Solving equations (3.83) and (3.84) and defining $\phi(x_P) = 0$, gives

$$\phi(x) = \begin{cases} V_{\text{bi}}, x \leq -x_N \\ V_{\text{bi}} - \dfrac{qN_{\text{D}}}{2\varepsilon}(x + x_N)^2, -x_N < x \leq 0 \\ \dfrac{qN_{\text{A}}}{2\varepsilon}(x - x_P)^2, 0 \leq x < x_P \\ 0, x \geq x_P \end{cases} \tag{3.86}$$

The electrostatic potential must be continuous at $x = 0$. Therefore, from equation (3.86),

$$V_{\text{bi}} - \frac{qN_{\text{D}}}{2\varepsilon}x_N^2 = \frac{qN_{\text{A}}}{2\varepsilon}x_P^2 \tag{3.87}$$

In the absence of any interface charge at the metallurgical junction, the electric field is also continuous at this point (really, it is the displacement field, $\vec{D} = \varepsilon\vec{E}$, but in this example, ε is independent of position), and

$$x_N N_{\text{D}} = x_P N_{\text{A}} \tag{3.88}$$

This is simply a statement that the total charge in either side of the depletion region exactly balance each other and therefore the depletion region extends furthest into the more lightly doped side.

Solving equations (3.87) and (3.88) for the depletion width, W_{D}, gives[3]

$$W_{\text{D}} = x_N + x_P = \sqrt{\frac{2\varepsilon}{q}\left(\frac{N_{\text{A}} + N_{\text{D}}}{N_{\text{A}}N_{\text{D}}}\right)V_{\text{bi}}}. \tag{3.89}$$

Under nonequilibrium conditions, the electrostatic potential difference across the junction is modified by the applied voltage, V, which is zero in thermal equilibrium. As a consequence, the depletion width is dependent on the applied voltage,

$$W_{\text{D}}(V) = x_N + x_P = \sqrt{\frac{2\varepsilon}{q}\left(\frac{N_{\text{A}} + N_{\text{D}}}{N_{\text{A}}N_{\text{D}}}\right)(V_{\text{bi}} - V)}. \tag{3.90}$$

As previously stated, the built-in voltage, V_{bi}, can be calculated by noting that under thermal equilibrium the net hole and electron currents are zero. The hole current density is

$$\vec{J}_p = q\mu_p p_o \vec{E} - qD_p \nabla p = 0. \tag{3.91}$$

[3] A somewhat more rigorous treatment of equation 3.89 would yield a factor of $2kT/q$ which is ~50 mV at 300 K, or

$$W_{\text{D}} = \sqrt{\frac{2\varepsilon}{q}\left(\frac{N_{\text{A}} + N_{\text{D}}}{N_{\text{A}}N_{\text{D}}}\right)(V_{\text{bi}} - 2kT/q)} \quad [3].$$