If r_1 is the radius of the radiator and r_2 is the distance from the center of the radiator to the concentrator input, then it is clear from Figure 11.20 that $\sin\theta_{\text{max,in}} = r_1/r_2$. If we assume that the radiator is radiating as a black body, then the total power density at the surface of the radiator is $P_{\text{rad}} = \sigma T_s^4$, where σ is the Stefan Boltzmann constant and T_s is the radiator source temperature. Simple geometry gives the power density at the entrance to the concentrator as $P_{\text{canc}} = (r_1/r_2)^2 P_{\text{rad}} = \sin^2\theta_{\text{max,in}} P_{\text{rad}}$. The power density at the receiver is simply that at the concentrator entrance, multiplied by the concentration ratio, so $P_{\text{rec}} = C \sin^2\theta_{\text{max,in}} P_{\text{rad}}$. Now, imagine that the receiver is a black body insulated so that its only heat loss is by thermal radiation back from the receiving surface. Then the receiver will heat up until it is losing power by thermal radiation at the same rate it is receiving it. This will occur when the receiver is at a temperature such that $P_{\text{rec}} = \sigma T_r^4$. Equating this to the received power above

$$P_{\text{rec}} = \sigma T_{\text{r}}^4 = C \sin^2\theta_{\text{max,in}} P_{\text{rad}} = C \sin^2\theta_{\text{max,in}} \sigma T_{\text{s}}^4$$

or

$$C = \frac{T_{\text{r}}^4}{T_{\text{s}}^4} \frac{1}{\sin^2\theta_{\text{max,in}}}$$

Now, here is the thermodynamic part. It must be that $T_{\text{r}} \leq T_{\text{s}}$, for otherwise heat would be transferred from the source at a lower temperature to the receiver at a higher temperature without the use of work, in violation of the second law of thermodynamics. In other words, if the receiver was hotter than the source, one could build a perpetual motion machine. Inserting this inequality into the above equation gives

$$C \leq C_{\text{max}} = \frac{1}{\sin^2\theta_{\text{max,in}}}$$

The origin of the n^2 in the concentration equation can be seen from the fact that in a dielectric medium, the Stefan Boltzmann law becomes $P = n^2\sigma T^4$. The factor n^2 comes from the fact that the three-dimensional electromagnetic mode density is increased by n^3 owing to the decrease in wavelength by $1/n$, but the speed of light is decreased by $1/n$. Therefore, in thermal equilibrium, the density of photons is increased by n^3, but the number of photons crossing a surface per unit time, and hence the power crossing this surface, is increased by only n^2.

If a concentrator can accept light from anywhere in the upper hemisphere, then $\theta_{\text{max}} = 90^\circ$ and $C_{\text{max}} = 1$, or $C_{\text{max}} = n^2$ if the receiver is immersed in a dielectric. The history of concentrators is replete with proposed concentrators that achieve high concentration without tracking the sun; that is, it will accept light from any point in the sky. In each case the concept is supposed to skirt the maximum concentration theorem by some means. Examples are the use of diffraction (as with holograms), or bent light fibers, or quantum dyes (as in luminescent concentrators[13]). Most derivations of the maximum concentration theorem are based on geometric optics, and so it is conceivable that it could be

[13] The case of luminescent concentrators requires more care because of the spectral shift. This can be done by incorporating spectral filters into the thermodynamic analysis, but the finding is that they have rather limited concentration and efficiency potential.