characterised by a steady-state temperature field and the liquid–solid phase boundary can be modelled by a fixed-temperature boundary condition, as the silicon melting temperature of 1410°C. In the case of the ingot casting, the phase boundary moves through the crystal and the release of latent heat can be modelled by an enthalpy formulation. By this, the release of the latent heat of finite elements can be taken into account directly after their total solidification, or the fraction of latent heat must be considered for partly solidified elements [87, 88]. The importance of an accurate modelling of the release of latent heat may become more clear by estimating the crystallised volume rate in typical ingot processes to be around 9000 cm^3 per hour, which means a latent heat source of more than 8 kW at the location of the phase boundary. Pulling one 10-cm-wide ribbon, the crystallised volume rate is about 30 cm^3 per hour with a latent heat release of 0.03 kW.

The Czochralski and TriSi crystal-pulling techniques can be classified between ribbon and ingot crystallisation. The temperature profile can be assumed to be stationary and the SV parameter, given as 1/crystal diameter, lies, for example, in the range of 0.066/cm.

In general, the heat flow in silicon during crystallisation can be described by the heat-transport equation [89–91]:

$$\rho c_p \frac{\partial T}{\partial t} = \lambda \nabla^2 T + L \frac{\partial f_c}{\partial t}$$

with the silicon data:

density of solid silicon	$\rho_{(1410°C)}$	=	2.30 [g/cm^3]
density of liquid silicon	$\rho_{(1411°C)}$	=	2.53 [g/cm^3]
heat capacity	$c_{p(20°C)}$	=	0.83 [J/g K]
	$c_{p(1410°C)}$	=	1.03 [J/g K]
heat conductivity	$\lambda_{(20°C)}$	=	1.68 [W/cm K]
	$\lambda_{(1410°C)}$	=	0.31 [W/cm K]
latent heat of phase change	L	=	3300 [J/cm^3]

The time t and the temperature T are variable and result from the simulation. For the moving boundary case the solid fraction f_c becomes important. This parameter depends on the finite element temperature and varies between zero for a completely liquid finite element and one for a solid element.

Additionally to these material properties and the heat flow mechanisms in the silicon material, the description of the internal furnace construction must be taken into account to perform simulations of crystallisation facilities. This includes the geometrical description and material properties of the internal set-up as well as the radiative heat exchange with heaters and cooling facilities.

6.6.3 Simulation of Bulk Silicon Crystallisation

As an example of the temperature simulation of silicon ingot crystallisation, the SOPLIN casting technique is selected. To simulate this process, a finite element mesh of about 230 000 elements was built to describe the furnace geometry. This mesh includes the silicon ingot, the mould, all insulation materials and active heating and cooling facilities,