minuunt. ut post ea ligb. St ū ī teg̅. vuii. sufficientes ad infini-
tā multiplicatione. quąₑ noīa singl̄is s̄t supiecta.

celentes zemenias zeṅis. chalcus. auunes. arbas. ornus. andjas. ioiti.

Verū minuitię sunt .xx. uii. subscriptę cū nominib; suis.

deunx. dextans dodrans bisse. septunx. semis ąncux. triens quadrans.

sixtans sexcuncia uncia. semuncia. duella sicilicus seatula. dragma.

emissosela tremissis scripulus. obulus. cerates. siliqua. chalcus.

his caracterib; firmiter memorię com mandatis ꝫ quasi oculis
cordis subiacentib; misteriis eąₑ. diuide potis ꝫ multipli-

Rara Mathematica;

OR,

A COLLECTION

OF

TREATISES ON THE MATHEMATICS

AND

SUBJECTS CONNECTED WITH THEM,

From ancient inedited Manuscripts.

EDITED BY

JAMES ORCHARD HALLIWELL, Esq., f.r.s., f.s.a.,

&c. &c. &c.

OF JESUS COLLEGE, CAMBRIDGE.

"I see not how the history of any science is to be conducted through the middle-age period, but by the assistanc of the works of science of the middle-age writers, though the contents of them, as works of science, may have now become of little value."
Rev. J. Hunter's Monastic Libraries, Pref. p. xi.

THE SECOND EDITION.

LONDON:

PREFACE.

THIS is a new impression rather than a new edition of a work which has been so favourably received, that the editor is induced to submit it once more to the notice of the reading public. The following notes on some of the tracts here printed, remain nearly the same as in the other edition.

I. *Johannes de Sacro-Bosco de Arithmetica.* Often occurs in MSS. without his name; MSS. Harl. 3647. 3843. 4350 ; Bib. Reg. 12 C. xvii ; Arund. 343. Cott. Cleop. B. vi. f. 234; Publ. Cantab. Ii. I. 15. (1692). An English translation—Ashm. 396. The present text is taken from a manuscript formerly in the Library of the Abbate Canonici of Venice.

III. *A Treatise on the Numeration of Algorism.*—This is taken from a single leaf of vellum, found loose in an old MS. on astronomy, formerly in Dr. Clarke's library.

IV. *Bourne's Treatise on Optical Glasses.*—The other work he mentions in his dedication, as having been inscribed to Lord Burghley, is in the British Museum.—MS. Sloan. 3651.

V. *Johannes Robyns de Cometis.*—From a MS. in my own library. Other copies, Bib. Reg. 12 B. xv. and Trin. Coll. Cantab. inter MSS. Gal. O. I. 11.

VIII. *A Merchant's Account Table.*—This table is exceedingly curious, conducted partly similar to an abacus, the cyphers at the bottom being used to guide the manual calculator.

IX. *Carmen de Algorismo.* — A MS. of the Massa Compoti in the British Museum (Harl. 3902), by Alexander de Villa Dei, possesses an introduction to the work by some other author: it is there stated that the same author composed *Doctrinale et Algorismum Metricum.* M. Chasles informs me that a MS. of this tract in the French King's Library (7420. A.) has the following colophon at the end : *Explicit Algorismus editus a᾽ Magistro Alexandro de Villa Dei.* This is, I think, quite sufficient to prove him to be the author.

MSS. of it are very numerous : I will mention a few for guidance. MSS. Bib. Reg. 8 C. iv. 12 E. i. Cot. Vitell. A. i. (first chapter). Trin. Coll. Cant. O. v. 4. ii. 45. i. 31. S. Joh. F. 18. Publ. Mm. iii. 11. (2310). Ii. i. 13. (1690.) i. 15. (1692). Bodl. 57. Fairf. 27. Digb. 15. 22. 81. 97. 98 104. 190. Bodl. 4to. D. 21. Jur.

Fragments MSS. Sloan. 513. 1620. 2397.—Many copies of it occur in the Catalogue of the Library of the Dover Monastery (MS. Bodl. 920.) made in 1389.— *Vid.* Monast. iv. p. 532.

MS. Digb. 104, has Ambrosinus for Algorismus in the first line. MS. Sloan. 513, has the following colophon—

" Explicit tractatus algorismi tum satis breve et bono commento secundum Saxton.

Qui scripsit carmen sit benedictus. Amen !

Nomen scriptoris Galfridus plenus Amoris."

Whose name is here latinized I know not, but I am not inclined to give much credit to it.

XII. *The Preface to an Almanac for* 1430.—It was pointed out to me by Professor De Morgan, that some portions of this are evidently plagiarized from Chaucer's preface to his tract on the Astrolabe.

35, *Alfred Place, London, Jan. 1st,* 1840.

CONTENTS.

CORRECTIONS AND ADDITIONS.

P. 2, *l.* 31.—*For* idem, *read* eandem.

P. 3, *l.* 13.—*For* muneri, *read* numeri.

P. 6, *l.* 20.—*For* nonenarii, *read* novenarii, *and in other places.*

P. 7, *l.* 18.—*For* supponatur, *read* supraponatur.

P. 11, *l.* 25, 29.—*For* vide, *read* vid.

P. 12, *l.* 22.—*For* quinam, *read* quando.

P. 15, *l.* 21.—*For* perdictis, *read* prædictis.

P. 19, *l.* 11.—*For* quidenarius, *read* quindenarius.

P. 19, *l.* 25.—*For* perambulum, *read* præambulum.

P. 26, *l.* 15.—Sibi ; *so in the manuscript, but evidently* sub.

P. 29, *l.* 4.—*For* 14, *read* 15.

P. 30, *l.* 2, 6, 7 ; P. 31. *l.* 1, 13.—It will be readily seen that *y* is in-serted in these places for *th*, the Saxon character having been mistaken by the printer.

P. 51, *l.* 17.—*For* ince, *read* vice. I may mention here, that it was pointed out to me by G. S. Venables, Esq., that a portion of this preface is plagiarised from Cicero.

P. 54, *l.* 18.—Cædere ; *so in the manuscript, but evidently* cedere.

P. 55.—This table has been previously noticed, and in part printed by J. W. Lubbock, Esq., in *Phil. Trans.*

P. 55, Second table, *l.* 1.—*For* 10, *read* 0.

P. 56. The poem quoted in the note has since been printed in my " Early History of Freemasonry in England."

P. 57, *l.* 17.—*For* Bathoniensum, *read* Bathoniensem.

P. 73, *l.* 21.—*For* nihil, *read* in his.

P. 73, *l.* 22.—*For* potuit, *read* patuit.

P. 84, *l.* 12.—*For* descriptos, *read* desuper.

P. 85, *l.* 16.—*For* principem, *read* patrem.

NOTE.

The following volume was originally published at three several times: the first part (pp. 1—48) appeared on the 1st of June, 1838; the second (pp. 49—96) on the 1st of October, 1838: and the remainder on the 1st of June, 1839. It ought to be particularly remarked, that in no instance has the regular correction of the errors in the MSS. from which these tracts are printed been attempted.

JOANNIS DE SACRO-BOSCO

TRACTATUS

DE ARTE NUMERANDI.

OMNIA quæ a primæva rerum origine pro-
cesserunt ratione numerorum formata sunt,[1] et
quemadmodum sunt, sic cognosci habent : unde
in universa rerum cognitione, ars numerandi est
operativa. Hanc igitur scientiam numerandi
compendiosam edidit philosophus nomine Algus,[2]
unde algorismus nuncupatur, vel ars numerandi,

[1] "Omnia quæcunque a primæva rerum natura constructa
sunt, numerorum videntur ratione formata."—*Boetii Arith.* lib. i.
cap. 2, *Edit. Par.* 1521, fol. 8. Vid. Hen. Welpii Arith. Prac-
tica, 4to. *Colon.* 1543. Enchiridion Algorismi per Joannem
Huswirt, 4to. *Colon.* 1501. Licht de Algorismo, 4to. *Leip.*
1500.

[2] "Rex quondam Castelliæ." *Johannis Norfolk progressionis
summula,* MS. Harl. Mus. Brit. 3742. "Cum hæc scientia de
numeris quæ algorismus ab inventore vel ab Algo, quæ est in-
ductio et rismus, quæ est numerus quasi inductio in numeros
appellatur."—*Tractatus de Algorismo,* MS. Arundel. M.B. 332,
fol. 68. Vid. Pref. a " Œuvre tresubtille et profitable de Arith-
metique et Geom." 4to. *Par.* 1515, Sig. B. 2.

vel introductio in numerum. Numerus quidem
dupliciter notificatur, formaliter et materialiter :
formaliter ut numerus est multitudo ex unita-
tibus aggregata : materialiter ut numerus est
unitates collectæ. Unitas autem est qua una-
quæque res una dicitur. Numerorum alius
digitus ;[3] alius articulus ; alius numerus com-
positus sive mixtus. Digitus quidem dicitur
omnis numerus minor denario ; articulus vero
est omnis numerus qui potest dividi in decem
partes æquales, ita quod nihil residuum sit ;
compositus vero sive mixtus est qui constat
ex digito et articulo. Et sciendum est quod
omnis numerus inter duos articulos proximos
est numerus compositus. Hujus autem artis
novem[4] sunt species ; scilicet, numeratio, ad-
ditio, subtractio, mediatio, duplatio, multipli-
catio, divisio, progressio, et radicum extractio ;
et hæc dupliciter in cubicis et quadratis ; inter

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[3] "Digitus est omnis numerus minor decem. Articulus est
omnis numerus qui digitum decuplat, aut digiti decuplum, aut
decupli decuplum, et sic in infinitum. Separantur autem digiti
et articuli in limites. Limes est collectio numerorum, qui aut
digiti sunt, aut digitorum æquæmultiplices, quilibet sui relativi.
Limes itaque primus digitorum, secundus primorum articulo-
rum. Tertius est secundorum articulorum. Et sic in infinitum.
Numerus compositus est qui constat ex numeris diversorum
limitum. Item numerus compositus est qui pluribus figuris
significativis representatur."—*Algorismus demonstratus Regio-
montani*, edit. 1534, Sig. A. iv.

[4] Prodocimus de Beldemando de Padua facit idem divisionem,
sed Lucas Paciolus de Burgo Sancti Sepulchri omittit duplatio-
nem et mediationem. Vid. Wallisii Opera, tom. iii. Art. *Alg.*

quas primo de numeratione et postea de aliis
per ordinem dicetur.

I.—*Numeratio.*

Est autem numeratio cujuslibet numeri per
figuras competentes artificialis representatio.
Figura vero differentia locus et limes idem
supponunt, sed diversis rationibus imponuntur.
Figura dicitur quantum ad lineæ pertractionem.
Differentia vero quantum per illam ostenditur
qualiter figura precedens differat a subsequenti.
Locus vero dicitur ratione spatii in quo scri-
bitur. Limes [5] quia est via ordinata ad cujus-
libet muneri representationem. Juxta igitur
novem limites inveniantur novem figuræ, no-
vem digitos representantes ; quæ tales sunt,
9 . 8 . 7 . 6 . 5 . 4 . 3 . 2 . 1. Decima figura di-
citur theta,[6] vel circulus, vel cifra,[7] vel figura
nihili quia nihil significat, sed locum tenens
dat aliis significare : nam sine cifra vel cifris
purus non potest scribi articulus. Cum igitur

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[5] "Numerorum diversi sunt limites. Primus enim limes restat
ab unitate usque ad denarium; a denario in centenarium: a cen-
tenario in millenarium : et sic per decuplum. Secundus limes
a binario usque ad vigenarium : a vigenario usque ad ducenta ;
a ducentis usque ad duo millia. Similiter et cæteri per cæteros
digitos sumuntur. Sunt autem tot limites quot digiti."—*M S. de
Algorismo*, Bib. Trin. Col. Cant. Gal. Collect. O 2. 45, fol. 37.

[6] In multis MSS. *teca* et *theca.*

[7] Vid. Noviomagus de Numeris, 12mo. *Par.* 1539, p. 40.
Vossius de Scien. Math. et Wallisii Algebra, Angl. edit. p. 9.

per has novem figuras significativas adjunctas
quandoque cifræ quandoque cifris contingat
quemlibet numerum representare, non fuit ne-
cesse plures invenire figuras significativas. No-
tandum igitur quod quilibet digitus una sola
figura sibi appropriata habet scribi. Omnis
vero articulus per cifram primo positam et per
figuram digiti a quo denominatur, ille articulus
habet representari vel denominari. Quilibet
articulus ab aliquo digito denominatur, ut de-
narius ab unitate, vigenarius a binario, et sic
de aliis. Omnis numerus in eo quod digitus
habet poni in prima differentia : omnis articulus
in secunda. Omnis vero numerus a decem us-
que ad centum, ut centenarius excludatur, scri-
batur duabus figuris ; si sit articulus, per cifram
primo loco positam et figuram scriptam versus
sinistram quæ significat digitum a quo deno-
minatur ille articulus ; si sit numerus com-
positus, scribatur digitus qui est pars illius
numeri compositi et sinistretur articulus. Item
omnis numerus a centum usque ad mille, ut
millenarius excludatur, per tres figuras habet
scribi. Item omnis numerus a mille usque ad
decem millia, per quatuor figuras habet scribi,
et sic deinceps. Notandum quod quælibet
figura primo loco posita significat suum digi-
tum : secundo decies suum digitum : tertio
centies suum digitum ; quarto loco millesies ;
quinto loco decies millesies ; sexto loco cen-
ties millesies ; septimo loco millesies millesies,

et sic usque ad infinitum multiplicando per hæc
tria, decem, centum, mille : quæ tamen omnes
continentur in hac maxima ; quælibet figura in
sequenti loco posita decies tantum significat
quantum in præcedenti. Item sciendum est quod
supra quamlibet figuram loco millenarii positam
componenter possunt poni quidam punctus ad
denotandum quod tot millenarios debet ultima
figura representare, quot fuerunt puncta pertran-
sita. Sinistrorsum autem scribimus in hac arte
more Arabum hujus scientiæ inventorum, vel hac
ratione ut in legendo, consuetum ordinem obser-
vantes numerum majorem proponamus.

II.—*Additio.*

Additio[8] est numeri vel numerorum aggregatio,
ut videatur summa excrescens. In additione
duo ordines figurarum et duo numeri adminus
sunt necessarii, numerus cui debet fieri additio,
et numerus qui recipit additionem alterius et
debet superscribi. Numerus vero addendus est
ille qui debet addi ad alium, debet subscribi, et
competentius est ut minor numerus subscribatur
et majori addatur. Si igitur velis numerum

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[8] " Additio est numeri ad numerum aggregatio, ut videatur
summa excrescens : vel aliter : additio est duorum numerorum
tertii inventio qui ambos contineat. In additione duo sunt
ordines et duo numeri sunt necessarii ; scilicet, numerus adden-
dus et numerus cui debet fieri additio."— *MS. de Arithmetica,*
Mus. Brit. Arundel. 332, fol. 68, b.

addere numero, scribe numerum cui debet fieri
additio in superiori ordine per suas differentias ;
numerum vero addendum in inferiori ordine per
suas differentias, ita quod prima inferioris ordinis
sit sub prima superioris ordinis, secunda sub
secunda, et sic de aliis. Hoc facto, addatur prima
inferioris ordinis primæ superioris ordinis; ex
tali ergo additione aut excrescat digitus, aut
articulus, aut numerus compositus. Si digitus,
loco superioris deletæ scribatur digitus excres-
cens. Si articulus, loco superioris deletæ scribatur
cifra et transferatur digitus a quo denominatur
ille articulus versus sinistram partem, et addatur
proximæ figuræ sequenti, si sit figura sequens ;
si nulla, sit figura ponatur in loco vacuo. Si
autem contingat quod figura sequens cui debet
fieri additio articuli sit cifra, loco illius deletæ
scribatur digitus articuli. Si vero contingat quod
sit figura nonenarii, et ei debeat fieri additio
unitatis ; loco illius nonenarii deletæ scribatur
cifra, et sinistretur articulus ut prius. Si autem
excrescat numerus compositus, loco superioris
deletæ scribatur digitus, pars illius numeri com-
positi, et sinistretur articulus ut prius. Hoc
facto, addatur secunda secundæ sibi suprapositæ,
et fiat ut prius.

III.—*Subtractio.*

Subtractio est, propositis duobus numeris, ma-
joris ad minorem excessus inventio : vel sic,

subtractio est numeri a numero ablatio, ut vide-
atur summa excrescens. Minor autem de majori
subtrahi potest, vel par de pari ; major vero de
minori nequaquam. Ille quidem numerus major
est qui plures habet figuras, dummodo ultima sit
significativa : si vero tot sint unitates in uno
quot in alio reliquo, videndum est per ultimas vel
penultimas, et sic deinceps. In subtractione vero
duo sunt numeri necessarii ; scilicet, numerus
subtrahendus, et numerus a quo debet fieri sub-
tractio. Numerus subtrahendus debet sibi scribi
per suas differentias, ita quod prima sub prima,
secunda sub secunda, et sic de aliis. Subtrahe
ergo primam figuram inferioris ordinis a figura
sibi supraposita, et illa aut erit par, aut major,
aut minor. Si par, ea deleta loco ejus ponatur
cifra, et hoc propter figuras sequentes ne minus
significent. Si major supponatur, tunc deleantur
tot unitates quot contineat inferior figura et re-
siduum loco ejus ponatur. Si minor quam, major
numerus de minori subtrahi non potest, mutuetur
unitas a figura proxima sequente quæ valet decem
respectu precedentis figuræ ; ab illo ergo denario
et a figura a qua debuit fieri subtractio simul
junctis subtrahatur figura inferior et residuum
ponatur in loco figuræ deletæ. Si vero figura
a qua mutuanda est unitas sit unitas, ea deleta
loco ejus scribatur cifra ne figuræ sequentes
minus significent deinde operare ut prius. Si
vero figura a qua mutuanda est unitas sit cifra,
accedet ad proximam figuram significativam et

ibi mutuat unitate et in redeundo in loco cujus-
libet cifræ pertransitæ ponatur figura nonenarii;
cum igitur perventum fuerit ad illam figuram de
qua intenditur, remanebit tibi denarius ab illo
ergo denario, et a figura non potest fieri subtractio
simul junctis subtrahatur figura sibi supposita ut
prius.   Ratio autem quare loco cujuslibet cifræ
pertransitæ relinquitur nonenarii figura hæc est.
Si autem tertio loco mutuetur unitas illa respectu
illius a qua debuit fieri subtractio, valuit centum;
sed loco cifræ pertransitæ, relinquitur nonenarius
qui valet nonaginta, unde remanet tantum dena-
rius, et eadem erit ratio si a quarto loco vel a
quinto vel deinceps mutuetur unitas.   Hoc facto,
subtrahe secundam inferioris ordinis a sua supe-
riori, et negociandum est ut prius.   Sciendum
est quod tam in additione quam in subtractione
possumus incipere operari a sinistra parte ten-
dendo versus dextram; sed, ut dicebatur, fuit
commodius.   Si autem probare volueris utrum
bene feceris, nec ne figuras quas prius subtraxisti
adde superioribus, et concurrent eædem figuræ
si recte feceris, quas prius habuisti.   Similiter
in additione, quum omnes figuras addideris, sub-
trahas quas prius addidisti et redibunt eædem
figuræ quæ prius habuisti, si feceris recte : est
enim subtractio additionis probatio, et converso.[9]

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[9] "Una species probat aliam; si itaque volueris explorare
veritatem calculi in additione, subtrahe partes conjungendas ab
aggregato : sique nihil remanet, vera est operatio. E contra
vero, in subtractione exploraturus calculum, adde relictum ad

IV.—*Mediatio.*

Mediatio est numeri propositi medietatis in-
ventio, ut videatur quæ et quanta sit illa me-
dietas. In mediatione autem tantum est unus
ordo figurarum et unus numerus necessarius, sci-
licet, numerus mediandus. Si ergo velis nume-
rum aliquem mediare, scribatur ille numerus per
suas differentias et incipe a dextris, scilicet, a
prima figura versus dextram tendendo ad sinis-
tram. Prima igitur figura aut erit significativa
aut non : si non sit significativa, cifra præmitta-
tur et fiat alterius : si vero fuerit significativa,
ergo representabit unitatem aut alium digitum :
si unitatem loco ejus deletæ ponatur cifra propter
figuras sequentes ne minus significent, et scriba-
tur illa unitas exterius in tabula, vel resolvatur
illa unitas in sexaginta minuta, et medietas illo-
rum sexaginta abjiciatur et reliqua medietas
reservetur exterius in tabula, vel scribatur figura
dimidii : sciendum tum quod nullum ordinis
locum obtineat aliquod, tamen significat quod
medietas duplata in suum locum recipiatur in
duplatione. Si autem prima figura significet
alium digitum ab unitate, ille numerus aut erit
par aut impar. Si par, loco ejus deletæ scribatur
medietas illius numeri paris. Si impar, sume

numerum subtrahendum, cumque redit is, a quo subtractio facta
est, justa est operatio."—*Winshemii Compendium Logisticæ
Astronomicæ*, 12mo. 1563, Sig. B. 3.

numerum proximum parem sub illo contentum,
et medietatem ejus pone in loco illius imparis
deleti ; de unitate autem quæ remanet medianda,
fac ut prius. Hoc facto, medianda est secunda
figura et negocianda est ut prius, si cifra præter-
mittatur intacta. Si autem figura sit significativa,
aut par aut impar erit : si par, loco ejus deletæ
scribatur ejus medietas ; si impar, aut erit unitas
aut alius digitus numerum imparem representans.
Si unitas, loco ejus deletæ scribatur cifra. Illa
autem unitas cum valet decem respectu figuræ
prioris de illis decem sumatur medietas : quina-
rius et addatur figuræ præcedenti. Si vero fuerit
alius digitus numerum imparem representans,
sume proximum parem sub illo contentum, et
medietas ejus loco illius imparis deletæ ponatur :
unitas autem quæ remanet medianda valet decem
respectu figuræ præcedentis : dividatur ergo ille
denarius in duos quinarios, et unus illorum ab-
jiciatur, et reliquus addatur figuræ præcedenti ut
prius. Si autem cifra fuerit cui debet addi qui-
narius, deleatur cifra, et loco ejus scribatur
quinarius, et sic operandum est donec totus
numerus mediatur qui scriptus fuerit.

V.—*Duplatio.*

Duplatio est numeri propositi ad seipsum ag-
gregatio, ut videatur summa excrescens. In
duplatione tantum unus ordo figurarum est
necessarius. In tribus speciebus precedentibus

inchoamus a dextra et a figura minori; in hac
autem specie et in omnibus sequentibus inchoa-
mus a sinistra et a figura majori : unde versus—

> Subtrahis aut addis a dextris vel mediabis ;
> A leva dupla, divide, multiplicaque :
> Extrahe radicem semper sub parte sinistra.[1]

Si enim velis incipere duplare a prima figura, con-
tinget idem bis duplare. Et licet aliquo modo
possumus operari incipiendo a dextris, tum diffi-
cilior erit operatio et doctrina. Si igitur velis
aliquem numerum duplare, scribatur ille nume-
rus per suas differentias, et dupletur ultima figura.
Ex illa igitur duplatione aut excrescit digitus,
aut articulus, aut numerus compositus. Si digi-
tus, loco illius deletæ scribatur digitus excrescens.
Si articulus, loco illius deletæ scribatur cifra, et
transferatur articulus, versus sinistram. Si nu-
merus compositus, loco illius deletæ scribatur
digitus qui est pars illius compositi et sinistretur
articulus. Hoc facto, duplanda est ultima figura,
et quicquid excreverit negociandum est, ut prius.
Si vero occurrerit cifra, relinquenda est intacta.
Sed si aliquis numerus cifræ debeat loco illius

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[1] In Dionysii algorismo. MS. Bib. Reg. Mus. Brit. 8. c. iv.
Vide Arithmeticæ Brevis Introductio, per A. Lonicerum, 12mo.
Francof. 1551:

> " Addas, subducas a dextris multiplicesque ;
>   Dividit ac mediat deinde sinistra manus."

Vide Cirveli Algorismus, 4to. *Par.* 1513. Buclæi Arith.
Memor. 12mo. *Cantab.* 1613 ; et Arithmetica Speculativa Bra-
vardini, 4to. *Par.* 1500.

deletæ scribatur numerus addendus, eodem modo
negociandum est ut prius de omnibus : probatio
hujus talis est : si recte mediaveris, dupla et
occurrent eædem figuræ quas prius habuisti. Est
enim duplatio mediationis probatio et converso.

VI.—*Multiplicatio.*

Multiplicatio est numeri per se vel per alium,
propositis duobus numeris, est tertii inventio
qui contineat alterum illorum quot continentur
unitates in reliquo. In multiplicatione duo sunt
numeri necessarii, scilicet, numerus multiplican-
dus et numerus multiplicans. Numerus multi-
plicans adverbialiter nuncupatur. Numerus vero
multiplicandus nominalem recipit appellationem :
potest et jam tertius numerus assignari qui pro-
ductus dicitur, perveniens ex ductione unius in
alterum. Notandum est quod de multiplicante
potest fieri multiplicandus et econverso, manente
semper eadem summa, omnis numerus in seipso
convertitur multiplicando. Sunt autem sex re-
gulæ multiplicationis, quarum prima talis est :
quinam digitus multiplicat digitum, subtrahendus
est minor digitus ab articulo suæ denominationis
per differentiam majoris digiti ad denarium,
denario simul computato. Verbi gratia, si vis
scire quot sunt quater in octo, vide quot sunt
unitates intra octo et decem, denario simul com-
putato, et patet quot sunt duo : subtrahatur
ergo quaternarius a quadraginta bis et remanent

32, et hæc est summa totius multiplicationis.
Similiter quando digitus multiplicat seipsum.
Quando autem digitus multiplicat numerum
compositum, ducendus est digitus in utramque
partem numeri compositi, ita quod digitus in
digitum per primam regulam, et digitus in articu-
lum per secundam regulam ; postea producta
conjungantur simul, et erit summa totius multi-
plicationis. Quando articulus multiplicat articu-
lum, ducendus est digitus a quo denominatur
ille articulus in digitum a quo denominatur
reliquus. Quando articulus multiplicat numerum
compositum, ducendus est digitus articuli in
utramque partem numeri compositi ; conjungan-
tur producta, et patebit summa totius. Quando
numerus compositus multiplicat numerum com-
positum, ducenda est utraque pars numeri multi-
plicantis in utramque partem numeri multipli-
candi et sic ducetur digitus bis, quia semel in
digitum et semel in articulum. Articulus similiter
bis, quia semel in digitum et iterum in articulum :
hic tamen ubique articulus non ad principales
extenditur articulos. Si igitur velis aliquem
numerum vel per se vel per alium multiplicare,
scribe numerum multiplicandum in superiori or-
dine per suas differentias, numerum vero multi-
plicantem in inferiori per suas differentias, ita
tamen quod prima figura inferioris ordinis sit
sub ultima superioris. Hoc facto, ducenda est
ultima multiplicantis in ultimam multiplicandi.
Ex illo igitur ductu aut excrescit digitus, aut

articulus, aut numerus compositus. Si articulus,
ex directo figuræ multiplicantis scribatur cifra,
et transferatur articulus versus sinistram. Si
digitus, ex directo super positionem figuræ multi-
plicantis scribatur digitus excrescens. Si nume-
rus compositus, ex directo figuræ multiplicantis
scribatur digitus illius numeri compositi, et sinis-
tretur articulus, ut prius. Hoc autem facto,
ducenda est ultima numeri multiplicantis in ulti-
mam multiplicandi, et quicquid inde excreverit,
negociandum est, ut prius ; et sic fiat de omni-
bus aliis numeri multiplicantis, donec perveniatur
ad primam multiplicantis, quæ ducenda est in
ultimam multiplicandi, et ex illo ductu aut ex-
crescit digitus, aut articulus, aut numerus com-
positus. Si digitus, loco superioris deletæ scri-
batur digitus excrescens ; si articulus, loco supe-
rioris deletæ scribatur cifra et sinistretur articulus :
si numerus compositus, loco superioris deletæ
scribatur digitus qui est pars illius numeri com-
positi et sinistretur articulus, ut prius. Hoc
autem facto, anteriorandæ sunt figuræ numeri
multiplicantis per unam differentiam, ita quod
prima multiplicantis sit sub penultima multipli-
candi, reliquis similiter per unum locum anterio-
ratis. Quo facto, ducenda est figura ultima
multiplicantis in illam figuram sub qua est prima
figura multiplicantis, et ex illo autem ductu aut
excrescit digitus, aut articulus, aut numerus com-
positus. Si digitus, ex directo figuræ suprapositæ
addatur : si articulus, transferatur versus sinis-

tram, et figura directi supraposita relinquatur
intacta ; si numerus compositus, addatur digitus
supraposita figura et sinistretur articulus. Si-
militer quælibet figura numeri multiplicantis in
penultimam multiplicandi donec perveniatur ad
primum multiplicantis, ubi operandum est ut
prius, vel quemadmodum determinatur de primis;
deinde anteriorandæ sunt figuræ per unicam
differentiam, ut prius. Nec cessandus est a tali
anterioratione nec a tali ductu. Quovis quæ-
libet figura numeri multiplicantis ducatur in
quamlibet figuram numeri multiplicandi. Si
autem contingat quod prima figura numeri
multiplicantis sit cifra et ei supponatur figura
significativa, loco illius superioris deletæ scri-
benda est cifra. Si autem contingat quod
cifra sit inter primam figuram et ultimam
multiplicandi, anteriorandus est ordo figura-
rum per duas differentias, quamvis ex duc-
tione alicujus figuræ in cifram nihil resultat.
Ex perdictis ergo patet quod si prima figura
numeri multiplicandi sit cifra, sub ea non debet
fieri anterioratio. Sciendum autem quod in mul-
tiplicatione et divisione et radicum extractione
competenter potest relinqui spacium vacuum
inter duos ordines figurarum, ut ibi scribatur,
quod pervenit addendum aut subtrahendum ne
aliquid memoriæ intercidatur.[2]

[2] Vid. Piscatoris Compendium Arithmeticæ, 12mo. *Lips.*
1592. Ursini Systema Arithmeticæ, 12mo. *Colon.* 1619. Frisii
Arith. Pract. Methodus facilis, 12mo. *Colon.* 1592. Tonstallus
de Arte Supputandi, 4to. *Lond.* 1522.

VII.—*Divisio.*

Divisio[3] numeri per numerum est, propositis
duobus numeris, majorem in tot partes distribuere
quot sunt unitates in minori.   Notandum ergo
quod in divisione, tres numeri sunt necessarii ;
scilicet, numerus dividendus ; numerus dividens,
sive divisor ; et numerus denotans quotiens.
Numerus autem dividendus semper debet esse
major, vel saltem par divisori, si debeat fieri
divisio per integra.   Si velis igitur aliquem
numerum per alium dividere, scribe numerum
dividendum in superiori ordine per suas differen-
tias, divisorem vero in inferiori ordine per suas
differentias, ita quod ultima divisoris sit sub
ultima dividendi, penultima sub penultima, et
sic de aliis, si competenter fieri possit.   Sunt
autem duæ causæ quare ultima sub ultima in-
ferioris ordinis non possit collocari, quia aut
ultima inferioris ordinis non possit subtrahi ab
ultima superioris quod est minor inferiori, aut
quæ licet ultima superioris possit subtrahi a sua
superiori : reliquo tamen non possunt subtrahi
a figuris sibi suprapositis, si ultima inferioris sit
par figuræ suprapositæ.   His itaque ordinatis,

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[3] "Numeratio conjuncta, est multiplicatio aut divisio : mul-
tiplicatio est, qua multiplicandus toties addatur, quoties unitas
in multiplicante continetur, et habetur factus : divisio est, qua
divisor subducitur a dividendo quoties in eo continetur, et
habetur quoties." — *Rami Arithmetica*, Edit. 1581, pp. 11
et 14.

incipiendum est operari ab ultima figura numeri
divisoris, et videndus est quotiens, possit illa
subtrahi a figura sibi supraposita, et reliquæ
a residuo sibi supraposito, si aliquid fuerit resi-
duum. Viso ergo quotiens, figuræ inferioris
ordinis possint subtrahi a suis superioribus,
scribendus est numerus denotans quotiens ex
directo supraposito illius figuræ sub qua est
prima figura numeri divisoris, et per illam divi-
dendæ sunt omnes figuræ inferioris ordinis a suis
superioribus. Si autem contingat post anterio-
rationem quod non quotiens, possit subtrahi
ultima figura divisoris a figura sibi supraposita
super figuram sub qua est prima divisoris, recte
scribenda est cifra in ordine numeri denotantis
quotiens, et anteriorandæ sunt figuræ, ut prius;
similiter faciendum est ubicunque contingit in
numero dividendo quod divisor non possit subtrahi
a numero dividendo, ponenda est cifra in ordine
numeri denotantis quotiens et anteriorandæ sunt
figuræ, ut prius: nec cessandum est a ductu
numeri denotantis quotiens in divisorem, nec
a ductu divisoris subtrahendæ donec prima
divisoris sit subtracta a prima dividendi: quo
facto, aut aliquid erit residuum aut nihil: si
aliquid sit residuum, reservetur exterius et scri-
batur in tabula et erit semper unius divisoris.
Cum igitur facta fuerit talis divisio, et probare
volueris utrum benefeceris, multiplica numerum
denotantem quotiens per divisorem et redibunt
eædem figuræ quas prius habuisti, si nihil fuerit

residuum; sed si aliquid fuerit residuum, tunc cum additione residui redibunt eædem figuræ quas prius habuisti; et ita probat multiplicatio divisionem, et econtrario. Sed si facta multiplicatione, dividatur productum per multiplicantem, exibunt in numero denotante quotiens figuræ numeri multiplicandi.

VIII.— *Progressio.*[4]

Progressio est numerorum secundum æquales excessus ab unitate vel binario sumptorum aggregatio ut universorum summa compendiose habeatur. Progressionum alia naturalis sive continua, alia intercisa sive discontinua. Naturalis est quando incipitur ab unitate et non omittitur in accensu aliquis numerus, ut 1 . 2 . 3 . 4 . 5 . 6 . et cætera; et sic numerus sequens superat numerum precedentem unitate. Intercisa est quando omittitur numerus aliquis, ut 1 . 3 . 5 . 7 . 9 . et cætera. Similiter a binario possunt incipi, ut 2 . 4 . 6 . 8 . et sic numerus sequens superat precedentem numerum in duobus unitatibus. Notandum quod progressionis naturalis duæ sunt regulæ, quarum prima est

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[4] "Progressio est numerorum æqualiter distantium in unam summam collectio. Progressio arithmetica continua sive naturalis est ubi post primum characterem nullus intermittitur. Progressio arithmetica discontinua sive intercisa, est figuris æqualiter interceptis numerorum ordo." Hudaldrichus de Arithmetica, 12mo. *Frid.* 1550, p. 70. Vid. Glareanus de Algorismo, 12mo. *Par.* 1558, p. 20.

talis; quando progressio naturalis terminatur
in numerum parem, per medietatem ipsius mul-
tiplica numerum proximum totali superiorem;
verbi gratia, 1 . 2 . 3 . 4 . multiplica quinarium
per binarium, et exibunt decem, summa totius
progressionis. Secunda regula talis est; quando
progressio naturalis terminatur in numerum im-
parem, sume majorem partem illius imparis et
per illam multiplica totalem numerum; verbi
gratia, 1 . 2 . 3 . 4 . 5 . multiplica quinarium per
ipsum trinarium, et resultabunt quidenarius,
summa totius progressionis. Similiter de pro-
gressione intercisa duæ dantur regulæ, quarum
prima talis est; quando progressio intercisa
terminatur in numerum parem, per medietatem
illius multiplica numerum proximum medietati
superiorem, ut, 2 . 4 . 6 . multiplica quaternariem
per ternarium, et resultabunt duodecim, summa
totius progressionis. Secunda regula talis est;
quando progressio intercisa terminatur in nu-
merum imparem, multiplica majorem portionem
per se ipsam; verbi gratia, 1 . 3 . 5 . multiplica
ternarium per se et exibit nonenarius, summa
totius progressionis.

IX.—*Perambulum ad Radicum Extractionem.*

Sequitur de radicum extractione, et primo in
quadratis : unde videndum est qui sit numerus
quadratus et quæ sit radix numeri, et quid sit
radicem ejus extrahere. Primo notandum tamen

est hæc divisio; numerorum alius linearis, alius
artificialis, alius solidus. Numerus linearis est
qui consideratur tamen penes processum, non
habito respectu ad ductionem numeri in nume-
rum, et dicitur linearis, quia unicum tantum
habet numerum, sicut linea unicam habet di-
mensionem, longitudinem sine latitudine. Nu-
merus superficialis est qui resultat ex ductu
numeri in numerum, et dicitur superficialis quia
habet duos numeros denotantes sive mensurantes
ipsum, sicut superficies duas habet divisiones;
scilicet, longitudinem et latitudinem. Sed scien-
dum est qui dupliciter potest numerus duci in
numerum aut semel aut bis : si numerus semel
ducatur in numerum hoc erit in seipsum vel in
alium. Sciendum quod si ducatur in se semel,
fit numerus quadratus, qui, diversum scriptus
per unitates, habet quatuor latera æqualia admo-
dum quadranguli. Si ducatur in alium, fit nume-
rus superficialis et non quadratus, ut binarius
ductus in ternarium constituit senarium, nume-
rum superficialem, et non quadratum; unde patet
quod omnis numerus quadratus est superficialis
et non convertitur. Radix autem numeri quad-
rati est ille numerus qui ita ducitur in se, ut bis
duo sunt quatuor. Quaternarius igitur est pri-
mus numerus quadratus, et binarius est ejus radix.
Si autem numerus bis ducatur in numerum, facit
numerum solidum, ut dicitur, sicut solidus corpus
tres habet dimensiones; scilicet, longitudinem et
latitudinem et spissitudinem. Ita numerus iste

tres habet numeros ducentes in se. Sed numerus
potest bis duci in numerum dupliciter, qui quot
in seipsum aut in alium. Si igitur numerus bis
ducatur in se vel semel in se et postea in suum
quadratum sit numerus cubicus, et dicitur nume-
rus cubicus ab nomine cubi quod est solidus. Est
autem cubus quidam corpus habens sex super-
ficies, solidos octo angulos, et duodecim latera.
Si autem aliquis numerus bis ducatur in alium
fit numerus solidus et non cubicus ; ut bis tria
bis constituerunt duodecim. Unde patet quod
omnis numerus cubicus est solidus, sed non con-
vertitur. Ex predictis igitur patet quod idem
numerus est radix numeri quadrati et cubici, non
tamen illius radicis idem est quadratus et cubicus.
Cum igitur ex ductu unitatis in se semel vel bis
nihil perveniat nisi unitas, sicut dicit Boetius in
arithmetica sua, quod omnis unitas potentialiter
est numerus omnis, nullus autem auctus. Notan-
dum autem quod inter quoslibet quadratos proxi-
mos continget reperire unicum medium propor-
tionale, quod pervenit ex ductu radicis numeri
quadrati in radicem alterius. Item inter quos-
libet duos cubicos proximos est reperire dicitur
medium proportionale, scilicet, minus medium
et majus. Minus medium pervenit ex ductu
radicis majoris cubici in quadratum minoris.
Majus medium est, si ducatur radix minoris
cubici in quadratum majoris. Cum igitur ultra
summam numerorum solidorum in arte præsenti,
non fiat processus, autem quatuor limites nume-

rorum distinguuntur : est enim limes numerorum ejusdem naturæ extremis contentorum terminis, continua ordinatio ; unde primus limes est novem digitorum continua progressio. Secundus novem articulorum principalium est tertius centenariorum. Quartus novem millenariorum tres limites et resultant incomposites per digitorum appositionem super quam cubum articulorum trium predictorum, ut si alter alteri proponatur. Sed per finalis termini rationem ex millenariorum receptione super se semel per modum quadratorum, aut bis per modum solidorum, resultat penultima et ultimus limes.

X.—*Extractio radicum in quadratis.*

Radicem numeri quadrati extrahere est, proposito aliquo numero, radicem quadrati invenire, si numerus propositus quadratus fuerit. Si numerus vero non fuerit quadratus, tunc radicem extrahere est maximi quadrati sub numero proposito contenti invenire. Si velis igitur radicem alicujus numeri quadrati extrahere, scribe numerum illum per suas differentias, et computa numerum figurarum, utrum sit par vel impar. Si par, incipiendum est operari sub penultima. Si impar, ab ultima ; et, ut breviter dicatur, incipiendum est a figura posita in ultimo loco impari ; sub ultima igitur figura in impari loco posita, inveniendus est quidam digitus, qui ductus in se deleat totum sibi suprapositum respectu sui vel inquan-

tum vicinius potest; tali igitur invento, ducto digito et a superiori subtracto, duplandus est ille digitus, et duplatum ponendum est sub proxima superioris versus dextram et ejus duplum sub illo. Quo facto, inveniendus est quidam digitus sub proxima figura ante duplatum, qui ductus in duplatum deleat totum suprapositum respectu duplati; deinde ductus in se deleat totum suprapositum respectu sui vel inquantum vicinius potest: vel potest ita subtrahi digitus ultimo inventus ut ducatur in duplatum vel duplata et postea in se; deinde illa duo producta simul addantur, ita quod prima figura ultimi producti ponatur ante primam primi producti et superiora addatur primæ, et sic de aliis, et subtrahatur simul a totali numero respectu digiti inventi. Si autem contingit quod non possit aliquis digitus inveniri post anteriorationem, ponenda est cifra sub tertia figura versus dextram, et anteriorandum est primum duplatum cum suo subduplo: non cessandum est a talis digiti inventione, nec a digiti inventi duplatione, nec a duplatorum anterioratione, nec a subdupli subduplo positione, donec sub prima figura inventus sit quidam digitus, qui ductus in omnes duplatos deleat totum suprapositum respectu sui vel inquantum vicinius potest. Quo facto, aut aliquid erit residuum aut nihil; si nihil constat, quamvis propositus fuerit quadratus et ejus radix est digitus ultimo inventus cum subduplo vel cum subduplis, ita quod proponatur: si vero aliquid fuerit residuum,

constat quod numerus propositus non fuerit
quadratus. Sed digitus ultimo inventus cum
subduplo vel subduplis est radix maximi quadrati
sub numero proposito contenti. Si velis ergo
probare utrum beneficeris necne, multiplica digi-
tum ultimo inventum cum subduplo vel cum
subduplis per eundem digitum, et redibunt eædem
figuræ quas prius habuisti, si non fuerit resi-
duum ; sed si aliquid fuerit residuum, tunc cum
additione illius redibunt eædem figuræ quas prius
habuisti.

XI.—*Extractio radicum in cubicis.*

Sequitur de radicum extractione in cubicis :
videndum est quid sit numerus cubicus et quæ
sit radix ejus, et quid sit radicem cubicam ex-
trahere. Est igitur numerus cubicus, sicut patet
ex predictis, qui pervenit ex ductu alicujus nu-
meri bis in se vel semel in suum quadratum :
radix numeri cubici est ille numerus qui. ita
ducitur bis in se vel semel in suum quadratum.
Unde patet quod numerus cubicus et quadratus
eandem habuit radicem sicut supra dictum est.
Radicem autem cubicam extrahere est numeri
propositi radicem invenire cubicam, si numerus
propositus sit cubicus : si vero non sit cubicus,
tunc radicem cubicam extrahere est maximi
cubici sub numero proposito contenti radicem
cubicam invenire. Proposito igitur aliquo nu-
mero, cujus radicem velis extrahere cubicam ;

primo computandæ sunt figuræ per quartas vel sub loco ultimo millenarii inveniendus est quidam digitus qui ductus in se deleat totum suprapositum respectu sui vel inquantum vicinius potest. Quo facto, triplandus est ille digitus et triplatum ponendum est sub proxima figura tertia versus dextram et ejus subtriplum sub subtriplo. Deinde inveniendus est quidam digitus sub proxima figura ante triplatum, qui cum subtriplo ductus in triplatum, deinde cum subtriplo ductus in productum deleat totum suprapositum respectu triplati. Deinde ductus in se deleat totum suprapositum respectu sui vel inquantum vicinius potest: hoc facto, triplandus est ille digitus iterum et ponendum est triplatum sub tertia figura ut prius, et ejus triplatum sub eo ; postea anteriorandum est primum triplatum cum subtriplo per duas diffe-rentias. Deinde inveniendus est quidam digitus ante triplatus sub proxima figura, qui cum sub-triplis ductus in triplata et postea sine subtriplis ductus in productum deleat totum suprapositum respectu sui vel inquantum vicinius potest. Nec cessandum est a talis digiti inventione, nec a digiti inventi triplatione, nec a triplatorum an-terioratione per suas differentias, nec a subtripli subtriplo positione, nec a tali multiplicatione, nec a subtractione, donec perventum fuerit ad primam figuram sub qua inveniendus est quidam digitus qui cum subtriplis ductus in triplata, deinde sine subtriplis ductus in productum deleat

totum suprapositum respectu sui vel inquantum
vicinius potest. Notandum est quod productum
perveniens ex ductu digiti inventi in se possunt
addi et similiter simul contrahi a tali numero
supraposito in respectu digiti inventi : hoc facto,
aut aliquid erit residuum aut nihil : si nihil, con-
stat quod numerus propositus fuit cubicus, et
ejus radix est digitus ultimo inventus propositus
cum subtriplis vel subtriplo, quæ radix si ducatur
in se et postea in productum erunt eædem figuræ
quas prius habuisti. Si vero fuerit residuum,
reservetur idem exterius in tabula, et constat
quod ille numerus non fuit cubicus. Sed digitus
ultimo inventus cum subtriplo vel subtriplis est
radix maximi cubici sibi numero proposito con-
tenti, quæ radix si ducatur in se et postea in
productum est maximus cubicus sub numero pro-
posito contentus, et si illo cubico addatur residuum
in tabula, erunt eædem figuræ quæ prius fuerunt.
Si autem aliquis digitus post anteriorationem,
non inveniri possit, tunc ponenda est cifra sub
quarta figura versus dextram et anteriorandæ
sunt figuræ ut prius. Notandum est, et quod si
in numero proposito non est aliquis locus mil-
lenarii, incipiendum est operari sub prima figura.
De extractione radicum dicta sufficiant.

A METHOD USED IN ENGLAND IN THE FIFTEENTH CENTURY FOR TAKING THE ALTITUDE OF A STEEPLE OR INACCESSIBLE OBJECT.[5]

HERE foloweth a rule howe a mann stondyng in a playne by a steple or such another thynge of height by lokyng vponn it shall knowe the certentie of the height thereof. First, let a mann consider by his estimacioun howe farre he stondeth from it be it xx, xxx, or xl fadam, And thereaboute as he demeth the certentie let hym stonde and there pitche a staffe the vpper poynte thereof to be juste with his yie, he stondyng upp righte therby. And thann let hym leye hym

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[5] It is scarcely necessary to observe that this method of proceeding could only have been practised by the more ignorant classes : the English mathematicians of that period were skilful in the application of the quadrant, and all other then known scientific instruments, as may be seen from their numerous works which still remain in manuscript in the various public and private collections of Great Britain. Fuller quaintly remarks, " I never did spring such a covye of mathematicians all at once, as I met with at this time."—*History of the Worthies of England.* Edit. 1811. Vol. II. p. 413.

downe alonge upp righte beyonde the staffe from
the steple warde his feet juste to the staffe, and
whann the staffe so stondeth he lying as is afore-
said, as his yie on the hyghest poynte of the
staffe is juste with the height of the heighest
poynte of the steple.   Than the juste space from
his yie as he lyith to the foote of the steple, that
is to saye to that parte of the foote which is as
litill as the top therof is the juste mesure of the
height of the said steple.   And if the staff stonde
not juste let hymm remeve it till his yie he lying
as is aforesaid with the highest poynte of the
staf accorde with the highest poynte of the
steple.

A TREATISE

NUMERATION OF ALGORISM,

FROM A MS. OF THE 14TH CENTURY.

———————

To alle suche even nombrys the most have
cifrys as to ten. twenty. thirtty. an hundred. an
thousand and suche other. but ye schal vnder-
stonde that a cifre tokeneth nothinge but he
maketh other the more significatyf that comith
after hym. Also ye schal vnderstonde that in
nombrys composyt and in alle other nombrys
that ben of diverse figurys ye schal begynne in
the ritht syde and so rekene backwarde and so
he schal be wryte as thus—1000. the cifre in
the ritht side was first wryte and yit he tokeneth
nothinge no the secunde no the thridde but theı
maken that figure of 1 the more signyficatyf that
comith after hem by as moche as he born oute of
his first place where he schuld yf he stode ther

tokene but one.   And there he stondith nowe in
the ferye place he tokeneth a thousand as by this
rewle.   In the first place he tokeneth but hym-
self.   In the secunde place he tokeneth ten times
hymself.   In the thridde place he tokeneth an
hundred tymes himself.   In the ferye he tokeneth
a thousand tymes himself.   In the fyftye place
he tokeneth ten thousand tymes himself.   In the
sexte place he tokeneth an hundred thousand
tymes hymself.   In the seveth place he tokeneth
ten hundred thousand tymes hymself, &c.   And
ye schal vnderstond that this worde nombre is
partyd into thre partyes.   Somme is callyd nom-
bre of digitys for alle ben digitys that ben
withine ten as ix, viii, vii, vi, v, iv, iii, ii, i.
Articules ben alle thei that mow be devyded
into nombrys of ten as xx, xxx, xl, and suche
other.   Composittys be alle nombrys that ben
componyd of a digyt and of an articule as
fourtene fyftene thrittene and   suche other.
Fourtene is componyd of four that is a digyt
and of ten that is an articule.   Fyftene is com-
ponyd of fyve that is a digyt and of ten that is
an articule and so of others . . . .   But as
to this rewle.   In the firste place he tokeneth
but himself that is to say he tokeneth but that
and no more.   If that he stonde in the secunde
place he tokeneth ten tymes himself as this figure
2 here 21. this is oon and twenty.   This figure 2
stondith in the secunde place and therfor he
tokeneth ten tymes himself and ten tymes 2 is

twenty and so forye of every figure and he
stonde after another toward the lest syde he
schal tokene ten tymes as moche more as he
schuld token and he stode in that place ther that
the figure afore him stondeth : lo an example as
thus 9634. This figure of foure that hath this
schape 4 tokeneth but himself for he stondeth in
the first place. The figure of thre that hath this
schape 3 tokeneth ten tyme himself for he ston-
deth in the secunde place and that is thritti.
The figure of sexe that hath this schape 6
tokeneth ten tyme more than he schuld and he
stode in the place yer the figure of thre stondeth
for ther he schuld tokene but sexty. And now
he tokeneth ten tymes that is sexe hundrid.
The figure of nyne that hath this schape 9
tokeneth ten tymes more than he schulde and
he stode in the place ther the figure of 6 ston-
deth inne for thanne he schuld tokene but nyne
hundryd. And in the place that he stondeth
inne nowe he tokeneth nine thousand. Alle
the hole nombre of these foure figurys. Nine
thousand sexe hundrid and foure and thritti.

A TREATISE

ON THE PROPERTIES AND QUALITIES OF GLASSES
FOR OPTICAL PURPOSES,

ACCORDING TO THE MAKING, POLISHING, AND GRINDING OF THEM.

BY WILLIAM BOURNE.

FROM MS. LANSD. MUS. BRIT. 121.

EPISTLE DEDICATORY.

To the Right Honorable, and hys singuler good
Lorde, Sir Wilyam Cicil, Baron of Burghley,
Knight of the moste noble order of the garter,
Lorde Highe Treasurer of Englande : Mr. of the
Courte of Wardes and Liverys, Chancelour of the
Vniversity of Cambridge, and one of the
Queens Majestie's Honorable Privy Counsell.

Right Honorable, fynding myself moste deepely
bounde, vnto youre Honour, in dyvers respects :
And allso youre Honours moste excellent and
worthy skilles, and knowledge in all notable,
laudable and noble experiences of learning in all
maner of causes : And allso, for that of late
youre honour hathe had some conference and

speache with mee, as concerning the effects and
qualityes of glasses, I have thought yt my duty
to furnish your desyer, according vnto suche
simple skill, as God hathe given me, in these
causes, Whiche ys muche inferiour vnto the
knowledg of those, that ys learned and hathe
red suche authors, as have written in those
causes, and also have better ability and tyme, to
seke the effects, and quality thereof, then I have,
eyther elles can, or may, by the meanes of my
small ability, and greate charge of children :
Whiche (otherwyse) yt ys possible that I shoulde
have bene better able to have done a nomber of
thinges, that now I must of force leave, that
perhapps shoulde have bene.   And allso aboute
seaven yeares passed, vppon occasyon of a cer-
tayne written Booke of myne, which I delivered
your honour, Wherein was set downe the nature
and qualitye of water : As tuchinge ye sinckinge
or swymminge of thinges.   In sort youre
Honoure had some speeche with mee, as touching
measuring the moulde of a shipp.   Whiche gave
mee occasyon, to wryte a litle Boke of Statick.[6]
Whiche Booke since that tyme, hath beene profit-
able, and helpped the capacityes, bothe of some
sea men, and allso shipp carpenters.   Therefore,
I have now written this simple, and breefe note,
of the effects, and qualityes of glasses, according
vnto the several formes, facyons, and makyngs

[6] The work here alluded to is probably his " Treasure for
Travellers," which was published at London in 1578.

of them, and allso the foylinges of them.   That ys
to saye, the foyle or using of them, that yow may
not looke thorowe the Glass: Whiche causeth
the Glass to cast a beame vnto your eye, acord-
ing vnto the shape, or forme of any thinge, yt
standeth against yt.   And allso the polishing and
grynding of glasse, whiche causeth sondry
effects: As in ye readinge hereafter dothe ap-
peare the merveylous nature and operation of
glasses &c.   I humbly desyring your Honour,
to take this simple rude matter in good parte:
And to accept yt as my good Will, allthoughe
that the matter ys of none importance.

By your Honoures, dutyfully to Commande,

W. BOURNE.

CHAPTER I.—*Introduction.*

Whereas the eye ys the principall member of
the body neyther the Body in respect, coulde not
moove any distance, but vnto perrill, yf yt were
not for the sighte of the Eye, whose quality ys
moste wonderfull, and hathe the largest prehemi-
nence of all the members of the body.   For that
the Eye ys able to discerne and see any thinge,
how farr down that the distance ys from yow :
Yf the thinge bee of magnitude or bignes, corres-
pondent vnto the distance.   Now, this quality of
the sighte of the Eye ys of no quantity or bignes:

But onely the quality of the eye ys to see, and
begynneth at a poynte, withoute any quantity or
bignes.   As for example you may knowe yt by
this : Pricke a hoale in any thinge, with the poynte
of a fine needle, and then holde that vnto your
Eye, and beholde any thinge thorowe the hole,
and you may see a greater thinge, if that yt bee
any distance from yow : yow may beholde a
whole towne, beeynge a greate distance from
you, &c.   And for that perspective ys the dis-
cerning of any thinge either substancyall or acci-
dentall, accordinge to the bignes, and distance, and
hath his boundes, betweene too righte lynes, from
a poynte : And so extending infinitely from the
sight of the Eye, yt showeth yt self according to
the quantity or bignes, correspondent vnto the
distance.   And for that perspective ys muche
amplifyed and furdered by the vertue and meanes
of Glasses, I do thinke yt good to shewe the pro-
perty of glasses : And suche, as touchinge the
nature and quality of glasses, commonly called
Lookinge Glasses.   Whiche are those sortes of
Glasses, that have a ffoyle, layde on the backe syde
thereof, that causeth the same glasse to cast from
yt a beame or shadowe, accordinge vnto the
forme of that thinge that standeth ageanst yt
shewynge yt vnto the sighte of the eye.   Whereof
there ys three severall sortes, accordinge vnto
the sundry makinge and polisshinge of these
glasses : I do not meane sondry sortes of stuffe,
ffor that theare ys some sortes of lookinge glasses

that are made of metalles, which are commonly
called steele glasses.   But I do meane three son-
dry sorts of forms of making them.   As in the
one sorte, the beame will shewe ytself accordinge
to the bignes, as yt ys: And in the other sorte
yt will shewe ytself less, as in the order of the
making of yt, the face of a man shall not shewe
ytself as broade as the nayle of youre hande.
And so accordingly you may make the glasse to
shewe the face, at what bignes you lyst, vntill
that yt shall shewe the face at inst the bignes
thereof.   And the thirde sorte of the making of
glasses as you may make the glasse, in sorte,
that you will make the face as bigg as the whole
glasse, howe broade soever that the glasse ys,
yow standing at some one assigned distance.

CHAPTER II.

In what forme to make Glasses, for to have yt
shewe yt self, according to the bignes of the
thinge.

For to have a Glasse (to shewe the beame vnto
the eye of that biggnes that the thinge ys) That
glasse must bee made flatt and playne ; and being
well polysshed, and smoothe, and well foyled on
the back syde.   Then you standinge righte with
the middle thereof, you shall receave a beame
vnto your eye of the trewe forme and shape of

your face, or any thinge, that standeth directly
right against yt.   But yf that you do stande any
thinge oblique, or awrye, so shall you receave
the beame, or see any thinge that maketh the
lyke triangle, eyther acute, or sharpp, and obtuse
or broade ; Accordinge vnto the angle : that
commeth from youre eye, vnto the glasse.   So
shall you receave, or see that beame, accordinge
vnto the angle on the other syde, whatsoever that
yt bee, howe farr soever that thinge ys in distance
from the place, or neare hande.

CHAPTER III.

In what forme to make glasses for to have them
shewe the forme or facyon of any thinge less in
bignes then yt ys.

To have a glasse to make any thinge
shewe smaller then yt ys, That must be in
the makinge thereof made hillye or bossy out-
wardes, and to have the foylle layde on the
hollowe or concave syde, and so yt may bee
made, that youre face showe in the beame
that commeth to the eye, as small as you
lyst, or any syse that ys less than your face
at your pleasure.   And for to have yt shewe
very smalle, then let yt be made half a globe, or
boawle, as smalle as a Tennys balle, and so the
foylle layde on the concave or hollowe syde.   So

shall the beame that ys cast vnto your eye, shewe
your whole face, not to be so bigg as the nayle
of your fingeres, and so you may make yt to bee
hilly, or bossy outewardes, to have any thinge to
shewe of what syse, that you lyst.   For if that
yt bee made as half a boawle or globe, Then of
what syde soever you doo stande, you shall see
youre owne face, or any thinge that standeth
right as you do stande : And to bee in swellenes
accordinge to the forme of the hylling or bossing
outwardes.   And allso as many tymes, that you
doo see looking glasses, which make the face
longer or broder then the forme or proportyon of
your face, the reason thereof ys, that yt hilleth or
bosseth more one way, then yt dothe another
way.   And that way that yt bendeth moste
outewardes, that way yt maketh moste narrowest,
and that way, that the glasse ys moste myghtest,
that way yt sheweth the face moste longest.
For yf that a glasse were made righte one way,
and rounde outwardes the other way, and the
foyle layde on the hollow syde, Then that Glasse
woulde make the face, the streighte way, the
inst lengthe of the face, and the other way
narrowe accordinge vnto the roundinge of the
glasse.   So that all sortes of Lookinge glasses,
that dothe bosse or hyll owtewardes, dothe shewe
the thinge less then yt ys, accordinge vnto the
bendinge hilling or bossinge outwarde.

CHAPTER IV.

In what maner of forme to make Lookinge
Glasses, to make any thinge shewe bigger then
yt ys.

To make lookinge Glasses for to shewe any
thinge bigger then yt ys, That Glasse muste bee
made very large : for elles yt will not conteyne
any quantitye in sighte ; and this glasse must
bee Concave inwardes, and well pollyshed of the
hollowe or concave syde : and then the foylle
must bee layde on that syde that doth swell, as
a hyll, and bosse outwarde, And then this glasse,
the property of yt ys, to make all thinges which
are seene in yt to seem muche bigger then yt ys
to the syghte of the Eye, and at some appoynted
distance, from the glasse, accordinge to the
forme of the hollowness, the thinge will seeme at
the biggest, and so yow standinge nearer the
thinge will seeme less, vnto the sighte of the
eye : so that, accordinge vnto the forme of the
concavity or hollownes, and at some appointed
distance from hym that looketh into the glasse,
And yf that the glasse were a yearde broade, the
beame that shoulde come vnto his eye, shall
showe his face as broade, as the whole glasse,
And to see his face in this glasse, hee must stande
righte with the middle of the glasse, &c. And
these sortes of glasses ys very necessary for

perspective : for that yt maketh a large beame, whereby that 'a small thinge may be seene, at a greate distance from you : and especially to bee amplified by the ayde of other glasses, &c.

CHAPTER V.

In what order to make a glass, that yow may looke thorow, that shall forther your sighte, and to have a small thynge to seem bigg, which ys very necessary for perspective : And yt may bee so made, that you may discerne a small thinge, a greate distance, and specyally by the ayde of other glasses.

And nowe furdermore, as I have shewed before, the forme, and facyon of glasses, that dothe reflect a beame from the glasse, commonly called Lookinge Glasses : So in lyke manner I will shewe you the makinge of Glasses called perspective glasses, that do helpe sighte, by the meanes of the beame, that pearceth commonly thorowe the glasse.   And first for makinge of the smallest sorte of them, commonly called spectacle glasses. These sortes of glasses ys grounde vppon a toole of Iron, made of purpose, somewhat hollowe, or concave inwardes.   And may be made of any kynde of glasse, but the clearer the better.   And so the Glasse, after that yt ys full rounde, ys made fast with syman vppon a smalle block,

and so grounde by hande, vntill that yt ys bothe
smoothe and allso thynne, by the edges, or sydes,
but thickest in the middle.   And then yt ys the
quality or property of the Glasse that ys cleare,
to shewe all thinge, that ys seene through yt, to
seeme bigger and perfecter, then that yow may
see yt withoute the Glasse, and the thynner vnto
the sydes and edges.   And the thicker that yt
ys in the middle, the bigger or larger any thinge
sheweth vnto the eye, and yf that the glasse bee
very cleare, the more perfecter, &c.   And now
allso in lyke manner for to make a glasse for
perspective, for to beholde, and see any thinge,
that ys of greate distance from yow, which ys
very necessary : for to viewe an army of men,
or any castle, or forte, or such other lyke causes.
Then they must prepare very cleare, and white
Glasse that may bee rounde, and beare a foote
in diameter ; as fyne and white Vennys Glasse.
And the larger, the better : and allso yt must bee
of a good thicknes, and then yt must bee grounde
vppon a toole fitt for the purpose.   Beynge sett
fyrst vppon a syman block, and firste, grynde
on the one syde, and then on ye other syde,
vntill that the sydes bee very thynn, and the
middle thicke.   And for that yf the glasse bee
very thicke, then yt will hynder the sighte.
Therefore yt must bee grounde vntill that the
myddle thereof bee not above a quarter of an
ynche in thickness : and the sydes or edges very
thynne, and so polysshed or cleared.   And so

sette in a frame meete for the purpose for use :
so that yt may not be broken. And so this glasse
being made in this forme, Then yt wille have
three marvellous operacyons, or qualityes, as
hereafter you shall see.

CHAPTER VI.

*The first, and the Principall quality of this Glass,
and ys, as touchinge perspectyve.*

The quality of the Glasse, (that ys made as
before ys rehearsed) ys, that in the beholding
any thinge thorowe the glasse, yow standinge
neare vnto the Glasse, yt will seeme thorow the
glasse to bee but little bigger, then the propor-
tion ys of yt : But as yow do stande further,
and further from yt, so shall the perspective
beame, that commeth through ye glasse, make the
thinge to seeme bigger and bigger, vntill suche
tyme, that the thinge shall seeme shall seeme [7] of
a marvellous bignes : Whereby that these sortes
of glasses shall muche proffet them, that desyer
to beholde those things that ys of great distance
from them : And especially yt will be much
amplifyed and furdered, by the receavinge of the
beame that commeth thorow the glasse, somewhatt

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[7] This repetition of the words " shall seeme " is evidently
a mistake, but being in the original MS. I have retained it in
the text.

concave or hollowe inwardes and well polysshed
as I will hereafter furder declare.

CHAPTER VII.

The seconde quality of this glass made in the
  forme before declared.

The quality of this Glass ys, if that the sunne
beames do pearce throughe yt, at a certayne
quantity of distance, and that yt will burne any
thinge, that ys apte for to take fyer : And this
burnynge beame, ys somewhat furder from the
glasse, then the perspective beame.

CHAPTER VIII.

The thyrde quality of this kynde of Glass, that ys
  grounde, and made in that forme before declared,
  ys to reverse, and turne that thyng that yow do
  beholde, thorowe ye glass, to stande the contrary
  way.

And yf that yow doo beholde any thinge
thorowe this Glasse, and sett the glasse furder
from yowe then the burninge beame, and so
extendinge after that what distance that yow list,
all suche thinges, that yow doo see or beholde,
thoroughe the glasse, the toppes ys turned
downwardes.   Whether that yt bee trees, hilles,
shippes on the water, or any other thinge what-

soever that yt be : As yf that yt were people,
yow shall see them thoroughe the Glasse, theyre
heades downwardes, and theyre feete vpwardes,
theire righte hande turned to theyre lefte hande,
&c.   So that this kynde of Glasse beynge thus
grounde hathe three marvellous qualityes.   For
at some assigned, or appoynted distance, accord-
inge vnto the gryndinge of the Glasse, bothe in
his diameter, and thicknes in the middle, and
thinnes towardes the sydes.   That (beholdinge
any thinge thorowe the glasse) yt shall make
the best perspective beame : So that the thinge
that yow doo see thorowe shall seeme very large
and greate : and more perfitter withall.   And
allso standing further from the glasse yow shall
discerne nothinge thorowe the glasse : But like
a myst, or water : And at that distance ys
the burninge beame, when that yow do holde
yt so that the sunne beames dothe pearce
thorowe yt.   And allso yf that yow do stande
further from the glasse, and beholde any thinge
thorowe the glasse, Then you shall see yt re-
versed and turned the contrary way, as before
ys declared.   So that accordinge vnto youre
severall standinge, nearer, or furder from the
Glasse, beholding any thinge thorowe yt. Suche,
yt hathe his perspective beame : and then stand-
inge furder from the glasse, and then all thinges
seen thorowe, shall shewe vnto the sighte of your
eye, cleene turned, and reversed another way,
whatsoever that yt bee.

CHAPTER IX.

The effects what may bee done with these two last
  sortes of glasses: The one concave with a
  foyle, vppon the hylly syde, and the other
  grounde and pollisshed smoothe, the thickest in
  the myddle, and thinnest towordes the edges or
  sydes.

For that the habillity of my purse ys not able
for to reache, or beare the charges, for to seeke
thorowly what may bee done with these two
sortes of Glasses, that ys to say, the hollowe or
concave glasse: and allso that glasse, that ys
grounde and polysshed rounde, and thickest on
the myddle, and thynnest towardes the sydes or
edges, Therefore I can say the lesse vnto the
matter.   For that there ys dyvers in this Lande,
that can say and dothe knowe muche more, in
these causes, then I : and specially Mr. Dee,
and allso Mr. Thomas Digges, for that by theyre
Learninge, they have reade and seene many moo
auctors in those causes : And allso, theyre ability
ys suche, that they may the better mayntayne
the charges : And also they have more leysure
and better tyme to practyze those matters, which
ys not possible for mee, for to knowe in a nombre
of causes, that thinge that they doo knowe.   But
notwithstanding upon the smalle proofe and ex-
peryence those that bee but vnto small purpose,

of the skylles and knowlledge of these causes, yet I am assured that the glasse that ys grounde, beynge of very cleare stuffe, and of a good largenes, and placed so, that the beame dothe come thorowe, and so reseaved into a very large concave lookinge glasse, That yt will shewe the thinge of a marvellous largenes, in manner vncredable to bee beleeved of the common people. Wherefore yt ys to bee supposed, and allso, I am of that opinyon, that havinge dyvers, and sondry sortes of these concave lookinge glasses, made of a great largeness, That suche the beame, or forme and facyon of any thinge beeynge of greate distance, from the place, and so reseaved fyrste into one glasse : and so the beame reseaved into another of these concave glasses : and so reseaved from one glasse into another, beeynge so placed at suche a distance, that every glasse dothe make his largest beame.   And so yt ys possible, that yt may bee helpped and furdered the one glasse with the other, as the concave lookinge glasse with the other grounde and polysshed glasse. That yt ys lykely yt ys true to see a smalle thinge, of very greate distance.   For that the one glasse dothe rayse and enlarge, the beame of the other so wonderfully.   So that those things that Mr. Thomas Digges hathe written[8] that his

8 "My father by his continual paynfull practises, assisted with demonstrations Mathematicall, was able, and sundrie times hath by proportionall Glasses duely situate in convenient angles, not onely discovered things farre off, read letters, numbred

father hathe done, may bee accomplisshed very well, withowte any dowbte of the matter : But that the greatest impediment ys, that yow can not beholde, and see, but the smaller quantity at a tyme.

peeces of money with the very coyne and superscription thereof, cast by some of his freends of purpose vppon Downes in open fieldes, but also seven myles of declared what hath been doon at that instante in private places."—*Pref. to Pantometria.* Edit. 1571. Sig. A. iii. b.

JOHANNIS ROBYNS

COMMENTARIA.

DEDICATIO.

Ad invictissimum principem Henricum ejusdem
nominis octavum, Serenissimum Anglorum Re-
gem, de cometis commentaria Johannis Robyns
sui Alumni, et socii Collegii Omnium Animarum,
Oxoniæ.

Quanquam omnes philosophiæ partes (Princeps
Illustrissime) tum utiles, tum jucundæ sint ; ea
tamen philosophiæ portio, quæ de cælis, cælo-
rumque motibus ordine ac influentiis agit, cæteris
longe præstantior, multo jucundior, et pene
divinum quid nobis esse videtur. Id quod
luculentius apparebit, si de his, quæ in aliis phi-
losophiæ partibus tractantur, pauca dixerimus.
Et primam a crassissimo, et infimo elemento,
scilicet terra, orationem inchoabimus, quæ in
media mundi sede collocatur, cernitur solida

globosa, et undique nutibus suis conglobata ;
floribus, herbis, arboribus, frugibus ornata,
quorum omnium incredibilis multitudo insatiabili
varietate distinguitur.   Adde etiam fontium geli-
das perhennitates ; liquores perlucidos amnium ;
riparum vestitus viridissimos ; speluncarum con-
cavas altitudines ; saxorum asperitates ; impen-
dentium montium sublimitates ; immensitates-
que camporum.   Adde etiam reconditas auri,
argenti, metallorumque venas, infinitamque vim
marmoris.   Quæ vero, et quam varia genera
bestiarum, vel cicurum, vel ferarum ! qui volu-
crum lapsus ! atque cantus ! qui pecudum pas-
tus ! quæ vita silvestruum !   Quid jam de homi-
num genere loquar ! qui quasi cultores terræ
constituti, nec patiuntur eam immanitate bellu-
arum efferari, nec stirpium asperitate vastari.
Quorumque operibus agri, insulæ, littoraque
collucent, distincta tectis et urbibus.   Jam de
liquidibus et fusilibus elementi (maris inquam)
pulchritudine pauca dicamus oportet.   Cujus
quidem maris speciem, et animantium quæ in
eodem continentur, satis admirari nequeo.   Nam
ipsum mare undique terram circumfluens, conti-
nentes, insulas, peninsulas, et isthmos efficit, et
quatenus diversas alluit regiones, et diversa sor-
titur nomina ; ut mare Britannicum, Gallicum,
Libicum, Punicum, et cætera.   Ac ita quidem
terram appetens circumquaque littoribus claudit,
et quasi una ex duabus naturis conflata videatur.
Quid de piscibus, belluisque marinis dicam ?

E

Quot genera, quamque disparia, partim submer-
sarum, partim fluitantium, partim innatantium
belluarum, partim ad saxa nativis testis inhæren-
tium? De quarum generatione, proprietatibus,
et natura, Aristoteles omnium philosophorum
facile princeps in historia naturali acutissime
disputat. Exinde mari finitimus est aer; cujus
suprema portio purior quidem, et sincerior, cœli
orbiculationem (sicuti et ipse ignis) subsequitur.
Media vero aeris plaga quæ (ut doctiores sen-
tiunt) non parum frigescit, aptior deputatur
locus, ubi vapores elevati in nubes concrescant.
Inibi etiam fulmina, fulgetra, et tonitruum sedes
sibi peculiares vendicant. At infima aeris regio,
quæ partim terræ, partim aquatico elemento
conterminata junctaque cernitur, avium volatus
sustinet, et salutarem, vitalemque spiritum ani-
mantibus quidem terrestribus ministrat. Jam ab
aere ad ignem postea nostra demigrabit oratio,
qui, ex elementis in supremo ac maxime sublimi
loco constitutus, aerem in sua concavitate fovens,
orbiculariter complectitur. Sicut aer aquam, et
aqua terram undique circuit, nisique una pars
terræ propter vitam quorundam animantium
tuendam aquis discooperta relinquitur. Neque
sentiendum est, qua deus et natura inaniter, ac
superflue hoc quartum corpus simplex scilicet
ignem superaddiderunt. Aeris enim caliditas ab
aquæ terræque frigiditate ilico consumeretur,
nisi superior ignea vis calefactiva aerem refocil-
laret. Præterea terræ siccitas ab aere et aqua

facillime obrueretur, nisi ignis vim suam arescen-
tem emittens, aeris humiditatem deliniret. Hac-
tenus succincte de elementis, cæterisque corpo-
ribus, quæ infra sphæram activorum et passivorum
continentur. Quarum rerum scientia, si (ut
revera est) jucunda et delectabilis fuerit, quid de
illorum corporum cognitione, quæ cœlestia æter-
naque esse feruntur sentiendum est? A quorum
influentiis et virtutibus tum elementata (ut ita
dicamus) suum regimen sortiuntur. Ab istis
enim orbis terrarum plagæ ac regiones habitatæ
quibusdam certis temporum vicibus suas magnas
hyemes et æstates (ut Aristotelico utar verbo)
recipiunt. Magna autem æstas dicitur cum
aliqua provincia ex nimia siccitate et fervore ita
sterilescit, ut diutius inhabitatores fovere ne-
queat. Versa ince, magnam hiemem alicujus
regionis appellamus, cum humectantes cœlorum
influentiæ sic in eadem regione sæviunt, ut terra
fluctibus et aquis penitus obruatur; id quod ab
aliquibus vocatur diluvium particulare: et non
solum istæ prodigiosæ mutationes cœlorum con-
stellationibus tanquam causis attribuuntur, verum
etiam quæ diversæ regiones diversa progignunt,
corporum cœlestium benignitati aut asperitati pri-
mitus ascribitur. Quælibet enim terræ habitabilis
portio, suos fructus et fruges, arbores, herbas,
ac frutices, lapides etiam pretiosos et mineralia,
pro cœli qualitate et influentiis producit. Quid
de fertilitate et abundantia, pace atque tranquilli-
tate, fame, bello, pestilentia, et brutorum animan-

tium strage dicam? quid de sectis, legibus,
civitatibus, imo et regnis integris loquar? Quo-
rum omnium fælicitas et infælicitas, mala, damna,
et infortunia, successiones, prosperitates, et bona,
dispositive saltem a cœlorum constellationibus
dependent. Nec minorem certe potestatem in
liquido elemento (inquam) in mari influentiæ
cœlestes exercent. Maris enim fluxus, et refluxus
æstuosi, in istis fretorum angustiis, a virtutibus
lunæ demanant. Præterea piscium et marinarum
belluarum ortus, et interitus, augmentationes,
diminutiones, et alterationes; necnon eorundem
copia, ac penuria, astrorum virtutibus (omnium
doctissimorum consensu) ascribuntur. At in aere
quam manifestum imperium habent sidera, quo-
rum præsentia vel absentia efficitur, ut idem aer
nunc fulgentissima luce splendescat, nunc obscu-
rus et tenebrosus relinquatur. Sol etenim exori-
ens, suis clarissimis radiis aerem illustrans, diem
conficit, et cum primum a nobis recesserit subter
cardinem occidentalem . . . . . . , subrepta nox,
quippe quæ nil aliud est, quam ut aer (propter
solis sub orizonte dilapsum) tenebris offundatur.
Rursus et a solis cursu sub zodiaco quatuor anni
tempora, ver, æstas, autumnus, et hyems pro-
ficiscuntur. In quibus equidem temporibus sol
aerem et cætera elementa tum tristitia quadam
contrahit, tum vicissim ita lætificat, ut cum ipso
cœlo exhilarata esse videantur. Nolim hic in
cæterarum stellarum (quæ erraticæ dicuntur)
ratione, ordine et influentiis multus videri, qua-

rum tantus est concentus ut cum summa Saturni
infrigidet, media Martis accendat, his interjecta
Jovis stella temperet et illuminet, infraque Martem
tres soli obediunt; Luna, Venus, et Mercurius.
Et hæ omnes stellæ quas errores appellamus,
iisdem vagantur spatiis, quibus et sol, et eodem
modo circa terram agitatæ oriuntur, et occidunt,
quarum motus nunc incitantur, nunc retardantur,
suisque aspectibus et influentiis, (secundum quod
sub variis discurrant signis) quatuor anni tempora
nunc calidiora, nunc frigidiora, nunc humidiora,
nunc sicciora redeunt : qua contemplatione nihil
pulchrius, nihil admirabilius esse potest. Quid
de gelu, nive, grandine, ventis, pluvia tonitruo,
fulmine, fulgetro, stellis item caducis et per
aerem discurrentibus dicam ? quæ omnia ab
astrorum constellationibus progrediuntur. Pos-
tremo et illa crinita sidera famosa quidem et
portentosa cœlestium stellarum vires, causam suæ
generationis, et, si rarius spectantur, æque natu-
raliter sibi, vendicant, atque ea quæ ab oculis
quotidie videntur. Sed plebei quamvis numerosa
rerum naturalium multitudo recitari possit, qua-
rum causæ secretiores et magis abditæ sunt,
tamen ob insolitam ipsius rei faciem maximopere
cometas admirantur. De quibus siquidem cometis
tua sacrosancta majestas voluit, ut a nobis quæ-
dam commentaria in lucem æderentur. Id quod
non hac quidem de causa factum erat (certissime
scio) ut in his nostris perlegendis, tua sacrosancta
majestas quidpiam eruditionis exhauriret, sed

aliorum consulens utilitati.   Ut qui harum rerum
(de quibus tractamus) minus essent periti ex
nostris commentariis aliquid saltem adipisci pos-
sent.   Non enim incognitum est (eruditissime
Princeps) quam singularis in scientiis mathe-
maticis tua fuerit doctrina.   Id quod ex illis
disputationibus manifestissime mihi perspectum
erat, quas de cometarum naturis et effectibus
tum Woydstokiæ, tum Bokynghamiæ nobiscum
habuisti, quis enim invenire poterat argutius,
quis colligere veruosius, quis explicare venustius ?
In aliis vero scientiis et potissime theologia
(serenissime Rex) quam præstantissima tua fuerit
cognitio, nihil dicturus sum.   Opera enim ea quæ
a tua sacrosancta majestate edita sunt, sufficienter
ostendunt.   In quibus equidem operibus (ut illud
quod sentio ingenue dicam) nec Augustino, nec
Hieronimo cædere videris.   Nam quam caste
quam nitide, quam polite omnia mihi disputantur,
solidi, succi, et nervorum plena, sed temperabo
me a laudibus.   Scio etenim, scio (laudatissime
Princeps) quam nolunt laudari hi, qui laudes
maximæ merentur omnium.

[*From what is here given the nature of the work may be
seen, and I do not think it as a whole necessary to be printed
in this place.*—Ed.]

TWO TABLES:

ONE SHEWING THE TIME OF HIGH WATER AT LONDON BRIDGE, AND THE OTHER THE DURATION OF MOON LIGHT.

FROM A MS. OF THE 13TH CENTURY.

Bib. Cott. Mus. Brit. Jul. D. vii. Fol. 45, b.

FLOD AT LONDON BRIGGE.   **QUANTUM LUNA LUCET IN NOCTE.**

Ætas Lunæ.	Horæ.	Minuta.	Ætas Lunæ.	Horæ.	Minuta.	Ætas Lunæ.	Horæ.	Minuta.	Ætas Lunæ.	Horæ.	Minuta.
1	3	48	16	3	48	1	10	48	16	11	12
2	4	36	17	4	36	2	1	36	17	10	24
3	5	24	18	5	24	3	2	24	18	9	36
4	6	12	19	6	12	4	3	12	19	8	48
5	7	0	20	7	0	5	4	0	20	8	0
6	7	48	21	7	48	6	4	48	21	7	12
7	8	36	22	8	36	7	5	36	22	6	24
8	9	24	23	9	24	8	6	24	23	5	36
9	10	12	24	10	12	9	7	12	24	4	48
10	11	0	25	11	0	10	8	0	25	4	0
11	11	48	26	11	48	11	8	48	26	3	12
12	12	36	27	12	36	12	9	36	27	2	29
13	1	24	28	1	24	13	10	24	28	1	36
14	2	12	29	2	12	14	11	12	29	0	48
15	3	0	30	3	0	15	12	0	30	0	0

A TREATISE

MENSURATION OF HEIGHTS AND DISTANCES.

FROM A MS. OF THE 14TH. CENTURY.

Bib. Sloan. Mus. Brit. 213. xiv. *Fol.* 120.

Nowe sues here a Tretis of Geometri wherby you may knowe the heghte, depnes, and the brede of mostwhat erthely thynges.

Geometri[9] es saide of þis greke worde *geos.* þat es erthe on englisch. and of þis greke worde

[9] Vid. MS. Bib. Reg. Mus. Brit. 17 A. 1. f. 2b–3.

> "The clerk Euclyde on þis wyse hit fonde
> Thys craft of gemetry yn Egypte londe
> Yn Egypte he tawghte hyt ful wyde,
> Yn dyvers londe on every syde.
> Mony erys afterwarde y vnderstonde
> Gher þat þe craft com ynto þys londe.
> Thys craft com ynto England, as y ghow say,
> Yn tyme of good kyng Adelstones day."

This notice of the introduction of Euclid's Elements into England, if correct, invalidates the claim of Adelard of Bath, who has always been considered the first that brought them from abroad into this country, and who flourished full two centuries

metros. þat es mesure on englisch. þan es geometri als erthly mesure. for it es one of þe sevene science techyng to mesure þe erth in heght. depnes and brede. and length. THIS TRETIS ES DEPARTED IN THRE. þat es to say. hegh mesure.

after the "good kyng Adlestone." Adelard translated the Elements from the Arabic into Latin, and early MSS. of the translation occur in so many libraries, that we may fairly conclude that it was in general circulation among mathematicians for a considerable time after it was written. Tiraboschi was the first who remarked, that the translation attributed to Campanus, was in reality Adelard's, with a commentary by the former, (Libri Hist. des Mat. tom. II. p. 48); there are also several MSS. in the titles to which this is expressly stated, (MS. Bib. Bodl. Seld. Arch. B. 29. MS. Bodl. 3623. MSS. Paris, Lat. 7213, 7214, 7215. MS. Harl. Mus. Brit. 5266 : "per Adelardum Bathoniensum de Arabico in Latinum translatus, cum Commento Campani Novariensis."). In the Library of Queens' College, Cambridge, there was formerly a MS. entitled "Geometria Euclidis cum Commentario Adelardi" (Lelandi Collect. tom. iii. p. 19), and Chasles (Aperçu Historique, p. 509,) mentions one in the library of the Dominicans of St. Marc at Florence, under the same title; this would appear to intimate that the commentary is also by Adelard, and many MSS. in which the name of Campanus does not appear, contain the commentary, (MS. Oxon. in Coll. S. Trin. 30, iv. MS. Harl. Mus. Brit. 5404. MS. Bib. Pub. Cant. Dd. 12, 61). Doctor John Dee, in the Catalogue of his Collection of MSS. (MS. Harl. 1879. MS. Bib. Trin. Coll. Cant. Collect. Gal. O. 4, 20. MS. Ashm. 1142.) gives the title of one, in which the books of Euclid on Optics and Catoptrics, as well as the Elements, appear under Adelard's name.

There have been two independent notices of the plagiarism of Campanus; one by Charles Butler, (the author of the "Introduction to the Mathematics," 8vo. 1814,) in some MS. papers in the Editor's possession : the other by the Author of the article *Geometry,* in the "Penny Cyclopædia."

playne mesure. and depe mesure.   First foryi
shewe we hegh mesure. þat es to say howe any
thynge þat has heght may be met howe hegh it
es. and þis may be done in many maneres. first
þerfor schewe howe it may be done by þe quad-
rant.   When you wille wite þe heghte of any
thyng þat you may negh. biholde þan þe heght
of þat thyng by bothe holes of þe quadrant and
come toward and go froward til þe perpendicle
þat es to say þe threde whereon þe plumbe henges
falle vpon þe mydel lyne of þe quadrant. þat es to
say þe 45 degre. þan take als mykel lande behynde
ye as fro þethen to þe erthe and marke wele þat
place. þan mete howe many fete are bytwene þi
mark and þe fondement of þat thyng whos heght
you sekest. and sekirly so many fote heght it es.
Also when you wilt wite þe heght of any thynge
by þe quadrant. biholde þe heght of þat thyng by
bothe þe holes. and byholde vpon what place of
þe quadrant þe perpendicle falles. for ouþer it
wille falle on þe vmbre toward or on þe vmbre
froward. and if it falle vpon þe vmbre toward
biholde vpon whilk poynte of þat vmbre þe
perpendicle falles. þan mesure þe distaunce þat
es to say þe space betwene ye and þat thyng
whos heght you sekes, and when you has so done
þan multiply you by 12 þat same mesure. þan al
þat comes of þat multiplyeng departe you by þe
nonmbre of þe poyntes of þe vmbre. and to alle
þat comes þereof set þe quantite of þin heghte.
and set al þis togydere. and þan you has þe

heght of þe thyng whos heght you sekes.   If
peraventure þe perpendicle falle vpon þe vmbre
froward biholde þan þe poyntes þereof and torne
þem into þe poyntes of þe vmbre toward and do
furth þan as we taght byfore and þus sekirly you
sale have þe heght of þe thyng whos heght you
sekes. namely so þe space be playne bitweene þe
and it.   Note you þat þe quadrat. þat es to say
4 square whilk es descryvede þat es to say
schewed in þe quadrant has tuo sides. þat es to
say þe side of þe vmbre toward. and þe side of þe
vmbre froward. and aþer of þese 2 sides es depar-
ted in 12 even parties.   When you holdes þe cone
of þe quadrant. þat es to say þe cornel of þe
quadrant even vpryght in whilk cornel es þe nayle
whereby þe perpendicle henges. þan þe circum-
ferens. þat es to say þe cumpasse es toward þe
erth. þan þat side of þe quadrat whilk es nere
ye es called þe vmbre toward and þat oþer side es
called þe vmbre froward. and þe 12 departynges
of aiþer of þo sides are called poyntes. þan es
a poynte þe twelft parte of any thyng. namely of
ouþer side of þe quadrat in þe quadrant.   Also
when þe heght of þe sonne es more þan 45 degres.
þan þe perpendicle falles vpon þe vmbre toward.
And ageynward when þe heght of þe sonne es less
þan 45 degrees. þan þe perpendicle falles vpon þe
vmbre froward.   When sothly þe heght of þe
sonne es even 45 degrees. þan þe perpendicle
falles even vpon þe 45 whilk es þe medil lyne.
If you may noght negh þe thyng þat you wolde

mesure for letting of water or summe oþer thyng
bitwene. þan biholde þe heght þereof by bothe þe
holes. and biholde þe nonmbre of þe poyntes of
þe vmbre toward. namely vpon whome þe per-
pendicle falles. þan set D for a mark in þat place
where you stondes þan. go ferre or nerre þat
thyng whos heght you sekes. and þat by an evene
lyne and beholde este þe forsaide heght by bothe
holes. þan þere stondyng seke þe nonmbre of þe
poyntes of þe vmbre toward. and set þere C for
anoþer mark. þan mesure howe many fete are
bitwene þise tuo markes D and C and kepe þat
wele in þi mynde. þan abate þe lesse nonmbre of
þese tuo in þe vmbre toward fro þe more and kepe
wele þe difference bytwene þo tuo nonmbres.
þan multiplie by twelve þe distaunce bitwene
þe forsaide D and C and alle þat comes of þat
multeplyeng departe you by þe distaunce of þe
poyntes. and to þat þat leves over. set als mykel
as fro þethen to þe erthe and þan sekirli you has
þe heght of þat thynge. bot loke þe holes of þi
quadrant be right straite and elles you may
lightly be deceyvede. Peraventure you standes
in an aley. and þe thyng þat you wolde mete
es vp on an hegh hille. first þan biholde þe
heghte of þe hille by both þe holes of þe
quadrant. and þat by tuo stondynges of D and
C as we taght nowe next bifore. and marke
þat wele in þi mynde. þan biholde in þe same
wise þe heght of the hille and of þat same
thyng togedire. þan abate þe heght of þe hille

fro al þat remenant and þan sekirly hast ou þe
heght of þat thyng. If peraventure þe contry
es hilly. þan do þat þe perpendicle falle even
vpon þe begynnyng of þe side of þe vmbre
froward. þan se by bothe holes in þe thyng to
be mesured þe poynte þat es called A and do
as we taght bifore. and þat leves after þi
wirkyng es þe heght fro A poynte to þe heght
of þe thyng : bot þan salt you noght set þereto
þe heght of þi stature. Parcas you woldest
mesure þe heght of a thyng by the schadowe
þereof. þan abyde til þe sonne be in þe heght
of fyve and fourti degrees. þan mesure þe
vmbre of þat thyng and þat es þe heght þereof.
If you wilt mesure þe heght of any thyng by
þe schadowe þat es to say þe vmbre þereof in
ilk houre þan do þus. mesure þe vmbre þat es
to say þe schadowe of þat thyng. and multiply
þat by 12 and al þat departe þan by þe poyntes
of þe vmbre toward. and þe nonmbre howe ofte
euer it be es þe heght of þat thyng if þe per-
pendicle falle vpon þe vmbre toward. If þe
perpendicle falle vp þe poyntes of þe vmbre
froward. þan multeply þe vmbre. þat es to say
þe schaddowe by þe poyntes of þat vmbre. and
al þat departe by 12 and þe nonmbre howe oft
it be. es þe heght of þat thyng. Or if you
wilt. lede þe poyntes of þe vmbre froward into
þe poyntes of þe vmbre toward. and þan multeply
þe vmbre. þat es to say þe schaddowe of þat
thyng by 12 and þan al þat departe by þe

poyntes of þe vmbre ladde. Whilk after þe
ledying are poyntes of þe vmbre toward. Take
þe poyntes of the vmbre þus. late þe sonne
benies passe by bothe holes. and mark where
þe perpendicle falles. þan counte þe poyntes
fro þe begynnyng of þe side of þe vmbre to
þe touche of þe perpendicle. and þo are þe
poyntes of þe vmbre. þat es to say þe scha-
dowe. What poyntes ever yei be. wheþer of
þe vmbre toward or froward. When any thynge
es whose hight you wilt mesure by þe schad-
dowe þereof and a ȝerde. þan rere even vp a
ȝerde vpon þe playne grounde were þe ende
of þe schadowe of þat thyng whilk you wille
mesure, so rere it þat þe one parte of þame
ȝerde falle vpon þat schadowe. and þe oþer part
of þat ȝerde falle withouten. and mark þe place
in þe ȝerde where þe schadowe begynnes to
touche it. and by þe quantite of the ȝerde
whilk es bitwene þe touche of þe schadowe
in þe ȝerde and þe playne. multeply þe quan-
tite of al þe schadowe whilk es bitwene þe
lower party of þe thyng to be mesured and
þe toppe of þe schadowe in þe playne. and
depart þan al þat by þe quantite of þe schadowe
whilk es bitwene þe toppe of þe schadowe and
þe ȝerde. and þe nonmbre þat þan comes es þe
heght of þat thyng. Also if you wilt mesure
þe heght of a thyng by þe schadowe in ilk
houre of þe day. take a ȝerde of two fote longe
or thre. and on a playne rayse it even vp. and

þan mesure þe schadowe of it. þan mesure þe
schadowe of þat thyng to be mesuride. and
multiply þat by þe length of þe ʒerde. and þan
departe al þat by þe schadowe of þe ʒerde and
þat nonmbre howe ofte it be es þe heght of
þat thyng.   If you wilt have þe heght of any
thyng wantyng grounde. as if you be in an
house and wolde wite howe ferre were any
thyng beyng in þe rofe.   Take a table and
rayse it vp a litel fro þe erthe. so þat you
may se þat ilk thyng bitwene þe erthe and þat
table. þan take a reulure and continu it to þe
table. þat es to say side to side. and þan se
by al þe reulure þe thyng to be mesurede.
and þan drawe a lyne in þe table by þe reu-
lure. este do þe same in anoþer site. þat es to
say place of þe table. and make anoþer lyne.
þan mesure þe heght whilk es bitwene þe hy-
here parte of þe table and þe erthe. and þat
mesure sale be callede þe heght kept. þan set
þe table on þe erthe and take tuo thredes and
put þat one in þat one lyne. and þat oþer threde
in þat oþer lyne. þan make a mark þere as þise
tuo thredes metes. þan mesure þe length bitwene
þe mark and þe table. and set þereto þe heght
kept. and þan hast ou þe heght of þat thyng.
If you wilt mesure þe heght of any thyng
withouten quadrant and withouten schadowe.
rayse evene vp a rodde on a playne ageyne
þat thyng and go toward it and froward it til
þi sight beme passe by þe heght of þe ʒerde

and of þat thyng. þan loke[1] howe mikil es bi-
twene þi fote and þe grounde of þat þyng in þe
tyme of beholdynge. and to þat length set þe
space fro þethen to þe erthe. þan multiply al þat
by þe length of þe ʒerde. and þan departe al þat
by þe space bitwene ye and þe ʒerde and þere-
with þe quantite fro þethen to þe erthe. and
þat es þe heght. Also when you wilt mesure
þe heght of any thing by two ʒerds, even cor-
neldly joyned, take a ʒerde even to þe length of
þi stature. and anoþer ʒerde, tuo so longe als
þat in þe myddel of þe lengere ʒerde, set þe
schortere even corneldely þan þis instrument þus
made layde by þe playne ground til by þe toppes
of bothe þe ʒerdes you se þe toppe of the þing
to be mesured, þan make þere a mark and set
þereto þi stature. and þan set þe marke þere.
and so heigh es þat thyng howe mekil length es
bitwene þe grounde of þat thyng and þe latter
mark. bot forgete noght þat perpendicle or equi-
pendy, þat es to say, even hangere lolle by þe
toppe of þe longer rodde to schewe when þin
instrument es even vpright, and when it bagges
Ensample, þe stature of þe matere be called AB,
þe ʒerde doubling it CD þe ʒerde evene cor-
neldy joyned to it AE, and the foundement of
þe þyng F, þan I say þe height of the þyng es BF,
with þe quantite of BC. When you standes by
a walle of a castelle or toure, and you wolde
mesure þe heght of it with outen defaute, make

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[1] MS. loloke.

a quadrat or quarterd. þat es to say a table even
foure square of wode or brasse of what quantite
you wilt, and ay þe more it be. þe better it es.
and loke it be over alle square. in þe manner of
ABCD and put a chippe of what length you wilt
in þe cornelle B and anoþer in þe cornelle C
and þe þird in þe cornel D and loke þei be fast
on þat quadrat þat evenly þei stande raysed vp.
and þe side of þe quadrat bitwene A and B mote
be persede reulefully. in whilk persyng put a
chippe like þe oþer thre. bot it sale be moveable
fro A to B and þis chippe sale hight E and wite
you that A es þe right cornel vpward. B þe left
cornel vpward. C þe left cornel donward. D þe
right cornel donward. When þe face of þe quad-
rat es torned toward ye. and þese chippes I calle
eighen as in þe quadrant, þan loke even vp by C
and B chippis or eighne to þe heyest of þe toure.
So þat þe quadrat joyne to þe walle. and þat
highest of þe toure sale hight F loke þe side AB
be departed on 30 or 40 or howe fele you wilt.
and in þe same manere departe þe side AD. þan
move þe chippe E hedire and þedire til you se þe
hiest F ageyne thurgh chippes D, E noght
chaunging þe raþer place BC þan biholde where
þe chippe E stondes bitwene A and B and loke
howe mekil þat part es EA to AD and so mekil
part es DC to CBF and howe ofte EA es in AB
so ofte es CD in CBF forþi multiply DA in AB
and þat comes þereof departe by AE and þat
nonmbre howe ofte it be es þe heght of CBF and

F

þat you have al the heght fro F to þe erthe. me-
sure þe length AC to þe erth. whilk length set
to þe heght CBF and þat comes is þe verey heght.
If you wilt mesure þe heght of any thyng by
a myrure. lay þe myrure in þe playne grounde,
and go toward and froward til you se þe toppe of
þat thing in þe mydel of þat myrure. þan mul-
tiply þe playne bitwene þe foundement of þat
thyng to be mesured and þe myrure by þe space
fro þethen to þe erthe. and þat comes þereof de-
parte by þe space bytwene þi fote and þe myroure
and þe nonmbre howe ofte it be es þe heght of
þat thyng.   Also als fro þethen to þe erthe has it
to þe space bytwene þi fote and þe myrure so þe
heght of þat thyng has it to þe playne. Whilk es
bitwene þe rote of þat thyng to be mesured and þe
myrure and so ageyne.

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*Nowe we have taght to mesure the heght of a
thynge whilk es the first parte of oure tretis.   We
wil teche to mesure the playne. for that es the se-
conde parte.*

When you wilt mesure þe length of any playne
with þe quadrant stonde in one ende of þe playne
and byholde þat oþer by bothe holes. and holde þe
cone. þat es to say þe cornelle of þe quadrant nere
þin eigh. and þe compas toward þe playne to be
mesured. þan when you sees þe oþer ende of þat
playne take þe nonmbre of þe poyntes of þe vmbre
froward whilk þe perpendicle kyttes. þan multeply

þe distaunce. þat es to say þe space fro þethen to
þe erthe by 12 and departe þat comes þereof by
þe nonmbre of þe vmbre of þe poyntes froward
raþer had. and þe nonmbre howe ofte it be es þe
quantite of þe length of þat playne.   Parcas þe
playne whose length you wolde mesure es noght
evene nor even distondyng to þin orisont. þat es to
say to þe ende of þin sight. bot þat playne es lift
vp and croked. þan biholde þe crokidnes þereof
by þe holynge. þat es to say þe eyghne of þe
quadrant. whilk þus you schalt do. set vpright
tuo ȝerdes of one length in þe endes of þe playne
to be mesurede. þan biholde þe toppes of þise
thynges by bothe þe eyghne of þe quadrant. þan
loke howe fele poyntes þe perpendicle kyttes.
and þat of þe vmbre froward. if parcas þe perpen-
dicle falle vpon þe poyntes of þe vmbre toward.
þan torne þem into þe poyntes of þe vmbre fro-
ward. þan kepe þise poyntes.   Est se þe ende of
þat playne by bothe þe eyghen of þe quadrant.
and loke howe fele poyntes of þe vmbre froward
þe perpendicle kyttes. þan set þese poyntes to þe
poyntes raþer kept. if you stonde in a lowere
place aftere þan you did bifore. and if you stonde
in an heighere. þan take away þe poyntes raþer
kept. þan do with þise poyntes as you did with þe
poyntes in þe next Chapitere bifore in mesuryng
of an evene playne. þat es to say multeply þe dis-
taunce fro þethen to þe erthe by 12 and depart þat
comes þereof by þe poyntes of þe vmbre fro-
ward. and þe nonmbre howe ofte it be es þe

quantite of þe length of þat playne.   When you
wilt mesure þe playne of lande or water withouten
þe quadrant. take þan tuo ȝerdes and rayse þat
one even vp right on þe playne. and calle þe
playne BE and þe ȝerde vp raysed AB in whilk
ȝerde set evene corneldly anoþer ȝerde even dis-
tonding to þe playne and þis ȝerde sale hight CD
þan beside þe ȝerde vp raised AB set þin eigh
and biholde þe ende of þe playne to be mesurede.
and mark by whilk place of þat oþer ȝerde CD þi
sight bem passes. and calle þat poynte E þan by
þe quantite of þe seconde ȝerde CD multeply þe
first ȝerde AB and departe þat comes þereof by þe
quantite of AC and þan comes þe length of þe
playne.   When you wilt wite whilk es þe brede
of a ryvere. kast a table vpon þe grounde nere þe
ryvere. þan biholde þe ende of þat oþer side þe
ryver by a reulure vpon þe table.   And drawe
a lyne by þe reulure on þe table.   Est se þe for-
saide ende by þe same reulure in an oþer place of
þe table. and þere make anoþer line. þan go in a
playne place and lay a threde on þat one lyne.
and anoþer threde on þat oþer lyne and continu
þem even til þei come to gidere. and howe mykel
distaunce þat es to say space es fro þe metyng of
þe thredes to þe table so brode es þe ryvere.
When you wilt mesure þe brede of a ryvere by a
quadrat, make þi quadrat ABCD as it es saide
bifore of mesuryng of heght by þe quadrant. þan
set þe quadrat beside þe ryvere and loke by CB
þe ende on þat oþẏer side þe ryvere. and calle þat

ende F and B es þe ende on þis side þe ryvere.
þan go fro B poynte by an evene lyne with þis
quadrat. and þat þe reulure of þat lyne be oc-
thogonyely. þat es to say even corneldly bytwene
B and F til yu se est þe ende F by þe poyntes or
chippes D, B in þe quadrat. and calle G þe mark
in þe place of þe seconde site. þat es to say
stondynge þan howe mekil length es bitwene B
and G so mekil as þe brede bytwene B and F.
Also when you stondes by a ryvere and wille me-
sure þe brede of it. put þis quadrat vpon þe
erche nere þe ryvere. and biholde by þe lyne CB
þe ende on þat oþer side þe ryvere whilk es called
F. þan move E hedire and þidere til you se þe
same ende F by DE noght changyng þe first
place of BC. þan biholde where E stondes bi-
twene A and B þan multiply DA into AB and de-
parte þat comes þereof by AE and þe nonmbre
þat comes howe oft it be es þe brede of þe ryvere
whilk es BF. When you wilt mesure a playne by
a mesure. þan rayse vp right on þe playne a rod
of þe length fro þin eigh to þe erche. and calle
þat rod AB. on þat rod hange a litel myrure. and
ay þe lesse þe better. and þe place on þe rod
where þe myroure hanges calle C and þe playne to
be mesurede sale hight AD. þan stonde you vpon
þo playne bitwene þe myrure and þe ende of þe
length of þe playne. and loke in þe myrure
movyng toward and froward þe myrure til you
se þe ende of þe playne in þe mydille of the
myrure. þat place of þe playne where you stondes

you sale calle it E. þan multiply þe length of þi
stondynge fro þe ʒerde by þe quantite bytwene
þe myrure and þe playne. and departe þat comes
þereof by þe distaunce of þe myrure fro þe hyere
parte of þe rod. and þat nonmbre howe ofte it be
es þe length of þat playne. as þus. multiply AE in
AC and depart þat comes þereof by þe lyne CB.
and þe nonmbre þat comes þereof es þe length of
þe playne.

———————

*Nowe we have taght to mesure the playnes of
ilk erthly thyng whilk es the secunde parte of
this tretyse. So we teche to mesure the depnes.
for that es the thrid parte and laste of this boke.*

When you wille mesure þe depnes of a welle.
loke fro þat one side of þe welle to þe ende of þat
oþer syde in þe bothome of þe same welle with þe
quadrant. and holde þe cornel of þe quadrant nere
þine eigh. and þe circumferens toward þe welle
and take hede what es þe nonmbre of þe poyntes
of þe vmbre toward vpon whilk þe perpendicle
falles. þan mesure þe diameter of þe pit or welle.
þat es to say þe mouthes brede. and multiply þat
by 12 and depart þat comes þereof by þe nonmbre
of þe forsaide poyntes and howe ofte þat nonmbre
be. it es þe depnes of þat pitte. Also for þe same.
rayse vp a table on þe mouthe of þe welle. þan put
vpon þe table a reulure whereby biholde you þe
ende on þe bothome þereof. þan make þer a lyne
by þat reulure. est put þe reulure in anoþer place

of þe table. and þan se by þat reulure þe forsaide
ende. and þere make anoþer lyne by þe reulure in
þat table. þan lay þe table vpon þe playne grounde.
and lay on þise tuo lynes tuo thredes even til þei
mete. þan mesure þe distaunce bitwene þe metyng
of þo thredes and þe table. and so depe es þe
welle. Also for þe same. rayse even vp on þe mouthe
of þe well swilk a quadrat as we spak of bifore.
þan by CB loke þe ende in þe bothome of þe
welle. and þat sale hight F. þan move E toward
and froward til by DE you se est F noght
chaungynge þe first place BC. þan biholde where
E stondes bytwene AB. þan multiply DA in AB
and departe þat comes þereof by AE. and þat
nonmbre howe oft it be es þe depnes of þe welle.
whilk es BF.

Nowe oure tretis of geometri es thus endid.

AN ACCOUNT TABLE

FOR THE

USE OF MERCHANTS.

FROM A MS. OF THE FOURTEENTH CENTURY.

Bib. Sloan. Mus. Brit. 213.

	0	0	0	0	0	0	0	0	0	0	0				
	11	10	9	8	7	6	5	4	3	2	1				
	penyes													0	3
	0	0	0	0	0	0	0	0	0	0	0			0	2
	11	10	9	8	7	6	5	4	3	2	1			0	1
0	12	schillynges										poundes			
0	13												ferthynges		
0	14	schyllynges	ẍ	ṁ	c̈	ẍ	ṁ	ċ	ẋ	m	c	x			
0	15		0	0	0	0	0	0	0	0	0	0	0	1	
0	16		0	0	0	0	0	0	0	0	0	0	0	2	
0	17		0	0	0	0	0	0	0	0	0	0	0	3	
0	18		0	0	0	0	0	0	0	0	0	0	0	4	
0	19		0	0	0	0	0	0	0	0	0	0	0	5	
			0	0	0	0	0	0	0	0	0	0	0	6	
			0	0	0	0	0	0	0	0	0	0	0	7	
			0	0	0	0	0	0	0	0	0	0	0	8	
			0	0	0	0	0	0	0	0	0	0	0	9	

This es tabil marchaunte for alle manere acountes.

CARMEN DE ALGORISMO.

Hæc algorismus ars præsens dicitur;[2] in qua
Talibus Indorum[3] fruimur bis quinque figuris.

  0. 9. 8. 7. 6. 5. 4. 3. 2. 1.

Primaque significat unum : duo vero secunda :
Tertia significat tria : sic procede sinistra
Donec ad extremam venias, quæ cifra vocatur ;
Quæ nil significat; dat significare sequenti.
Quælibet illarum si primo limite ponas,
Simpliciter se significat : si vero secundo,

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[2] " Hæc præsens ars dicitur algorismus ab Algore rege ejus
inventore, vel dicitur ab *algos* quod est ars, et *rodos* quod est
numerus; quæ est ars numerorum vel numerandi, ad quam artem
bene sciendum inveniebantur apud Indos bis quinque (id est
decem) figuræ." *Comment. Thomæ de Novo-Mercatu.* MS. Bib.
Reg. Mus. Brit. 12 E. 1.

[3] " Hæ necessariæ figuræ sunt Indorum characteros." *MS. de*
numeratione. Bib. Sloan. Mus. Brit. 513, fol. 58. " Cum vidissem
Yndos constituisse ix literas in universo numero suo propter dis-
positionem suam quam posuerunt, volui patefacere de opere quod
sit per eas aliquidque esset levius discentibus, si Deus voluerit. Si
autem Indi hoc voluerunt et intentio illorum nihil novem literis
fuit, causa quæ mihi potuit. Deus direxit me ad hoc. Si vero
alia dicam præter eam quam ego exposui, hoc fecerunt per hoc
quod ego exposui, eadem tam certissime et absque ulla dubitatione
poterit inveniri. Levitasque patebit aspicientibus et discentibus."
MS. Bibl. Publ. Cant. 1869, Ii. vi. 5.

Se decies: sursum procedas multiplicando.
Post prædicta scias breviter quod tres numerorum
Distinctæ species sunt; nam quidam digiti sunt;
Articuli quidam; quidam quoque compositi sunt.
Sunt digiti numeri qui semper infra decem sunt;
Articuli decupli digitorum; compositi sunt
Illi qui constant ex articulis digitisque.
Ergo, proposito numero tibi scribere, primo
Respicias quis sit numerus; quia si digitus sit,
Una figura satis sibi; sed si compositus sit,
Primo scribe loco digitum post articulum; atque
Si sit articulus, in primo limite cifram,
Articulum vero tu in limite scribe sequenti.
Quolibet in numero, si par sit prima figura,
Par erit et totum, quicquid sibi continuatur;
Impar si fuerit, totum sibi fiet et impar.
Septem[4] sunt partes, non plures, istius artis;
Addere, subtrahere, duplareque dimidiare;
Sextaque dividere est, sed quinta est multiplicare;
Radicem extrahere pars septima dicitur esse.
Subtrahis aut addis a dextris vel mediabis;
A leva dupla, divide, multiplicaque;

[4] En argorisme devon prendre
  Vii especes .   .   .   .
  Adision subtracion
  Doubloison mediacion
  Monteploie et division
  Et de radix enstracion
  A chez vii especes savoir
  Doit chascun en memoire avoir
  Letres qui figures sont dites
  Et qui excellens sont ecrites.
  MS. *Seld. Arch.* B. 26.

Extrahe radicem semper sub parte sinistra.[5]
Addere si numero numerum vis, ordine tali
Incipe; scribe duas primo series numerorum
Primam sub prima recte ponendo figuram,
Et sic de reliquis facias, si sint ibi plures.
Inde duas adde primas hac conditione:
Si digitus crescat ex additione primorum,
Primo scribe loco digitum, quicumque sit ille;
Sed si compositus, in limite scribe sequenti
Articulum, primo digitum; quia sic jubet ordo.
Articulus si sit, in primo limite cifram,
Articulum vero reliquis inscribe figuris;
Et per se scribas si nulla figura sequatur.
Si tibi cifra superveniens occurrerit, illam
Dele suppositam; post illic scribe figuram:
Postea procedas reliquas addendo figuras.
A numero numerum si sit tibi demere cura,
Scribe figurarum series, ut in additione;
Majori numero numerum suppone minorem,
Sive pari numero supponatus numerus par.
Postea si possis a prima demere primam,
Scribas quod remanet, cifram si nil remanebit.
Sed si non possis a prima demere primam;
Procedens, unum de limite deme sequenti;
Quod demptum pro denario reputabis ab illo,
Subtrahe totalem numerum quem proposuisti.
Quo facto, scribe super quicquid remanebit,
Facque nonenarios de cifris, cum remeabis,
Occurrant si forte cifræ, dum demeris unum;
Postea procedas reliquas demendo figuras.

[5] Vide p. 11.

An subtractio sit bene facta probare valebis,
Quas subtraxisti primas addendo figuras.
Nam, subtractio si bene sit, primas retinebis,
Et subtractio facta tibi probat additionem.
Si vis duplare numerum, sic incipe; solam
Scribe figurarum seriem, quamcumque voles tu;
Postea procedas primam duplando figuram;
Inde quod existit, scribas, ubi jusserit ordo,
Juxta præcepta quæ dantur in additione.
Nam si sit digitus, in primo limite scribe;
Articulus si sit, in primo limite cifram,
Articulum vero reliquis inscribe figuris;
Vel per se scribas, si nulla figura sequatur:
Compositus si sit, in limite scribe sequenti
Articulum, primo digitum; quia sic jubet ordo:
Et sic de reliquis facias, si sint ibi plures.
Incipe sic, si vis aliquem numerum mediare:
Scribe figurarum seriem solam, velut ante;
Postea procedas medians, et prima figura
Si par aut impar videas; quia si fuerit par,
Dimidiabis eam, scribens quicquid remanebit;
Impar si fuerit, unum demas mediare,
Quod non præsumas, sed quod superest mediabis;
Inde supertactum, fac demptum quod notat unum;
Si monos, dele; sit ibi cifra post nota supra.
Postea procedas hac conditione secunda:
Impar[6] si fuerit, hinc unum deme priori,
Inscribens quinque, nam denos significabit
Monos prædictam: si vero secunda sit una,
Illa deleta, scribatur cifra; priori

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[6] *i. e.* figura secundo loco posita.

Tradendo quinque pro denario mediato ;
Nec cifra scribatur, nisi deinde figura sequatur :
Postea procedas reliquas mediando figuras,
Ut supra docui, si sint tibi mille figuræ.
Si mediatio sit bene facta probare valebis,
Duplando numerum quem primo dimidiasti.
Si tu per numerum numerum vis multiplicare,
Scribe duas, quascunque velis, series numerorum ;
Ordo tum servetur, ut ultima multiplicandi
Ponatur super anteriorem multiplicantis ;
A leva reliquæ sunt scriptæ multiplicantis.
In digitum cures digitum si ducere, major
Per quantum distat a denis respice, debes
Namque suo decuplo tocies delere minorem ;
Sicque tibi numerus veniens exinde patebit.
Postea procedas postremam multiplicando,
Recte multiplicans per cunctas inferiores,
Conditione tamen tali ; quod multiplicantem
Scribas in capite, quicquid processerit inde ;
Sed postquam fuerit hæc multiplicata, figuræ
Anteriorentur seriei multiplicantis ;
Et sic multiplica, velut istam multiplicasti,
Quæ sequitur numerum scriptum quibusque figuris.
Sed cum multiplicas, primo sic est operandum,
Si dabit articulum tibi multiplicatio solum ;
Proposita cifra, summam transferre memento.
Sin autem digitus excreverit articulusve,
Articulus supraposito digito salit ultra ;
Si digitus autem, ponas ipsum super ipsam,
Subdita multiplicans hanc quæ super incidit illi
Delebit penitus, et scribens quod venit inde ;

Sed cum multiplicat aliam positam super ipsam,
Adjunges numerum quem probet ductus earum ;
Si supraposita cifra debet multiplicare,
Prorsus eam delet, scribi quod loco cifra debet,
Sed cifra multiplicans aliam posita super ipsam,
Sitque locus supra vacuis super hanc cifra fiet ;
Si supra fuerit cifra semper pretereunda est ;
Si dubites, an sit bene multiplicatio facta,
Divide totalem numerum per multiplicantem,
Et reddet numerum emergens inde priorem.
Si vis dividere numerum, sic incipe primo ;
Scribe duas, quascunque velis, series numerorum ;
Majori numero numerum suppone minorem,
Nam docet ut major teneat bis terve minorem ;
Et sub supprima supprimam pone figuram,
Sic reliquis reliquas a dextra parte locabis ;
Postea de prima primam sub parte sinistra
Subtrahe, si possis, quocies potes adminus illud,
Scribens quod remanet sub tali conditione ;
Ut toties demas demendas a remanente,
Quæ serie recte ponuntur in inferiori,
Unica si, tum sit ibi decet hac operari ;
Sed si non possis a prima demere primam,
Procedas, et eam numero suppone sequenti ;
Hanc uno retrahendo gradu comites retrahantur,
Et, quoties poteris, ab eadem deme priorem,
Ut toties demas demendas a remanente,
Nec plusquam novies aliquem tibi demere debes,
Nascitur huic numerus quociens supraque sequentem
Hunc primo scribas, retrahens exinde figuras,
Dum fuerit major suppositus inferiori,

Et rursus fiat divisio more priori ;
Et numerum quotiens supra scribas pereunti,
Si fiat saltus retrahendo cifra locetur,
Et pereat numero quotiens, proponas eidem
Cifram in numerum pereat vis, dum locus illic
Restat, et expletus divisio non valet ultra :
Dum fuerit numerus minor inferiore seorsum
Illum servabis ; hunc multiplicando probabis,
Si bene fuisti, divisor multiplicetur
Per numerum quotiens ; cum multiplicaveris, adde
Totali summæ, quod servatum fuit ante,
Redditurque tibi numerus quem proposuisti ;
Et si nil remanet, hunc multiplicatio reddet.
Cum ducis numerum per se, qui pervenit inde
Sit tibi quadratus, ductus radix erit ejus,
Nec numeros omnes quadratos dicere debes,
Est autem omnis numerus radix alicujus.
Cum voles numeri radicem quærere, scribi
Debet ; deinde notes si sit locus ultimus impar,
Estque figura loco talis scribenda sub illo,
Quæ, per se ducta, numerum tibi destruit illum,
Vel quantum poteris ex huic delebis eadem ;
Et retrahendo duples retrahens duplando sub illa
Quæ primo sequitur, duplicatur per duplacatam,
Post per se minuens sub ea saliendo.
Post his propones digitum, qui, more priori
Per precedentes post per se multiplicatus,
Destruit in quantum poterit numerum remanentem,
Et sic procedas retrahens duplando figuram,
Reponendo novam donec totum peragatur,
Subdupla propriis servare docetque duplatis ;

Si det compositum numerum duplacio, debet
Inscribi digitus a dextra parte propinqua,
Articulusque loco quo non cedebat duplicando ;
Si dabit articulum, sit cifra loco pereunte
Articulusque locum tenet unum, duplanda recessit ;
Si donet digitum, sub prima pone sequente,
Si supraposita fuerit duplicata figura
Major proponi debet tantum modo cifra,
Has retrahens solito propones more figurarum,
Usque sub extrema ita fac retrahendo figuras,
Si totum debes numerum quem proposuisti,
Quadratus fuerit de dupla quem duplicasti,
Sicque tibi radix illius certa patebit,
Si de duplatis sit juncta supprima figura ;
Si radicem per se multiplices habeasque
Propositum primo, bene te fuisse probasti ;
Non est quadratus, si quis restat, sed habuentur
Radix quadrati qui stat major sub eodem ;
Vel quicquid remanet tabula servare memento ;
Hoc casu radix per se quoque multiplicetur,
Et sic quadratus sub primo major habetur,
Huic addas remanens, et prius debes habere ;
Si locus extremus fuerit par, scribe figuram
Sub pereunte loco per quam debes operari,
Quæ quantum poterit supprimas destruat ambas,
Vel penitus legem teneas operando priorem,
Si suppositum digitus in fine repertus,
Omnino delet illic scribi cifra debet,
A leva si qua sit ei sociata figura ;
Si cifræ remanent in fine pares decet harum
Radices, numero mediam propone partem,

Tali quesita radix patebit arte reperta.
Per numerum recte si nosti multiplicare
Ejus quadratum, numerus qui pervenit inde
Dicetur cubicus; primus erit radix ejus;
Nec numeros omnes cubicatos dicere debes,
Est autem omnis numerus radix alicujus;
Si cures cubici radicem quærere, primo
Inscriptum numerum distinguere per loca debes;
Quæ tibi mille notant a mille notante supprima
Junctum, summes operandi parte sinistra,
Illic et scribas digitum, qui multiplicatus
In semet cubicæ suprapositum sibi perdat,
Et si quid fuerit adjunctum parte sinistra
Si non omnino quantum poteris inveniendo,
Hunc triplans retrahe saltem, faciendo sub illo
Quod manet a digito deleto; terna figura
Sibi propones quæ sub triplo asocietur,
Et cum subtriplo per eam tripla multiplicatur;
Hinc per eam solam productum multiplicabis,
Postea totalem numerum, qui pervenit inde
A suprapositis respectu tolle triplatæ
Addita supprimo cubicæ tunc multiplicetur,
Respectu cujus, numerus qui progredietur
Ex cubito ductu suprapositis adimetur;
Tunc ipsam dele triples saltum faciendo,
Semper sub terna, retrahens alias triplicatas
Ex hinc triplatis aliam propone figuram,
Quæ per triplatas ducatur more priori;
Primo sub triplis sibi junctis, postea per se,
In numerum ducta, productum de triplicatis:
Utque prius dixi numerus qui prevenit inde

G

A suprapositis has respiciendo trahatur,
Huic cubicæ junctum supprimo multiplicabis,
Respectuque sui, removebis de remanente,
Et sic procedas retrahendo triplando figuras.
Et proponendo nonam, donec totum peragatur,
Subtripla sub propriis servare decet triplicatis ;
Si nil in fine remanet, numerus datus ante
Est cubicus; cubicam radicem sub tripla probent,
Cum digito juncto quem sub primo posuisti,
Huic cubicæ ducta, numerum reddant tibi primum.
Si quid erit remanens non est cubicus, sed habetur
Major sub primo qui stat radix cubicati,
Servari debet quicquid radice remansit,
Extracto numero, decet hoc addi cubicato.
Quo facto, nnmerus reddi debet tibi primus.
Nam debes per se radicem multiplicare
Ex hinc in numerum duces, quod pervenit inde
Sub primo cubicus major sic invenietur ;
Illi jungatur remanens, et primus habetur,
Si per triplatum numerum nequeas operari ;
Cifram propones, nil vero per hanc operari
Sed retrahens illam cum saltu deinde triplatam,
Propones illi digitum sub lege priori,
Cumque cifram retrahes saliendo, non triplicabis,
Namque nihil cifræ triplacio dicitur esse ;
Aut tu cum cifram pertraxeris autem triplicatam,
Huic cum subtriplo semper servare memento :
Si det compositum, digiti triplacio debet
Illius inscribi, digitus saliendo super ipsam ;
Quæ manet a digito deleto, terna figura ;
Articulus jungitur cum triplata pereunte,

Sed facit hunc scribi per se triplacio prima,
Quæ si det digitum per se facit scribi illum;
Consumpto numero, si solæ fuit tibi cifræ
Triplatæ, propone cifram saltum faciendo,
Cumque cifram retrahe triplatam, scribendo figuram,
Propones cifræ, sic procedens operare,
Si tres vel duo series sint, pone sub una,
A dextris digitum servando prius documentum.

PREFATIO

DANIELIS DE MERLAI

DE NATURIS SUPERIORUM ET INFERIORUM.

Philosophia magistri Danielis de Merlai ad Johannem Norwicensem episcopum.

Cum dudum ab Anglia me causa studii excepissem et Parisius aliquandiu moram fecissem, videbam quosdam bestiales in scholiis gravi auctoritate sedes occupare, habentes coram se scamna duo vel tria et descriptos codices importabiles aureis literis Ulpiani traditiones representantes: necnon et tenentes stilos plumbeos in manibus, cum quibus asteriscos et obelos in libris suis quadam reverentia depingebant: qui dum propter inscientiam suam locum statue tenerent, tamen volebant sola taciturnitate videri sapientes: sed tales cum aliquid dicere conabantur infantissimos ripperiebam. Cum hoc, inquam, in hunc modum se habere Deprehenderem, ne et ego simile damnum incurrerem, artes que scripturas illuminant

non in transitu salutandas vel sub compendio
pretereundas mecum sollicita deliberatione trac-
tabam.   Sed quoniam doctrina Arabum, que in
quadruvio fere tota existit, maxime his diebus
apud Toletum celebratur, illuc ut sapientiores
mundi philosophos audirem, festinantur properavi.
Vocatus vero tandem ab amicis et invitatus ut
ab Hyspania redirem, cum pretiosa multitudine
librorum in Angliam veni.   Cumque nunciatum
esset mihi quod in partibus illis discipline libe-
rales silentium haberent, et pro Ticio et Seio
penitus Aristotiles et Plato oblivioni darentur,
vehementer indolui, et tamen ne ego solus inter
Romanos Græcus remanerem, ubi hujusmodi stu-
dium florere didiceram, iter arripui.   Sed in ipso
itinere obviam honorem Dominum meum et prin-
cipem spiritualem Johannem Norwicensem epis-
copum qui me honorifice ut eum decebat reci-
piens ; valde meo congratulabatur adventui.

PROPOSALS

INVENTIONS IN THE MECHANICAL ARTS.

MS. Lansd. 101.

A Note of sundry sorts of Engynes. 1583.

1. First a carriedge with his properties to carry or drawe fyve hundreth weight with one mans strength.

2. An ingen of wonderfull strength to pull downe parcullices or irone gatts.

3. A chaine of yron non licke it in strength of his bidgnes.

4. A paire of gripes to the same chaing belonging of strange fashone.

5. A gine to hoyste or pull vp earth to make rampiors.

6. A scaffolde to be removed.

7. A device to remove any burden of 10 tonne weight without horse or beast.

8. An ingen to lanch shippes.

9. A float to pase men over waters.

10. A bridg to be carried for passing an army of men ordenaunce and such licke carriges over any ryvers, &c.

11. A myll to grine by water winde or men for forte castell or towne of warr.

12. A crane to hoyst vp 10 or 20 tonne weight.

13. A gynne to hoyst vp any cannon and laie him in his carriage by one man onley.

14. An ingen for clensing or taking away of any shelves or shallow places in the river of Tems or any such river the same device maie serve for clensing of diches about citties or towns pondes or any such licke standing waters.

15. A water myll to rune longer then before tyme.

16. A winde myll and not to turne the howse about.

17. To make water workes for fountains cunditts and suche licke.

18. To make pipes of lead 6 or 7 foot long without sauder.

19. To make a boat to goe fast one the water without ower or saile.[7]

[7] This was a favourite project about this time. In a few "speciall breife remembrances" of some "pleasante serviceable and rare inventions as I have by longe studie and chargeable practice founde out," addressed to Queen Elizabeth by Ralphe Rabbard, I find one headed "The rarest engyne that ever was invented for sea service;" and this is described as follows:— "A vessell in manner of a Tally or Talliote to passe upon the seas and ryvers without oars or sayle against wynde and tyde, swifter than any that ever hath bynne seene, of wonderfull

20. To preserve a boat from drowning and the people that be therein.

effect bothe for intelligence and many other admirable exploytes almost beyonde the expectation of man :" MS. Lansd. 121. Now see Bourne's *Inventions or Devises,* a book deserving the particular attention of all those who are interested in the history of mechanical inventions.   Edmund Jentill, in 1594, proposed a "device wonderfull strange," similar to the above : it was one of the inventions which he offered to discover to Lord Burghley, on his release from prison, for counterfeiting foreign coin :—MS. Lansd. 77 and 113.

THE PREFACE

A CALENDAR OR ALMANAC

FOR THE YEAR 1430.

MS. Harl. 937.

My soverayne maistres. certen evydens have done
me to vnderstonde ʒour abylyte to lerne scyens
partyculere, and als wele consyder I ʒour desyre
in specyal to lerne a certen conclusyons of þe new
kalender. I say a certen conclusyons for þys cawse
for sum of þaim profundely to be expressyd or
lerned for defawte of termes convenyent in ower
moder langage beyn to stronge to a tendyr wytte
to comprehende þat is not elevate be processe and
cowrse of scoles. þerefor als myche as þe grete
phylosophyr sayth he wrappyth hym in hys frende
þat condescendyth to þe ryʒtfull prayer of hys
frende . . . . . me and mor mevyth me syth I . . . .
bondene to make satisfactyon to ʒour desyre
prayng ever dyscrete persone þat þis redyth or
heryth to have my rude endytyng excusyd and

my superfluyte of wordes for two cawses. þe
furste cawse for so curyos and harde sentens in
obscure termes is full tedyos to syth a tender wytte
to cousayne. þe secunde cawse is for sothely me
semythe better to wryte and twyse teche one
gode sentens þen ones forgoten. levyng þerfore
all vayn preambles of superfluyte þat papyr ful‐
fyllyȝte with owtyn fruyte þis lesone I ȝyfe ȝow
fyrste þat in ȝour ȝerus beyn xii moneth. January
February March Aprile May June July Auguste
Septembre Octobre Novembre Decembre. And
ccc and sexty days and 5 and sex odde howres
qwyche odde howres gedyrd togeder 4 ȝerus
makyth 24 howres þat is a day naturale so þat
in þis ȝere þat is clepyt lyp ȝere beyng ccc and
sexty and sex days. In everych ȝere we vse
a new letter þat Sonday gothe by. In þe lyp
ȝere we occupy twene þe furste servyth fro new
ȝerus day to seynt mathye day. þe secunde tellyth
us owre soneday þe remnande of þe ȝere.

  ȝour pryme schal be to ȝow a specyal doctour
of dyverse conclusyons querfore in latyn he is
clepyt þe golden nownbur and begynnyȝt at one
and rennyth to xix and turnyth aȝayn to one, and
so in case þat one were pryme þis ȝere next ȝere
schulde 2 be pryme and þe 3 ȝere 3. so rennyng
ȝere be ȝere to xix and aȝayn to one and so
abowte with owten ende. In þe fyrst table of
ȝowre kalendere by þe reede letters in þe firste
lyne joyned to þe blake letters in þe secunde lyne
schal ȝe knowe þe lyp ȝere qwen it fallyth. In

þe 3 lyne beyn wryten primes frome one to xix
and turnyth aȝayn. Seche þen ȝour prime of þe
ȝere þat ȝe be in jonyd to þe domynycale letter
of þe same ȝere and þen may ȝe se by þe rede
letters in þe fyrst lyne qwenus it be lyp ȝere, or
how nere ȝe be, þis ensample. I ȝyffe ȝow þe
ȝere of owre lorde a.mccccxxx in qwych ȝere þis
kalendere to ȝow was wryten. Went prime by 6
and ȝowre letter domynycale was A qwyche beyn
jonyd and wryten to geder in þe hede and þe
begynnyng of þis sayd table. þen behalde how
þe next ȝere folowyng schal 7 be prime and G
domynycale. And þe next ȝere after schal 8 be
prime and be lyp ȝere as ȝe may se by þe reede
letter F jonyd to þe blak letter E so þat in þis
lyp ȝere schal F be domynycale letter fro þe
Circumscycyon þat is clepyd newȝerus day vnto
seynt Mathye day. and þen schal E be ȝour
sonday letter to þe ȝerus ynde as I sayd beforne.
þis doctryne kepe þurghe alle þe forsayd table.
Amonge þe blak letters in þe same lyne beyn
wryten reede letters qwyche teches þe indyccyon
þat is a terme ful necessary to þaim þat lyste knaw
þe verray and certene date of þe Pope bulles and
of þe olde Imperyale wrytynge of Rome. ȝyf it
lyke ȝow to wytte in qwat Jndyccyon ȝe be in
Take ȝour prime and ȝour domynycale letter of
þe same ȝere þat ȝe beyn in qwyche letter in
case þat it be rede calle þat þe first Jndyccyon
and þe next letter þe secunde jndiccion and so
rennyng vnto xv and ȝen aȝayn to one at þe

next reede letter. ʒif it be a blake letter þat ʒour prime fallyth one cownte fro þe next goyng beforus and so mony is þe jndiccyon þat ʒe seche forʒete not þis lesone.

A table þat next folowyʒt is callyd þe table of þe 5 festes moveyabylle þat is to say qwen septuagesime commeyʒt in þat is þe sonday before lentyn qwen Allia is closyd vp. Also þis table tellyʒt qwen lentyn fallyth qwen Eysterday. qwen þe Rogacyons and qwen qwytesoneday. It techyʒt also how mony weykes be fro Crystemesday to lentyn how mony wekes and days fro qwytesonday to mydsommerday And how mony fro Whytesonday to þe advente on þis maner. Tak þe prime of þe ʒere þat ʒe be in. in þe fyrste lyne wryten with reede. and þe domynycale letter next folowyng in þe secunde lyne. and folowe stryght forthe in to þe lyne of þat thynge wryten in þe heyde þat ʒe desyre. and by þe nownbur þat ʒe fynde schalle ʒe knaw þat ʒe desyre. Vnderstonde wele þis ensample. Aftyr crystynmesse þe ʒere of owre lorde a.m.cccc.xxx. I was aferde of lentyn and lokyd in my kalender for septuagesime and lentyn how nere þai were and soghte my prime in þe fyrste lyne þat happynd to be 7 þe same tyme. and toke my domynycale letter þat was G next following. þo turnyd I forth fro þis G streght in to nexte lyne þat haythe Septuagesime writyn in þe hede and I fonde 28 þere wryten þereby demyd I þat septuagesime sonday schal falle ʒe 28 day

of þe moneyth of Januare wrytyn in þe hede.
þo loked I forthermore in þe next lyne that hayth
lentyn wryten in þe hede and þere I fonde 18
wryten and þereby I demyd þat first sonneday
of lentyn schulde be on þe 18 day of þe moneyth
of February nexte wrytyn above ȝyt I loked for-
thermore in þe next lyne for Esterday and fonde
þere 1 wryten and þen wyste I þat Esterday
schulde be on þe firste day of þe moneythe of
Aprile wrytyn next above. and ȝow take ensample
as I have do and loke ȝe kepe þe same thurghe
alle þe table.

JOHANNIS NORFOLK

IN ARTEM PROGRESSIONIS SUMMULA.

In artem progressionis continue et discontinue secundum magistrum Johannem Norefolk incipit summula.

Non invenientes sed doctrinam tradita inde numerorum progressione ab Algore rege quondam Castellie suo in Algorismo de integris perficere curantes. occultas aliorum tradiciones si que fuerint penitus omittentes dummoda tales nostra assidua percunctacione non obstante re maxima nobis miranda ante nostram edicionem in nostras manus visuras nos nusquam recepisse fatemur testimonium tamen fide dignorum nostrorum amicorum nobis possibilitatem operis concludebat eo quod tales regulas utilem artem progressionis concernentes ante nostra tempora fuisse editas testabantur. Oportunum igitur apud nos indicavimus cum ars tam preciosa arithmeticeque sciencie prefulgida margarita conspectibus hominum se non presentabat sed quorundam forte inpericia incarcerata atque porcis oblata sciolos

latebat circa possibilitatem operis novarumque
regularum edicionem divino auxilio primitus in-
vocato petitisque auctoritatibus et favoribus
eorum quibus incumbit hujusmodi summulas
corrigere operas exiles soleritur impendere et
eas sic collectas nostro calamo rudique stilo de-
scriptas. Primo ad honorem Dei beatissime
virginis Marie et omnium suorum sanctorum
ac ad profectum animarum omnium fidelium
defunctorum quarum collegio Oxonie primus
vicecustos exciteramus et ad ceteras hujusmodi
scienciam diligentes plenius promulgare cura-
vimus. Reverendam ergo tuam benevolenciam
quiscunque es o carissime has nostras regulas
exiles exiliterque collectas placito vultu bonoque
animo ut acceptare digneris benigne rogamus.
Est namque opus harum regularum omnium nu-
merorum proportionalitate aliqua comperatorum
sive plures paucioresve fuerunt ut in his exem-
plis. 1. 2. 4. 8. 16. 32. vel sic. 5. 6. 7. 8. vel sic.
4. 8. 12. 16. vel sic. 1. 2. 3. 4. 5. 6. et sic ad quo-
titatem unitatum sive percussionum horologii
quas sonat horologium ad 12 horas vel ad quar-
tem partem diei naturalis et similibus quid re-
sultans fuerit levi arificio explicare. Fiat obse-
cramus legitimum kalendare ergo in anima tua
o lector honorande hoc scriptum exiguum magna
tamen solertia et industria compilatum nec desit
tibi spes enarrandi quid ex predictis numeris re-
sultat et omnibus consimilibus proportionalitate
aliqua comperatis cum regule satis sufficienter

sunt tradite et nomen auctoris publice prefato
est monstratum preambulo. Sic ergo karissime
incipe et mente tua recollige tria documenta
generalia que tractatus presentis processimi ple-
nissime facilitabunt. Primum documentum est
quod scias quid progressio sit et quod sectionem
patitur in continuam et discontinuam prout pos-
terius apercius clarescet. Secundum documentum
est ut bene consideres quod omnium numerorum
ordinate se in progressione habencium aliqua
proportionalitas reperta nec quid ex talibus ar-
tificiose poterit certificari propter infinite vari-
abilitatis talem. Tercium est quod non deficiat
tibi spes perveniendi in finem premissum per
hanc artem facillimam cum de omnibus numeris
proportionalitate geometrica ordinatis ac de qui-
busdam arithmetica proportionalitate magis fa-
mose reperibilibus dantur regule sequentes solum
omittentes numeros inusitatos arithmetica tamen
proportionalitate constitutos ac omnes numeros
musica proportionalitate collectos tum facilius
sit enarrare qui ex talibus numeris resultat quam
hujusmodi numeros artificiose reperire et com-
ponere et hec tria documenta nota diligentissime.
Specificantes modo predicta tria documenta ge-
neralia dicamus primo quod progressio est nu-
merorum diversorum ordinata collectio et dicitur
ordinata pro tanto quod si aliqua collectione
numerus minor sequatur majorem ut hic. 12
65. 413. ibi non est ratio progressio judicanda
quamvis enim metaphorica progressio nuncupanda

est omnis numerorum collectio a minore inci-
piendo terminando in majorem numerum ipsa
tamen proprie dicitur progressio cum numeri
quadam proportionalitate colliguntur istud nam-
que evidentissimum est cum apud omnes maxime
indicatur progressionem naturalem esse incipiendo
computare ab unitate ascendendo ad alios nume-
ros naturali ordine succedentes et ibi aliqua pro-
portionalitas inter terminos reperta est et que
postea apparebit.   Progressio sic proprie sumpta
dividitur in continuam et discontinuam progressio
continua est numerorum diversorum ordinata
collectio nullo numero alium superexcrescente
nisi sola unitate et ista potest indrunter contin-
gere vel incipiendo ab unitate vel ultra unitatem
ut sic. 1. 2. 3. 4. 5. vel sic. 3. 4. 5. 6. et cætera.
Progressio vero discontinua est numerorum di-
versorum ordinata collectio uno alio supraex-
crescente plusquam unitate servata tamen ut
prius in progressione continua proportionalitate
geometrica et hoc indrunter incipiendo ab uni-
tate vel altro numero quocunque ut hic. 1. 4.
7. 10. 13. et cætera. vel sic. 5. 7. 9. 11. 13. 15.
et cætera.   Et quia progressio proprie dicta ut
dictum est constat ex numeris geometrica pro-
portionalitate covintis ne tractatus iste neces-
sariis videatur deficere superfluisve habundare
quorum utrumque collectoris impericie ascri-
bendum esset videndum est de proportionalitate
quid sit et ejus speciebus quæ etiam species ejus.
ad hanc artem magis pertinet que vero abji-

cienda. Est namque proportionalitas duarum
proportionum aut plurimum in simul compera-
torum habitudo hujus autem tres sunt species
scilicet geometrica arithmetica arismonica sive
musica proportionalitas geometrica est quando
sunt tres termini aut plures et equales est ex-
cessus secundi ad primum sicut tertii ad se-
cundum et quarti ad tercium et cætera si tanti
fuerint ut hic. 1. 2. 3. 4. 5. 6. vel sic. 2. 4. 6. 8.
vel sic. 2. 7. 12. 17. 22. et cætera. Proportio-
nalitas vero arithmetica est quando sunt tres ter
mini vel plures et equalis est proportio inter
primum et secundum et secundum ad tertium et
tercium ad quartum ut hic. 1. 2. 4. 8. 16. vel sic.
1. 3. 9. 27. et cætera. Sed proporcionalitas mu-
sica sive arsmonica est quando sunt tres termini
insimul proporcionalitati et equalis est proporcio
tercium ad primum sicut est proporcio excessus
tercii ad secundum ad excessum secundi ad
primum ut hic. 3. 4. 6. vel sic. 6. 8. 12. vel
sic. 12. 16. 24. hiis sic brevissime recitatis per
nos dicamus quod in omnibus numeris propor-
cionalitate geometrica proporcionalitatis hec ars
habet locum sive plures fuerint sic pauciores
eciam in numeris famosioribus arithmetica pro-
porcionalitate comperatis ut in numeris propor-
cionalitate duplica collectis omittentes alias spe-
cies ejusdem proporcionalitatis sicut omittimus
omnes species arsmonica proporcionalitate inte-
gritas ob id quod superius docetur documento
tertio. Hiis sic specificatis inducamus primam

regulam primo de numero geometrica proporcion-
alitate adunatis et ubi progressio continua est que
hec est. Numero locorum secundum progres-
sionem continuam exeunte sub numero pari per
minus medium multiplicetur numerus locorum et
habetur quod queritur hæc namque regula gene-
ralis est ut ostendent exempla posita in prima
tabula in fine tractatus hujus et quamvis hæc re-
gula viris maturis aliqualiter arithmetice sciencie
noticia inbutis satis lucida est ut tamen juvenes
et minus provecti per eam sapiant sic eam delu-
cidamus ut ejus aliarumque regularum recitan-
darum noticie perfectus habeantur primo notanda
sunt quid nominis horum quinque terminorum
scilicet numerus locorum vel loca numerorum
numerus par numerus impar majus medium
minus medium eiis quinque terris nostro more
declaratis facilime perquiritur sensus regule pre-
cedentis ac cujusque quatuor regularum sequen-
cium. Sic vero incipientes dicamus quod non
probabile aliquem pro eadem morula sive eodem
.... plures numeros recitare et sic numerum
primo recitatum dicimus esse in primo loco et
secundo recitatum numerum dicimus esse in
secundo loco et sic de aliis quibuscumque reci-
tandis conformiter sicut si numeri diversi scribi
debeant oportet quod variis locis scribantur et
de eiis idem indicare quoad loca quare citissime
cognoscitur numerus locorum secundum numera-
cionem numerorum enumeratorum. De secundo
et tercio ternimis in simul licet annotare quia

cognito quid sit numerus par tanquam suum
privatum cognoscitur numerus inpar qui aliam
cognicionem non habet quam per suum positivum
ut docet aristolus. Est autem numerus par ut
notat Boycius in sua arithmetica et est sua
prima definitio illius quilibet numerus qui potest
in duo equalia nullo medio intercedente dividi ab
hac namque condicione deficit numerus impar ut
constat quia quamvis quinarius dividi potest in
duos binarios etiam in duo equalia tamen aliqua
unitas integrans quinarium intercidit cum duo
et tria faciunt quinque. Ceterum restat declarare
quid minus medium quidve majus medium sit
promissum ut impleatur pro quorum noticia est
primo animadvertendum quid sit medium. Est
namque medium quod equaliter distat ab extremis
et sicut facilime ymaginatur medium ubi loca
numerorum numerantur numero impari ut hic
1. 2. 3. 4. 5. 3. est medium vel sic ubi loca
numerorum numerantur a numero pari saltem
ultra binarium inveniendi sunt duo tales numeri
quorum minor numerus mius medium dicitur et
major numerus majus medium nuncupatur ut
hic. 2. 3. 4. 5. numerus locorum est quatuor
minus medium. 3. majus medium. 4. aut sic.
5. 8. 11. 14. 4. est numerus locorum ut prius
minus medium est. 8. majus medium est. 11. hiis
sic cognitis evidens est prima regula prius recitata
de progressione continua et sequitur secunda
regula de progressione continua eadem facilitate
que hec est. Numero locorum secundum pro-

gressionem continuam existente sub numero
impari per medium multiplicetur numerus lo-
corum et habetur quod queritur sine aliqua sub-
tractione vel additione et hec regula certissima
est perfectus a nobis jam duabus regulis ad
progressionem continuam pertinentibus succe-
dunt. 3. regule spectantes ad progressionem
discontinuam et quia in tali progressione minor
erat difficultas nobis certitudinem regularum uti-
ıium invenire ut tamen posteri nostri eas regulas
citissime cognoscant quas nostris magnis laboribus
a diu percuntavimus licet ipse aliquibus obscuris
terminis a nobis primo dentur declarationibus
tamen nostris levissime conceptis quilibet me-
diocri ingenii capacitate fultus ipsas poterit faci-
lime experiri et est prima regula talis. In pro-
gressione discontinua numero locorum exeunte
sub numero pari et si eorum numerorum excessus
nominatus fuerit a numero impari per majus
medium intelligibile inter numeros mediales mul-
tiplicetur numerus locorum et subtrahatur medie-
tas numeri locorum et habetur quod queritur
hanc autem regulam assumpto hoc exemplo.
5. 10. 15. 20. sic delucidamus nam in exemplo
dato numerorum loca esse sub numero pari nul-
lius ambigit et unumquemque numerum exce-
dere suum numerum in medietate precedentem per
5. 3. qui est numerus imper nulli dubium est
tunc constat quod inter duos numeros media-
les qui sunt 10. et 15. quatuor numeros ordine
naturali intelligi posse videlicet 11. 12. 13. 14.

eorumque numerorum subintellectorum majus
medium esse 13 hic vero numerus 13. multi-
plicandus est per numerum locorum scilicet 4.
et subtrahatur 2. qui est medietas numeri lo-
corum et patet summa ideo vero dicitur 13.
medium intelligibile quia non expresse inter
numeros exemplo positos invenitur sed quadam
solertia per intellectus indagacionem experitur
ac eciam quod majus medium sit consimiliter
indicatur. Hiis sic declaratis sequitur secunda
regula que hec est. Si progressione discon-
tinua numerus locorum fuerit par eorumque
numerorum excessu exeunte sub numero pari
per medium intelligibilem inter numeros medi-
ales multiplicetur numerus locorum et habetur
summa quesita hanc autem regulam duobus
exemplis declaramus sic primum exemplum.
2. 4. 6. 8. in quo exemplo medium intelligi-
bile est 5. qui ductus in numerum locorum
exurgunt 20 ut evidet qui est summa totalis.
Exemplum secundum prefate regule in quo
exemplo numerus excessus est 4. medium vero
intelligibilem 11. cum inter duos numeros
mediales scilicet 9. et 13. ordine naturali in-
tercipiuntur hii tres numeri scilicet 10. 11. 12.
aut intelligi possunt quorum medium est 11.
ut dictum est per quod multiplicetur numerus
locorum et habetur numerus ex omnibus resul-
tans. Ultima regula secundum genus propor-
cionalitatis geometrice et habita in ordine est hec
progressione discontinua numero locorum ex-

eunte sub numero impari per medium multi-
plicetur numerus locorum et habetur quod que-
ritur exemplum 6. 8. 10. per 8. multiplicetur
numerus locorum qui est tercia et exurgunt.
24. Et quia prius promissum est regulas fieri
de quibusdam numeris arithmetica proporcio-
nalitate comparatis hanc solam regulam dispo-
suimus pro numeris magis famosis et sunt illi
numeri qui proporcione dupla inter se comperan-
tur ut hic 1. 2. 4. 8. 16. 32. et cetera et est regula
talis. Si plures numeri proporcionalitate arith-
metica comperati ubi posterior suum inmediate
priorem proporcione dupla excedit insimul com-
perentur dupletur ultimus et a resultante sub-
trahatur primus et habetur quod queritur evidet
hec regula in exemplo posito nam dupatur 32.
resultant 64. a quibus subtrahatur unitas que
est primus numerus et resultant 63. et hec regula
generalis est in omnibus talibus sive ab unitate
incipiatur sive non sive vero loca numerorum
paria fuerint sive non de aliis autem numeris
secundum hoc genus proporcionalitatis dicimus
nunc ut dictum est prius documento tertio. *et sic
perfectus est iste tractatus brevissimus in collegio
animarum Oxonie anno domini millesimo quadrin-
gentesimo quadragesimo quinto quod Norfolk scrip-
tor ac compilator hujus tractatus.*

<h2 style="text-align:center">*Explicit.*</h2>

Nil amplius restat ad perfeccionem hujus sum-
mule colligere et scribere quam quod anteadictum

est sed solum regulas prescriptas ad facilitatem prospicere volencium recolligere cum tabulis exemplarium ac consulere quod lectores ejusdem perfecte sint in arte multiplicationis per intellectum ideo sequitur tabula subscripta et deo gratias.

Prima regula.

Numero locorum secundum progressionem continuam exeunte sub numero pari per minus medium multiplicetur numerus locorum et addatur medietas numeri locorum et habetur quod queritur.

Exemplum prime regule.

1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10 . 11 . 12.

Vel sic $\begin{cases} 2.3.4.5.6.7. \text{ vel sic } 6.7.8.9.10.11. \\ 3.4.5.6.7.8. \text{ vel sic } 19.20.21.22. \end{cases}$

Secunda regula.

Numero locorum secundum progressionem continuam exeunte sub numero impari per medium multiplicetur numerus locorum et habetur quod queritur sine aliqua subtraccione vel addicione.

Exemplum secunde regule.

1 . 2 . 3 . 4 . 5 . vel sic 5 . 6 . 7 . 8 . 9 . 10 . 11.
vel sic 12 . 13 . 14 . 15 . 16.
vel sic 21 . 22 . 23 . 24 . 25 . 26 . 27.

Tercia regula.

In progressione discontinua numero locorum exeunte sub numero pari et si eorum numerorum excessus nominatus fuerit a numero impari per majus medium intelligibilem inter numeros mediales multiplicetur numerus locorum et subtrahatur medietas numeri locorum et habetur quod queritur.

Exemplum tercie regule.

. 5 . 10 . 15 . 20 . 25 . 30 . vel sic 2 . 5 . 8 . 11.
vel sic 3 . 10 . 17 . 24 . 31 . 38.
vel sic 3 . 6 . 9 . 12.

Quarta regula.

Numerus locorum si progressione discontinua fuerit par eorumque numerorum excessu exeunte sub numero pari per medium intelligibilem inter numeros mediales multiplicetur numerus locorum et habetur quod queritur.

Exemplum quarte regule.

5 . 9 . 13 . 17. vel sic 3 . 5 . 7 . 9.
vel sic 2 . 4 . 6 . 8 . 10 . 12. vel sic 3 . 7 . 11 . 15.

Quinta regula.

Numero locorum secundum progressionem discontinuam exeunte sub numero impari per medium multiplicetur numerus locorum et habetur quod queritur.

Exemplum quinte regule.

4 . 6 . 8. vel sic 3 . 5 . 7 . 9 . 11 . 13 . 15.
vel sic 12 . 14 . 16. vel sic 3 . 7 . 11 . 15 . 19 . 23 . 27.

Sexta regula sed est de numeris arithmetica proportionalitate collectis.

Si plures numeri ubi posterior suum immediate precedentem proportione dupla excedat insimul comparentur dupletur ultimus et a resultante subtrahitur primus et habetur quod queritur.

Exemplum sexte regule.

12 . 4 . 8 . 16 . 32. vel sic 2 . 4 . 8 . 16.

Explicit tabula exemplarium summule super progressione continua et discontinua secundum Norefolk.

APPENDIX.

I.

A few Observations on the Numerical Contractions found in some manuscripts of the Treatise on Geometry by Boetius.

THE remarks which follow have not, as far as I am aware, found a place in any prior publication: I intend them to form an appendix to the interesting chapter on the same subject presented to the literary world by M. Chasles.

I could not have connected the following pages in the form of a continuous history, without introducing much that is already known; I have, therefore, considered it advisable to place my notes under distinct articles, without any attempt at arrangement.

1. It is very probable that the well-known passage on the *Abacus*, in the first book of the Geometry of Boetius, is an interpolation. For in a MS. once belonging to Mr. Ames, no such passage appears; and in another, now in the library of Trinity College, it is also wanting: again, no such contractions occur in any copy of the Treatise on

Arithmetic * by the same author; although, in the library†
just mentioned, there is a list of them, on a fly-leaf to
a MS. of that work, in a hand-writing of the fourteenth
century, which is thus headed:

> Primus igin; andras; ormis; quarto subit arbas;
>
> Quinque quinas; termas; zenis; temenias; celentis.

and over these names the contractions are written, as
well as Roman numerals explaining them.

2. There are two MSS. in the Bodleian Library which
merit particular attention. One, MS. Hatton. 112, pos-
sesses two distinct treatises on arithmetic on this system:
the first is very extensive, but anonymous; the rubrication
to the preface of the other is as follows: *Incipit pre-
fatio libri Abaci quæ junior Berhelinus edidit Parisiis,
Domino suo Amulio.* In both these treatises, as well as
in the other MS., local position is clearly pointed out.

3. Vossius ‡ attributes them to a Grecian origin; Huet §
derives them from the Hebrew; and the Bodleian MSS.
refer them to Syria and Chaldea. It is scarcely neces-
sary to observe, that there is no connection between these
numerals and those among the contractions of Tyro and
Seneca.

4. M. Chasles has confused the sipos and celentis, the
latter of which was seldom used as a cipher. In the
second Bodleian MS. we read, *inscribitur et in ultimo
figura* 0, *sipos nomine. Que licet numerum nullum
significet: tum ad alia quædam utilis est.* In the

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* Most of the MSS. of this work that I have examined are very old,—
generally prior to the thirteenth century. Only one MS. that I am ac-
quainted with (Bib. Burn. 275.) contains Arabic numerals. It may also be
remarked here, that a treatise on arithmetic in verse, by one Leopald (Bib.
Arund. 339.) possesses numerals whose forms are, as far as I know, unique.
But this tract will receive its due attention in a proper place.

† R. xv. 16. There are also a few pages on arithmetic, which contain the
following account of its rise among the ancients: *Hanc igitur artem nume-
randi apud Grecos Samius Pitagoras et Aristoteles scripserunt, diffusiusque
Nicomachus et Euclides; licet et alii in eadem floreunt, ut Erastosthenes et
Crisippus. Apud Latinos primus Apuleius, deinde Boecius.*

‡ Observationes ad Pomponiam Melam. 4to. Hag. Com. 1658, p. 64.

§ Demonstratio Evangelica. Prop. iv. p. 173. E.

Lansdown collection (842) in the British Museum, is a very beautiful MS. of the whole works of Boetius: what renders it more interesting in the present inquiry, is the contraction for the sipos without the drawing of an abacus, which curiously illustrates the difficulty of the transition from numerical operations, by means of that instrument, to local position without distinguishing boundaries.

5. The Metz MS. in the Arundel collection, referred to by Mr. Hallam, is probably an abridgment of one or more extensive treatises on the subject: the author says, *quicquid ab abacistis excerpere potui compendiose collegi, componens inde mihi certas regulas, que volentibus ad hanc disciplinam attingere non inutiles.* He quotes Boetius.

6. The following verses occur in a MS.* of the fourteenth century on arithmetic:

> Unus adest igin; andras duo; tres reor armin;
> Quatuor est arbas; et per quinque fore quinas;
> Sex calcis; septem zenis; octo zenienias;
> Novem celentis; per deno sume priorem.

And a list of the contractions is given on the preceding page of the same volume.

7. The fractional notation appears to be as curious as the integral, but the contractions are not quite so arbitrary, and a regular system is evidently followed up throughout. It is for the most part merely an adapta-

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* Bib. Trin. Coll. Cant. inter MSS. Gal. O. 2. 45. f. 33b. Chasles has given verses to the same import, but he does not mention the source whence he has obtained them. There is every reason to think that the sipos was a later improvement, and the Metz MS. contains no allusion to it. Mr. Barnwell, of the British Museum, informs me, that a short tract on celentis was once pointed out to him in a MS. volume belonging to that establishment, but no reference to it being in any of the catalogues, I have not been fortunate enough to find it.

tion of the Roman weights to numerical computation ; for instance, taking as for unity, we have—

$\frac{11}{12}$ deunx $\qquad$ $\frac{10}{12}$ dextans

$\frac{9}{12}$ dodrans $\qquad$ $\frac{8}{12}$ bisse

$\frac{7}{12}$ septunx $\qquad$ $\frac{6}{12}$ semis

$\frac{5}{12}$ quicunx $\qquad$ $\frac{4}{12}$ triens

$\frac{3}{12}$ quadrans $\qquad$ $\frac{2}{12}$ sextans

$\frac{1}{12}$ uncia

The *uncia* was also divided into twelve portions, but differently—

$\frac{1}{2}$ semiuncia $\qquad$ $\frac{1}{3}$ duella

$\frac{1}{4}$ sicilius $\qquad$ $\frac{1}{6}$ sexcula

$\frac{1}{8}$ dragma $\qquad$ $\frac{1}{12}$ hemissecla

$\frac{1}{18}$ tremissis $\qquad$ $\frac{1}{24}$ scrupulus

$\frac{1}{48}$ obulus $\qquad$ $\frac{1}{72}$ bissiliqua

$\frac{1}{96}$ ceraces $\qquad$ $\frac{1}{144}$ siliqua

To these was added the $\frac{1}{192}$th part of the uncia : *ut usque ad minimum extremum diatessaron et diapente symphoniarum tonorum semitonorumque intervallis distinclarum, harum fractionum denominatio conscenderet vel contenderet.* Zambertus gives the contractions, *quæ sepissime inveniuntur in antiquis libris.** In the Metz MS. the contraction for the *as* is omitted. Several examples of reckoning *time* by this method occur in the classical writings, especially in Pliny,† as also in some MSS. of the eleventh and twelfth centuries in the British Museum. Bede wrote a tract on this fractional notation, and he adds, *hæc ponderum vocabula, vel characteres non modo ad pecuniam mensurandam, verum ad quævis corpora, sive tempora dimetienda, conveniunt.* A MS. in the Public Library (Kk. v. 32) contains an explanation in

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* Euclidis Elementa ex Campano a Zamberto, fol. Par. 1516, p. 248.

† Archæologia, vol. xxvi. p. 159. Vid. Bedæ Opera. Edit. Bas. 1563, t. i., col. 101, 141, et 182. MS. Arund. Mus. Brit. 25, f. 124, et 356, f. 45.

Saxon, which very much coincides with that given by Bede, and would appear to be taken from it.

It would be impossible, with the few materials yet brought to light, to conjecture with any great probability how far these Boetian contractions may have influenced the introduction, or co-operated with the Arabic system to the formation, of our present numerical notation. It appears to me highly probable that the two systems became united, because the middle age forms of the figure five coincide with the Boetian mark for the same numeral, and those of two others are very similar. The idea of local position, again, may have had an independent European origin; the inconveniences of the abacus on paper would have suggested it by destroying the distinguishing boundaries, and inventing an arbitrary hieroglyphic for the representation of an empty square.

II.

NOTES ON EARLY ALMANACS.*

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THE following short paper has been compiled from notes
collected at various times, and without any intention of
placing a dissertation on the subject before the public.
I mention this merely to suggest to the reader that no
connected history of almanacs has been attempted, and that
it will be unfair to view what is here placed before him as
any other than an attempt to abridge the labour of a suc-
cessor who might wish, at some future period, to dive more
deeply into the subject.

The early history of almanacs is involved in much ob-
scurity. The Egyptians, indeed, possessed instruments
answering most of the same purposes : but the log calendars
are the most ancient almanacs, properly so called. Ver-
stegan † derives their name from a Saxon origin, viz.
al-mon-aght, or the observation of all the moons, that being
the purpose for which they were originally made : an
eastern origin would appear to me to be more probable.
They are doubtless of high antiquity, and, if we can be
guided by the errors of the more modern ones in their*

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* This has been printed in the " Companion to the British Almanac " for
1839 ; to the kindness of Mr. Knight, the publisher of that useful perio-
dical, I am indebted for power to reprint it here.
          † Restitution of decayed intelligence, p. 64.

ecclesiastical computation, we might refer them to the second or third century.* Gruter has delineated one at Rome, and which is said to have been used by the Goths and Vandals : this was cut in elm, though most are in box, and some few in fir, brass, horn, &c. Each of these calendars contains four sides, for the four quarters of the year, and gives the golden numbers, epacts, dominical letter, &c. The numerical notation is imperfect but curious ; dots are put for the first four digits, a mark similar to the Roman numeral V, for five ; this mark, and additional dots for the next four, and the algebraical sign + for ten. Specimens of these logs may be seen in the British Museum† ; and, as they are not uncommon, it is unnecessary to enter into further detail.

Before I commence with written almanacs, it will be necessary to remark the distinction between astronomical and ecclesiastical calendars, the first of which contain astronomical computations, and the other lists of saints' days, and other matters relative to the church ; sometimes, indeed, both are found united, although the latter claim a higher antiquity, being prefixed to most ancient Latin manuscripts of the Scriptures.

The folding calendars were, perhaps, the most ancient forms of them, and merit particular attention. Several of these are in the British Museum,‡ and at Oxford ; one of them was written in the year 1430, and is in English ; but the writer confesses his inability to find suitable expressions

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* MS. Harl., Mus. Brit., 5958.

† MSS. Harl., 197, 198. The last of these is a modern one used in Derbyshire or Staffordshire, and cut, probably, in the latter part of the 17th century : the other one is much earlier, though perhaps not of very high antiquity. Others may be seen in the Ashmolean Museum, and in St. John's College, Cambridge. I refer to Dr. Plott's History of Staffordshire for a very good description of them. (See also Brady's *Clavis Calendaria*.)

‡ Cotton Rolls, viii. 26 ; MSS. Harl. 937, 3812, 5311 ; MSS. Sloan. 996, 2250 ; which last is the calendar of John Somers, afterwards mentioned. There is also one in the Ashmolean collection at Oxford in singularly fine preservation.

for the technical terms which were derived for the most part from Arabic, *for defawte of terms convenyent in our moder langage.* In the Pepysian library at Cambridge there is one printed by Wynkin de Worde, in octo-decimo, which, in its original form, folds up from a small folio sheet of vellum; it bears the date of 1523.[*]

The standard almanacs emanated from Oxford, the seat of British science throughout the middle ages: in fact, before Newton's time, Cambridge was a blank, and the only scientific names that cheer the pages of the history of its early literature are Holbroke of St. Peter's College, Buckley of King's, and Dee of St. John's: the first known by his astronomical tables, the second by a plagiarism of a method of extracting the roots of fractions from Robert Record, and the third a memorable instance of one of the greatest men of his time uniting the pure truths of science with the grossest absurdities. All three were astrologers,[†] owing, perhaps, more to the place of their education than to the individuals themselves.

There has been some dispute relative to the authenticity of Roger Bacon's calendar, of which there is a MS. in the British Museum: the following is an exact transcript of the commencement :—

"Kalendarium sequens extractum est a tabulis tholetanis. anno domini . 1292 . factus ad meridiem civitatis tholeti que in Hispania scita est cujus meridianus non multum distat a meridiano medii puncti Hibernie in quo . 3 . continentur." f. 2.

If we retain *factus*, it cannot be translated, but, fortunately, the other MS. at Oxford has *factum*, and this must evidently be the true reading. Professor Peacock

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writes *factis,* but there is not, as far as I know, any MS. authority for it. With respect to the author of it, the Bodleian MS., in a coeval rubric, states the calendar to have been written *à fratre Rogero Bacon;* while the Cotton MS., not having any original title, is ascribed to Roger Bacon, in a hand of the 17th century : both of the MSS. belong to the 14th century. In the Harleian collection (No. 941) is a MS. on the length of the days throughout the year, stated to have been *made at Oxynforde be the new kalendere and proved in all the university:* this *new kalendere* may possibly refer to Roger Bacon's ; but there are not sufficient data to enable us to attain an approach to certainty.

The calendar of John Somers, of Oxford, written in 1380, was one of the most popular of the time : there is generally appended to it, *Tabula docens algorismum legere, cujus utilitas est in brevi satis spatio numerum magnum comprehendere. Et quia numeri in kalendario positi vix excedunt sexaginta, ultra illam summam non est protensa.** Several English translations of this tract are among the Ashmolean MSS.

We have likewise in MS. *Almanach Profacii Judei,* which is very ancient. Walter de Elvendene wrote a calendar in 1327,† and Nicholas de Lynna published another in 1386.‡ Sometimes these calendars are found in rolls.

In the library at Lambeth Palace is a very curious calendar in the English language, written in 1460 ; at the end is a table of eclipses from 1460 to 1481 ; but a very perfect volvelle is most worthy of notice, because those instruments are generally found imperfect. In the Cottonian collection is another English calendar, written about

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* MS. Bib. Cott. Mus. Brit. Vespas. E. vii. f. 4.
† MS. Sloan. Mus. Brit. 286. ‡ MS. Ashm. Oxon. 5.

1450, but so much damaged by the fire that the nature of it cannot be seen.   In Trinity College, Cambridge, there is a MS., said to have been composed in 1347, and entitled, *An Almanak, translated in perpetuite, out of Arabike into Latin;* and in the same library I find *The Effemerides of John of Mounte Riol,* a German *Prince of Astronomyers.*   Professor Leslie mentions a very beautiful calendar in the library of the university of Edinburgh, with the date of 1482: he does not appear to be aware that they were common in MS. libraries, and he greatly overrates its value.

There was printed at Hackney, in 1812, a small octavo volume, containing an account of an English almanac for the year 1386: it contains a very large portion of astronomical and medical matter, but appears to be of little interest, save that it is the earliest one in English I have ever heard of.   The contents of this calendar are as follow :—

    1. The houses of the planets and their properties.

    2. The exposition of the signs.

    3. Chronicle of events from the birth of Cain.

In 1325 there was a *grete hungur* in England ; in 1333 a great tempest ; in 1349 the first, in 1361 the second, and in 1369 the third pestilence.   It is curious to remark the clumsy method of expressing numbers consisting of more than two figures: for instance, we have 52mcc20 put for 52,220.   This shows that the Arabic notation was even then but imperfectly understood among the common people.

    4. To find the prime numbers.

    5. Short notes on medicine.

    6. On blood-letting.

    7. A description of the table of the signs, and moveable feasts.

    8. Quantitates diei artificialis.

The extracts from this calendar are wretchedly transcribed, and evidently by one who was totally unacquainted with MSS.

The clock or albion of Richard de Walingford, of St. Alban's, answered the purpose of a calendar. * This clock made, says Bale, who appears to have seen it, *magno labore, majore sumptu, arte vero maxima,* was considered the greatest curiosity of its time. In his account of it, which still remains in manuscript, we have the following definitions:—*Albion est geometricum instrumentum: almanac autem arismetricum.* Peter Lightfoot's celebrated astronomical clock at Glastonbury may have been something of the same sort.

Peter de Dacia, about 1300, published a calendar, of which there is a very early MS. in the Savilian library at Oxford: the *condiciones planetarum* are thus stated—

> Jupiter atque Venus boni, Saturnusque malignus ;
> Sol et Mercurius cum Luna sunt mediocres.

The *homo signorum,* so common in later calendars, probably originated with him.

The earliest almanac printed in England was the *Sheapeheard's Kalender,* translated from the French, and printed by Richard Pynson, in 1497. It contains a vast portion of extraneous matter. The following verses on the planets will, at the same time, give a good idea of the nature

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* In one almanac of the commencement of the 17th century (MS. Harl. 5937. Bagford Collect., s. 139) is a very singular method for finding the hour of the day, if in the country and without any watch. I refer to it merely out of charity to those right-hearted enthusiastic antiquaries who do not *stick at trifles* in pursuing researches that can in any way illustrate the customs of our ancestors in the *good old times.* To those who only value the researches of antiquaries in proportion as they are likely to furnish some evident tangible utility, I would willingly spare their time in recurring to a method which, though it might excite their ridicule, could never, from its nature, be brought into practice in the present age.

of the astrological information in this and other calendars
of the period :—

> " Some hot, some colde, some moyst, some dry,
> If three be good, foure be worse at the most.
> Saturne is hyest and coldest, being full old,
> And Mars, with his bluddy swerde, ever ready to kyll ;
> Jupiter very good, and Venus maketh lovers glad,
> Sol and Luna is half good and half ill,
> Mercury is good and will verily
> And hereafter shalt thou know;
> Whiche of the seven most worthy be,
> And who reigneth hye, and who a lowe ;
> Of every planets propertie,
> Which is the best among them all,
> That causeth welth, sorrowe, or sinne,
> Tarry and heare sone thou shalt,
> Speake softe, for now I beginne."

Afterwards follow some prognostications of the weather.
The following method *to knowe what wether shall be all
the yere after the chaunge of every moone by the prime
dayes*, is taken from a MS. in Lambeth Palace :—

> " Sondaye pryme, drye wether.
> Mondaye pryme, moyst wether.
> Teusdaye pryme, cold and wynde.
> Wenesdaye pryme, mervelous.
> Thursdaye pryme, sonne and clere.
> Frydaye pryme, fayre and fowle.
> Saturdaye pryme, rayne."

Prognostications of the weather were early matters of
reproach—

> " Astronomyers also aren at ere whittes ende,
> Of that was calculed of the clymat the contrye thei fyndeth.

And in Heber's library was a little tract of three leaves,
entitled ' A Mery Prognostication '—

> " For the yere of Chryste's incarnacyon,
> A thousande fyve hundreth fortye and foure.
> This to prognosticate I may be bolde,
> That whan the new yere is come, gone is the olde."

Henry VIII. issued a proclamation against such false prognostications as this tract was intended to ridicule, but still no printer ventured to put his name to it. Not long after to believe them was a crime; "as for astrologicall and other like vaine predictions or abodes," says Thomas Lydiat, "I thanke God I was never addicted to them." *

Johannes de Monte-Regio, in 1472, composed the earliest European almanac that issued from the press; and, before the end of that century, they became common on the Continent. In England they were not in general use until the middle of the sixteenth century. Most of the best mathematicians of the time were employed in constructing them; but, before the end of the following century, almanac-makers began to form a distinct body, and, though they often styled themselves "studentes in the artes mathematicall," very few of them were at all celebrated in the pure sciences.

It may not be wholly irrelevant here to make some few observations on the memory-rhymes found in some almanacs of the present day, and which date their origin to a much earlier period. The well-known lines, used by many for recalling to their recollection the number of days in each month, I find in Winter's Cambridge Almanac for 1635, under the following slightly-varied form—

> " Aprill, June, and September,
> Thirty daies have as November;
> Ech month else doth never vary
> From thirty-one, save February;
> Wich twenty-eight doth still confine,
> Save on Leap-yeare, then twenty-nine."

And the nursery-rhymes, commencing " Multiplication is my vexation," were certainly made before 1570.†

* MS. Bodl. 662.
† Professor Davies's Key to Hutton's Mathematics, p. 17.

The early history of ecclesiastical computation is inti-
mately connected with that of calendars. Dionysius Exiguus
was one of the first who wrote on the subject : after him,
Bede, Gerlandus, Alexander de Villa Dei, and Johannes
de Sacro-Bosco, were the most celebrated. The Massa
Compoti of Alexander de Villa Dei, so common in MS.,
is perhaps the most singular tract on the subject that
has come down to us : his reason for the title of the
book is exceedingly curious :—*Sicut de multis laminis
æris in conflatorio massa una efficitur, ideo librum' istum
vocari volui massam compoti.*

I cannot conclude without mentioning the ' Almanac
and Prognostication' of Leonard Digges, which was so
often reprinted in the latter half of the sixteenth century :
it is filled with the most extravagant astrological absur-
dities, and a table of weather predictions. With respect
to the latter, however, I have had the curiosity to test
its accuracy for some months in comparison with our two
celebrated weather almanacs, and, on the average, have
found it to be quite as " neare the marke " as either of
them.

<div align="center">THE END.</div>