An introduction to nuclear physics

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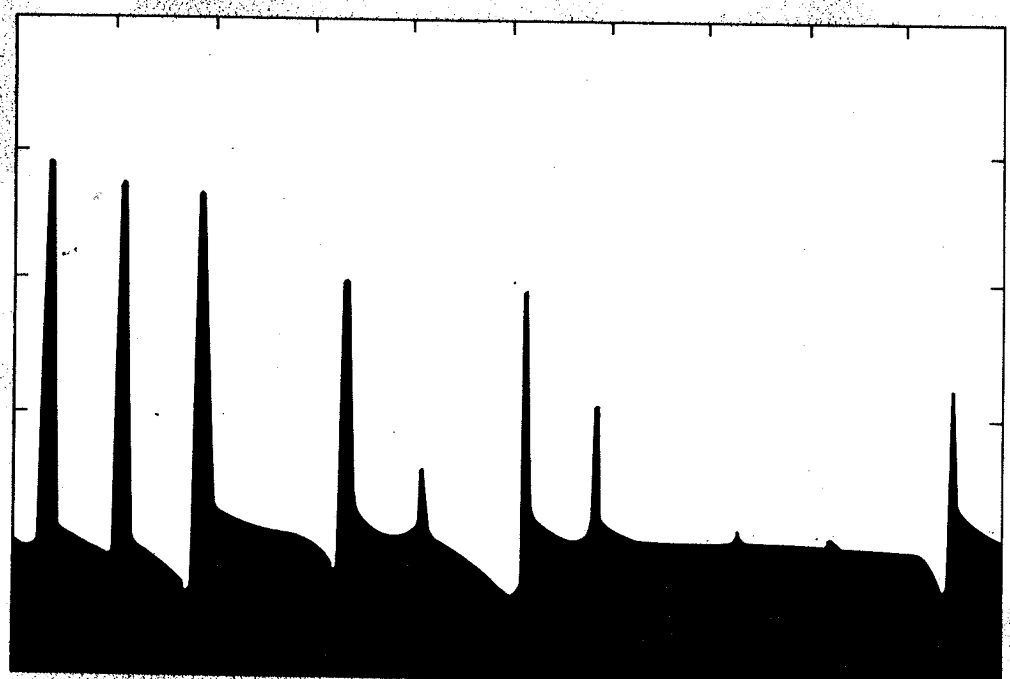

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! educational value.
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ıs, energy levels and the quantisation of
which may not be covered in elementary
treated in appendices. We consider the
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to a first course in nuclear physics.
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Constants of nature and conversion factors

Velocity of light	c	2.99792×10^{8} m s^{-1}
Planck's constant	$\hbar = h/2\pi$	1.05459×10^{-34} J s
Proton charge	e	1.60219×10^{-19} C
Boltzmann's constant	k_B	1.3807×10^{-23} J K^{-1} $= 8.617 \times 10^{-5}$ eV K^{-1}
Gravitational constant	G	6.67×10^{-11} m^3 kg^{-1} s^{-2}
Fermi coupling constant	G_F	$1.136 \times 10^{-11} (\hbar c)^3$ MeV^{-2}
Electron mass	m_e	9.1095×10^{-31} kg $= 0.51100$ MeV/c^2
Proton mass	m_p	1.007276 amu $= 938.28$ MeV/c^2
Neutron mass	m_n	1.00866 amu $= 939.57$ MeV/c^2
Atomic mass unit	(mass ^{12}C atom)/12	1.66057×10^{-27} kg $= 931.50$ MeV/c^2
Bohr magneton	$\mu_B = e\hbar/2m_e$	5.78838×10^{-5} eV T^{-1}
Nuclear magneton	$\mu_N = e\hbar/2m_p$	3.15245×10^{-8} eV T^{-1}
Bohr radius	$a_0 = 4\pi\varepsilon_0\hbar^2/m_e e^2$	0.529177×10^{-10} m
Fine-structure constant	$e^2/4\pi\varepsilon_0\hbar c$	1/137.036

$\hbar c = 197.329$ MeV fm, $e^2/4\pi\varepsilon_0 = 1.43998$ MeV fm
1 MeV $= 1.60219 \times 10^{-13}$ J

1 fm $=10^{-15}$ m, 1 barn $=10^{-28}$ m$^2=10^2$ fm^2

(Source: Review of particle properties (1984), *Rev. Mod. Phys.* **56**, No. 2, Part II)

Notation

r, **k**, etc., denote vectors (x, y, z), (k_x, k_y, k_z), and $r=|\mathbf{r}|$, $k=|\mathbf{k}|$,

$d^3\mathbf{r}=dx\,dy\,dz$, $d^3\mathbf{k}=dk_x\,dk_y\,dk_z$.

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}=\frac{1}{r}\frac{\partial^2}{\partial r^2}r+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2},$$

$d\Omega=\sin\theta\,d\theta\,d\phi$ denotes an infinitesimal element of solid angle.

Glossary of some important symbols

A	nuclear mass number $(=N+Z)$
$\mathbf{A}(\mathbf{r}, t)$	electromagnetic vector potential
a	§4.1 nuclear surface width; §4.5 bulk binding coefficient
$B(Z, N)$	binding energy of nucleus
$\mathbf{B}(\mathbf{r}, t)$	magnetic field
b	§4.5 surface tension coefficient; §13.1 impact parameter
$\mathbf{E}(\mathbf{r}, t)$	electric field
E	energy; E_n, E_p neutron energy, proton energy; E_n^F, E_p^F neutron, proton Fermi energy, measured from the bottom of the shell-model neutron potential well; E_G §8.3
$F(Z, E_e)$	§12.3 Coulomb correction factor in β-decay
$f(Z, E_0)$	§12.3 kinematic factor in total β-decay rate
G	§6.2 exponent in the tunnelling formula
g	§D.2 statistical factor in Breit–Wigner formula
g_L, g_s	§5.6 orbital and intrinsic magnetic moment coefficients
g_A, g_V, g_L	§12.6 axial, vector, lepton coupling constants
$\mathcal{G}(r_s/r_c)$	§6.2 tunnelling integral
$\mathbf{J}$	§C.3 total angular momentum operator
j	quantum number associated with $\mathbf{J}^2$
j_z	quantum number of J_z
$\mathbf{k}$	wave vector
k_F	value of $k=|\mathbf{k}|$ at the Fermi energy
$\mathbf{L}$	§C.1 orbital angular momentum operator

l quantum number associated with $\mathbf{L}^2$; Chapter 9, Chapter 13 mean free path
m quantum number of L_z; reduced mass
m_s quantum number of s_z
m_α mass of α-particle; m_a, m_{nuc} mass of atom, nucleus
N number of neutrons in nucleus
$n(E)$ density of states
$\mathcal{N}(E)$ integrated density of states
$\mathbf{P}$ momentum
Q § 5.7 nuclear electric quadrupole moment; § 6.1 kinetic energy release in nuclear reaction
q § 9.4 fission probability
R § 4.3 nuclear radius; § 12.3 reaction rate
r_s', r_c § 6.2 potential barrier parameters
$S_n(N,Z)$ § 5.2 neutron separation energy
$S(E)$ § 8.3 parameter of nuclear reaction cross-section for energies below the Coulomb barrier
$S_0(E), S_c(E)$ § 12.3 electron (positron) energy spectrum without and with Coulomb correction
$\mathbf{s}$ § C.2 intrinsic angular momentum operator
s quantum number associated with $\mathbf{s}$; § 4.5 symmetry energy coefficient
T kinetic energy
$T_{1/2}$ decay half life
t_{nuc} § 5.2 nuclear time scale
t_p § 9.4 prompt neutron life
U potential energy; $\bar{U}$ mean proton–neutron potential energy difference in nucleus
$u_l(r)$ radial wave-function
V normalisation volume; § 3.3 $V(r)$ nucleon–nucleon potential
v velocity
Z atomic number (number of protons in nucleus)
Γ, Γ_i width, partial width, of an excited state
γ § 13.1 relativistic factor $(1-v^2/c^2)^{-\frac{1}{2}}$
δ § 2.4 coefficient of pairing energy
ε_0 permittivity of free space
ε_F § 11.1 Fermi energy of electron gas
$\boldsymbol{\mu}$ § 5.5 magnetic dipole operator
μ_n, μ_p neutron, proton magnetic moment
μ § 5.5 magnetic dipole moment; § 11.1 stellar mass per electron; § 13.3 photon linear attenuation coefficient
μ_0 permeability of free space
$\nu, d\nu$ § 9.3 mean number of prompt neutrons, delayed neutrons, per fission
ρ § 2.1 electric charge density; § 13.2 mass density

ρ_{ch} § 4.1 electric charge density in units of e
ρ_0 § 4.3 nucleon *number* density in nuclear matter
$\rho_{nuc}, \rho_n, \rho_p$ *number* density of nuclei, neutrons, protons
$\boldsymbol{\sigma}$ § C.2 Pauli spin matrices
σ cross-section; σ_{tot}, σ_e, σ_f total, elastic, fission, cross-section
τ mean life; τ_{E1}, τ_{M1} electric, magnetic, dipole transition mean life; § 7.4 $(\tau_i)^{-1}$ partial decay rate
Φ § 3.4 meson field
ϕ electromagnetic scalar potential
$\psi(\mathbf{r})$ single particle wave-function
ψ_m § D.1 general wave-function
Ω_{S0}, Ω_T § 3.3 angular terms in the nucleon–nucleon potential
ω angular frequency

1 Prologue

The world is made up of some 92 chemical elements, distinguished from each other by the electric charge Ze on the atomic nucleus. This charge is balanced by the charge carried by the Z electrons which together with the nucleus make up the neutral atom. The elements are also distinguished by their mass, more than 99% of which resides in the nucleus. Are there other distinguishing properties of nuclei? Have the nuclei been in existence since the beginning of time? Are there elements in the Universe which do not exist on Earth? What physical principles underlie the properties of nuclei? Why are their masses so closely correlated with their electric charges, and why are some nuclei radio-active? Radio-activity is used to man's benefit in medicine. Nuclear fission is exploited in power generation. But man's use of nuclear physics has also posed the terrible threat of nuclear weapons.

This book aims to set out the basic concepts which have been developed by nuclear physicists in their attempts to understand the nucleus. Besides satisfying our appetite for knowledge, these concepts must be understood if we are to make an informed judgment on the benefits and problems of nuclear technology.

After the discovery of the neutron by Chadwick in 1932, it was accepted that a nucleus of atomic number Z was made up of Z protons and some number N of neutrons. The proton and neutron were then thought to be elementary particles, although it is now clear that they are not but rather are themselves structured entities. We shall also see that in addition to neutrons and protons several other particles play an important, if indirect, role in the physics of nuclei. In this and the following two chapters, to provide a background to our subsequent study of the nucleus, we shall describe the elementary particles of nature, and their interactions, as they are at present understood.

1.1 Fermions and bosons

Elementary particles are classified as either *fermions* or *bosons*. Fermions are particles which satisfy the Pauli exclusion principle: if an assembly of identical fermions is described in terms of single-particle wave-functions, then no two fermions can have the same wave-function. For example, electrons are fermions. This rule explains the shell structure of atoms and hence underlies the whole of chemistry. Fermions are so called because they obey the Fermi–Dirac statistics of statistical mechanics.

Bosons are particles which obey Bose–Einstein statistics, and are characterised by the property that *any* number of particles may be assigned the same single-particle wave-function. Thus, in the case of bosons, coherent waves of macroscopic amplitude can be constructed, and such waves may to a good approximation be described classically. For example, photons are bosons and the corresponding classical field is the familiar electromagnetic field **E** and **B**, which satisfies Maxwell's equations.

At a more fundamental level, these properties are a consequence of the possible symmetries of the wave-function of a system of identical particles when the coordinates of any two particles are interchanged. In the case of fermions, the wave-function changes sign; it is completely antisymmetric. In the case of bosons the wave-function is unchanged; it is completely symmetric.

There is also an observed relation between the intrinsic angular momentum, or spin, of a particle and its statistics. The intrinsic spin **s** is quantised, with spin quantum number s (see Appendix C). For a fermion, s takes one of the values $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$; for a boson, s takes one of the values $0, 1, 2, \ldots$. A theoretical explanation of this relationship can be given within the framework of relativistic quantum field theory.

1.2 The particle physicist's picture of nature

Elementary particle physics describes the world in terms of elementary fermions, interacting through fields of which they are sources. The particles associated with the interaction fields are bosons. To take the most familiar example, an electron is an elementary fermion; it carries electric charge $-e$ and this charge produces an electromagnetic field **E**, **B**, which exerts forces on other charged particles. The electromagnetic field, quantised according to the rules of quantum mechanics, corresponds to an assembly of *photons*, which are bosons. Indeed, Bose–Einstein statistics were first applied to photons.

Four types of interaction field may be distinguished in nature (see Table 1.1). All of these interactions are relevant to nuclear physics, though the gravitational field becomes important only in densely aggregated matter, such as stars. Gravitational forces act on all particles and are important for the physics on the large scale of macroscopic bodies. On the small scale of most terrestrial atomic and nuclear physics, gravitational forces are insignificant and except in Chapter 10 and Chapter 11 we shall ignore them.

Nature provides an even greater diversity of elementary fermions than of bosons. It is convenient to divide the fermions into two classes: *leptons*, which are not sources of the strong fields and hence do not participate in the strong interaction; and *hadrons*, which take part in all interactions. The leptons and their interactions are described in Chapter 2. The elementary hadrons, and the proton and the neutron, form the subject matter of Chapter 3.

1.3 Conservation laws and symmetries; parity

The total energy of an isolated system is constant in time. So also are its linear momentum and angular momentum. These conservation laws are derivable from Newton's laws of motion and Maxwell's equations, or from the laws of quantum mechanics, but they can also, at a deeper level, be regarded as consequences of 'symmetries' of space and time. Thus the law of conservation of linear momentum follows from the homogeneity of space, the law of conservation of angular momentum from the isotropy of space; it does not matter where we place the origin of our coordinate axes, or in which direction they are oriented.

These conservation laws are as significant in nuclear physics as elsewhere, but there is another symmetry and conservation law which is of

Table 1.1. *Types of interaction field*

Interaction field	Boson	Spin
Gravitational field	'Gravitons' postulated	2
Weak field	W^+, W^-, Z particles	1
Electromagnetic field	Photons	1
Strong field	'Gluons' postulated	1

particular importance in quantum systems such as the nucleus: reflection symmetry and parity. By reflection symmetry we mean reflection about the origin, $\mathbf{r} \to \mathbf{r}' = -\mathbf{r}$. A single-particle wave-function $\psi(\mathbf{r})$ is said to have parity $+1$ if it is even under reflection, i.e.

$$\psi(-\mathbf{r}) = \psi(\mathbf{r}),$$

and parity -1 if it is odd under reflection, i.e.

$$\psi(-\mathbf{r}) = -\psi(\mathbf{r}).$$

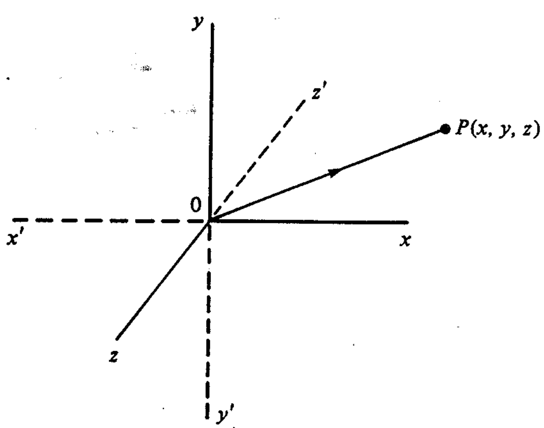
More generally, a many-particle wave-function has parity $+1$ if it is even under reflection of all the particle coordinates, and parity -1 if it is odd under reflection.

Parity is an important concept because the laws of the electromagnetic and of the strong interaction are of exactly the same form if written with respect to a reflected left-handed coordinate system $(0x', 0y', 0z')$ as they are in the standard right-handed system $(0x, 0y, 0z)$ (Fig. 1.1). We shall see in Chapter 2 that this is not true of the weak interaction. Nevertheless, for many properties of atomic and nuclear systems the weak interaction is unimportant and wave-functions for such systems can be chosen to have a definite parity which does not change with time.

1.4 Units

Every branch of physics tends to find certain units particularly congenial. In nuclear physics, the size of the nucleus makes 10^{-15} m $= 1$ fm (femtometre) convenient as a unit of length, usually called a *fermi*. However,

1.1 The point P at $\mathbf{r}$ with coordinates (x, y, z) has coordinates $(-x, -y, -z)$ in the primed, reflected coordinate axes. $(0x', 0y', 0z')$ make up a *left-handed* set of axes.



nuclear cross-sections, which have the dimensions of area, are measured in *barns*; $1\ \mathrm{b} = 10^{-28}\ \mathrm{m}^2 = 100\ \mathrm{fm}^2$. Energies of interest are usually of the order of MeV. Since mc^2 has the dimensions of energy, it is convenient to quote masses in units of MeV/c^2.

For order-of-magnitude calculations, the masses m_e and m_p of the electron and proton may be taken as

$$m_e \approx 0.5\ \mathrm{MeV}/c^2$$

$$m_p \approx 938\ \mathrm{MeV}/c^2$$

and it is useful to remember that

$$\hbar c \approx 197\ \mathrm{MeV\ fm}, \quad e^2/4\pi\varepsilon_0 \approx 1.44\ \mathrm{MeV\ fm},$$

$$e^2/4\pi\varepsilon_0 \hbar c \approx 1/137, \quad c \approx 3 \times 10^{23}\ \mathrm{fm\ s}^{-1}.$$

The student will perhaps be surprised to find how easily many expressions in nuclear physics can be evaluated using these quantities.

Problems

1.1 Show that the ratio of the gravitational potential energy to the Coulomb potential energy between two electrons is $\approx 2.4 \times 10^{-43}$.

1.2(*a*) Show that in polar coordinates (r, θ, ϕ) the reflection

$$\mathbf{r} \to \mathbf{r}' = -\mathbf{r} \quad \text{is equivalent to} \quad r \to r' = r$$

$$\theta \to \theta' = \pi - \theta, \quad \phi \to \phi' = \phi + \pi.$$

(*b*) What are the parities of the following electron states of the hydrogen atom:

(*i*) $$\psi_{100} = \frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0}\right)^{\frac{3}{2}} e^{-r/a_0},$$

(*ii*) $$\psi_{210} = \frac{1}{4\sqrt{(2\pi)}} \left(\frac{1}{a_0}\right)^{\frac{3}{2}} \frac{r}{a_0} e^{-r/2a_0} \cos\theta,$$

(*iii*) $$\psi_{21-1} = \frac{1}{8\sqrt{\pi}} \left(\frac{1}{a_0}\right)^{\frac{3}{2}} \frac{r}{a_0} e^{-r/2a_0} \sin\theta\, e^{-i\phi}\ ?$$

($a_0 = (4\pi\varepsilon_0)\hbar^2/m_e e^2$ is the Bohr radius.)

1.3(*a*) Show that the wavelength of a photon of energy 1 MeV is ≈ 1240 fm.

(*b*) The electrostatic self-energy of a uniformly charged sphere of total charge e, radius R, is $U = (3/5)e^2/(4\pi\varepsilon_0 R)$. Show that if $R = 1$ fm, $U = 0.86$ MeV.

2

Leptons and the electromagnetic and weak interactions

2.1 The electromagnetic interaction

The electromagnetic field is most conveniently described by a vector potential **A** and a scalar potential ϕ. For simplicity, we consider only the potential $\phi(\mathbf{r}, t)$. Using Maxwell's equations, this may be chosen to satisfy the wave equation

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho(\mathbf{r}, t)}{\varepsilon_0}. \tag{2.1}$$

Here $\rho(\mathbf{r}, t)$ is the electric charge density due to the charged particles, which in atomic and nuclear physics will usually be electrons and protons, and c is the velocity of light.

In regions where $\rho=0$, equation (2.1) has solutions in the form of propagating waves; for example, the plane wave

$$\phi(\mathbf{r}, t) = (\text{constant})\, e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}. \tag{2.2}$$

This satisfies

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 0 \tag{2.3}$$

provided

$$\omega^2 = c^2k^2. \tag{2.4}$$

The wave velocity is therefore c, as we should expect. In quantum theory, unlike classical theory, the total energy and momentum of the wave are quantised, and can only be integer multiples of the basic quantum of energy and momentum given by the de Broglie relations:

$$E = \hbar\omega, \quad \mathbf{p} = \hbar\mathbf{k}. \tag{2.5}$$

Such a quantum of radiation is called a *photon*. A macroscopic wave can be considered to be an assembly of photons, and we can regard photons as particles, each carrying energy E and momentum $\mathbf{p}$.

Using (2.4) and (2.5), E and $\mathbf{p}$ are related by

$$E^2 = p^2c^2. \tag{2.6}$$

For a particle of mass m, the Einstein equation gives

$$E^2 = p^2c^2 + m^2c^4.$$

We therefore infer that the photon is a particle having zero mass.

A second important type of solution of (2.1) exists when charged particles are present. If these are moving slowly compared with the velocity of light, so that the term $\partial^2\phi/(c^2\,\partial t^2)$ can be neglected, the solution is approximately the Coulomb potential of the charge distribution. For a particle with charge density ρ_1, we can take

$$\phi(\mathbf{r}, t) \approx \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_1(\mathbf{r}', t)}{|\mathbf{r}-\mathbf{r}'|}\, d^3\mathbf{r}'. \tag{2.7}$$

Another charged particle with charge density ρ_2 will have a potential energy given by

$$U_{12} = \int \rho_2(\mathbf{r}, t)\phi(\mathbf{r}, t)\, d^3r$$

$$= \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_1(\mathbf{r}', t)\rho_2(\mathbf{r}, t)}{|\mathbf{r}-\mathbf{r}'|}\, d^3\mathbf{r}\, d^3\mathbf{r}'. \tag{2.8}$$

Electric potential energy is basically responsible for the binding of electrons in atoms and molecules. We shall see that, in nuclear physics, it is responsible for the instability of heavy nuclei. If magnetic effects due to the motion of the charges are included, equation (2.8) is modified to

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_1'\rho_2 + (1/c^2)\mathbf{j}_1'\cdot\mathbf{j}_2}{|\mathbf{r}-\mathbf{r}'|}\, d^3\mathbf{r}\, d^3\mathbf{r}', \tag{2.9}$$

where $\mathbf{j} = \rho\mathbf{v}$ is the current associated with the charge distribution which has velocity $\mathbf{v}(\mathbf{r})$. Thus this magnetic contribution to the energy is of relative order v^2/c^2.

The electromagnetic interaction also gives rise to the scattering of charged particles. For example, if ρ_1 and ρ_2 represent the charge distributions of two electrons approaching each other the interaction gives a mutual repulsion which leads to a transfer of momentum between the particles. The process can be represented by a diagram such as Fig. 2.1. In quantum electrodynamics, these diagrams, invented by Feynman, have a precise technical interpretation in the theory. We shall use them only to help visualise the physics involved. The scattering of the two electrons may be thought of as caused by the emission of a 'virtual' photon by one electron and its absorption by the other electron. In a virtual process the photon does not actually appear to an observer, though it appears in the mathematical formalism that describes the process.

2.2 The weak interaction

There are three *weak interaction* fields associated with the W^+, W^- and Z particles. Each one, like the electromagnetic field, is described by a vector and a scalar potential. However, the bosons associated with the weak fields all have mass, and the W^- and W^+ bosons are electrically charged. The Z boson is neutral, and most similar to the photon, but it has a mass

$$M_Z = (92.9 \pm 1.6)\ \mathrm{GeV}/c^2 \sim 100 \text{ proton masses},$$

which is very large by nuclear physics standards.

The interactions between leptons and the electromagnetic and weak fields were combined into a unified 'electro-weak' theory by Weinberg and by Salam. The existence of the Z and $W^\pm$ bosons was predicted by the theory, and the theory also suggested values for their masses. These predictions were confirmed by experiments at CERN in 1983.

The wave equation satisfied by the scalar potential ϕ_Z associated with the Z boson is a generalisation of (2.1) and includes a term involving M_Z:

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \left(\frac{M_Z c}{\hbar}\right)^2\right]\phi_Z(\mathbf{r}, t) = -\frac{\rho_Z(\mathbf{r}, t)}{\varepsilon_0}, \qquad (2.10)$$

where ρ_Z is the neutral weak-charge density. There is a close, but not exact, analogy between weak-charge density and electric-charge density, and particles carry weak charge somewhat as they carry electric charge.

In free space where $\rho_Z = 0$ there exist plane wave solutions of (2.10),

$$\phi_Z(\mathbf{r}, t) = (\text{constant})\, e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)},$$

but now to satisfy the wave equation we require

$$\omega^2 = c^2k^2 + c^2(M_Z c/\hbar)^2,$$

and with the de Broglie relations (2.5) for the field quanta we obtain the Einstein energy–momentum relation for the Z boson:

$$E^2 = p^2c^2 + M_Z^2 c^4.$$

The static solution of (2.10) which corresponds to a point unit weak charge at the origin is

$$\phi_Z(r) = \frac{1}{4\pi\varepsilon_0}\frac{e^{-\kappa r}}{r}, \quad \text{writing } \kappa = \frac{M_Z c}{\hbar}. \qquad (2.11)$$

At points away from the origin where $\nabla^2\phi_Z - \kappa^2\phi_Z = 0$, this satisfies equation (2.10), as may be easily checked by substitution, using the formula $\nabla^2\phi_Z = (1/r)\,\mathrm{d}^2(r\phi_Z)/\mathrm{d}r^2$. Close to the origin the solution (2.11) behaves like the corresponding Coulomb potential $1/(4\pi\varepsilon_0 r)$ of a unit point electric charge, and hence has the correct point source behaviour. The generalisation of (2.11) to a distribution of weak charge gives the quasi-static solution (cf. (2.7))

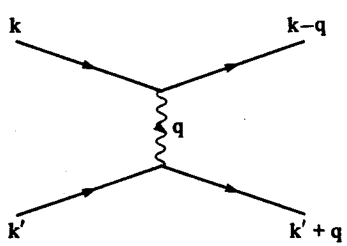
$$\phi_Z(\mathbf{r}, t) \approx \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_Z(\mathbf{r}', t)\, e^{-\kappa|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\,\mathrm{d}^3\mathbf{r}'. \qquad (2.12)$$

The exponential factor in the integral effectively vanishes for $|\mathbf{r}-\mathbf{r}'|$ greater than a few times $\kappa^{-1} = \hbar/M_Z c$ and

$$\hbar/M_Z c \approx 2 \times 10^{-3}\ \mathrm{fm}.$$

This is a very small distance in the context of nuclear physics: by the uncertainty principle, low momentum sources must be spread over distances much greater than this. Hence in the integral in (2.12) the factor ρ_Z is slowly varying over the range of the exponential and may be taken

2.1 The scattering of two electrons of momenta $\hbar\mathbf{k}$, $\hbar\mathbf{k}'$ by the exchange of a virtual photon carrying momentum $\hbar\mathbf{q}$. Time runs from left to right in these diagrams. (In principle, the exchange of a *Z* particle (§2.2) also contributes to electron–electron scattering, but the very short range and weakness of the weak interaction makes this contribution almost completely negligible: the electrons are in any case kept apart by the Coulomb repulsion induced by the photon exchange.)



outside the integral (which is then elementary):

$$\phi_Z(\mathbf{r},t) \simeq \frac{1}{4\pi\varepsilon_0}\rho_Z(\mathbf{r},t)\int \frac{e^{-\kappa|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\,d^3\mathbf{r}'$$
$$= \frac{1}{4\pi\varepsilon_0}\rho_Z(\mathbf{r},t)\int_0^\infty \frac{e^{-\kappa R}}{R}4\pi R^2\,dR \qquad (2.13)$$
$$= \frac{1}{\varepsilon_0}\left(\frac{\hbar}{M_Z c}\right)^2\rho_Z(\mathbf{r},t).$$

The potential energy between two particles associated with the scalar field ϕ_Z is, by analogy with (2.8),

$$U^Z_{12} = \int \rho_{Z2}(\mathbf{r},t)\phi_{Z1}(\mathbf{r},t)\,d^3\mathbf{r}$$
$$\approx \frac{1}{\varepsilon_0}\left(\frac{\hbar}{M_Z c}\right)^2\int \rho_{Z1}(\mathbf{r},t)\rho_{Z2}(\mathbf{r},t)\,d^3\mathbf{r},$$

and there is also a contribution from the vector part of the field, analogous to the magnetic contribution in (2.9), of the form

$$\frac{1}{\varepsilon_0 c^2}\left(\frac{\hbar}{M_Z c}\right)^2\int \mathbf{j}_{Z1}(\mathbf{r},t)\cdot\mathbf{j}_{Z2}(\mathbf{r},t)\,d^3\mathbf{r},$$

where $\mathbf{j}_Z$ is the weak-current density.

The physical consequences of these expressions are quite different from the electromagnetic interaction. U^Z_{12} is very much suppressed by the large mass factor in the denominator, and it is this which largely accounts for the 'weakness' of the weak interaction. Also the interaction at low energies appears as a 'contact interaction', effectively having zero range.

The electrically charged W^+ and W^- boson fields give rise to the most important weak interactions, and in particular to β-decay. They obey equations similar to those of the Z field, but the masses of the associated particles are somewhat smaller;

$$M_{W^+} = M_{W^-} = (80.8 \pm 2.7)\ \text{GeV}/c^2.$$

2.3 Mean life and half life

Not all particles are stable: some, for example the $W^\pm$ and Z bosons, have only a transient existence. Suppose that an unstable particle exists at some instant $t=0$; its *mean life* is the mean time it exists in isolation, before it undergoes radio-active decay. If we denote by $P(t)$ the probability that the particle survives for a time t, and make the basic assumption that the particle has a *constant* probability $1/\tau$ per unit time of decaying, then

$$P(t+dt) = P(t)(1-dt/\tau),$$

since $(1-dt/\tau)$ is the probability it survives the time interval dt. Hence

$$\frac{1}{P}\frac{dP}{dt} = -\frac{1}{\tau},$$

and integrating,

$$P(t) = P(0)e^{-t/\tau}.$$

Since $P(0)=1$ we have

$$P(t) = e^{-t/\tau}. \qquad (2.14)$$

Equation (2.14) is the familiar exponential-decay law for unstable particles. It is well verified experimentally.

The probability that the particle decays between times t, $t+dt$ is clearly $P(t)\times(dt/\tau)$, so that the mean life is

$$\int_0^\infty tP(t)(dt/\tau) = \int_0^\infty te^{-t/\tau}\,dt/\tau = \tau.$$

The 'half life' $T_{\frac{1}{2}}$ is the time at which there is a 50% probability that the particle has decayed, i.e.

$$P(T_{\frac{1}{2}}) = e^{-T_{\frac{1}{2}}/\tau} = \tfrac{1}{2}$$

Hence

$$T_{\frac{1}{2}} = \tau \log 2 = 0.693\tau.$$

In this book we have preferred to quote mean lives rather than half lives. We refer to $(1/\tau)$ as the *decay rate*.

2.4 Leptons

Leptons are spin $\frac{1}{2}$ fermions which interact through the electromagnetic and weak interactions, but not through the strong interaction. The known leptons are listed in Table 2.1.

Table 2.1. *Known leptons*

	Mass (MeV/c^2)	Mean life (s)	Charge
Electron e^-	0.5110	∞	$-e$
Electron neutrino ν_e	$<46\times10^{-6}$	∞	0
Muon μ^-	105.659	2.197×10^{-6}	$-e$
Muon neutrino ν_μ	<0.5	∞ ?	0
Tau τ^-	1784	$(3.4\pm0.5)\times10^{-13}$	$-e$
Tau neutrino ν_τ	<164	∞ ?	0

The electrically charged leptons all have magnetic moments of magnitude $\approx -e\hbar/2$ (mass) aligned with their spins.

Of these charged leptons, only the familiar electron is stable. Electrons are structureless particles that are described by the Dirac relativistic wave-equation. This equation explains the spin and magnetic moment of the electron, and has the remarkable feature that it predicts the existence of anti-particles: these are particles of the same mass and spin, but of opposite charge and magnetic moment to the particle. The anti-particle of the electron is called the *positron*. Positrons were identified experimentally by Anderson in 1932 soon after their theoretical prediction.

Since leptons do not interact with the strong interaction field, electrons and positrons interact principally through the electromagnetic field. A positron will eventually annihilate with an electron, usually to produce two or three photons, so that all the lepton energy appears as electromagnetic radiation. We write these processes as

$$e^- + e^+ \rightarrow 2\gamma$$
$$e^- + e^+ \rightarrow 3\gamma.$$

The converse processes of *pair-production* by photons are also possible, and pair-production from a single photon is possible provided another (charged) particle is present to take up momentum. Quantum electrodynamics, based on the Dirac and Maxwell equations, describes all processes involving electrons, positrons and photons to a high degree of accuracy.

It is a curious fact that nature provides us also with the electrically charged *muon* μ^- and *tau* τ^- and their anti-particles the μ^+ and τ^+. Apart from their greater masses and finite lifetimes, muons and taus seem to be just copies of the electron, and like the electron they are accurately described by Dirac equations. We shall see that the μ^- can be used as a probe of nuclear charge density, but otherwise neither the muons nor the taus play any significant role in nuclear physics.

The remaining leptons are the *neutrinos* ν and their corresponding anti-neutrinos denoted by $\bar{\nu}$. All the experimental evidence is consistent with their mass being zero, so that, like photons, they move with the speed of light. However, neutrinos are fermions with spin $\frac{1}{2}$.

It is exceedingly difficult and expensive to carry out experiments with neutrinos, but there is evidence that the electron, muon and tau have different neutrinos, ν_e, ν_μ, ν_τ associated with them.

2.5 The instability of the heavy leptons: muon decay

The W^+ and W^- bosons lead to processes called β-decay, which neither photons nor Z bosons can induce. In this chapter we illustrate this with the example of the β-decay of the muon; in the next chapter we shall describe β-decay processes involving hadrons.

The muon decays to a muon neutrino, together with an electron and an electron anti-neutrino:

$$\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e.$$

The W fields play the mediating role in this decay through the two virtual processes illustrated in Fig. 2.2. Again, in a virtual process actual W bosons do not appear to an observer.

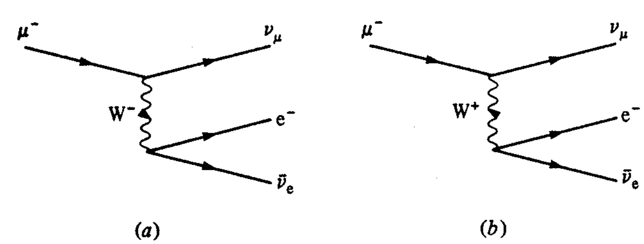
The W bosons can in principle produce any charged lepton and its anti-neutrino or an anti-lepton and its neutrino, but energy must be conserved overall. Hence in the case of muon decay the charged lepton must be an electron. A tau decay can produce a muon or an electron (and indeed it is sufficiently massive to decay alternatively to hadrons, as we shall see later).

It is of fundamental significance that electric charge is conserved at every stage of a decay. It is also believed to be true of all interactions that a single lepton can only change to another of the same type, and a lepton and an anti-lepton of the same type can only be created or destroyed together. There is thus a conservation law, the '*conservation of lepton number*' (anti-leptons being counted negatively), for each separate type of lepton.

2.6 Parity violation in muon decay

It is observed experimentally that in the decay of the negative muon, the electron momentum $\mathbf{p}_e$ is strongly biased to be in the direction opposite to that of the muon spin $\mathbf{s}_\mu$. To explain the implication of this

2.2 The decay $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$. In (*a*) the muon changes to its neutrino and a 'virtual' W^- boson, which then decays to the electron and the electron anti-neutrino. In (*b*) a 'virtual' W^+ is created from the vacuum with the electron and the electron anti-neutrino. The W^+ then transforms the muon into a muon neutrino.



observation for parity violation, we must first point out that there are two types of vector.

Under the reflection in the origin (Fig. 1.1), the position vector **r** of a particle and its momentum **p** transform:

$$\mathbf{r} \rightarrow \mathbf{r}' = -\mathbf{r} \quad \text{and} \quad \mathbf{p} = m\frac{d\mathbf{r}}{dt} \rightarrow \mathbf{p}' = m\frac{-d\mathbf{r}}{dt} = -\mathbf{p}, \tag{2.15}$$

r and **p** are both *true vectors.*

The angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ has many of the attributes of a vector, but under reflection

$$\mathbf{L} \rightarrow \mathbf{L}' = (-\mathbf{r}) \times (-\mathbf{p}) = +\mathbf{L}.$$

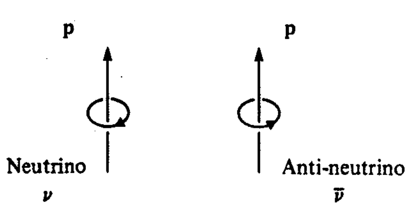
Thus **L** does not have the reflection property (2.14) of the true vectors **r** and **p**. It is called an *axial vector* or *pseudo-vector*. The intrinsic angular momentum **s** of a particle is likewise an axial vector.

Returning to muon decay, in the reflected coordinate system, $\mathbf{p}_e \rightarrow -\mathbf{p}_e$, $\mathbf{s}_\mu \rightarrow +\mathbf{s}_\mu$, so that the momentum would be said to be biased in the same direction as the muon spin! It appears that the equations of the theory are only valid in the original right-handed frame, and would have to be rewritten to hold in the left-handed reflected frame. Thus the laws are not invariant under reflection and hence parity is not conserved in muon decay. More generally, parity is not conserved in any process involving the weak interaction fields.

The inequivalence of right-handedness and left-handedness is most extreme in the case of neutrinos. Neutrinos produced in a weak interaction process are always 'left-handed', with their spin anti-parallel to their direction of motion, and anti-neutrinos are always 'right-handed' (Fig. 2.3). There is no evidence that right-handed neutrinos (or left-handed anti-neutrinos) exist at all.

The breakdown of parity conservation may be expressed slightly differently. The reflection in the origin $\mathbf{r} \rightarrow \mathbf{r}' = -\mathbf{r}$ is easily seen to be equivalent to mirror reflection in a plane, followed by a rotation through π

2.3 The relation between momentum **p** and spin for a neutrino ν and an anti-neutrino $\bar{\nu}$.



about an axis perpendicular to that plane (e.g. the *xy*-plane and the *z*-axis, cf. Problem 1.2). There is no evidence that the laws of physics break down under rotations, so the breakdown is in the mirror reflection: the assumption that the mirror image of a physical process is also a possible physical process is wrong, in so far as the weak interaction is involved.

Problems

2.1 Plane wave solutions of the relativistic wave-equation for a free particle of mass m are of the form

$\psi(\mathbf{r}, t) = (\text{constant})e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$

where

$\omega^2 = c^2k^2 + (m^2c^4/\hbar^2).$

Show that the group velocity of a wave-packet representing a particle of total energy $E = \hbar\omega$ is the same as the velocity of a relativistic classical particle having the same total energy.

2.2 The weak charge density of an electron bound in an atom has a similar magnitude to the electric charge density and has, similarly, a probability distribution over the atomic dimensions of the electron's wave-function. Show that the ratio of the weak interaction energy to the electrostatic interaction energy between two electrons bound in an atom is of order of magnitude $4\pi(\hbar/(a_0 M_Z c))^2 \sim 10^{-15}$, where a_0 is the Bohr radius. (Compare this result with Problem 1.1.)

2.3 An electron–positron pair bound by their Coulomb attraction is called *positronium*. Show that when positronium decays from rest to two photons, the photons have equal energy.

2.4 Use energy and momentum conservation to show that pair creation by a single photon, $\gamma \rightarrow e^+ + e^-$, is impossible in free space.

2.5 Show that a muon in free space with a kinetic energy of 1 MeV will travel a mean distance of about 90 m before it decays.

2.6 An electron and a μ^+ bound by their Coloumb attraction is called *muonium*. Which of the following decays can occur:

(*a*) $(\mu^+ e^-) \rightarrow \gamma + \gamma$

(*b*) $(\mu^+ e^-) \rightarrow \nu_e + \bar{\nu}_\mu$

(*c*) $(\mu^+ e^-) \rightarrow e^+ + e^- + \nu_e + \bar{\nu}_\mu$?

2.7 The masses of the electron and neutrinos from a muon decay are negligible compared with the muon mass. Show that if the muon decays from rest and the kinetic energy released is divided equally between the final leptons then the angle between the paths of any two of them is approximately 120°.

2.8 Starting from the Coulomb law and the Biot–Savart law, show that the electric field **E** is a true vector field, but that the magnetic field **B** is an axial vector field.

3 Nucleons and the strong interaction

We turn now to the hadrons, the particles which interact by the *strong interaction*, as well as by the weak and electromagnetic interactions. In particular we shall describe the *nucleons*, that is to say, the *proton* and the *neutron*, the forces between nucleons, and the effect of the weak interaction on the stability of nucleons.

3.1 Properties of the proton and the neutron

Nucleons, like leptons, are fermions with spin $\frac{1}{2}$. The mass of the neutron is 0.14% greater than that of the proton:

$$\begin{aligned} m_{\mathrm{n}} &= 939.57\ \mathrm{MeV}/c^2, \\ m_{\mathrm{p}} &= 938.28\ \mathrm{MeV}/c^2. \end{aligned} \tag{3.1}$$

Thus the mass difference $m_{\mathrm{n}} - m_{\mathrm{p}} = 1.29\ \mathrm{MeV}/c^2$ (≈ 2 electron masses).

The neutron has no net electric charge. The proton has the opposite charge to the electron: protons are responsible for exactly cancelling the charge of the electrons in electrically neutral atoms.

The electric charge on a proton is not concentrated at a point, but is symmetrically distributed about the centre of the proton. By the experimental methods to be discussed in Chapter 4, the mean radius R_{p} of this charge distribution is found to be $R_{\mathrm{p}} \approx 0.8$ fm. An extended charge distribution is also found in the neutron, positive charge in the central region being cancelled by negative charge at greater distances. The matter distribution in nucleons also extends to a distance of about R_p.

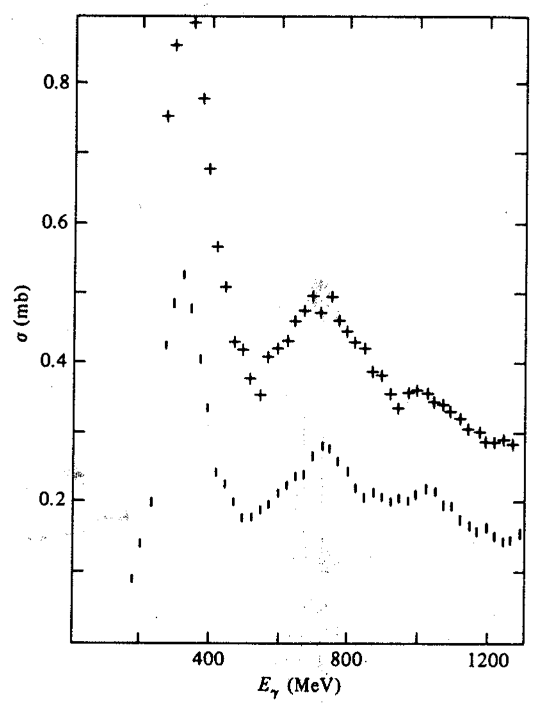
Both the proton and the neutron have a magnetic dipole moment, aligned with their spin:

$$\mu_p = 2.79284(e\hbar/2m_p),$$
$$\mu_n = -1.91304(e\hbar/2m_p). \tag{3.2}$$

Clearly neither magnetic moment is simply related to the value $e\hbar/[2(\text{nucleon mass})]$ expected from a simple Dirac equation, and this is a clear indication that the nucleons are not themselves fundamental particles.

Compelling evidence that the nucleons are the ground states of a composite system is given by data of which that in Fig. 3.1 is an example. This shows the cross-section for absorption of photons by protons and by deuterons (see § 3.3), as a function of photon energy up to 1300 MeV. The

3.1 The total photon cross-section for hadron production on protons (dashes) and deuterons (crosses). The difference between these cross-sections is approximately the cross-section on neutrons. (After Armstrong, T. A. *et al.* (1972), *Phys. Rev.* **D5**, 1640; *Nuc. Phys.* **B41**, 445.)



cross-sections vary rapidly with energy. A precise definition of cross-section is given in Appendix A, but for our immediate purpose it is sufficient to remark that the peaks are due to photons being preferentially absorbed to create an excited state when the photon energy matches the excitation energy of that state. Perhaps a more familiar example of photons being absorbed by a composite system is that of atomic absorption. Similar peaks in atomic absorption cross-sections, but at energies of a few electron volts, correspond to the excitation of the atom to higher energy states. The nucleon peaks have a similar interpretation, albeit on a very different energy scale. The first peak in the proton cross-section is at a photon energy of about 294 MeV, and corresponds to the formation of a state called the Δ^+. The Δ^+ is a fermion with mass of about $(938+294)$ MeV ≈ 1232 MeV; its spin has been determined to be $\frac{3}{2}$.

Data for the neutron show that it has a sequence of excited states of the same spins and almost identical energies as has the proton. The electrical energies associated with the charge distributions of the proton and neutron are of order of magnitude $e^2/(4\pi\varepsilon_0 R_p) \approx 2$ MeV (taking $R_p = 0.8$ fm), which is small compared with the nucleon self-energies and excitation energies. We shall see that, in all strong interactions, protons and neutrons behave in the same way to a good approximation. The near independence of the strong interaction on nucleon type is an important fact for our understanding of the properties of the nucleus.

3.2 The quark model of nucleons

Any composite system with spin $\frac{1}{2}$ must contain an odd number of fermion constituents. (An even number would give integral spin.) The highly successful quark model postulates that nucleons contain three fundamental fermions called *quarks*. We cannot here present the particle physics which establishes the validity of the quark model, but since particle physics does have implications for the concepts of nuclear physics we give – without attempting justification – some of the most relevant results.

As is the case with the elementary leptons, there are several types of quark, with a curious and so far unexplained mass hierarchy. For nucleons and nuclear physics only the two least-massive quarks are involved, the up quark u and the down quark d. The proton basically contains two up quarks and a down quark (uud) and the neutron two downs and one up (ddu). These quarks are bound by the fundamental strong interaction field, called by particle physicists the *gluon* field. The fact that the strong interactions of neutrons are almost the same as those of protons is explained by the gluon field having the same coupling to all quarks, independent of their type.

What are the properties of these quarks? They have mass, but the mass of a particle is generally determined by isolating it and measuring its acceleration in response to a known force. Because a single quark has never been isolated, this procedure has not been possible, and our knowledge of the quark masses is indirect. The consensus is that much of the nucleon mass resides in the gluon force fields that bind the quarks, and only a few MeV/c^2 need be assigned to the u and d quark masses. It is well established that the d quark is heavier than the u quark, since in all cases where two particles differ only in that a d quark is substituted for a u quark, the particle with the d quark is heavier. The principal example of this is the difference in mass between the neutron and proton. The mass, $\sim 2\ \text{MeV}/c^2$, associated with the electrical energy of the charged proton is far greater than that associated with the (overall neutral) charge distribution of the neutron, so that one might expect the proton to be heavier. However, the extra d quark in the neutron more than compensates for this, and makes the neutron heavier than the proton.

The electric charges carried by quarks are well verified by measurements of the electromagnetic transitions between the nucleon ground states and excited states. The u has charge $\frac{2}{3}e$ and the d has charge $-\frac{1}{3}e$. Thus the proton (uud) has net charge e and the neutron (ddu) has net charge zero. Again, since a quark has never been isolated, the evidence for these assignments is all indirect.

The differences between neutrons and protons, other than their electric and weak charges, are due to the u–d mass difference. This has only a small effect on the basic strong interactions, so that the resulting strong interaction between nucleons is almost independent of nucleon type. This independence may be expressed mathematically by introducing the concept of 'isotopic spin symmetry', but for our purposes this elaboration is unnecessary.

3.3 The nucleon–nucleon interaction: the phenomenological description

We shall see in later chapters that the kinetic energies and potential energies of nucleons bound together in a nucleus are an order of magnitude smaller than the energies (~ 290 MeV) required to excite the quarks in an individual nucleon. It is, therefore, reasonable to regard a nucleus as an assembly of nucleons interacting with each other, but basically remaining in their ground states. To understand the physics of nuclei it is therefore

important to be able to describe the interactions between nucleons. Since nucleons are composite particles, we can anticipate that their interactions with each other will not be simple. In fact they are rather complicated. Nevertheless, after 50 years of experimental and theoretical effort a great deal is known empirically about the forces between two nucleons, especially at the low energies relevant to nuclear physics.

The empirical approach is to construct a possible potential which incorporates our limited theoretical knowledge (which we shall discuss in §3.4) and has adjustable features, mainly to do with the short-range part of the interaction. The Schrödinger equation for two nucleons interacting through this potential is then solved numerically and the adjustable features are varied to fit the experimental facts, namely the properties of the *deuteron* and the low-energy scattering data.

The deuteron is a neutron–proton bound state with:

$$\begin{aligned} &\text{binding energy} = 2.23\ \text{MeV},\\ &\text{angular momentum } j = 1,\\ &\text{magnetic moment} = 0.8574(e\hbar/2m_p),\\ &\text{electric quadrupole moment} = 0.286\ \text{fm}^2. \end{aligned} \tag{3.3}$$

Neither proton–proton nor neutron–neutron bound states exist.

The scattering data provide much more information. Nucleons have spin $\frac{1}{2}$, which may be 'flipped' in the scattering. It can be shown that there are five independent differential cross-sections for spin-polarized proton–proton and proton–neutron scattering which can, in principle, be measured. Neutron–neutron cross-sections have never been measured directly because there are no targets of free neutrons.

As has been explained, the strong neutron–neutron interaction should be almost the same as the strong proton–proton interaction, and both these should be almost the same as the proton–neutron interaction for the same states of relative motion. However, we must remember here the Pauli exclusion principle: the neutron and proton can exist together in states which are not allowed for two protons or two neutrons. This is why the neutron and proton can have a bound state, whereas two protons or two neutrons do not bind, without any contradiction of the principle that the strong interaction is almost independent of nucleon type.

A large amount of careful and accurate data has been accumulated, and the most sophisticated and accurate empirical potential has been constructed by a group of scientists working in Paris. Two expressions are needed: one for the (anti-symmetric) states allowed for two protons or two neutrons, as well as a proton and a neutron, and one for symmetric states accessible only to the neutron–proton system. For both cases, when the spins of the two nucleons are coupled to give a total spin $S=0$ (see Appendix C) the nucleons only experience a central potential.

When the spins couple to $S=1$ there are four contributions to these potentials, which are then each of the form

$$V(r)=V_{C1}(r)+V_T(r)\Omega_T+V_{SO}(r)\Omega_{SO}+V_{SO2}(r)\Omega_{SO2},$$

where

$$\Omega_T=3\frac{(\sigma_1\cdot\mathbf{r})(\sigma_2\cdot\mathbf{r})}{r^2}-\sigma_1\cdot\sigma_2 \tag{3.4}$$

$$\hbar\Omega_{SO}=(\sigma_1+\sigma_2)\cdot\mathbf{L}$$

$$\hbar^2\Omega_{SO2}=(\sigma_1\cdot\mathbf{L})(\sigma_2\cdot\mathbf{L})+(\sigma_2\cdot\mathbf{L})(\sigma_1\cdot\mathbf{L}).$$

In these expressions $\sigma(\hbar/2)$ is the nucleon spin operator, $\mathbf{L}$ is the orbital angular momentum operator of the nucleon pair, and the subscripts 1 and 2 refer to the two nucleons present.

V_{C1} is essentially an ordinary central potential. $V_T\Omega_T$ is called the tensor potential. It has the same angular structure as the potential between two magnetic dipoles and it is also interesting because it is the only part of the potential which mixes states of different orbital angular momentum. $V_{SO}\Omega_{SO}$ and $V_{SO2}\Omega_{SO2}$ give rise to different terms for the different couplings of spin and orbital angular momenta. Spin orbit coupling is well known in atomic physics, where it is due to magnetic interactions. However, these terms in the nuclear potential, which are of major importance, arise out of the strong interaction.

In Fig. 3.2 we show the four potentials that are most important at low energies of interaction (< 100 MeV) and in particular are important for nucleons in nuclei.

The potential $V_{C0}(r)$ is appropriate for low-energy proton–proton and neutron–neutron interactions. The attractive tail is not, however, sufficiently deep to bind two nucleons. The potentials $V_{C1}(r)$, $V_{SO}(r)$ and $V_T(r)$ are responsible for binding the deuteron: note the deeply attractive part of $V_T(r)$, which is associated also with the large electric quadrupole moment of the deuteron.

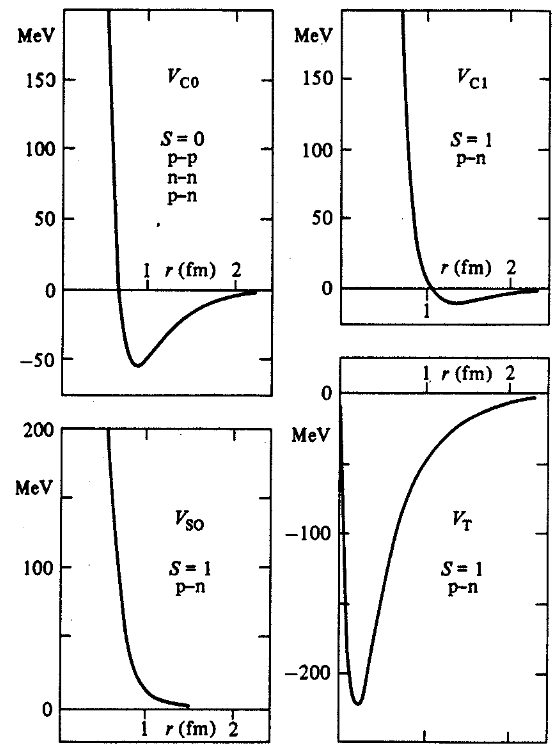
The central potentials have the important feature of a repulsive core at ~0.8 fm, which stops nuclei collapsing. The attractive part of these potentials binds nucleons together in nuclei. The tensor potential is particularly important for binding the deuteron, but since it is zero on averaging over all directions it becomes less important in heavier nuclei. This last remark presupposes that the potential established for the interaction of two nucleons in isolation is relevant when many nucleons are

interacting in an atomic nucleus. We shall discuss this assumption further in Chapter 4.

3.4 Mesons and the nucleon–nucleon interaction

Like all fermions, quarks have corresponding anti-particles. Anti-protons and anti-neutrons can exist, made up of anti-quarks, ($\bar{u}\bar{u}\bar{d}$) and ($\bar{d}\bar{d}\bar{u}$); the excited states of nucleons have images of identical mass but opposite charge in anti-quark matter. In fact the electromagnetic and strong interactions of anti-matter seem to be identical to those of matter. It is possible to contemplate the existence of stable anti-atoms, and macroscopic bodies, made up of anti-matter, but as electrons annihilate with positrons, so do nucleons annihilate with anti-nucleons; matter and anti-matter, though stable in isolation, cannot coexist. To study anti-particles we must create them in laboratories.

3.2 The most important components of the 'Paris potential'. (After Lacombe, M. *et al.* (1980), *Phys. Rev.* **C21**, 861.)



As well as binding three quarks or three anti-quarks together to make nucleons and anti-nucleons, the strong gluon field can bind a quark and an anti-quark together to form a short-lived particle called a *meson*. Like nucleons, such bound pairs have a sequence of excited states.

Of most importance for nuclear physics are the π-mesons. The electrically charged π^+ and π^- are made up of $(u\bar{d})$ and $(d\bar{u})$ pairs respectively, and the neutral π^0 is a superposition $(u\bar{u}-d\bar{d})/\sqrt{2}$ of quark anti-quark pairs. (The orthogonal combination $(u\bar{u}+d\bar{d})/\sqrt{2}$ belongs to a meson called the η.)

The masses of the π-mesons are:

$$\begin{aligned} &\text{mass of } \pi^+ = \text{mass of } \pi^- = 139.57\ \text{MeV}/c^2, \\ &\text{mass of } \pi^0 = 134.96\ \text{MeV}/c^2. \end{aligned} \tag{3.5}$$

(The η has mass ≈ 549 MeV/c^2.)

The quark anti-quark pairs in these mesons have orbital angular momentum zero and intrinsic spins coupled to give total angular momentum zero. The first excited states also have orbital angular momentum zero, but the intrinsic spins are coupled to give a total spin with quantum number $S=1$. These states are called the ρ^+, ρ^- and ρ^0 mesons; they have masses ~ 750 MeV/c^2.

For reasons that are not yet understood, the force between nucleons at distances $\gtrsim 1$ fm is not mediated by the basic gluon field (which is responsible for holding quarks together in a nucleon), but it is apparent that it is due to the exchange of mesons. Although mesons are composite particles, their motion as a whole is still described by a wave-function $\Phi(\mathbf{r}, t)$, obeying in free space the wave-equation for massive particles:

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \left(\frac{mc}{\hbar}\right)^2\right]\Phi(\mathbf{r}, t) = 0, \tag{3.6}$$

where m is the mass of the particle (cf. equations (2.10)–(2.12).)

One solution of this equation describes the π-meson field associated with a nucleon of spin $\boldsymbol{\sigma}_1(\hbar/2)$ at $\mathbf{r}_1$:

$$\Phi(\mathbf{r}, t) = g_\pi(\boldsymbol{\sigma}_1 \cdot \nabla_1)\frac{e^{-mc|\mathbf{r}-\mathbf{r}_1|/\hbar}}{|\mathbf{r}-\mathbf{r}_1|}, \tag{3.7}$$

where g_π is a measure of the meson source strength of the nucleon. The gradient operator ∇_1 acts only on $\mathbf{r}_1$, so that (3.7) is evidently a solution of (3.6) (cf. (2.11)).

The 'dipole-like' nature of the field (3.7) is well understood by particle physicists, and the interaction energy between two nucleons associated with it is of 'dipole–dipole' form:

$$U_{12} \propto g_\pi^2(\boldsymbol{\sigma}_2 \cdot \nabla_2)(\boldsymbol{\sigma}_1 \cdot \nabla_1)\frac{e^{-mc|\mathbf{r}_2-\mathbf{r}_1|/\hbar}}{|\mathbf{r}_2-\mathbf{r}_1|}. \tag{3.8}$$

The π mesons are the mesons of smallest mass and hence give the largest contribution to the interaction at large distances. The appropriate length scale, from the exponential in (3.7), is

$$\hbar/mc \approx 1.4\ \text{fm}.$$

Explicit differentiation shows that (3.8) includes a potential of the tensor form $V_T(r)\Omega_T$. It is empirically established that π meson exchange is responsible for most of the tensor potential of (3.4), and is the dominant contribution to the whole potential at distances $|\mathbf{r}_2-\mathbf{r}_1| > 1.4$ fm. At smaller distances other meson exchange processes become important, including the exchange of ρ mesons. However, the potentials at distances <0.8 fm and, in particular, the short-range repulsion, are empirical and so far have no established explanation.

3.5 The weak interaction; β-decay

Hadrons are subject to the weak interaction as well as to the electromagnetic and strong interactions, and it is through the weak interaction that quarks, like leptons, are coupled to the W and Z bosons. For example, one quark can change to another by emitting or absorbing a virtual W boson. The phenomena of β-decay, in which a neutron becomes a proton or a proton becomes a neutron, proceed in this way.

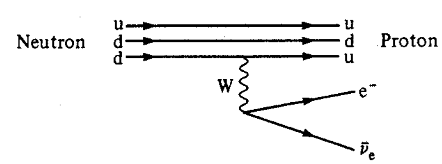
In free space, the proton is the only stable three-quark system. The neutron in free space has enough excess mass over the proton to decay to it by the process shown in Fig. 3.3.

The mean life of the neutron in free space is 15.0 minutes. However, a neutron bound in a nucleus will be stable if the nuclear binding energies make decay energetically forbidden. Conversely, a proton bound in a nucleus may change into a neutron

$$p \to n + e^+ + \nu_e,$$

if the nuclear binding energies involved allow the process to occur. The

3.3 The decay $n \to p + e^- + \bar{\nu}_e$. As with muon decay, parity is not conserved in this weak interaction.



energetics of β-decay will be dealt with in detail in Chapter 4, and a more quantitative theory of β-decay will be given in Chapter 12.

3.6 More quarks

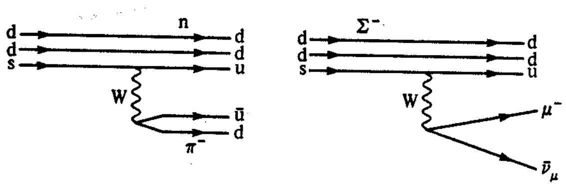
The u and d quarks are merely the two least massive of a sequence of types, or 'flavours' of quark, and to set the discussion of β-decay above into this wider context we list in Table 3.1 all the presently known flavours.

The existence of the more massive quarks in this table is revealed by the observation of states similar to the nucleon states and meson states we have already discussed, but which are apparently formed by substituting any of the 'new' quarks for the u or d quarks. Thus, for example, substituting an s quark for a d quark, there exists a K^+ meson ($u\bar{s}$) (mass 493.67 MeV/c^2) like the π^+ meson ($u\bar{d}$) but heavier, and a Σ^0 baryon (uds) (mass 1193 MeV/c^2) like the neutron (udd) but heavier. *Baryon* and *anti-baryon* are the generic names for particles essentially made up of three quarks or three anti-quarks. Again, since no quark has ever been isolated, the masses given in Table 3.1 are effective masses and have no precise significance.

Were it not for the weak interaction a heavy quark would be stable and there would be more absolute conservation laws, for example, the conservation of strangeness and the conservation of charm. Such laws hold for processes involving only the electromagnetic and strong interactions,

Table 3.1. *Properties of quarks*

Quark	Approximate mass (GeV/c^2)	Electric charge (e)
Down d	small	$-\frac{1}{3}$
Up u	small	$\frac{2}{3}$
Strange s	0.3	$-\frac{1}{3}$
Charm c	1.5	$\frac{2}{3}$
Bottom b	5.2	$-\frac{1}{3}$
Top t	~40.0?	$\frac{2}{3}$



3.4 The decays $\Sigma^- \to n + \pi^-$, $\Sigma^- \to n + \mu^- + \bar{\nu}_\mu$.

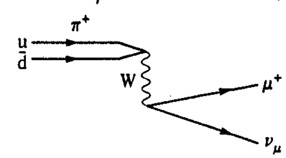
but are not absolute since all quarks couple to the $W^\pm$ and Z weak interaction fields, and a quark changes its flavour (but remains a quark!) when it emits or absorbs a virtual $W^\pm$ boson. Thus, for example, the s quark in the Σ^- baryon can decay through processes like those shown in Fig. 3.4. We shall see that nuclear binding energies are not sufficiently large to make a baryon containing a heavy quark stable even in a nucleus.

The weak interaction makes *all* mesons unstable. Mesons containing a heavy quark can decay by the heavy quark changing into a lighter quark. Another possible process is illustrated in Fig. 3.5, in which a quark and an anti-quark annihilate through the weak interaction into an anti-muon and a muon neutrino. This latter process is the predominant type of decay of the charged pions. The mean life of charged pions is 2.60×10^{-8} s.

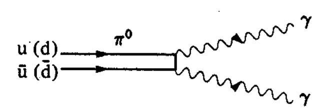
The π^0 usually decays into two photons by the direct annihilation of the quarks with their own anti-quarks, in a way rather similar to the decay of positronium (an electron–positron pair e^+e^- in a bound state). Such a decay (Fig. 3.6) takes place through the electromagnetic interaction, and is therefore much quicker: the mean life of the π^0 is 0.83×10^{-16} s.

A baryon and an anti-baryon are always created or destroyed together. All the available experimental evidence is consistent with there being a law of '*conservation of baryon number*': the total number of baryons (anti-baryons being counted negatively) is conserved in all interactions.

3.5 The decay $\pi^+ \to \mu^+ + \nu_\mu$. The charged pion was discovered by Powell and co-workers in Bristol in 1947 by the observation of this decay.



3.6 The electromagnetic decay. $\pi^0 \to \gamma + \gamma$.



Problems

3.1 The spins of the neutron and the proton in the deuteron are aligned. Show that the magnetic moment of the deuteron is within 3% of the sum of the neutron and proton moments. What might be the origin of the discrepancy?

3.2(*a*) Show that the magnetic interaction energy between two magnetic dipoles $\mu\boldsymbol{\sigma}_1$ and $\mu\boldsymbol{\sigma}_2$ is of the form $V_T(r)\Omega_T$ with $V_T(r) = -(\mu_0/4\pi)\mu^2/r^3$. ($\mu_0$ is the permeability of the vacuum.)

(*b*) Verify that equation (3.8) includes terms in the nucleon–nucleon potential of tensor form.

3.3 The Coulomb self-energy of a hadron with charge +e or −e is about 1 MeV. The quark content and rest energies (in MeV) of some hadrons are:

n(udd) 940, p(uud) 938

Σ^-(dds) 1197, Σ^0(uds) 1192, Σ^+(uus) 1189

K^0(d$\bar{s}$) 498, K^+(u$\bar{s}$) 494.

The u and d quarks make different contributions to the rest energy. Estimate this difference.

3.4 Which of the following processes are allowed by the conservation laws:

(*a*) $n \to p + \gamma$,

(*b*) $p \to e^+ + \gamma$,

(*c*) $p \to \pi^+ + \gamma$,

(*d*) $\bar{p} + n \to \pi^- + \pi^0$?

3.5 The decay Ξ^{*-} initiates the sequence of decays shown below:

$\Xi^{*-} \to K^- + \Sigma^0$
- $\Sigma^0 \to \Lambda^0 + \gamma$
  - $\Lambda^0 \to p + e^- + \bar{\nu}_e$
- $K^- \to \pi^- + \pi^0$
  - $\pi^0 \to \gamma + \gamma$
  - $\pi^- \to \mu^- + \bar{\nu}_\mu$
    - $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$

The quark content of the hadrons involved is:

Ξ^{*-}(ssd), Σ^0(sud), Λ^0(sud), p(uud),

K^-(s, $\bar{u}$), π^-(d$\bar{u}$), π^0(u$\bar{u}$ − d$\bar{d}$).

Classify the decays as strong, electromagnetic, or weak.

4
Nuclear sizes and nuclear masses

We now turn to the study of the nucleus. A nucleus is a bound assembly of neutrons and protons. ${}^A_Z X$ denotes a nucleus of an atom of the chemical element X containing A nucleons, of which Z are protons and $N = (A - Z)$ are neutrons. For example, ${}^{35}_{17}$Cl denotes a chlorine nucleus with 18 neutrons and ${}^{37}_{17}$Cl a chlorine nucleus with 20 neutrons. Since the chemical symbol determines the atomic number Z, ^{35}Cl or ^{37}Cl is identification enough, but the addition of the Z label is often useful. $A = (N + Z)$ is called the *mass number* of the nucleus. Nuclei which differ only in the number of neutrons they contain are called *isotopes*. Nuclei of the same A but different Z are called *isobars*.

4.1 Electron scattering by the nuclear charge distribution

Rutherford's famous analysis in 1911 of the scattering of α-particles by matter established that the size of the nucleus of an atom is small compared with the size of the atom. Whereas the electronic distribution extends to a distance of the order of angströms (1 Å = 10^{-10} m) from the nucleus, these and later experiments showed that the distribution of nucleons is confined to a few fermis (1 fm = 10^{-15} m). Early theories of α-decay and nuclear binding energies gave estimated values for nuclear radii of a similar magnitude.

Precise information came in the 1950s, with experiments using the elastic scattering of high-energy electrons to probe the nuclear charge distribution.

There is an obvious advantage in using charged leptons (electrons or muons) to probe nuclear matter, since leptons interact with nucleons primarily through electromagnetic forces: the complications of the strong nuclear interaction are not present, and the weak interaction is negligible for the scattering process. The most significant interaction between a charged lepton, which can be regarded as a structureless point object, and the nuclear charge, is the Coulomb force, and this is well understood. If the nucleus has a magnetic moment, the magnetic contribution to the scattering becomes important at large scattering angles, but this also is well understood.

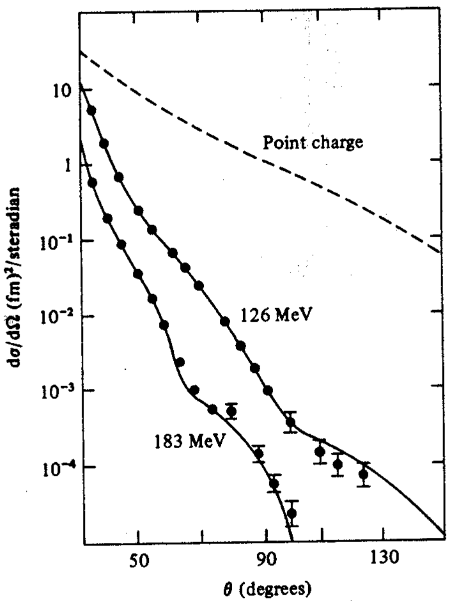
If scattering experiments are to give detailed information on the nuclear charge distribution, it is clear that the de Broglie wavelength λ of the incident particle must be less than, or at least comparable with, the distances over which the nuclear charge density changes. An electron with $(\lambda/2\pi) \sim 1$ fm has momentum $p = 2\pi\hbar/\lambda$ and hence energy $E = (p^2c^2 + m^2c^4)^{\frac{1}{2}} \sim 200$ MeV. At these energies, the electrons are described by the Dirac relativistic wave equation, rather than by the Schrödinger equation. The experiments yield a differential cross-section $d\sigma(E, \theta)/d\Omega$ (Appendix A) for elastic scattering from the nucleus through an angle θ, which depends on the energy E of the incident electrons. Typical experimental data are shown in Fig. 4.1.

The incident electrons are, of course, also scattered by the atomic electrons in the target. However, this scattering is easily distinguished from the nuclear scattering by the lower energy of the scattered electrons. Whereas the recoil energy taken up by the heavy nucleus is very small, the recoil energy taken up by the atomic electrons is appreciable, except for scattering in the forward direction. (See Problem 4.1.)

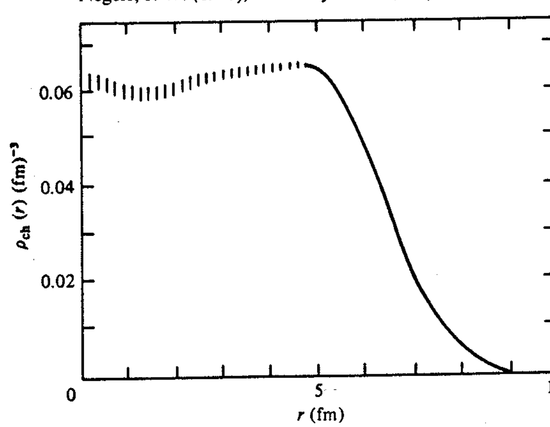
The nuclear charge density will be described by some density function $e\rho_{ch}(\mathbf{r})$. (The proton charge e is put in as a factor for convenience.) This function is not necessarily spherically symmetric – we shall mention this later – but for nuclei which are spherically symmetric, or nearly so, we can assume the charge density depends only on the distance r from the centre of the nucleus. Then, using the Dirac wave equation for the electron, $d\sigma/d\Omega$ is in principle completely determined by $\rho_{ch}(r)$, though the calculations are not trivial. The inverse problem, that of finding $\rho_{ch}(r)$ from a knowledge of $d\sigma/d\Omega$, is even more difficult (see Problem 4.2). The restricted amount of experimental information available means that, at best, only a partial resolution of the problem can be made. Some idea of the results of a direct inversion of scattering data is given by Fig. 4.2.

It has been more usual to assume a plausible shape for $\rho_{ch}(r)$, describe this

4.1 Experimental elastic electron-scattering differential cross-section from gold $^{197}_{79}$Au at energies of 126 MeV and 183 MeV. The fitted curves are calculated with an assumed charge distribution of the form given by equation (4.1), with $R = 6.63$ fm, $a = 0.45$ fm. The cross-section to be expected, at 126 MeV, if the gold nucleus had a point charge is shown for comparison. (Data and theoretical curves taken from Hofstadter, R. (1963), *Electron Scattering and Nuclear and Nucleon Structure*, New York: Benjamin.)



4.2 The electric charge density of $^{208}_{82}$Pb from a model-independent analysis of electron scattering data. The bars indicate the uncertainty. (Friar, J. L. & Negele, J. W. (1973), *Nuc. Phys.* **A212**, 93.)



by a simple mathematical expression involving a few parameters, and then determine the parameters by fitting to the scattering data. A form which has been widely adopted is

$$\rho_{ch}(r) = \frac{\rho^0_{ch}}{1+e^{(r-R)/a}}, \qquad (4.1)$$

where the parameters to be determined are R and a, and ρ^0_{ch} is a normalisation constant chosen so that

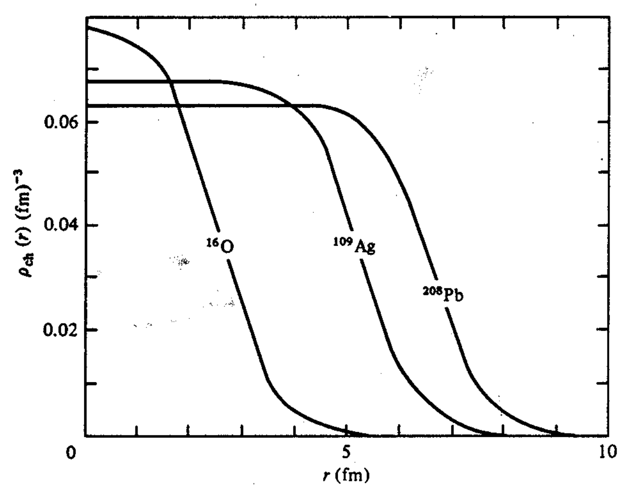
$$\int \rho_{ch}(r)\, d^3\mathbf{r} = 4\pi \int_0^\infty \rho_{ch}(r) r^2\, dr = Z.$$

It should be stressed that the choice of this expression has no fundamental significance; it just conveniently describes a charge distribution which extends almost uniformly from the centre of the nucleus to a distance R, and falls to zero over a well-defined surface region of thickness $\sim a$. This picture is consistent with the results of direct inversion.

In Fig. 4.3 we show nuclear charge distributions for a light ($^{16}_{8}O$), a medium ($^{109}_{47}Ag$) and a heavy ($^{208}_{82}Pb$) nucleus obtained from experimental scattering data, using this parametrisation of the charge density. The corresponding values of R and a are given in Table 4.1.

As the examples in the table indicate, it appears that there is a well-defined 'surface region' which has much the same width for all nuclei, even light ones.

4.3 The electric charge density of three nuclei as fitted by $\rho_{ch}(r) = \rho^0_{ch}/[1+\exp((r-R)/a)]$. The parameters are taken from the compilation in Barrett, R. C. & Jackson, D. F. (1977), *Nuclear Sizes and Structure*, Oxford: Clarendon Press.



4.2 Muon interactions

The negative muon is another leptonic probe of nuclear charge. Its properties, other than its mass of $m_\mu \approx 207\, m_e$ and its mean life of 2.2×10^{-6} s, are similar to those of the electron. However, the radius of its lowest Bohr orbit in an atom of charge Z is $(4\pi\varepsilon_0)\hbar^2/m_\mu Ze^2$, and this is smaller than the corresponding electron orbit by a factor (m_e/m_μ). For $Z=50$ the radius is only 5 fm. Hence the wave-functions of the lowest muonic states will lie to a considerable extent within the distribution of nuclear charge, particularly in heavy nuclei, and the energies of these states will therefore depend on the details of the nuclear charge distribution.

Experimentally, negative muons are produced in the target material by the decay of a beam of negative pions, and are eventually captured in outer atomic orbitals. Before they decay, many muons fall into lower orbits, emitting X-rays in the transitions. The measured energies of these X-rays may be compared with those calculated with various choices of parameters for $\rho_{ch}(r)$. Values of R and a, found in this way, agree well with results from electron scattering.

4.3 The distribution of nuclear matter in nuclei

From the distribution of charge in a nucleus, which as we have seen can be determined by experiment, we can form some idea of the distribution of nuclear matter. If the proton were a point object, we could identify the proton number density $\rho_p(r)$ with $\rho_{ch}(r)$. Since the strong nuclear forces which bind nucleons together are charge independent and of short range, we can assume that to a good approximation the ratio of neutron density ρ_n to proton density ρ_p is the same at all points in a nucleus, i.e. $\rho_n(r)/\rho_p(r) = N/Z$. Then the total density of nucleons $\rho = \rho_n + \rho_p$ can be expressed as $\rho =$

Table 4.1. *Nuclear radii* (R) *and nuclear surface widths* (a)

Nucleus	R (fm)	a (fm)	$R/A^{1/3}$ (fm)
$^{16}_{8}O$	2.61	0.513	1.04
$^{109}_{47}Ag$	5.33	0.523	1.12
$^{208}_{82}Pb$	6.65	0.526	1.12

$(A/Z)\rho_{ch}$, where $A=N+Z$. The resulting nuclear matter densities for the same nuclei we took in Fig. 4.3 are plotted in Fig. 4.4. These densities are only approximate, since we have neglected the finite size of both proton and neutron and the effect of Coulomb forces, but they indicate that at the centre of a nucleus the nuclear-matter density ρ is roughly the same for all nuclei. It increases with A, but appears to tend to a limiting value ρ_0 of about 0.17 nucleons fm^{-3} for large A. The existence of this limiting value ρ_0, known as the 'density of nuclear matter', is an important result. Consistently with this, we find (Table 4.1), that the 'radius' R of a nucleus is very closely proportional to $A^{\frac{1}{3}}$, and, approximately, $(4\pi/3)R^3\rho_0=A$. We shall take

$$\rho_0=0.17 \text{ nucleons fm}^{-3}$$

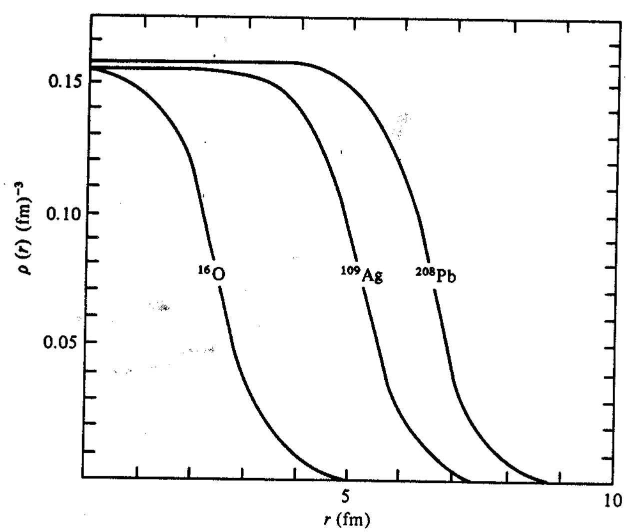
and

$$R=1.1A^{\frac{1}{3}}\text{ fm}. \tag{4.2}$$

4.4 The masses and binding energies of nuclei in their ground states

It thus appears that a nucleus is rather like a spherical drop of liquid, of nearly uniform density. How are we to understand its properties? A nucleus is a quantum-mechanical system. We shall see later that its excited states are generally separated by energies ~ 1 MeV from its ground state, so that to all intents and purposes nuclei in matter at temperatures

4.4 The nucleon density of the nuclei of Fig. 4.3, with $\rho(r)=(A/Z)\rho_{ch}(r)$.



that are accessible on Earth are in their ground states. Like any other finite system, a nucleus in its ground state has a well-defined energy and a well-defined angular momentum. In this chapter we shall be concerned with the ground-state energy. Other ground-state properties of a nucleus will be discussed in the next chapter.

Since a nucleus is a bound system, an energy $B(Z,N)$ is needed to pull it completely apart into its Z protons and N neutrons. From the Einstein relation between mass and energy, the *binding energy* $B(Z,N)$ is related to the mass $m_{nuc}(Z,N)$ of the nucleus by

$$m_{nuc}(Z,N)=Zm_p+Nm_n-B(Z,N)/c^2, \tag{4.3}$$

and $B(Z,N)$ must be positive for the nucleus to be formed. We shall see that nuclear binding energies are of the order of 1% of the rest-mass energy $m_{nuc}c^2$.

Experimentally, the masses of atomic ions, rather than the masses of bare nuclei, are the quantities usually measured directly. If $m_a(Z,N)$ is the mass of the neutral atom,

$$m_a(Z,N)=Z(m_p+m_e)+Nm_n-B(Z,N)/c^2-b_{electronic}/c^2, \tag{4.4}$$

where $b_{electronic}$ is the binding energy of the atomic electrons. These electronic contributions are, for many purposes, negligible. (The simple Thomas–Fermi statistical model of a neutral atom gives the total electronic binding energy $\approx 20.8Z^{\frac{7}{3}}$ eV.)

Atomic masses are known very accurately, and published tables give atomic masses rather than nuclear masses. Measurements in 'mass spectrometers' depend on the deflection of charged ions in electric and magnetic fields. Instruments of great ingenuity have been developed, giving relative masses accurate to about one part in 10^7. The unit employed is the *atomic mass unit*, which is defined to be $\frac{1}{12}$ of the mass of the neutral ^{12}C atom:

$$1\text{ amu}=931.5016\pm0.0026\text{ MeV}/c^2.$$

Differences between the masses of stable atoms and unstable, radioactive, atoms (for which mass spectrometers may be inappropriate) can be determined by measuring the energy release in the unstable atom decay, again using the Einstein mass–energy relation.

Table 4.2 shows the experimental binding energies for some of the lighter nuclei, those formed by successively adding a proton followed by a neutron to an original neutron. Note that all the binding energies are positive: this reflects the basic long-range attraction of the nucleon–nucleon interaction.

Also given in the table is the average binding energy per nucleon,

$B(Z, N)/A$. For the heavier nuclei in the table, the average binding energy appears to be gradually increasing to around 8 MeV, but the numbers fluctuate somewhat from nucleus to nucleus. The fluctuation is more dramatically exhibited in the binding energy difference between a nucleus and the one preceding it, also shown in the table. This energy can be interpreted as the binding energy of the last nucleon added to the nucleus in the given sequence. It is particularly large for the 'even–even' nuclei $^4_2$He, $^8_4$Be, $^{12}_6$C and $^{16}_8$O, and particularly small for the nuclei immediately following, growing steadily as the next three nucleons are added to form the next even–even nucleus. Clearly we see here some extra binding energy associated with neutron–neutron and proton–proton pairing. The effect stems from the attractive character of the nucleon–nucleon interaction, and is associated with the pairing of angular momenta discussed in Chapter 5. Table 4.2 also gives the spins and parities of the nuclei for later reference; it will be seen that the even–even nuclei have spin zero.

As we shall see in Chapter 6, because of its low mass, low electric charge and relatively large binding energy, the first even–even nucleus $^4_2$He is particularly important in the nuclear physics of heavy nuclei. Indeed, $^4_2$He

Table 4.2. *Energies of some light nuclei (MeV)*

Nucleus	Binding energy (MeV)	Binding energy of last nucleon (MeV)	Binding energy per nucleon (MeV)	Spin and parity
$^2_1$H	2.22	2.2	1.1	1^+
$^3_1$H	8.48	6.3	2.8	$\frac{1}{2}^+$
$^4_2$He	28.30	19.8	7.1	0^+
$^5_2$He	27.34	−1.0	5.5	$\frac{3}{2}^-$
$^6_3$Li	31.99	4.7	5.3	1^+
$^7_3$Li	39.25	7.3	5.6	$\frac{3}{2}^-$
$^8_4$Be	56.50	17.3	7.1	0^+
$^9_4$Be	58.16	1.7	6.5	$\frac{3}{2}^-$
$^{10}_5$B	64.75	6.6	6.5	3^+
$^{11}_5$B	76.21	11.5	6.9	$\frac{3}{2}^-$
$^{12}_6$C	92.16	16.0	7.7	0^+
$^{13}_6$C	97.11	5.0	7.5	$\frac{1}{2}^-$
$^{14}_7$N	104.66	7.6	7.5	1^+
$^{15}_7$N	115.49	10.8	7.7	$\frac{1}{2}^-$
$^{16}_8$O	127.62	12.1	8.0	0^+
$^{17}_8$O	131.76	4.1	7.8	$\frac{5}{2}^+$

played an important role in the early history of nuclear physics and before it was properly identified it was given a special name, the α-particle, a name still in use today.

Some of the large binding energy of the nuclei $^4_2$He, $^{12}_6$C and $^{16}_8$O can be associated with their 'shell structure', which will be discussed in Chapter 5. As for $^8_4$Be, its binding energy is less than that of two α-particles by 0.1 MeV, and so the nucleus $^8_4$Be is unstable. It does have a transient existence for a time long compared with the 'nuclear time-scale' (§ 5.2), but if it is formed it will eventually fall apart into two α-particles.

Another interesting special case in Table 4.2 is that of $^5_2$He. The binding energy of the last nucleon is here negative; if $^5_2$He is formed it, too, has only a transient existence before falling apart into a neutron and an α-particle. The other nuclei in Table 4.2 are all stable.

4.5 The semi-empirical mass formula

The features of 'pairing energies' and shell-structure effects, superposed on a slowly varying binding energy per nucleon, can be discerned throughout the range of nuclei for which data are available. We saw in § 4.3 that the density of nuclear matter is approximately constant, and also that nuclei have a well-defined surface region. It appears as if a nucleus behaves in some ways rather like a drop of liquid. This analogy is made more precise in the '*semi-empirical mass formula*', a remarkable formula which, with just a few parameters, fits the binding energies of all but the lightest nuclei to a high degree of accuracy. There are several versions of the mass formula. The one which is sufficiently accurate for the purposes of this book gives for the total binding energy of a nucleus of A nucleons, made up of Z protons and N neutrons,

$$B(N,Z) = aA - bA^{\frac{2}{3}} - s\frac{(N-Z)^2}{A} - \frac{dZ^2}{A^{\frac{1}{3}}} - \frac{\delta}{A^{\frac{1}{2}}}. \tag{4.5}$$

The parameters a, b, s, d and δ are found by fitting the formula to measured binding energies. Wapstra (*Handbuch der Physik*, XXXVIII/1) gives

$a = 15.835$ MeV

$b = 18.33$ MeV

$s = 23.20$ MeV

$d = 0.714$ MeV

and

$$\delta = \begin{cases} +11.2 \text{ MeV for odd–odd nuclei (i.e., odd } N\text{, odd } Z) \\ 0 \text{ for even–odd nuclei (even } N\text{, odd } Z\text{, or even } Z\text{, odd } N) \\ -11.2 \text{ MeV for even–even nuclei (even } N\text{, even } Z). \end{cases}$$

It is the first two terms in this formula which have an analogue in the theory of liquids. The term (aA) represents a constant bulk-binding energy per nucleon, like the cohesive energy of a simple liquid. The second term represents a surface energy, in particular the surface energy of a sphere. The surface area of a sphere is proportional to the two-thirds power of its volume and hence, at constant density of nucleons, to $A^{\frac{2}{3}}$. As in a liquid, this term subtracts from the bulk binding since the particles in the surface are not in the completely enclosed attractive environment of those in the bulk. In liquids this term is identified with the energy of surface tension, and is responsible for drops of liquid being approximately spherical when gravitational effects are small. In nuclei, gravitational effects are always small, and indeed nuclei do tend to be spherical.

The term $-dZ^2/A^{\frac{1}{3}}$, called the Coulomb term, also has a simple explanation; it is the electrostatic energy of the nuclear charge distribution. If the nucleus were a uniformly charged sphere of radius $R_0A^{\frac{1}{3}}$ (equation (4.2)) and total charge Ze, it would have energy

$$E_c = \frac{3}{5}\frac{(Ze)^2}{(4\pi\varepsilon_0)R_0A^{\frac{1}{3}}}. \tag{4.6}$$

With $R_0 = 1.1$ fm this gives an estimate of d, $d = 0.79$ MeV, close to the value found empirically.

The term $-s(N-Z)^2/A$ is the simplest expression which, by itself, would give the maximum binding energy, for fixed A, when $N = Z$ (A even) or $N = Z \pm 1$ (A odd). It is called the symmetry energy, since it tends to make nuclei symmetric in the number of neutrons and protons. As was exemplified in the case of the deuteron discussed in Chapter 3, the average neutron–proton attraction in a nucleus is greater than the average neutron–neutron or proton–proton attraction, essentially as a consequence of the Pauli exclusion principle. Thus for a given A it is energetically advantageous to maximise the number of neutron–proton pairs which can interact: this is achieved by making Z and N as near equal as possible. Since the forces are short range, the term must correspond to a 'bulk' effect, like the cohesive energy. Hence there must be a factor A in the denominator, so that overall the term is proportional to A for a fixed ratio of neutrons to protons. One can also argue (see Problem 5.2) that the kinetic energy contribution to the energy results in a similar term, which is absorbed in the coefficient s.

The final term in the semi-empirical mass formula is the pairing energy $\delta/A^{\frac{1}{2}}$, manifest in the light nuclei included in Table 4.2. It is purely phenomenological in form and the $A^{-\frac{1}{2}}$ dependence is empirical. For the larger nuclei the pairing energy is small but, as we shall see, it does give rise to important physical effects.

More sophisticated versions of the formula include also 'shell structure' effects (Chapter 5), but for nuclei heavier than neon ($A = 20$) for which our formula is appropriate these extra terms are of less significance than the five terms of equation (4.5).

We have in the semi-empirical mass formula a description and an understanding of the binding energies of the nuclei. We shall see that it gives a simple but profound explanation of the masses of the chemical elements and of why there is only a finite number of stable atoms in chemistry.

4.6 The β-stability valley

Using equations (4.3) and (4.5), the mass of the neutral atom with its nucleus having Z protons and N neutrons is given by

$$m_a(N,Z)c^2 = (Nm_n + Z(m_p + m_e))c^2 - aA + bA^{\frac{2}{3}} + \frac{dZ^2}{A^{\frac{1}{3}}} + \frac{s(N-Z)^2}{A} + \frac{\delta}{A^{\frac{1}{2}}}, \tag{4.7}$$

(neglecting the electron binding energies).

For a fixed number of nucleons A, we can write this as a function of Z, replacing N by $A - Z$:

$$\begin{aligned} m_a(A,Z)c^2 &= (Am_nc^2 - aA + bA^{\frac{2}{3}} + sA + \delta A^{-\frac{1}{2}}) \\ &\quad - (4s + (m_n - m_p - m_e)c^2)Z + (4sA^{-1} + dA^{-\frac{1}{3}})Z^2 \\ &= \alpha - \beta Z + \gamma Z^2, \quad \text{say.} \end{aligned} \tag{4.8}$$

Consider first the case A odd, so that $\delta = 0$. The plot of $m_a(A, Z)$ against Z is a parabola, with a minimum at

$$Z = \beta/2\gamma = \frac{(4s + (m_n - m_p - m_e)c^2)A}{2(4s + dA^{\frac{2}{3}})} \tag{4.9}$$

Thus the atom with the lowest rest-mass energy for given A has Z equal to the integer Z_{min} closest to $\beta/2\gamma$. From the form of the expression (4.9) and the values of the parameters, it is evident that $Z_{min} \leqslant A/2$, so that $N \geqslant Z$ for this nucleus.

Now β-decay, described in § 3.5, is a process whereby the Z of a nucleus changes while A remains fixed, if the process is energetically allowed. Thus if a nucleus has $Z < Z_{min}$ the process

$$(A, Z) \rightarrow (A, Z+1) + e^- + \bar{\nu}_e$$

is possible if

$$m_{\text{nuc}}(A, Z) > m_{\text{nuc}}(A, Z+1) + m_e, \quad (4.10)$$

since the mass of the anti-neutrino (if indeed it has mass) is exceedingly small. Adding Zm_e to each side of this inequality, the condition may be written in terms of atomic masses:

$$m_a(A, Z) > m_a(A, Z+1). \quad (4.11)$$

More precisely, conditions (4.10) and (4.11) differ by a few (electron volts)/c^2, associated with the electronic binding energy differences, and since β-decay usually takes place in an atomic environment (4.11) is the more suitable form. The energy released in nuclear β-decay is never large enough to produce particles other than electrons or positrons, and neutrinos.

As an example, ${}^{77}_{32}\text{Ge}$ decays by a series of β-decays to ${}^{77}_{34}\text{Se}$, Z increasing by one at each stage:

$$\begin{aligned} {}^{77}_{32}\text{Ge} \to {}^{77}_{33}\text{As} &+ e + \bar{\nu}_e + 2.75\ \text{MeV} \\ &\downarrow \\ {}^{77}_{34}\text{Se} &+ e + \bar{\nu}_e + 0.68\ \text{MeV}. \end{aligned}$$

${}^{77}_{34}\text{Se}$ is the only stable nucleus with $A = 77$.

A nucleus with $Z > Z_{\min}$ can decay by emitting a positron and a neutrino. For example, another sequence of decays ending in ${}^{77}_{34}\text{Se}$ is:

$$\begin{aligned} {}^{77}_{36}\text{Kr} \to {}^{77}_{35}\text{Br} &+ e^+ + \nu_e + 2.89\ \text{MeV} \\ &\downarrow \\ {}^{77}_{34}\text{Se} &+ e^+ + \nu_e + 1.36\ \text{MeV}. \end{aligned}$$

For the process of β-decay by positron emission to be possible the condition is

$$m_{\text{nuc}}(A, Z) > m_{\text{nuc}}(A, Z-1) + m_e,$$

or, in terms of atomic masses,

$$m_a(A, Z) > m_a(A, Z-1) + 2m_e. \quad (4.12)$$

In an atomic environment, a β-decay process competing with positron emission is *electron capture*, in which the nucleus absorbs one of its cloud of atomic electrons, emitting only a neutrino. For example,

$${}^{77}_{35}\text{Br} + e \to {}^{77}_{34}\text{Se} + \nu_e + 2.38\ \text{MeV}.$$

Such processes are often referred to as K-capture, since the electron is most likely to come from the innermost 'K-shell' of atomic electrons. The condition for K-capture to be possible is less restrictive than (4.12):

$$m_{\text{nuc}}(A, Z) + m_e > m_{\text{nuc}}(A, Z-1),$$

or

$$m_a(A, Z) > m_a(A, Z-1). \quad (4.13)$$

For example, ${}^{7}_{4}\text{Be}$ decays by K-capture:

$${}^{7}_{4}\text{Be} + e \to {}^{7}_{3}\text{Li} + \nu_e + 0.86\ \text{MeV},$$

whereas it cannot decay by positron emission. When both processes are possible, the energy release in K-capture will be $2m_ec^2 \approx 1$ MeV greater than in the corresponding positron emission.

Thus odd-A nuclei decay to the value of Z closest to $\beta/2\gamma$. It is clearly highly unlikely that there will be two values of Z giving exactly the same atomic masses; we expect there to be only one β-stable Z value for odd-A nuclei, and such is the case.

Nuclei with even A must have Z and N both even numbers, or Z and N both odd numbers. In the semi-empirical mass formula, the even–even nuclei have a lower energy than the odd–odd nuclei by $2\delta A^{-\frac{1}{2}}$. This quantity varies from 5 MeV when $A = 20$ to 1.4 MeV when $A = 250$. Thus there are two mass parabolas with relative vertical displacement $2\delta A^{-\frac{1}{2}}/c^2$, as in Fig. 4.5, for each even value of A.

In Fig. 4.5, the values $Z = 28$ and $Z = 30$ on the lower even–even parabola are both stable with respect to β-decay, since processes in which two electrons or two positrons are emitted simultaneously have not been observed. The figure is characteristic of nuclei with even A, and pairs of stable nuclei with different (even) Z but the same A are common. The only odd–odd nuclei which are stable are the four lightest: ${}^{2}_{1}\text{H}$, ${}^{6}_{3}\text{Li}$, ${}^{10}_{5}\text{B}$ and ${}^{14}_{7}\text{N}$ – but for $A < 20$ the semi-empirical mass formula is less accurate.

The nuclei which are observed to be β-stable are plotted in Fig. 4.6 as points in the (N, Z) plane. Nuclei of constant A lie on the diagonal lines $N + Z = A$. The bottom of the 'β-stability valley' where the β-stable nuclei are found is given remarkably well by the approximation (equation (4.9)).

$$Z = \beta/2\gamma = \frac{(4s + (m_n - m_p - m_e)c^2)A}{2(4s + dA^{\frac{2}{3}})}. \quad (4.14)$$

4.7 The masses of the β-stable nuclei

With the approximation $Z = \beta/2\gamma$, the binding energies of the β-stable nuclei can be calculated from equation (4.5). Neglecting the pairing energy, the resulting binding energy per nucleon $B(A)/A$ is plotted against A in Fig. 4.7 and the various contributions to $B(A)/A$ are displayed in Fig. 4.8.

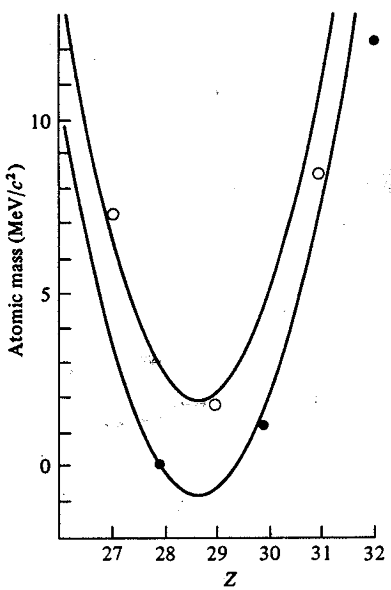
It should be noted that apart from pairing effects the bulk term is the only positive contribution to the binding energy. The initial rise of B/A with A is simply due to the negative surface contribution diminishing in magnitude relative to the bulk contribution as the size of the nucleus increase However, as A and therefore Z increase further, the quadratic Coulom term becomes important and produces a maximum on the curve.

The curve gives the observed nuclear-binding energies quite well. Th small deviations of the experimental values from the smooth curve are fo the most part due to the quantum mechanical 'shell' effects, which ar considered in the next chapter. The maximum binding energies lie in th neighbourhood of ^{56}Fe.

4.8 The energetics of α-decay and fission

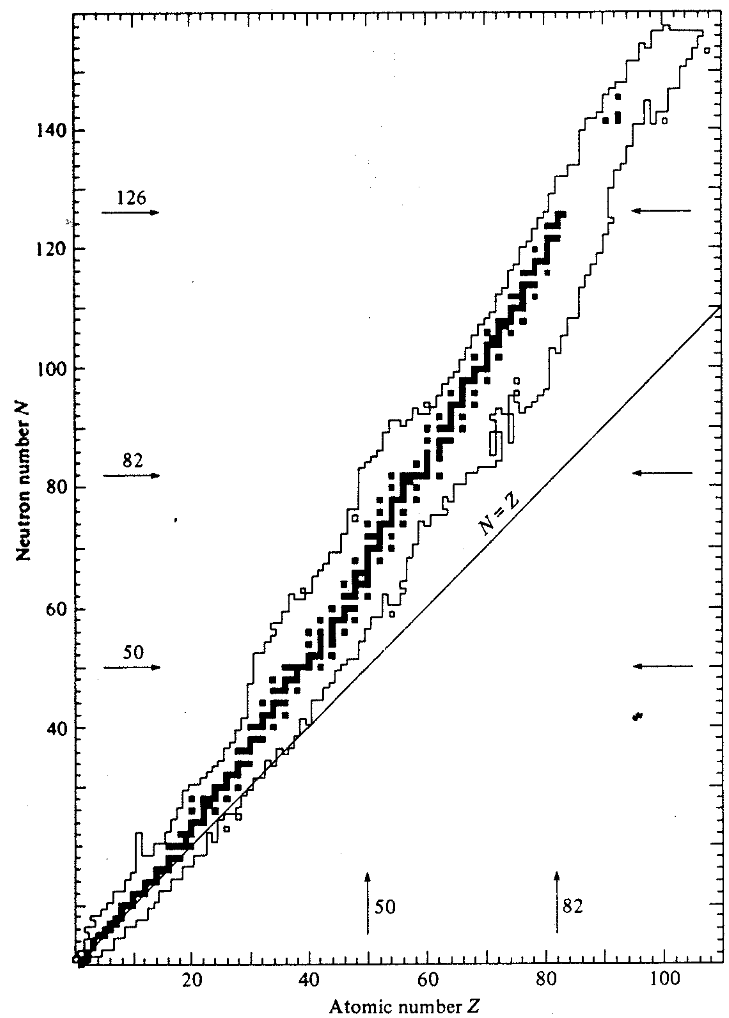
The peak in the binding energy curve makes possible other mode of decay for a heavy nucleus which is stable against β-decay. Since there is gradual decrease of (B/A) with A for the heavier nuclei, it may b energetically advantageous for a heavy nucleus to split into two smalle nuclei, which together have a greater net binding energy. The most commo

4.5 The atomic masses of atoms with $A=64$ relative to the atomic mass of $^{64}_{28}$Ni. Open circles ○ are odd–odd nuclei, filled circles ● are even–even nucle The theoretical even–even and odd–odd parabolas are drawn using the parameters of equation (4.5). Note the odd–odd nucleus $^{64}_{29}$Cu, which can β^--decay to $^{64}_{30}$Zn or β^+-decay to $^{64}_{28}$Ni, both of which are stable, naturally occurring, isotopes. These decays are discussed in detail in Chapter 12.



such process is the emission of an α-particle. As Table 4.2 shows, $^4_2$He has the comparatively large binding energy of 28.3 MeV. The condition for α-emission to be possible from a nucleus (A, Z) to give a nucleus $(A-4, Z-2)$

4.6 The β-stability valley. Filled squares denote the stable nuclei and long-lived nuclei occurring in nature. Neighbouring nuclei are unstable. Those for which data on masses and mean lives are known fill the area bounded by the lines. For the most part these unstable nuclei have been made artificially. (Data taken from *Chart of the Nuclides* (1977), Schenectady: General Electric Company.)

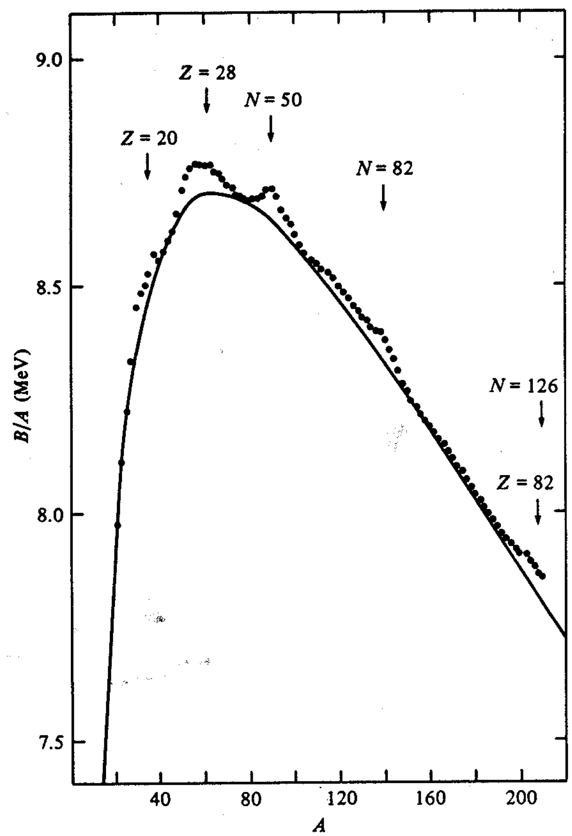


is

$$B(A,Z) < B(A-4, Z-2) + 28.3\ \text{MeV}. \qquad (4.15)$$

For (A, Z) on the line of β-stability, this condition is always satisfied fo sufficiently large A, $A \gtrsim 165$, and all such nuclei are, in principle, able to emi α-particles. However, we shall see in Chapter 6, where the physica mechanism of α-decay is analysed, that decay rates are so slow that the stable nuclei can also be regarded as α-stable up to $^{209}_{83}$Bi. Beyond, onl some isotopes of Th and U are sufficiently long-lived to have survived

4.7 The binding energy per nucleon of β-stable (odd-A) nuclei. Note the displaced origin. The smooth curve is from the semi-empirical mass formula with Z related to A by equation (4.14). Experimental values for odd-A nuclei are shown for comparison; the main deviations ($<1\%$) are due to 'shell' effects not included in our formula.

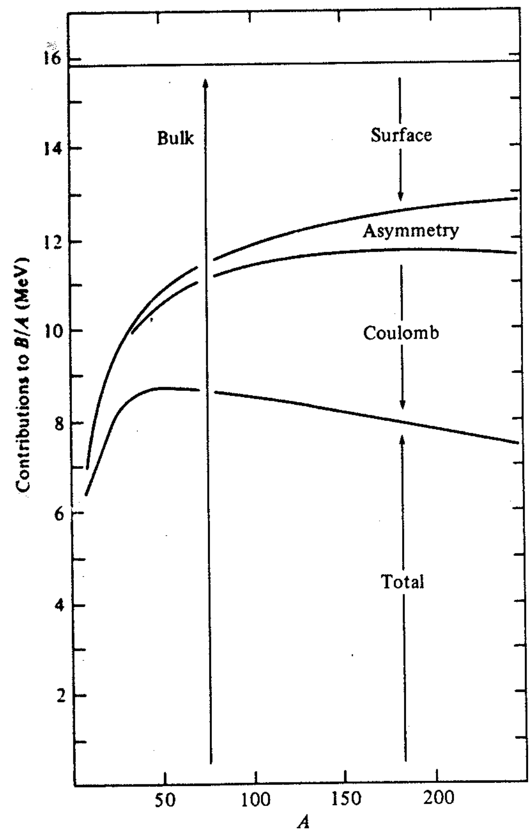


Earth since its formation; other unstable heavy elements are produced either from the decay of these, or artificially.

Another energetically-favourable process which is possible when A is large is the splitting of a nucleus into two more nearly equal parts. This is called *fission*. The energetics of fission may be explored using the semi-empirical mass formula, and in Chapter 6 we shall investigate the rate of spontaneous fission processes.

Beyond the heavy elements of the actinide group, α-decay and fission bring the Periodic Table to an end.

4.8 The contributions to B/A. Note that the surface, asymmetry and Coulomb terms all subtract from the bulk term.



4.9 Nuclear binding and the nucleon–nucleon potential

To what extent do the nuclear properties discussed in this chapter follow from the nucleon–nucleon potential introduced in Chapter 3? Much theoretical effort has been expended on this question. In a nucleus containing three or more nucleons, the nuclear potential energy need not be the simple sum of two-body potentials over all pairs of nucleons: since the nucleons are composite particles, there may well be additional interactions.

Even if the possibility of additional interactions is not considered, the computations are not easy but it appears that the two-body potentials are the dominant contribution to the nuclear potential energy. For nuclear matter the Paris potential gives a value of 16 MeV/nucleon for the binding energy per nucleon, in good agreement with values found for the parameter a in the semi-empirical mass formula (4.5). However, the calculated density of nuclear matter is somewhat too high. The Paris potential gives 0.94 fm rather than the empirical 1.1 fm for the parameter in (4.2).

Similar semi-quantitative agreement is found when the two-nucleon potential is applied to particular light nuclei. For example, the binding energy of ${}^3_1H$ is calculated to be 7.38 MeV, and the experimental value (Table 4.2) is 8.48 MeV.

Problems

4.1 A relativistic electron whose rest mass can be neglected has energy E. It scatters elastically from a particle of mass M at rest and after the collision has turned through an angle θ and has energy E'.

(a) Show that the total energy of the struck particle after the collision is

$E_M = E - E' + Mc^2$.

(b) Show that its momentum is

$P_M = [E^2 + E'^2 - 2EE' \cos\theta]^{\frac{1}{2}}/c$.

(c) Hence (using $E_M^2 = P_M^2c^2 + M^2c^4$) show that the fraction of energy lost by the electron is

$$\frac{E-E'}{E} = \frac{1}{1 + Mc^2/[E(1-\cos\theta)]}.$$

For $E \sim$ a few hundred MeV, show that this is small if the struck particle is a heavy nucleus, and is large (except for $\theta \approx 0$) if the struck particle is an electron.

4.2 In quantum mechanics, the differential cross-section for the elastic scattering of a relativistic electron with energy $E \gg m_ec^2$ from a fixed electrostatic potential $\phi_c(r)$ is given in Born approximation, and

neglecting the effects of electron spin, by

$$\frac{d\sigma}{d\Omega} = \left(\frac{E}{2\pi}\right)^2 \left(\frac{1}{\hbar c}\right)^4 \left(e\int \phi_c(r) e^{i\mathbf{q}\cdot\mathbf{r}}\, d^3\mathbf{r}\right)^2$$

where $\mathbf{q}$ is the difference between the final and the initial wave vectors of the electron.

(a) Show that $q = |\mathbf{q}| = (2E/\hbar c)\sin(\theta/2)$, where θ is the scattering angle.

(b) Poisson's equation relates the potential $\phi_c(r)$ to the charge density $e\rho_{ch}(r)$ by $\nabla^2\phi_c = -e\rho_{ch}/\varepsilon_0$. Noting $\nabla^2 e^{i\mathbf{q}\cdot\mathbf{r}} = -q^2 e^{i\mathbf{q}\cdot\mathbf{r}}$, and integrating by parts, show that

$$\frac{d\sigma}{d\Omega} = \left(\frac{E}{2\pi}\right)^2 \left(\frac{1}{\hbar c}\right)^4 \frac{1}{q^4} \left(\frac{e^2}{\varepsilon_0}\int \rho_{ch}(r) e^{i\mathbf{q}\cdot\mathbf{r}}\, d^3\mathbf{r}\right)^2.$$

For light nuclei (for which the Born approximation has a greater validity) a measured cross-section can be used to infer the Fourier transform of the charge distribution, as this example indicates.

4.3 Show that the characteristic velocity v of a lepton of mass m bound in an atomic orbit is given by $v/c \approx \hbar/amc = \frac{1}{137}$, where $a = (4\pi\varepsilon_0)\hbar^2/me^2$ is the appropriate Bohr radius for that lepton. Hence show that the muon mean life is long compared with the characteristic timescale a/v for its motion in an atomic orbit.

4.4 The ground state wave-function of a lepton of mass m in a Coulomb potential $-Ze^2/(4\pi\varepsilon_0 r)$ is

$$\psi(r) = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a}\right)^{\frac{3}{2}} e^{-Zr/a}$$

where $a = (4\pi\varepsilon_0)\hbar^2/me^2$, and the corresponding binding energy E is $Z^2\hbar^2/2ma^2$.

The finite size of the nucleus modifies the Coulomb energy for $r < R$, the nuclear radius, by adding a term of the approximate form

$$V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 R}\left[\frac{3}{2} - \frac{r^2}{2R^2} - \frac{R}{r}\right].$$

(a) Show that the volume integral of this potential is

$$\int V(r)\, d^3\mathbf{r} = \frac{Ze^2R^2}{10\varepsilon_0}.$$

(b) Show that the first-order correction to the binding energy due to this term, $\Delta E = \int \psi^*(r)V(r)\psi(r)\, d^3\mathbf{r}$, is

$$\Delta E \approx \frac{e^2}{10\pi\varepsilon_0}\frac{Z^4R^2}{a^3}.$$

(Note that the lepton wave-function can be taken to be constant over nuclear dimensions.)

(c) For the nucleus $^{66}_{30}$Zn show that

$$\frac{\Delta E}{E} \approx 5\times 10^{-6} \quad \text{for electrons.}$$

$$\frac{\Delta E}{E} \approx 0.2 \quad \text{for muons.}$$

4.5 Using Table 4.2 show that $^{8}_{4}$Be can decay to two α-particles with an energy release of 0.1 MeV, but that $^{12}_{6}$C cannot decay to three α-particles. Show that the energy released (including the energy of the photon) in the reaction $^{2}_{1}\text{H}+^{4}_{2}\text{He}\rightarrow ^{6}_{3}\text{Li}+\gamma$ is 1.5 MeV.

4.6 Consider nuclei with small nucleon number A and such that $Z=N=A/2$. Neglecting the pairing term, show that the semi-empirical mass formula then gives the binding energy per nucleon

$B/A = a - bA^{-\frac{1}{3}} - (d/4)A^{\frac{2}{3}}$.

Show that this expression reaches a maximum for $Z=A/2=26$ (iron).

4.7 Using the formula (4.14) calculate Z for $A=100$ and $A=200$. Compare your results with Fig. 4.6 and comment.

4.8 The carbon isotope $^{14}_{6}$C is produced in nuclear reactions of cosmic rays in the atmosphere. It is β-unstable,

$^{14}_{6}\text{C}\rightarrow ^{14}_{7}\text{N}+e^{-}+\bar{\nu}_e+0.156$ MeV,

with a mean life of 8270 years.

It is found that a gram of carbon, newly extracted from the atmosphere, gives on average 15.3 such radio-active decays per minute. What is the proportion of ^{14}C isotope in the carbon?

4.9 On the basis of the different properties of nuclei with even A and with odd A, explain why there are about 300 β-stable nuclei with masses up to that of $^{209}_{83}$Bi. What is the average number of isotopes per element?

5

Ground state properties of nuclei; the shell model

5.1 Nuclear potential wells

In the last chapter, we set out a semi-empirical theory for the binding energy of an atomic nucleus, and quantum-mechanical considerations came in only rather indirectly. Experimental atomic masses show deviations from the smooth curve given by the semi-empirical mass formula, deviations which we said were of quantum-mechanical origin. Since a nucleus in its ground state is a quantum system of finite size, it has angular momentum **J**, with quantum number j which is some integral multiple of $\frac{1}{2}$. If $j\neq 0$ the nucleus will have a magnetic dipole moment, and it may have an electric quadrupole moment as well.

The nuclear angular momentum and magnetic moment manifest themselves most immediately in atomic spectroscopy, where the interaction between the nuclear magnetic moment and the electron magnetic moments gives rise to the hyperfine structures of the electronic energy levels. In favourable cases both j and the magnetic moment may be deduced from this hyperfine splitting.

The observed values of nuclear angular momenta give strong support to the validity of a simple quantum-mechanical model of the nucleus: the nuclear shell model. In this model, each neutron moves independently in a common potential well that is the spherical average of the nuclear potential produced by all the other nucleons, and each proton moves independently in a common potential well that is the spherical average of the nuclear