* magnification $m = -s'/s$ in Fig. 34.51b is *infinite* because the virtual image is at infinity, but that doesn't mean that the inchworm looks infinitely large through the magnifier! (That's why we didn't attempt to draw an infinitely large inchworm in Fig. 34.51b.) When dealing with a magnifier, M is useful but m is not.

To find the value of M, we first assume that the angles are small enough that each angle (in radians) is equal to its sine and its tangent. Using Fig. 34.451a and drawing the ray in Fig. 34.51b that passes undeviated through the center of the lens, we find that θ and θ' (in radians) are

$$\theta = \frac{y}{25\text{ cm}} \qquad \theta' = \frac{y}{f}$$

Combining these expressions with Eq. (34.21), we find

$$M = \frac{\theta'}{\theta} = \frac{y/f}{y/25\text{ cm}} = \frac{25\text{ cm}}{f} \quad \text{(angular magnification for a simple magnifier)} \tag{34.22}$$

It may seem that we can make the angular magnification as large as we like by decreasing the focal length f. In fact, the aberrations of a simple double-convex lens set a limit to M of about 3× to 4×. If these aberrations are corrected, the angular magnification may be made as great as 20×. When greater magnification than this is needed, we usually use a compound microscope, discussed in the next section.

Test Your Understanding of Section 34.7 You are examining a gem using a magnifier. If you change to a different magnifier with twice the focal length of the first one, (i) you will have to hold the object at twice the distance and the angular magnification will be twice as great; (ii) you will have to hold the object at twice the distance and the angular magnification will be $\frac{1}{2}$ as great; (iii) you will have to hold the object at $\frac{1}{2}$ the distance and the angular magnification will be twice as great; (iv) you will have to hold the object at $\frac{1}{2}$ the distance and the angular magnification will be $\frac{1}{2}$ as great.

34.8 Microscopes and Telescopes

ActivPhysics Online
15.12 Two-Lens System
15.3 The Telescope and Angular Magnification

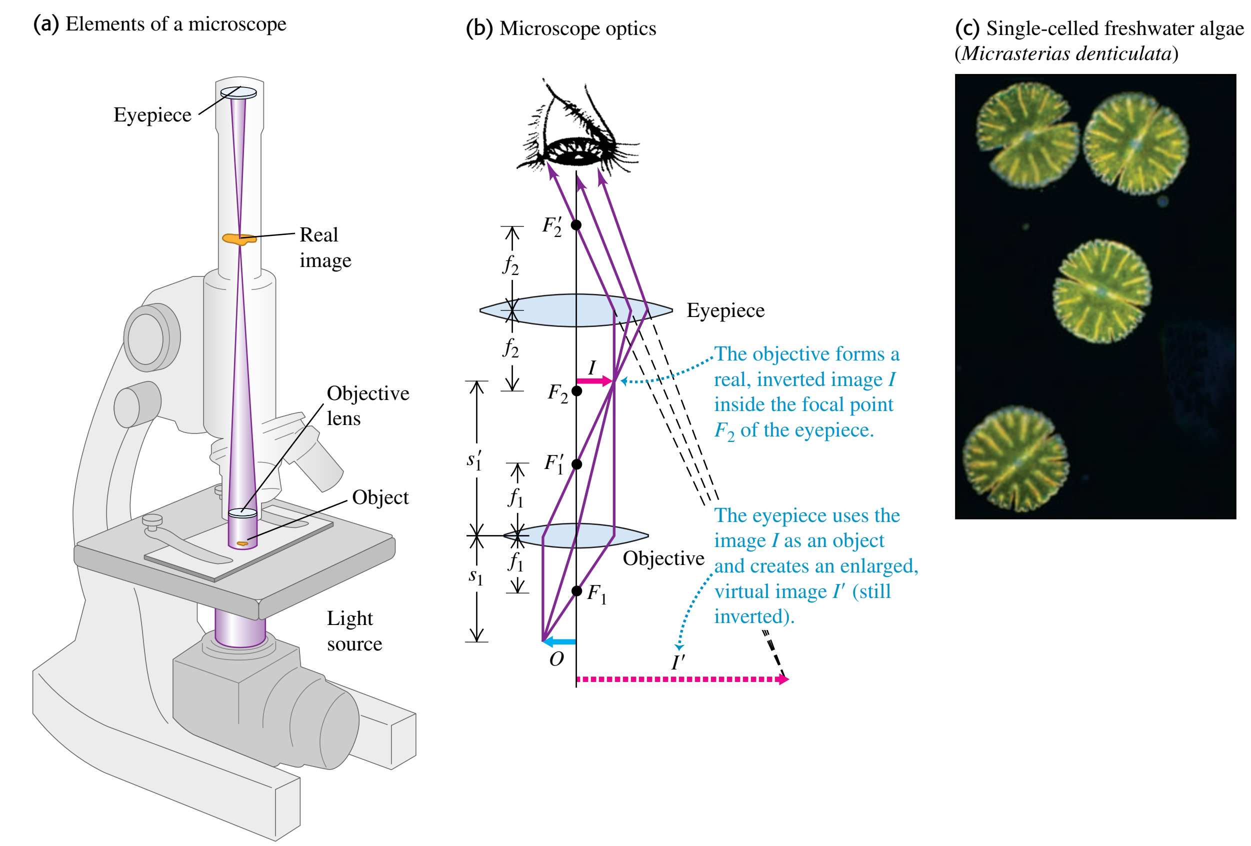
Cameras, eyeglasses, and magnifiers use a single lens to form an image. Two important optical devices that use *two* lenses are the microscope and the telescope. In each device a primary lens, or *objective,* forms a real image, and a second lens, or *eyepiece,* is used as a magnifier to make an enlarged, virtual image.

Microscopes

When we need greater magnification than we can get with a simple magnifier, the instrument that we usually use is the **microscope,** sometimes called a *compound microscope.* The essential elements of a microscope are shown in Fig. 34.52a. To analyze this system, we use the principle that an image formed by one optical element such as a lens or mirror can serve as the object for a second element. We used this principle in Section 34.4 when we derived the thin-lens equation by repeated application of the single-surface refraction equation; we used this principle again in Example 34.11 (Section 34.4), in which the image formed by a lens was used as the object of a second lens.

The object O to be viewed is placed just beyond the first focal point F_1 of the **objective,** a converging lens that forms a real and enlarged image I (Fig. 34.52b). In a properly designed instrument this image lies just inside the first focal point F_1' of a second converging lens called the **eyepiece** or *ocular.* (The reason the image should lie just *inside* F_1' is left for you to discover; see Problem 34.108.) The eyepiece acts as a simple magnifier, as discussed in Section 34.7, and forms a final virtual image I' of I. The position of I' may be anywhere between the near

34.52 (a) Elements of a microscope. (b) The object O is placed just outside the first focal point of the objective (the distance s_1 has been exaggerated for clarity). (c) This microscope image shows single-celled organisms about 2×10^{-4} m (0.2 mm) across. Typical light microscopes can resolve features as small as 2×10^{-7} m, comparable to the wavelength of light.



and far points of the eye. Both the objective and the eyepiece of an actual microscope are highly corrected compound lenses with several optical elements, but for simplicity we show them here as simple thin lenses.

As for a simple magnifier, what matters when viewing through a microscope is the *angular* magnification M. The overall angular magnification of the compound microscope is the product of two factors. The first factor is the *lateral* magnification m_1 of the objective, which determines the linear size of the real image I; the second factor is the *angular* magnification M_2 of the eyepiece, which relates the angular size of the virtual image seen through the eyepiece to the angular size that the real image I would have if you viewed it *without* the eyepiece. The first of these factors is given by

$$m_1 = -\frac{s_1'}{s_1} \qquad (34.23)$$

where s_1 and s_1' are the object and image distances, respectively, for the objective lens. Ordinarily, the object is very close to the focal point, and the resulting image distance s_1' is very great in comparison to the focal length f_1 of the objective lens. Thus s_1 is approximately equal to f_1, and we can write $m_1 = -s_1'/f_1$.

The real image I is close to the focal point F_1' of the eyepiece, so to find the eyepiece angular magnification, we can use Eq. (34.22): $M_2 = (25\text{ cm})/f_2$, where f_2 is the focal length of the eyepiece (considered as a simple lens). The overall angular magnification M of the compound microscope (apart from a negative sign, which is customarily ignored) is the product of the two magnifications:

$$M = m_1 M_2 = \frac{(25\text{ cm})s_1'}{f_1 f_2} \qquad \text{(angular magnification for a microscope)} \qquad (34.24)$$

where s_1', f_1, and f_2 are measured in centimeters. The final image is inverted with respect to the object. Microscope manufacturers usually specify the values of m_1 and M_2 for microscope components rather than the focal lengths of the objective and eyepiece.

Equation (34.24) shows that the angular magnification of a microscope can be increased by using an objective of shorter focal length f_1, thereby increasing m_1 and the size of the real image I. Most optical microscopes have a rotating "turret" with three or more objectives of different focal lengths so that the same object can be viewed at different magnifications. The eyepiece should also have a short focal length f_2 to help to maximize the value of M.

To take a photograph using a microscope (called a *photomicrograph* or *micrograph*), the eyepiece is removed and a camera placed so that the real image I falls on the camera's CCD array or film. Figure 34.52c shows such a photograph. In this case what matters is the *lateral* magnification of the microscope as given by Eq. (34.23).

Telescopes

The optical system of a **telescope** is similar to that of a compound microscope. In both instruments the image formed by an objective is viewed through an eyepiece. The key difference is that the telescope is used to view large objects at large distances and the microscope is used to view small objects close at hand. Another difference is that many telescopes use a curved mirror, not a lens, as an objective.

Figure 34.53 shows an *astronomical telescope.* Because this telescope uses a lens as an objective, it is called a *refracting telescope* or *refractor.* The objective lens forms a real, reduced image I of the object. This image is the object for the eyepiece lens, which forms an enlarged, virtual image of I. Objects that are viewed with a telescope are usually so far away from the instrument that the first

34.53 Optical system of an astronomical refracting telescope.

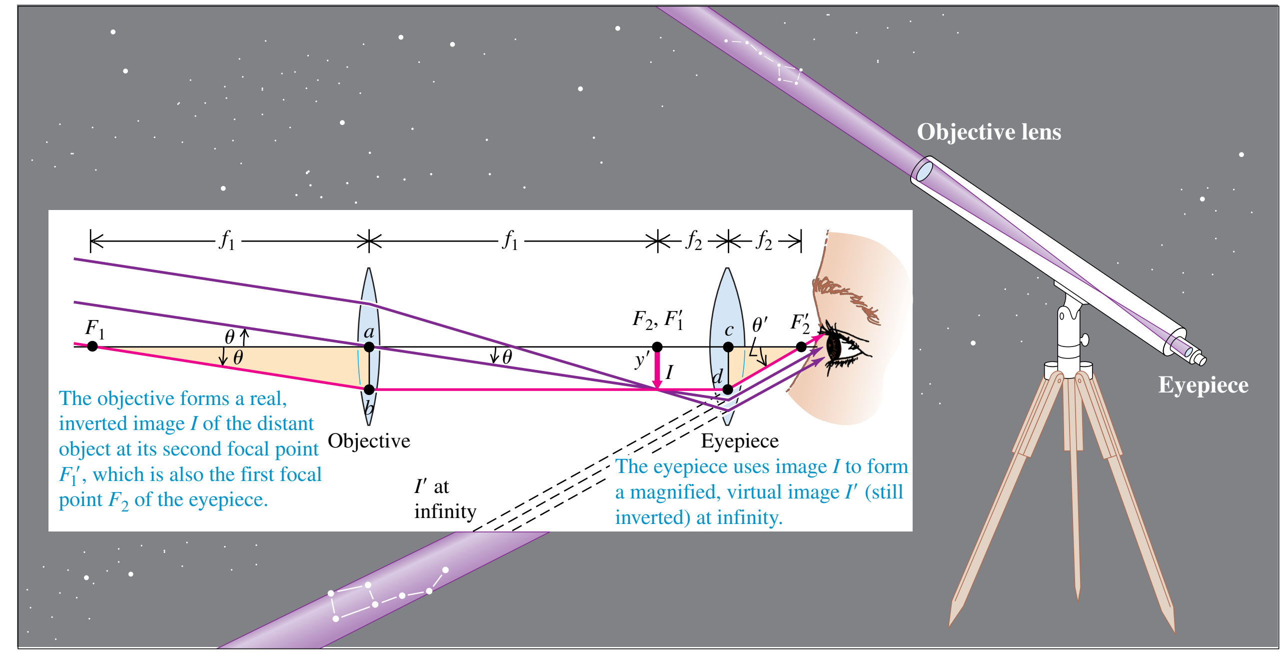


image I is formed very nearly at the second focal point of the objective lens. If the final image I' formed by the eyepiece is at infinity (for most comfortable viewing by a normal eye), the first image must also be at the first focal point of the eyepiece. The distance between objective and eyepiece, which is the length of the telescope, is therefore the *sum* of the focal lengths of objective and eyepiece, $f_1 + f_2$.

The angular magnification M of a telescope is defined as the ratio of the angle subtended at the eye by the final image I' to the angle subtended at the (unaided) eye by the object. We can express this ratio in terms of the focal lengths of objective and eyepiece. In Fig. 34.53 the ray passing through F_1, the first focal point of the objective, and through F_2', the second focal point of the eyepiece, is shown in red. The object (not shown) subtends an angle θ at the objective and would subtend essentially the same angle at the unaided eye. Also, since the observer's eye is placed just to the right of the focal point F_2', the angle subtended at the eye by the final image is very nearly equal to the angle θ'. Because bd is parallel to the optic axis, the distances ab and cd are equal to each other and also to the height y' of the real image I. Because the angles θ and θ' are small, they may be approximated by their tangents. From the right triangles F_1ab and $F_2'cd$,

$$\theta = \frac{-y'}{f_1} \qquad \theta' = \frac{y'}{f_2}$$

and the angular magnification M is

$$M = \frac{\theta'}{\theta} = -\frac{y'/f_2}{y'/f_1} = -\frac{f_1}{f_2} \qquad \text{(angular magnification for a telescope)} \qquad (34.25)$$

The angular magnification M of a telescope is equal to the ratio of the focal length of the objective to that of the eyepiece. The negative sign shows that the final image is inverted. Equation (34.25) shows that to achieve good angular magnification, a *telescope* should have a *long* objective focal length f_1. By contrast, Eq. (34.24) shows that a *microscope* should have a *short* objective focal length. However, a telescope objective with a long focal length should also have a large diameter D so that the f-number f_1/D will not be too large; as described in

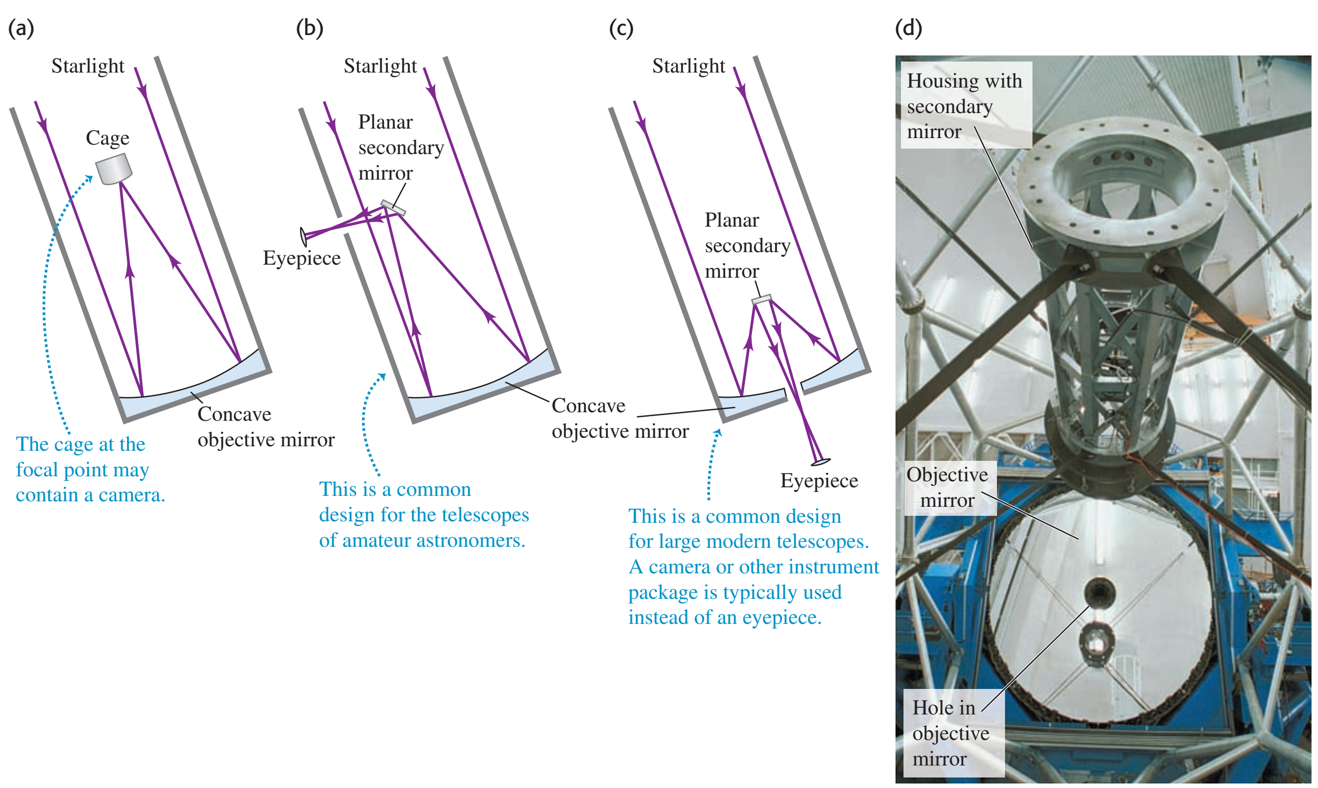
Section 34.5, a large f-number means a dim, low-intensity image. Telescopes typically do not have interchangeable objectives; instead, the magnification is varied by using different eyepieces with different focal lengths f_2. Just as for a microscope, smaller values of f_2 give larger angular magnifications.

An inverted image is no particular disadvantage for astronomical observations. When we use a telescope or binoculars—essentially a pair of telescopes mounted side by side—to view objects on the earth, though, we want the image to be right-side up. In prism binoculars, this is accomplished by reflecting the light several times along the path from the objective to the eyepiece. The combined effect of the reflections is to flip the image both horizontally and vertically. Binoculars are usually described by two numbers separated by a multiplication sign, such as 7×50. The first number is the angular magnification M, and the second is the diameter of the objective lenses (in millimeters). The diameter helps to determine the light-gathering capacity of the objective lenses and thus the brightness of the image.

In the *reflecting telescope* (Fig. 34.54a) the objective lens is replaced by a concave mirror. In large telescopes this scheme has many advantages, both theoretical and practical. Mirrors are inherently free of chromatic aberrations (dependence of focal length on wavelength), and spherical aberrations (associated with the paraxial approximation) are easier to correct than with a lens. The reflecting surface is sometimes parabolic rather than spherical. The material of the mirror need not be transparent, and it can be made more rigid than a lens, which has to be supported only at its edges.

The largest reflecting telescopes in the world, the Keck telescopes atop Mauna Kea in Hawaii, each have an objective mirror of overall diameter 10 m made up of 36 separate hexagonal reflecting elements.

34.54 (a), (b), (c) Three designs for reflecting telescopes. (d) This photo shows the interior of the Gemini North telescope, which uses the design shown in (c). The objective mirror is 8 meters in diameter.



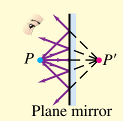
One challenge in designing reflecting telescopes is that the image is formed in front of the objective mirror, in a region traversed by incoming rays. Isaac Newton devised one solution to this problem. A flat secondary mirror oriented at 45° to the optic axis causes the image to be formed in a hole on the side of the telescope, where it can be magnified with an eyepiece (Fig. 34.54b). Another solution uses a secondary mirror that causes the focused light to pass through a hole in the objective mirror (Fig. 34.54c). Large research telescopes, as well as many amateur telescopes, use this design (Fig. 34.54d).

Like a microscope, when a telescope is used for photography the eyepiece is removed and a CCD array or photographic film is placed at the position of the real image formed by the objective. (Some long-focal-length "lenses" for photography are actually reflecting telescopes used in this way.) Most telescopes used for astronomical research are never used with an eyepiece.

Test Your Understanding of Section 34.8 Which gives a lateral magnification of greater absolute value: (i) the objective lens in a microscope (Fig. 34.52); (ii) the objective lens in a refracting telescope (Fig. 34.53); or (iii) not enough information is given to decide?

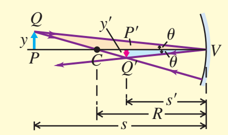
CHAPTER 34 SUMMARY

Reflection or refraction at a plane surface: When rays diverge from an object point P and are reflected or refracted, the directions of the outgoing rays are the same as though they had diverged from a point P' called the image point. If they actually converge at P' and diverge again beyond it, P' is a real image of P; if they only appear to have diverged from P', it is a virtual image. Images can be either erect or inverted.

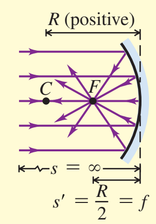


Lateral magnification: The lateral magnification m in any reflecting or refracting situation is defined as the ratio of image height y' to object height y. When m is positive, the image is erect; when m is negative, the image is inverted.

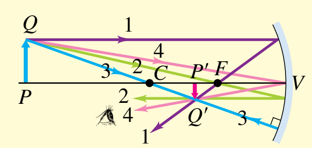
$$m = \frac{y'}{y} \qquad (34.2)$$



Focal point and focal length: The focal point of a mirror is the point where parallel rays converge after reflection from a concave mirror, or the point from which they appear to diverge after reflection from a convex mirror. Rays diverging from the focal point of a concave mirror are parallel after reflection; rays converging toward the focal point of a convex mirror are parallel after reflection. The distance from the focal point to the vertex is called the focal length, denoted as f. The focal points of a lens are defined similarly.



Relating object and image distances: The formulas for object distance s and image distance s' for plane and spherical mirrors and single refracting surfaces are summarized in the table. The equation for a plane surface can be obtained from the corresponding equation for a spherical surface by setting $R = \infty$. (See Examples 34.1–34.7.)



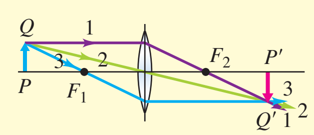
	Plane Mirror	Spherical Mirror	Plane Refracting Surface	Spherical Refracting Surface
Object and image distances	$\frac{1}{s} + \frac{1}{s'} = 0$	$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} = \frac{1}{f}$	$\frac{n_a}{s} + \frac{n_b}{s'} = 0$	$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$
Lateral magnification	$m = -\frac{s'}{s} = 1$	$m = -\frac{s'}{s}$	$m = -\frac{n_a s'}{n_b s} = 1$	$m = -\frac{n_a s'}{n_b s}$

Object-image relationships derived in this chapter are valid only for rays close to and nearly parallel to the optic axis; these are called paraxial rays. Nonparaxial rays do not converge precisely to an image point. This effect is called spherical aberration.

Thin lenses: The object-image relationships, given by Eq. (34.16), is the same for a thin lens as for a spherical mirror. Equation (34.19), the lensmaker's equation, relates the focal length of a lens to its index of refraction and the radii of curvature of its surfaces. (See Examples 34.8–34.11.)

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad (34.16)$$

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (34.19)$$

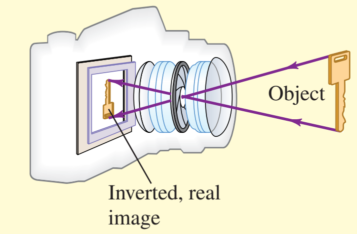


Sign rules: The following sign rules are used with all plane and spherical reflecting and refracting surfaces.

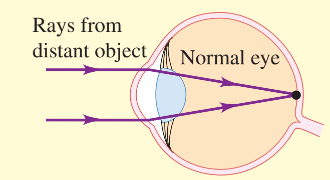
- $s > 0$ when the object is on the incoming side of the surface (a real object); $s < 0$ otherwise.
- $s' > 0$ when the image is on the outgoing side of the surface (a real image); $s' < 0$ otherwise.
- $R > 0$ when the center of curvature is on the outgoing side of the surface; $R < 0$ otherwise.
- $m > 0$ when the image is erect; $m < 0$ when inverted.

Cameras: A camera forms a real, inverted, reduced image of the object being photographed on a light-sensitive surface. The amount of light striking this surface is controlled by the shutter speed and the aperture. The intensity of this light is inversely proportional to the square of the f-number of the lens. (See Example 34.12.)

$$f\text{-number} = \frac{\text{Focal length}}{\text{Aperture diameter}} = \frac{f}{D} \qquad (34.20)$$

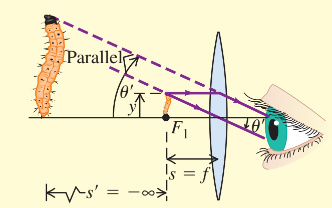


The eye: In the eye, refraction at the surface of the cornea forms a real image on the retina. Adjustment for various object distances is made by squeezing the lens, thereby making it bulge and decreasing its focal length. A nearsighted eye is too long for its lens; a farsighted eye is too short. The power of a corrective lens, in diopters, is the reciprocal of the focal length in meters. (See Examples 34.13 and 34.14.)

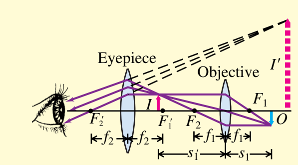


The simple magnifier: The simple magnifier creates a virtual image whose angular size θ' is larger than the angular size θ of the object itself at a distance of 25 cm, the nominal closest distance for comfortable viewing. The angular magnification M of a simple magnifier is the ratio of the angular size of the virtual image to that of the object at this distance.

$$M = \frac{\theta'}{\theta} = \frac{25\text{ cm}}{f} \qquad (34.22)$$



Microscopes and telescopes: In a compound microscope, the objective lens forms a first image in the barrel of the instrument, and the eyepiece forms a final virtual image, often at infinity, of the first image. The telescope operates on the same principle, but the object is far away. In a reflecting telescope, the objective lens is replaced by a concave mirror, which eliminates chromatic aberrations.



Key Terms

Answer to Chapter Opening Question ?

A magnifying lens (simple magnifier) produces a virtual image with a large angular size that is infinitely far away, so you can see it in sharp focus with your eyes relaxed. (A surgeon doing microsurgery would not appreciate having to strain his eyes while working.) The object should be at the focal point of the lens, so the object and lens are separated by one focal length.

Answers to Test Your Understanding Questions

34.1 Answer: (iv) When you are a distance s from the mirror, your image is a distance s on the other side of the mirror and the distance from you to your image is $2s$. As you move toward the mirror, the distance $2s$ changes at twice the rate of the distance s, so your image moves toward you at speed $2v$.

34.2 Answers: (a) concave, (b) (ii) A convex mirror always produces an erect image, but that image is smaller than the object (see

Fig. 34.16b). Hence a concave mirror must be used. The image will be erect and enlarged only if the distance from the object (your face) to the mirror is less than the focal length of the mirror, as in Fig. 34.20d.

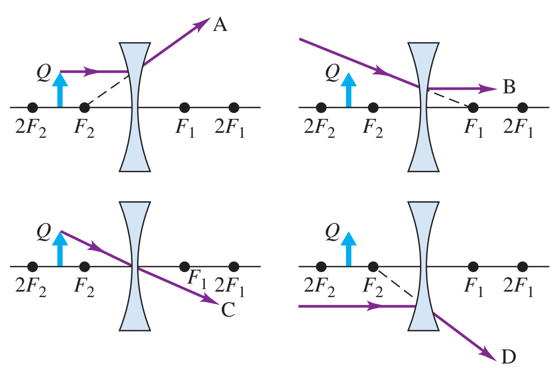
34.3 Answer: no The sun is very far away, so the object distance is essentially infinite: $s = \infty$ and $1/s = 0$. Material a is air $(n_a = 1.00)$ and material b is water $(n_b = 1.33)$, so the image position s' is given by

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \quad \text{or} \quad 0 + \frac{1.33}{s'} = \frac{1.33 - 1.00}{R}$$

$$s' = \frac{1.33}{0.33}R = 4.0R$$

The image would be formed 4.0 drop radii from the front surface of the drop. But since each drop is only a part of a complete sphere, the distance from the front to the back of the drop is less than $2R$. Thus the rays of sunlight never reach the image point, and the drops do not form an image of the sun on the leaf. While the rays are not focused to a point, they are nonetheless concentrated and can cause damage to the leaf.

34.4 Answers: A and C When rays A and D are extended backward, they pass through focal point F_2; thus, before they passed through the lens, they were parallel to the optic axis. The figures



show that ray A emanated from point Q, but ray D did not. Ray B is parallel to the optic axis, so before it passed through the lens, it was directed toward focal point F_1. Hence it cannot have come from point Q. Ray C passes through the center of the lens and hence is not deflected by its passage; tracing the ray backward shows that it emanates from point Q.

34.5 Answer: (iii) The smaller image area of the CCD array means that the angle of view is decreased for a given focal length. Individual objects make images of the same size in either case; when a smaller light-sensitive area is used, fewer images fit into the area and the field of view is narrower.

34.6 Answer: (iii) This lens is designed to correct for a type of astigmatism. Along the vertical axis, the lens is configured as a converging lens; along the horizontal axis, the lens is configured as a diverging lens. Hence the eye is hyperopic (see Fig. 34.46) for objects that are oriented vertically but myopic for objects that are oriented horizontally (see Fig. 34.47). Without correction, the eye focuses vertical objects behind the retina but horizontal objects in front of the retina.

34.7 Answer: (ii) The object must be held at the focal point, which is twice as far away if the focal length f is twice as great. Equation (24.22) shows that the angular magnification M is inversely proportional to f, so doubling the focal length makes $M\ \frac{1}{2}$ as great. To improve the magnification, you should use a magnifier with a *shorter* focal length.

34.8 Answer: (i) The objective lens of a microscope is designed to make enlarged images of small objects, so the absolute value of its lateral magnification m is greater than 1. By contrast, the objective lens of a refracting telescope is designed to make *reduced* images. For example, the moon is thousands of kilometers in diameter, but its image may fit on a CCD array a few centimeters across. Thus $|m|$ is much less than 1 for a refracting telescope. (In both cases m is negative because the objective makes an inverted image, which is why the question asks about the absolute value of m.)

PROBLEMS

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Discussion Questions

Q34.1. A spherical mirror is cut in half horizontally. Will an image be formed by the bottom half of the mirror? If so, where will the image be formed?

Q34.2. For the situation shown in Fig. 34.3, is the image distance s' positive or negative? Is the image real or virtual? Explain your answers.

Q34.3. The laws of optics also apply to electromagnetic waves invisible to the eye. A satellite TV dish is used to detect radio waves coming from orbiting satellites. Why is a curved reflecting surface (a "dish") used? The dish is always concave, never convex; why? The actual radio receiver is placed on an arm and suspended in front of the dish. How far in front of the dish should it be placed?

Q34.4. Explain why the focal length of a *plane* mirror is infinite, and explain what it means for the focal point to be at infinity.

Q34.5. If a spherical mirror is immersed in water, does its focal length change? Explain.

Q34.6. For what range of object positions does a concave spherical mirror form a real image? What about a convex spherical mirror?

Q34.7. When a room has mirrors on two opposite walls, an infinite series of reflections can be seen. Discuss this phenomenon in terms of images. Why do the distant images appear fainter?

Q34.8. For a spherical mirror, if $s = f$, then $s' = \infty$, and the lateral magnification m is infinite. Does this make sense? If so, what does it mean?

Q34.9. You may have noticed a small convex mirror next to your bank's ATM. Why is this mirror convex, as opposed to flat or concave? What considerations determine its radius of curvature?

Q34.10. A student claims that she can start a fire on a sunny day using just the sun's rays and a concave mirror. How is this done? Is the concept of image relevant? Can she do the same thing with a convex mirror? Explain.

Q34.11. A person looks at his reflection in the concave side of a shiny spoon. Is it right side up or inverted? Does it matter how far his face is from the spoon? What if he looks in the convex side? (Try this yourself!)

Q34.12. In Example 34.4 (Section 34.2), there appears to be an ambiguity for the case $s = 10$ cm as to whether s' is $+\infty$ or $-\infty$ and whether the image is erect or inverted. How is this resolved? Or is it?

Q34.13. Suppose that in the situation of Example 34.7 of Section 34.3 (see Fig. 34.26) a vertical arrow 2.00 m tall is painted on the side of the pool beneath the water line. According to the calculations in the example, this arrow would appear to the person shown in Fig. 34.26 to be 1.50 m long. But the discussion following Eq. (34.13) states that the magnification for a plane refracting surface is $m = 1$, which suggests that the arrow would appear to the person to be 2.00 m long. How can you resolve this apparent contradiction?

Q34.14. The bottom of the passenger side mirror on your car notes, "Objects in mirror are closer than they appear." Is this true? Why?

Q34.15. How could you very quickly make an approximate measurement of the focal length of a converging lens? Could the same method be applied if you wished to use a diverging lens? Explain.

Q34.16. The focal length of a simple lens depends on the color (wavelength) of light passing through it. Why? Is it possible for a lens to have a positive focal length for some colors and negative for others? Explain.

Q34.17. When a converging lens is immersed in water, does its focal length increase or decrease in comparison with the value in air? Explain.

Q34.18. A spherical air bubble in water can function as a lens. Is it a converging or diverging lens? How is its focal length related to its radius?

Q34.19. Can an image formed by one reflecting or refracting surface serve as an object for a second reflection or refraction? Does it matter whether the first image is real or virtual? Explain.

Q34.20. If a piece of photographic film is placed at the location of a real image, the film will record the image. Can this be done with a virtual image? How might one record a virtual image?

Q34.21. According to the discussion in Section 34.2, light rays are reversible. Are the formulas in the table in this chapter's Summary still valid if object and image are interchanged? What does reversibility imply with respect to the *forms* of the various formulas?

Q34.22. You've entered a survival contest that will include building a crude telescope. You are given a large box of lenses. Which two lenses do you pick? How do you quickly identify them?

Q34.23. You can't see clearly underwater with the naked eye, but you *can* if you wear a face mask or goggles (with air between your eyes and the mask or goggles). Why is there a difference? Could you instead wear eyeglasses (with water between your eyes and the eyeglasses) in order to see underwater? If so, should the lenses be converging or diverging? Explain.

Q34.24. You take a lens and mask it so that light can pass through only the bottom half of the lens. How does the image formed by the masked lens compare to the image formed before masking?

Exercises

Section 34.1 Reflection and Refraction at a Plane Surface

34.1. A candle 4.85 cm tall is 39.2 cm to the left of a plane mirror. Where is the image formed by the mirror, and what is the height of this image?

34.2. The image of a tree just covers the length of a plane mirror 4.00 cm tall when the mirror is held 35.0 cm from the eye. The tree is 28.0 m from the mirror. What is its height?

34.3. As shown in Fig. 34.9, mirror 1 uses the image P_2' formed by mirror 2 as an object and forms an image of it. Show that this image is at point P_3' in the figure.

Section 34.2 Reflection at a Spherical Surface

34.4. A concave mirror has a radius of curvature of 34.0 cm. (a) What is its focal length? (b) If the mirror is immersed in water (refractive index 1.33), what is its focal length?

34.5. An object 0.600 cm tall is placed 16.5 cm to the left of the vertex of a concave spherical mirror having a radius of curvature of 22.0 cm. (a) Draw a principal-ray diagram showing the formation of the image. (b) Determine the position, size, orientation, and nature (real or virtual) of the image.

34.6. Repeat Exercise 34.5 for the case in which the mirror is convex.

34.7. The diameter of Mars is 6794 km, and its minimum distance from the earth is 5.58×10^7 km. When Mars is at this distance, find the diameter of the image of Mars formed by a spherical, concave, telescope mirror with a focal length of 1.75 m.

34.8. An object is 24.0 cm from the center of a silvered spherical glass Christmas tree ornament 6.00 cm in diameter. What are the position and magnification of its image?

34.9. A coin is placed next to the convex side of a thin spherical glass shell having a radius of curvature of 18.0 cm. An image of the 1.5-cm-tall coin is formed 6.00 cm behind the glass shell. Where is the coin located? Determine the size, orientation, and nature (real or virtual) of the image.

34.10. You hold a spherical salad bowl 90 cm in front of your face with the bottom of the bowl facing you. The salad bowl is made of polished metal with a 35-cm radius of curvature. (a) Where is the image of your 2.0-cm-tall nose located? (b) What are the image's size, orientation, and nature (real or virtual)?

34.11. (a) Show that Eq. (34.6) can be written as $s' = sf/(s - f)$ and hence that the lateral magnification, given by Eq. (34.7), can be expressed as $m = f/(f - s)$. (b) Use these formulas for s' and m to graph s' as a function of s for the case $f > 0$ (a concave mirror). (c) For what values of s is s' positive, so that the image is real? (d) For what values of s is s' negative, so that the image is virtual? (e) Where is the image if the object is just inside the focal point (s slightly less than f)? (f) Where is the image if the object is at infinity? (g) Where is the image if the object is next to the mirror $(s = 0)$? (h) Graph m as a function of s for the case of a concave mirror. (i) For which values of s is the image erect and larger than the object? (j) For what values of s is the image inverted? (k) For which values of s is the image smaller than the object? (l) What happens to the size of the image when the object is placed at the focal point?

34.12. Using the formulas for s' and m obtained in part (a) of Exercise 34.11, graph s' as a function of s, and graph m as a function of s, for the case $f < 0$ (a convex mirror), so that $f = -|f|$. (a) For which values of s is s' positive? (b) For what values of s is s' negative? (c) Where is the image if the object is at infinity? (d) Where is the image if the object is next to the mirror $(s = 0)$? For which values of s is the image (e) erect; (f) inverted; (g) larger than the object; (h) smaller than the object?

34.13. Dental Mirror. A dentist uses a curved mirror to view teeth on the upper side of the mouth. Suppose she wants an erect image with a magnification of 2.00 when the mirror is 1.25 cm from a tooth. (Treat this problem as though the object and image lie along a straight line.) (a) What kind of mirror (concave or convex) is needed? Use a ray diagram to decide, without performing any calculations. (b) What must be the focal length and radius of curvature of this mirror? (c) Draw a principal-ray diagram to check your answer in part (b).

34.14. A spherical, concave, shaving mirror has a radius of curvature of 32.0 cm. (a) What is the magnification of a person's face when it is 12.0 cm to the left of the vertex of the mirror? (b) Where is the image? Is the image real or virtual? (c) Draw a principal-ray diagram showing the formation of the image.

Section 34.3 Refraction at a Spherical Surface

34.15. A speck of dirt is embedded 3.50 cm below the surface of a sheet of ice $(n = 1.309)$. What is its apparent depth when viewed at normal incidence?

34.16. A tank whose bottom is a mirror is filled with water to a depth of 20.0 cm. A small fish floats motionless 7.0 cm under the surface of the water. (a) What is the apparent depth of the fish when viewed at normal incidence? (b) What is the apparent depth of the image of the fish when viewed at normal incidence?

34.17. A Spherical Fish Bowl. A small tropical fish is at the center of a water-filled, spherical fish bowl 28.0 cm in diameter. (a) Find the apparent position and magnification of the fish to an observer outside the bowl. The effect of the thin walls of the bowl may be ignored. (b) A friend advised the owner of the bowl to keep it out of direct sunlight to avoid blinding the fish, which might swim into the focal point of the parallel rays from the sun. Is the focal point actually within the bowl?

34.18. The left end of a long glass rod 6.00 cm in diameter has a convex hemispherical surface 3.00 cm in radius. The refractive index of the glass is 1.60. Determine the position of the image if an object is placed in air on the axis of the rod at the following distances to the left of the vertex of the curved end: (a) infinitely far, (b) 12.0 cm; (c) 2.00 cm.

34.19. The glass rod of Exercise 34.18 is immersed in oil $(n = 1.45)$. An object placed to the left of the rod on the rod's axis is to be imaged 1.20 m inside the rod. How far from the left end of the rod must the object be located to form the image?

34.20. The left end of a long glass rod 8.00 cm in diameter, with an index of refraction 1.60, is ground and polished to a convex hemispherical surface with a radius of 4.00 cm. An object in the form of an arrow 1.50 mm tall, at right angles to the axis of the rod, is located on the axis 24.0 cm to the left of the vertex of the convex surface. Find the position and height of the image of the arrow formed by paraxial rays incident on the convex surface. Is the image erect or inverted?

34.21. Repeat Exercise 34.20 for the case in which the end of the rod is ground to a *concave* hemispherical surface with radius 4.00 cm.

34.22. The glass rod of Exercise 34.21 is immersed in a liquid. An object 14.0 cm from the vertex of the left end of the rod and on its axis is imaged at a point 9.00 cm from the vertex inside the liquid. What is the index of refraction of the liquid?

Section 34.4 Thin Lenses

34.23. An insect 3.75 mm tall is placed 22.5 cm to the left of a thin planoconvex lens. The left surface of this lens is flat, the right surface has a radius of curvature of magnitude 13.0 cm, and the index of refraction of the lens material is 1.70. (a) Calculate the location and size of the image this lens forms of the insect. Is it real or virtual? Erect or inverted? (b) Repeat part (a) if the lens is reversed.

34.24. A lens forms an image of an object. The object is 16.0 cm from the lens. The image is 12.0 cm from the lens on the same side as the object. (a) What is the focal length of the lens? Is the lens converging or diverging? (b) If the object is 8.50 mm tall, how tall is the image? Is it erect or inverted? (c) Draw a principal-ray diagram.

34.25. A converging meniscus lens (see Fig. 34.32a) with a refractive index of 1.52 has spherical surfaces whose radii are 7.00 cm and 4.00 cm. What is the position of the image if an object is placed 24.0 cm to the left of the lens? What is the magnification?

34.26. A converging lens with a focal length of 90.0 cm forms an image of a 3.20-cm-tall real object that is to the left of the lens. The image is 4.50 cm tall and inverted. Where are the object and image located in relation to the lens? Is the image real or virtual?

34.27. A converging lens forms an image of an 8.00-mm-tall real object. The image is 12.0 cm to the left of the lens, 3.40 cm tall, and erect. What is the focal length of the lens? Where is the object located?

34.28. A photographic slide is to the left of a lens. The lens projects an image of the slide onto a wall 6.00 m to the right of the slide. The image is 80.0 times the size of the slide. (a) How far is the slide from the lens? (b) Is the image erect or inverted? (c) What is the focal length of the lens? (d) Is the lens converging or diverging?

34.29. A double-convex thin lens has surfaces with equal radii of curvature of magnitude 2.50 cm. Looking through this lens, you observe that it forms an image of a very distant tree at a distance of 1.87 cm from the lens. What is the index of refraction of the lens?

34.30. Six lenses in air are shown in Fig. 34.32. Each lens is made of a material with index of refraction $n > 1$. Considering each lens individually, imagine that light enters the lens from the left. Show that the three lenses shown in Fig. 34.32a have *positive* focal lengths and hence are *converging* lenses. In addition, show that the three lenses in Fig. 34.32b have *negative* focal lengths and hence are *diverging* lenses.

34.31. Exercises 34.11 and 34.12 deal with spherical mirrors. (a) Show that the equations for s' and m derived in part (a) of Exercise 34.11 also apply to a thin lens. (b) A concave mirror is used in Exercise 34.11. Repeat these exercises for a converging lens. Are there any differences in the results when the mirror is replaced by a lens? Explain. (c) A convex mirror is used in Exercise 34.12. Repeat these exercises for a diverging lens. Are there any differences in the results when the mirror is replaced by a lens? Explain.

34.32. A converging lens with a focal length of 12.0 cm forms a virtual image 8.00 mm tall, 17.0 cm to the right of the lens. Determine the position and size of the object. Is the image erect or inverted? Are the object and image on the same side or opposite sides of the lens? Draw a principal-ray diagram for this situation.

34.33. Repeat Exercise 34.32 for the case in which the lens is diverging, with a focal length of -48.0 cm.

34.34. An object is 16.0 cm to the left of a lens. The lens forms an image 36.0 cm to the right of the lens. (a) What is the focal length of the lens? Is the lens converging or diverging? (b) If the object is 8.00 mm tall, how tall is the image? Is it erect or inverted? (c) Draw a principal-ray diagram.

Section 34.5 Cameras

34.35. A camera lens has a focal length of 200 mm. How far from the lens should the subject for the photo be if the lens is 20.4 cm from the film?

34.36. When a camera is focused, the lens is moved away from or toward the film. If you take a picture of your friend, who is standing 3.90 m from the lens, using a camera with a lens with a 85-mm focal length, how far from the film is the lens? Will the whole image of your friend, who is 175 cm tall, fit on film that is 24×36 mm?

34.37. Figure 34.41 shows photographs of the same scene taken with the same camera with lenses of different focal length. If the object is 200 m from the lens, what is the magnitude of the lateral magnification for a lens of focal length (a) 28 mm; (b) 105 mm; (c) 300 mm?

34.38. A photographer takes a photograph of a Boeing 747 airliner (length 70.7 m) when it is flying directly overhead at an altitude of 9.50 km. The lens has a focal length of 5.00 m. How long is the image of the airliner on the film?

34.39. Choosing a Camera Lens. The picture size on ordinary 35-mm camera film is 24 mm $\times$ 36 mm. Focal lengths of lenses available for 35-mm cameras typically include 28, 35, 50 (the "normal" lens), 85, 100, 135, 200, and 300 mm, among others. Which of these lenses should be used to photograph the following objects, assuming that the object is to fill most of the picture area? (a) a building 240 m tall and 160 m wide at a distance of 600 m, and (b) a mobile home 9.6 m in length at a distance of 40.0 m.

34.40. Zoom Lens. Consider the simple model of the zoom lens shown in Fig. 34.43a. The converging lens has focal length $f_1 = 12$ cm, and the diverging lens has focal length $f_2 = -12$ cm. The lenses are separated by 4 cm as shown in Fig. 34.43a. (a) For a distant object, where is the image of the converging lens? (b) The image of the converging lens serves as the object for the diverging lens. What is the object distance for the diverging lens? (c) Where is the final image? Compare your answer to Fig. 34.43a. (d) Repeat parts (a), (b), and (c) for the situation shown in Fig. 34.43b, in which the lenses are separated by 8 cm.

34.41. A camera lens has a focal length of 180.0 mm and an aperture diameter of 16.36 mm. (a) What is the f-number of the lens? (b) If the correct exposure of a certain scene is $\frac{1}{30}$ s at $f/11$, what is the correct exposure at $f/2.8$?

34.42. Recall that the intensity of light reaching film in a camera is proportional to the effective area of the lens. Camera A has a lens with an aperture diameter of 8.00 mm. It photographs an object using the correct exposure time of $\frac{1}{30}$ s. What exposure time should be used with camera B in photographing the same object with the same film if this camera has a lens with an aperture diameter of 23.1 mm?

34.43. Photography. A 35-mm camera has a standard lens with focal length 50 mm and can focus on objects between 45 cm and infinity. (a) Is the lens for such a camera a concave or a convex lens? (b) The camera is focused by rotating the lens, which moves it on the camera body and changes its distance from the film. In what range of distances between the lens and the film plane must the lens move to focus properly over the 45 cm to infinity range?

34.44. You wish to project the image of a slide on a screen 9.00 m from the lens of a slide projector. (a) If the slide is placed 15.0 cm from the lens, what focal length lens is required? (b) If the dimensions of the picture on a 35-mm color slide are 24 mm $\times$ 36 mm, what is the minimum size of the projector screen required to accommodate the image?

Section 34.6 The Eye

34.45. (a) Where is the near point of an eye for which a contact lens with a power of $+2.75$ diopters is prescribed? (b) Where is the far point of an eye for which a contact lens with a power of -1.30 diopters is prescribed for distant vision?

34.46. Curvature of the Cornea. In a simplified model of the human eye, the aqueous and vitreous humors and the lens all have a refractive index of 1.40, and all the refraction occurs at the cornea, whose vertex is 2.60 cm from the retina. What should be the radius of curvature of the cornea such that the image of an object 40.0 cm from the cornea's vertex is focused on the retina?

34.47. Corrective Lenses. Determine the power of the corrective contact lenses required by (a) a hyperopic eye whose near point is at 60.0 cm and (b) a myopic eye whose far point is at 60.0 cm.

Section 34.7 The Magnifier

34.48. A thin lens with a focal length of 6.00 cm is used as a simple magnifier. (a) What angular magnification is obtainable with the lens if the object is at the focal point? (b) When an object is examined through the lens, how close can it be brought to the lens? Assume that the image viewed by the eye is at the near point, 25.0 cm from the eye, and that the lens is very close to the eye.

34.49. The focal length of a simple magnifier is 8.00 cm. Assume the magnifier is a thin lens placed very close to the eye. (a) How far in front of the magnifier should an object be placed if the image is formed at the observer's near point, 25.0 cm in front of her eye? (b) If the object is 1.00 mm high, what is the height of its image formed by the magnifier?

34.50. You want to view an insect 2.00 mm in length through a magnifier. If the insect is to be at the focal point of the magnifier, what focal length will give the image of the insect an angular size of 0.025 radian?

34.51. You are examining an ant with a magnifying lens that has focal length 5.00 cm. If the image of the ant appears 25.0 cm from the lens, how far is the ant from the lens? On which side of the lens is the image located?

Section 34.8 Microscopes and Telescopes

34.52. Resolution of a Microscope. The image formed by a microscope objective with a focal length of 5.00 mm is 160 mm from its second focal point. The eyepiece has a focal length of 26.0 mm. (a) What is the angular magnification of the microscope? (b) The unaided eye can distinguish two points at its near point as separate if they are about 0.10 mm apart. What is the minimum separation between two points that can be observed (or resolved) through this microscope?

34.53. The focal length of the eyepiece of a certain microscope is 18.0 mm. The focal length of the objective is 8.00 mm. The distance between objective and eyepiece is 19.7 cm. The final image formed by the eyepiece is at infinity. Treat all lenses as thin. (a) What is the distance from the objective to the object being viewed? (b) What is the magnitude of the linear magnification produced by the objective? (c) What is the overall angular magnification of the microscope?

34.54. A certain microscope is provided with objectives that have focal lengths of 16 mm, 4 mm, and 1.9 mm and with eyepieces that have angular magnifications of 5$\times$ and 10$\times$. Each objective forms an image 120 mm beyond its second focal point. Determine (a) the largest overall angular magnification obtainable and (b) the least overall angular magnification obtainable.

34.55. The Yerkes refracting telescope of the University of Chicago has an objective 1.02 m in diameter with an f-number of 19.0. (This is the largest-diameter refracting telescope in the world.) What is its focal length?

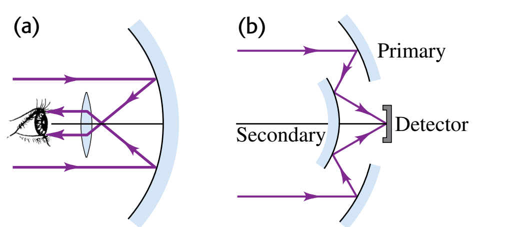
34.56. The eyepiece of a refracting telescope (see Fig. 34.53) has a focal length of 9.00 cm. The distance between objective and eyepiece is 1.80 m, and the final image is at infinity. What is the angular magnification of the telescope?

34.57. A telescope is constructed from two lenses with focal lengths of 95.0 cm and 15.0 cm, the 95.0-cm lens being used as the objective. Both the object being viewed and the final image are at infinity. (a) Find the angular magnification for the telescope. (b) Find the height of the image formed by the objective of a building 60.0 m tall, 3.00 km away. (c) What is the angular size of the final image as viewed by an eye very close to the eyepiece?

34.58. Saturn is viewed through the Lick Observatory refracting telescope (objective focal length 18 m). If the diameter of the image of Saturn produced by the objective is 1.7 mm, what angle does Saturn subtend from when viewed from earth?

34.59. A reflecting telescope (Fig. 34.55a) is to be made by using a spherical mirror with a radius of curvature of 1.30 m and an eyepiece with a focal length of 1.10 cm. The final image is at infinity. (a) What should the distance between the eyepiece and the mirror vertex be if the object is taken to be at infinity? (b) What will the angular magnification be?

Figure 34.55 Exercises 34.59 and 34.60 and Problem 34.112.



34.60. A Cassegrain telescope is a reflecting telescope that uses two mirrors, the secondary mirror focusing the image through a hole in the primary mirror (similar to that shown in Fig. 34.55b). You wish to focus the image of a distant galaxy onto the detector shown in the figure. If the primary mirror has a focal length of 2.5 m, the secondary mirror has a focal length of -1.5 m and the distance from the vertex of the primary mirror to the detector is 15 cm. What should be the distance between the vertices of the two mirrors?

Problems

34.61. If you run away from a plane mirror at 2.40 m/s, at what speed does your image move away from you?

34.62. An object is placed between two plane mirrors arranged at right angles to each other at a distance d_1 from the surface of one mirror and a distance d_2 from the other. (a) How many images are formed? Show the location of the images in a diagram. (b) Draw the paths of rays from the object to the eye of an observer.

34.63. What is the size of the smallest vertical plane mirror in which a woman of height h can see her full-length image?

34.64. A light bulb is 4.00 m from a wall. You are to use a concave mirror to project an image of the bulb on the wall, with the image 2.25 times the size of the object. How far should the mirror be from the wall? What should its radius of curvature be?

34.65. A concave mirror is to form an image of the filament of a headlight lamp on a screen 8.00 m from the mirror. The filament is 6.00 mm tall, and the image is to be 36.0 cm tall. (a) How far in front of the vertex of the mirror should the filament be placed? (b) What should be the radius of curvature of the mirror?

34.66. Rear-View Mirror. A mirror on the passenger side of your car is convex and has a radius of curvature with magnitude 18.0 cm. (a) Another car is seen in this side mirror and is 13.0 m behind the mirror. If this car is 1.5 m tall, what is the height of the image? (b) The mirror has a warning attached that objects viewed in it are closer than they appear. Why is this so?

34.67. Suppose the lamp filament shown in Example 34.1 (Section 34.2) is moved to a position 8.0 cm in front of the mirror. (a) Where is the image located now? Is it real or virtual? (b) What is the height of the image? Is it erect or inverted? (c) In Example 34.1, the filament is 10.0 cm in front of the mirror, and an image of the filament is formed on a wall 3.00 m from the mirror. If the filament is 8.0 cm from the mirror, can a wall be placed so that an image is formed on it? If so, where should the wall be placed? If not, why not?

34.68. Where must you place an object in front of a concave mirror with radius R so that the image is erect and $2\frac{1}{2}$ times the size of the object? Where is the image?

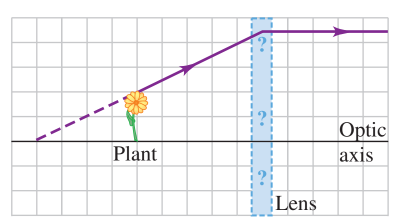
34.69. Virtual Object. If the light incident from the left onto a convex mirror does not diverge from an object point but instead converges toward a point at a (negative) distance s to the right of the mirror, this point is called a *virtual object.* (a) For a convex mirror having a radius of curvature of 24.0 cm, for what range of virtual-object positions is a real image formed? (b) What is the orientation of this real image? (c) Draw a principal-ray diagram showing the formation of such an image.

34.70. A layer of benzene $(n = 1.50)$ 2.60 cm deep floats on water $(n = 1.33)$ that is 6.50 cm deep. What is the apparent distance from the upper benzene surface to the bottom of the water layer when it is viewed at normal incidence?

34.71. Sketch the various possible thin lenses that can be obtained by combining two surfaces whose radii of curvature are 4.00 cm and 8.00 cm in absolute magnitude. Which are converging and which are diverging? Find the focal length of each if the surfaces are made of glass with index of refraction 1.60.

34.72. Figure 34.56 shows a small plant near a thin lens. The ray shown is one of the principal rays for the lens. Each square is 2.0 cm along the horizontal direction, but the vertical direction is not to the same scale. Use information from the diagram to answer the following questions: (a) Using only the ray shown, decide what type of lens (converging or diverging) this is. (b) What is the focal length of the lens? (c) Locate the image by drawing the other two principal rays. (d) Calculate where the image should be, and compare this result with the graphical solution in part (c).

Figure 34.56 Problem 34.72.



34.73. You are in your car driving on a highway at 25 m/s when you glance in the passenger side mirror (a convex mirror with radius of curvature 150 cm) and notice a truck approaching. If the image of the truck is approaching the vertex of the mirror at a speed of 1.5 m/s when the truck is 2.0 m away, what is the speed of the truck relative to the highway?

34.74. A microscope is focused on the upper surface of a glass plate. A second plate is then placed over the first. To focus on the bottom surface of the second plate, the microscope must be raised 0.780 mm. To focus on the upper surface, it must be raised another 2.50 mm. Find the index of refraction of the second plate.

34.75. Three-Dimensional Image. The *longitudinal* magnification is defined as $m' = ds'/ds$. It relates the longitudinal dimension of a small object to the longitudinal dimension of its image. (a) Show that for a spherical mirror, $m' = -m^2$. What is the significance of the fact that m' is *always* negative? (b) A wire frame in the form of a small cube 1.00 mm on a side is placed with its center on the axis of a concave mirror with radius of curvature 150.0 cm. The sides of the cube are all either parallel or perpendicular to the axis. The cube face toward the mirror is 200.0 cm to the left of the mirror vertex. Find (i) the location of the image of this face and of the opposite face of the cube; (ii) the lateral and longitudinal magnifications; (iii) the shape and dimensions of each of the six faces of the image.

34.76. Refer to Problem 34.75. Show that the longitudinal magnification m' for refraction at a spherical surface is given by

$$m' = -\frac{n_b}{n_a}m^2$$

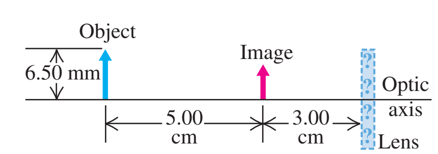
34.77. Pinhole Camera. A pinhole camera is just a rectangular box with a tiny hole in one face. The film is on the face opposite this hole, and that is where the image is formed. The camera forms an image *without* a lens. (a) Make a clear ray diagram to show how a pinhole camera can form an image on the film without using a lens. (*Hint:* Put an object outside the hole, and then draw rays passing through the hole to the opposite side of the box.) (b) A certain pinhole camera is a box that is 25 cm square and 20.0 cm deep, with the hole in the middle of one of the 25 cm × 25 cm faces. If this camera is used to photograph a fierce chicken that is 18 cm high and 1.5 m in front of the camera, how large is the image of this bird on the film? What is the magnification of this camera?

34.78. A Glass Rod. Both ends of a glass rod with index of refraction 1.60 are ground and polished to convex hemispherical surfaces. The radius of curvature at the left end is 6.00 cm, and the radius of curvature at the right end is 12.0 cm. The length of the rod between vertices is 40.0 cm. The object for the surface at the left end is an arrow that lies 23.0 cm to the left of the vertex of this surface. The arrow is 1.50 mm tall and at right angles to the axis. (a) What constitutes the object for the surface at the right end of the rod? (b) What is the object distance for this surface? (c) Is the object for this surface real or virtual? (*Hint:* See Problem 34.69.) (d) What is the position of the final image? (e) Is the final image real or virtual? Is it erect or inverted with respect to the original object? (f) What is the height of the final image?

34.79. The rod in Problem 34.78 is shortened to a distance of 25.0 cm between its vertices; the curvatures of its ends remain the same. As in Problem 34.78, the object for the surface at the left end is an arrow that lies 23.0 cm to the left of the vertex of this surface. The arrow is 1.50 mm tall and at right angles to the axis. (a) What is the object distance for the surface at the right end of the rod? (b) Is the object for this surface real or virtual? (c) What is the position of the final image? (d) Is the final image real or virtual? Is it erect or inverted with respect to the original object? (e) What is the height of the final image?

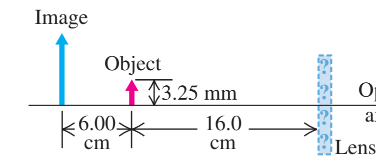
34.80. Figure 34.57 shows an object and its image formed by a thin lens. (a) What is the focal length of the lens, and what type of lens (converging or diverging) is it? (b) What is the height of the image? Is it real or virtual?

Figure 34.57 Problem 34.80



34.81. Figure 34.58 shows an object and its image formed by a thin lens. (a) What is the focal length of the lens, and what type of lens (converging or diverging) is it? (b) What is the height of the image? Is it real or virtual?

Figure 34.58 Problem 34.81



34.82. A transparent rod 30.0 cm long is cut flat at one end and rounded to a hemispherical surface of radius 10.0 cm at the other end. A small object is embedded within the rod along its axis and halfway between its ends, 15.0 cm from the flat end and 15.0 cm from the vertex of the curved end. When viewed from the flat end of the rod, the apparent depth of the object is 9.50 cm from the flat end. What is its apparent depth when viewed from the curved end?

34.83. A solid glass hemisphere of radius 12.0 cm and index of refraction $n = 1.50$ is placed with its flat face downward on a table. A parallel beam of light with a circular cross section 3.80 mm in diameter travels straight down and enters the hemisphere at the center of its curved surface. (a) What is the diameter of the circle of light formed on the table? (b) How does your result depend on the radius of the hemisphere?

34.84. A thick-walled wine goblet sitting on a table can be considered to be a hollow glass sphere with an outer radius of 4.00 cm and an inner radius of 3.40 cm. The index of refraction of the goblet glass is 1.50. (a) A beam of parallel light rays enters the side of the empty goblet along a horizontal radius. Where, if anywhere, will an image be formed? (b) The goblet is filled with white wine $(n = 1.37)$. Where is the image formed?

34.85. Focus of the Eye. The cornea of the eye has a radius of curvature of approximately 0.50 cm, and the aqueous humor behind it has an index of refraction of 1.35. The thickness of the cornea itself is small enough that we shall neglect it. The depth of a typical human eye is around 25 mm. (a) What would have to be the radius of curvature of the cornea so that it alone would focus the image of a distant mountain on the retina, which is at the back of the eye opposite the cornea? (b) If the cornea focused the mountain correctly on the retina as described in part (a), would it also focus the text from a computer screen on the retina if that screen were 25 cm in front of the eye? If not, where would it focus that text: in front of or behind the retina? (c) Given that the cornea has a radius of curvature of about 5.0 mm, where does it actually focus the mountain? Is this in front of or behind the retina? Does this help you see why the eye needs help from a lens to complete the task of focusing?

34.86. A transparent rod 50.0 cm long and with a refractive index of 1.60 is cut flat at the right end and rounded to a hemispherical surface with a 15.0-cm radius at the left end. An object is placed on the axis of the rod 12.0 cm to the left of the vertex of the hemispherical end. (a) What is the position of the final image? (b) What is its magnification?

34.87. What should be the index of refraction of a transparent sphere in order for paraxial rays from an infinitely distant object to be brought to a focus at the vertex of the surface opposite the point of incidence?

34.88. A glass rod with a refractive index of 1.55 is ground and polished at both ends to hemispherical surfaces with radii of 6.00 cm. When an object is placed on the axis of the rod, 25.0 cm to the left of the left-hand end, the final image is formed 65.0 cm to the right of the right-hand end. What is the length of the rod measured between the vertices of the two hemispherical surfaces?

34.89. Two thin lenses with focal lengths of magnitude 15.0 cm, the first diverging and the second converging, are placed 12.00 cm apart. An object 4.00 mm tall is placed 5.00 cm to the left of the first (diverging) lens. (a) Where is the image formed by the first lens located? (b) How far from the object is the final image formed? (c) Is the final image real or virtual? (d) What is the height of the final image? Is the final image erect or inverted?

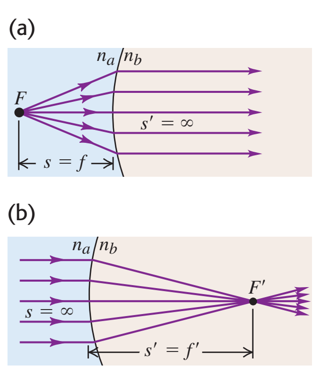
34.90. The radii of curvature of the surfaces of a thin converging meniscus lens are $R_1 = +12.0$ cm and $R_2 = +28.0$ cm. The index of refraction is 1.60. (a) Compute the position and size of the image of an object in the form of an arrow 5.00 mm tall, perpendicular to the lens axis, 45.0 cm to the left of the lens. (b) A second converging lens with the same focal length is placed 3.15 m to the right of the first. Find the position and size of the final image. Is the final image erect or inverted with respect to the original object? (c) Repeat part (b) except with the second lens 45.0 cm to the right of the first.

34.91. An object to the left of a lens is imaged by the lens on a screen 30.0 cm to the right of the lens. When the lens is moved 4.00 cm to the right, the screen must be moved 4.00 cm to the left to refocus the image. Determine the focal length of the lens.

34.92. For refraction at a spherical surface, the first focal length f is defined as the value of s corresponding to $s' = \infty$, as shown in Fig. 34.59a. The second focal length f' is defined as the value of s' when $s = \infty$, as shown in Fig. 34.59b. (a) Prove that $n_a/n_b = f/f'$. (b) Prove that the general relationship between object and image distance is

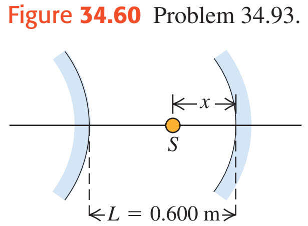
$$\frac{f}{s} + \frac{f'}{s'} = 1$$

Figure 34.59 Problem 34.92.



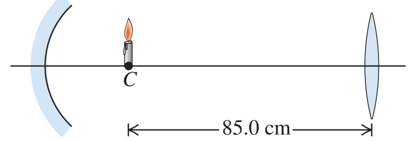
34.93. A convex mirror and a concave mirror are placed on the same optic axis, separated by a distance $L = 0.600$ m. The radius of curvature of each mirror has a magnitude of 0.360 m. A light source is located a distance x from the concave mirror, as shown in Fig. 34.60. (a) What distance x will result in the rays from the source returning to the source after reflecting first from the convex mirror and then from the concave mirror? (b) Repeat part (a), but now let the rays reflect first from the concave mirror and then from the convex one.

Figure 34.60 Problem 34.93.



34.94. As shown in Fig. 34.61 the candle is at the center of curvature of the concave mirror, whose focal length is 10.0 cm. The converging lens has a focal length of 32.0 cm and is 85.0 cm to the right of the candle. The candle is viewed looking through the lens from the right. The lens forms two images of the candle. The first is formed by light passing directly through the lens. The second image is formed from the light that goes from the candle to the mirror, is reflected, and then passes through the lens. (a) For each of these two images, draw a principal-ray diagram that locates the image. (b) For each image, answer the following questions: (i) Where is the image? (ii) Is the image real or virtual? (iii) Is the image erect or inverted with respect to the original object?

Figure 34.61 Problem 34.94.



34.95. One end of a long glass rod is ground to a convex hemispherical shape. This glass has an index of refraction of 1.55. When a small leaf is placed 20.0 cm in front of the center of the hemisphere along the optic axis, an image is formed inside the glass 9.12 cm from the spherical surface. Where would the image be formed if the glass were now immersed in water (refractive index 1.33) but nothing else were changed?

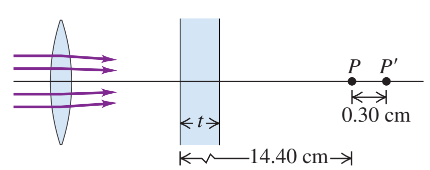
34.96. Two Lenses in Contact. (a) Prove that when two thin lenses with focal lengths f_1 and f_2 are placed *in contact,* the focal length f of the combination is given by the relationship

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

(b) A converging meniscus lens (see Fig. 34.32a) has an index of refraction of 1.55 and radii of curvature for its surfaces of 4.50 cm and 9.00 cm. The concave surface is placed upward and filled with carbon tetrachloride (CCl_4), which has $n = 1.46$. What is the focal length of the CCl_4–glass combination?

34.97. Rays from a lens are converging toward a point image P located to the right of the lens. What thickness t of glass with index of refraction 1.60 must be interposed between the lens and P for the image to be formed at P', located 0.30 cm to the right of P? The locations of the piece of glass and of points P and P' are shown in Fig. 34.62.

Figure 34.62 Problem 34.97.



34.98. A Lens in a Liquid. A lens obeys Snell's law, bending light rays at each surface an amount determined by the index of refraction of the lens and the index of the medium in which the lens is located. (a) Equation (34.19) assumes that the lens is surrounded by air. Consider instead a thin lens immersed in a liquid with refractive index n_{liq}. Prove that the focal length f' is then given by Eq. (34.19) with n replaced by n/n_{liq}. (b) A thin lens with index n has focal length f in vacuum. Use the result of part (a) to show that when this lens is immersed in a liquid of index n_{liq}, it will have a new focal length given by

$$f' = \left[\frac{n_{liq}(n-1)}{n - n_{liq}}\right] f$$

34.99. When an object is placed at the proper distance to the left of a converging lens, the image is focused on a screen 30.0 cm to the right of the lens. A diverging lens is now placed 15.0 cm to the right of the converging lens, and it is found that the screen must be moved 19.2 cm farther to the right to obtain a sharp image. What is the focal length of the diverging lens?

34.100. A convex spherical mirror with a focal length of magnitude 24.0 cm is placed 20.0 cm to the left of a plane mirror. An object 0.250 cm tall is placed midway between the surface of the plane mirror and the vertex of the spherical mirror. The spherical mirror forms multiple images of the object. Where are the two images of the object formed by the spherical mirror that are closest to the spherical mirror, and how tall is each image?

34.101. A glass plate 3.50 cm thick, with an index of refraction of 1.55 and plane parallel faces, is held with its faces horizontal and its lower face 6.00 cm above a printed page. Find the position of the image of the page formed by rays making a small angle with the normal to the plate.

34.102. A symmetric, double-convex, thin lens made of glass with index of refraction 1.52 has a focal length in air of 40.0 cm. The lens is sealed into an opening in the left-hand end of a tank filled with water. At the right-hand end of the tank, opposite the lens, is a plane mirror 90.0 cm from the lens. The index of refraction of the water is $\frac{4}{3}$. (a) Find the position of the image formed by the lens–water–mirror system of a small object outside the tank on the lens axis and 70.0 cm to the left of the lens. (b) Is the image real or virtual? (c) Is it erect or inverted? (d) If the object has a height of 4.00 mm, what is the height of the image?

34.103. You have a camera with a 35.0-mm focal length lens and 36.0-mm-wide film. You wish to take a picture of a 12.0-m-long sailboat but find that the image of the boat fills only $\frac{1}{4}$ of the width of the film. (a) How far are you from the boat? (b) How much closer must the boat be to you for its image to fill the width of the film?

34.104. An object is placed 18.0 cm from a screen. (a) At what two points between object and screen may a converging lens with a 3.00-cm focal length be placed to obtain an image on the screen? (b) What is the magnification of the image for each position of the lens?

34.105. Three thin lenses, each with a focal length of 40.0 cm, are aligned on a common axis; adjacent lenses are separated by 52.0 cm. Find the position of the image of a small object on the axis, 80.0 cm to the left of the first lens.

34.106. A camera with a 90-mm-focal-length lens is focused on an object 1.30 m from the lens. To refocus on an object 6.50 m from the lens, by how much must the distance between the lens and the film be changed? To refocus on the more distant object, is the lens moved toward or away from the film?

34.107. The derivation of the expression for angular magnification, Eq. (34.22), assumed a near point of 25 cm. In fact, the near point changes with age as shown in Table 34.1. In order to achieve an angular magnification of 2.0×, what focal length should be used by a person of (a) age 10; (b) age 30; (c) age 60? (d) If the lens that gives $M = 2.0$ for a 10-year-old is used by a 60-year-old, what angular magnification will the older viewer obtain? (e) Does your answer in part (d) mean that older viewers are able to see more highly magnified images than younger viewers? Explain.

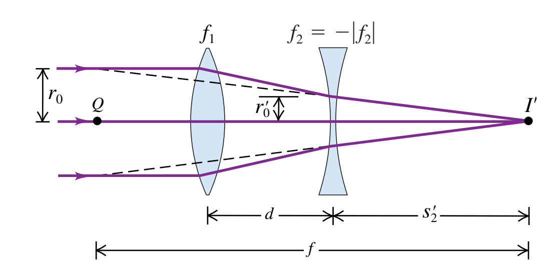
34.108. Angular Magnification. In deriving Eq. (34.22) for the angular magnification of a magnifier, we assumed that the object is placed at the focal point of the magnifier so that the virtual image is formed at infinity. Suppose instead that the object is placed so that the virtual image appears at an average viewer's near point of 25 cm, the closest point at which the viewer can bring an object into focus. (a) Where should the object be placed to achieve this? Give your answer in terms of the magnifier focal length f. (b) What angle θ' will an object of height y subtend at the position found in part (a)? (c) Find the angular magnification M with the object at the position found in part (a). The angle θ is the same as in Fig. 34.51a, since it refers to viewing the object *without* the magnifier. (d) For a convex lens with $f = +10.0$ cm, what is the value of M with the object at the position found in part (a)? How many times greater is M in this case than in the case where the image is formed at infinity? (e) In the description of a compound microscope in Section 34.8, it is stated that in a properly designed instrument, the real image formed by the objective lies *just inside* the first focal point F_1' of the eyepiece. What advantages are gained by having the image formed by the objective be just inside F_1', as opposed to precisely at F_1'? What happens if the image formed by the objective is *just outside* F_1'?

34.109. In one form of cataract surgery the person's natural lens, which has become cloudy, is replaced by an artificial lens. The refracting properties of the replacement lens can be chosen so that the person's eye focuses on distant objects. But there is no accommodation, and glasses or contact lenses are needed for close vision. What is the power, in diopters, of the corrective contact lenses that will enable a person who has had such surgery to focus on the page of a book at a distance of 24 cm?

34.110. A Nearsighted Eye. A certain very nearsighted person cannot focus on anything farther than 36.0 cm from the eye. Consider the simplified model of the eye described in Exercise 34.46. If the radius of curvature of the cornea is 0.75 cm when the eye is focusing on an object 36.0 cm from the cornea vertex and the indexes of refraction are as described in Exercise 34.46, what is the distance from the cornea vertex to the retina? What does this tell you about the shape of the nearsighted eye?

34.111. Focal Length of a Zoom Lens. Figure 34.63 shows a simple version of a zoom lens. The converging lens has focal length f_1, and the diverging lens has focal length $f_2 = -|f_2|$. The two lenses are separated by a variable distance d that is always less than f_1. Also, the magnitude of the focal length of the diverging lens satisfies the inequality $|f_2| > (f_1 - d)$. To determine the effective focal length of the combination lens, consider a bundle of parallel rays of radius r_0 entering the converging lens. (a) Show that the radius of the ray bundle decreases to $r_0' = r_0(f_1 - d)/f_1$ at the point that it enters the diverging lens. (b) Show that the final image I' is formed a distance $s_2' = |f_2|(f_1 - d)/(|f_2| - f_1 + d)$ to the right of the diverging lens. (c) If the rays that emerge from the diverging lens and reach the final image point are extended backward to the left of the diverging lens, they will eventually expand to the original radius r_0 at some point Q. The distance from the final image I' to the point Q is the *effective focal length* f of the lens combination; if the combination were replaced by a single lens of focal length f placed at Q, parallel rays would still be

Figure 34.63 Problem 34.111.



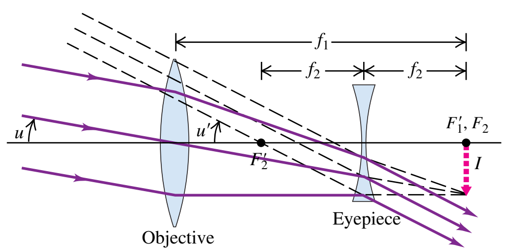
brought to a focus at I'. Show that the effective focal length is given by $f = f_1|f_2|/(|f_2| - f_1 + d)$. (d) If $f_1 = 12.0$ cm, $f_2 = -18.0$ cm, and the separation d is adjustable between 0 and 4.0 cm, find the maximum and minimum focal lengths of the combination. What value of d gives $f = 30.0$ cm?

34.112. A certain reflecting telescope, constructed as shown in Fig. 34.55a, has a spherical mirror with a radius of curvature of 96.0 cm and an eyepiece with a focal length of 1.20 cm. If the angular magnification has a magnitude of 36 and the object is at infinity, find the position of the eyepiece and the position and nature (real or virtual) of the final image. (*Note:* $|M|$ is *not* equal to $|f_1/f_2|$, so the image formed by the eyepiece is *not* at infinity.)

34.113. A microscope with an objective of focal length 8.00 mm and an eyepiece of focal length 7.50 cm is used to project an image on a screen 2.00 m from the eyepiece. Let the image distance of the objective be 18.0 cm. (a) What is the lateral magnification of the image? (b) What is the distance between the objective and the eyepiece?

34.114. The Galilean Telescope. Figure 34.64 is a diagram of a *Galilean telescope,* or *opera glass,* with both the object and its final image at infinity. The image I serves as a virtual object for the eyepiece. The final image is virtual and erect. (a) Prove that the angular magnification is $M = -f_1/f_2$. (b) A Galilean telescope is to be constructed with the same objective lens as in Exercise 34.57. What focal length should the eyepiece have if this telescope is to have the same magnitude of angular magnification as the one in Exercise 34.57? (c) Compare the lengths of the telescopes.

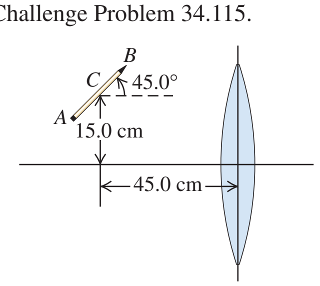
Figure 34.64 Problem 34.114.



Challenge Problems

34.115. An Object at an Angle. A 16.0-cm-long pencil is placed at a 45.0° angle, with its center 15.0 cm above the optic axis and 45.0 cm from a lens with a 20.0-cm focal length as shown in Fig. 34.65. (Note that the figure is not drawn to scale.) Assume that the diameter of the lens is large enough for the paraxial approximation to be valid. (a) Where is the image of the pencil? (Give the location of the images of the points A, B, and C on the object, which are located at the eraser, point, and center of the pencil, respectively.) (b) What is the length of the image (that is, the distance between the images of points A and B)? (c) Show the orientation of the image in a sketch.

Figure 34.65 Challenge Problem 34.115.

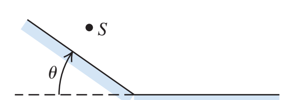


34.116. *Spherical aberration* is a blurring of the image formed by a spherical mirror. It occurs because parallel rays striking the mirror far from the optic axis are focused at a different point than are rays near the axis. This problem is usually minimized by using only the center of a spherical mirror. (a) Show that for a spherical concave mirror, the focus moves toward the mirror as the parallel rays move toward the outer edge of the mirror. (*Hint:* Derive an analytic expression for the distance from the vertex to the focus of the ray for a particular parallel ray. This expression should be in terms of (i) the radius of curvature R of the mirror and (ii) the angle θ between the incident ray and a line connecting the center of curvature of the mirror with the point where the ray strikes the mirror.) (b) What value of θ produces a 2% change in the location of the focus, compared to the location for θ very close to zero?

34.117. (a) For a lens with focal length f, find the smallest distance possible between the object and its real image. (b) Graph the distance between the object and the real image as a function of the distance of the object from the lens. Does your graph agree with the result you found in part (a)?

34.118. Two mirrors are placed together as shown in Fig. 34.66. (a) Show that a point source in front of these mirrors and its two images lie on a circle. (b) Find the center of the circle. (c) In a diagram, show where an observer should stand so as to be able to see both images.

Figure 34.66 Challenge Problem 34.118.



34.119. People with normal vision cannot focus their eyes underwater if they aren't wearing a face mask or goggles and there is water in contact with their eyes (see Discussion Question Q34.23). (a) Why not? (b) With the simplified model of the eye described in Exercise 34.46, what corrective lens (specified by focal length as measured in air) would be needed to enable a person underwater to focus an infinitely distant object? (Be careful—the focal length of a lens underwater is *not* the same as in air! See Problem 34.98. Assume that the corrective lens has a refractive index of 1.62 and that the lens is used in eyeglasses, not goggles, so there is water on both sides of the lens. Assume that the eyeglasses are 2.00 cm in front of the eye.)