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DOE FUNDAMENTALS HANDBOOK

NUCLEAR PHYSICS AND REACTOR THEORY

Volume 2 of 2

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ABSTRACT

The *Nuclear Physics and Reactor Theory* Handbook was developed to assist nuclear facility operating contractors in providing operators, maintenance personnel, and the technical staff with the necessary fundamentals training to ensure a basic understanding of nuclear physics and reactor theory. The handbook includes information on atomic and nuclear physics; neutron characteristics; reactor theory and nuclear parameters; and the theory of reactor operation. This information will provide personnel with a foundation for understanding the scientific principles that are associated with various DOE nuclear facility operations and maintenance.

Key Words: Training Material, Atomic Physics, The Chart of the Nuclides, Radioactivity, Radioactive Decay, Neutron Interaction, Fission, Reactor Theory, Neutron Characteristics, Neutron Life Cycle, Reactor Kinetics

FOREWORD

The *Department of Energy (DOE) Fundamentals Handbooks* consist of ten academic subjects, which include Mathematics; Classical Physics; Thermodynamics, Heat Transfer, and Fluid Flow; Instrumentation and Control; Electrical Science; Material Science; Mechanical Science; Chemistry; Engineering Symbology, Prints, and Drawings; and Nuclear Physics and Reactor Theory. The handbooks are provided as an aid to DOE nuclear facility contractors.

These handbooks were first published as Reactor Operator Fundamentals Manuals in 1985 for use by DOE category A reactors. The subject areas, subject matter content, and level of detail of the Reactor Operator Fundamentals Manuals were determined from several sources. DOE Category A reactor training managers determined which materials should be included, and served as a primary reference in the initial development phase. Training guidelines from the commercial nuclear power industry, results of job and task analyses, and independent input from contractors and operations-oriented personnel were all considered and included to some degree in developing the text material and learning objectives.

The *DOE Fundamentals Handbooks* represent the needs of various DOE nuclear facilities' fundamental training requirements. To increase their applicability to nonreactor nuclear facilities, the Reactor Operator Fundamentals Manual learning objectives were distributed to the Nuclear Facility Training Coordination Program Steering Committee for review and comment. To update their reactor-specific content, DOE Category A reactor training managers also reviewed and commented on the content. On the basis of feedback from these sources, information that applied to two or more DOE nuclear facilities was considered generic and was included. The final draft of each of the handbooks was then reviewed by these two groups. This approach has resulted in revised modular handbooks that contain sufficient detail such that each facility may adjust the content to fit their specific needs.

Each handbook contains an abstract, a foreword, an overview, learning objectives, and text material, and is divided into modules so that content and order may be modified by individual DOE contractors to suit their specific training needs. Each handbook is supported by a separate examination bank with an answer key.

The *DOE Fundamentals Handbooks* have been prepared for the Assistant Secretary for Nuclear Energy, Office of Nuclear Safety Policy and Standards, by the DOE Training Coordination Program. This program is managed by EG&G Idaho, Inc.

OVERVIEW

The *Department of Energy Fundamentals Handbook* entitled *Nuclear Physics and Reactor Theory* was prepared as an information resource for personnel who are responsible for the operation of the Department's nuclear facilities. Almost all processes that take place in a nuclear facility involves the transfer of some type of energy. A basic understanding of nuclear physics and reactor theory is necessary for DOE nuclear facility operators, maintenance personnel, and the technical staff to safely operate and maintain the facility and facility support systems. The information in this handbook is presented to provide a foundation for applying engineering concepts to the job. This knowledge will help personnel understand the impact that their actions may have on the safe and reliable operation of facility components and systems.

The *Nuclear Physics and Reactor Theory* handbook consists of four modules that are contained in two volumes. The following is a brief description of the information presented in each module of the handbook.

Volume 1 of 2

Module 1 - Atomic and Nuclear Physics

Introduces concepts of atomic physics including the atomic nature of matter, the chart of the nuclides, radioactivity and radioactive decay, neutron interactions and fission, and the interaction of radiation with matter.

Module 2 - Reactor Theory (Nuclear Parameters)

Provides information on reactor theory and neutron characteristics. Includes topics such as neutron sources, neutron flux, neutron cross sections, reaction rates, neutron moderation, and prompt and delayed neutrons.

OVERVIEW (Cont.)

Volume 2 of 2

Module 3 - Reactor Theory (Nuclear Parameters)

Explains the nuclear parameters associated with reactor theory. Topics include the neutron life cycle, reactivity and reactivity coefficients, neutron poisons, and control rods.

Module 4 - Reactor Theory (Reactor Operations)

Introduces the reactor operations aspect of reactor theory. Topics include subcritical multiplication, reactor kinetics, and reactor operation.

The information contained in this handbook is not all-encompassing. An attempt to present the entire subject of nuclear physics and reactor theory would be impractical. However, the *Nuclear Physics and Reactor Theory* handbook presents enough information to provide the reader with the fundamental knowledge necessary to understand the advanced theoretical concepts presented in other subject areas, and to understand basic system and equipment operation.

Department of Energy
Fundamentals Handbook

NUCLEAR PHYSICS AND REACTOR THEORY
Module 3
Reactor Theory (Nuclear Parameters)

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TERMINAL OBJECTIVE

1.0 Using appropriate references, **DESCRIBE** the neutron life cycle discussed in this module.

ENABLING OBJECTIVES

1.1 **DEFINE** the following terms:

a. Infinite multiplication factor, k_{∞}
b. Effective multiplication factor, k_{eff}
c. Subcritical
d. Critical
e. Supercritical

1.2 **DEFINE** each term in the six factor formula using the ratio of the number of neutrons present at different points in the neutron life cycle.

1.3 Given the macroscopic cross sections for various materials, **CALCULATE** the thermal utilization factor.

1.4 Given microscopic cross sections for absorption and fission, atom density, and ν, **CALCULATE** the reproduction factor.

1.5 Given the numbers of neutrons present at the start of a generation and values for each factor in the six factor formula, **CALCULATE** the number of neutrons that will be present at any point in the life cycle.

1.6 **LIST** physical changes in the reactor core that will have an effect on the thermal utilization factor, reproduction factor, or resonance escape probability.

1.7 **EXPLAIN** the effect that temperature changes will have on the following factors:

a. Thermal utilization factor
b. Resonance escape probability
c. Fast non-leakage probability
d. Thermal non-leakage probability

1.8 Given the number of neutrons in a reactor core and the effective multiplication factor, **CALCULATE** the number of neutrons present after any number of generations.

ENABLING OBJECTIVES (Cont.)

1.9 **DEFINE** the term reactivity.

1.10 **CONVERT** between reactivity and the associated value of k_{eff}.

1.11 **CONVERT** measures of reactivity between the following units:

a. $\Delta k/k$
b. $\%\Delta k/k$
c. $10^{-4}\ \Delta k/k$
d. Percent millirho (pcm)

1.12 **EXPLAIN** the relationship between reactivity coefficients and reactivity defects.

TERMINAL OBJECTIVE

2.0 From memory, **EXPLAIN** how reactivity varies with the thermodynamic properties of the moderator and the fuel.

ENABLING OBJECTIVES

2.1 **EXPLAIN** the conditions of over moderation and under moderation.

2.2 **EXPLAIN** why many reactors are designed to be operated in an under moderated condition.

2.3 **STATE** the effect that a change in moderator temperature will have on the moderator to fuel ratio.

2.4 **DEFINE** the temperature coefficient of reactivity.

2.5 **EXPLAIN** why a negative temperature coefficient of reactivity is desirable.

2.6 **EXPLAIN** why the fuel temperature coefficient is more effective than the moderator temperature coefficient in terminating a rapid power rise.

2.7 **EXPLAIN** the concept of Doppler broadening of resonance absorption peaks.

2.8 **LIST** two nuclides that are present in some types of reactor fuel assemblies that have significant resonance absorption peaks.

2.9 **DEFINE** the pressure coefficient of reactivity.

2.10 **EXPLAIN** why the pressure coefficient of reactivity is usually negligible in a reactor cooled and moderated by a subcooled liquid.

2.11 **DEFINE** the void coefficient of reactivity.

2.12 **IDENTIFY** the moderator conditions under which the void coefficient of reactivity becomes significant.

TERMINAL OBJECTIVE

3.0 Without references, **DESCRIBE** the use of neutron poisons.

ENABLING OBJECTIVES

3.1 **DEFINE** the following terms:

a. Burnable poison
b. Non-burnable poison
c. Chemical shim

3.2 **EXPLAIN** the use of burnable neutron poisons in a reactor core.

3.3 **LIST** the advantages and disadvantages of chemical shim over fixed burnable poisons.

3.4 **STATE** two reasons why fixed non-burnable neutron poisons are used in reactor cores.

3.5 **STATE** an example of a material used as a fixed non-burnable neutron poison.

TERMINAL OBJECTIVE

4.0 Without references, **DESCRIBE** the effects of fission product poisons on a reactor.

ENABLING OBJECTIVES

4.1 **LIST** two methods of production and two methods of removal for xenon-135 during reactor operation.

4.2 **STATE** the equation for equilibrium xenon-135 concentration.

4.3 **DESCRIBE** how equilibrium xenon-135 concentration varies with reactor power level.

4.4 **DESCRIBE** the causes and effects of a xenon oscillation.

4.5 **DESCRIBE** how xenon-135 concentration changes following a reactor shutdown from steady-state conditions.

4.6 **EXPLAIN** the effect that pre-shutdown power levels have on the xenon-135 concentration after shutdown.

4.7 **STATE** the approximate time following a reactor shutdown at which the reactor can be considered "xenon free."

4.8 **EXPLAIN** what is meant by the following terms:

a. Xenon precluded startup
b. Xenon dead time

4.9 **DESCRIBE** how xenon-135 concentration changes following an increase or a decrease in the power level of a reactor.

4.10 **DESCRIBE** how samarium-149 is produced and removed from the reactor core during reactor operation.

4.11 **STATE** the equation for equilibrium samarium-149 concentration.

4.12 **DESCRIBE** how equilibrium samarium-149 concentration varies with reactor power level.

ENABLING OBJECTIVES (Cont.)

4.13 **DESCRIBE** how samarium-149 concentration changes following a reactor shutdown from steady-state conditions.

4.14 **DESCRIBE** how samarium-149 concentration changes following a reactor startup.

4.15 **STATE** the conditions under which helium-3 will have a significant effect on the reactivity of a reactor.

TERMINAL OBJECTIVE

5.0 Without references, **DESCRIBE** how control rods affect the reactor core.

ENABLING OBJECTIVES

5.1 **DESCRIBE** the difference between a "grey" neutron absorbing material and a "black" neutron absorbing material.

5.2 **EXPLAIN** why a "grey" neutron absorbing material may be preferable to a "black" neutron absorbing material for use in control rods.

5.3 **EXPLAIN** why resonance absorbers are sometimes preferred over thermal absorbers as a control rod material.

5.4 **DEFINE** the following terms:

a. Integral control rod worth
b. Differential control rod worth

5.5 **DESCRIBE** the shape of a typical differential control rod worth curve and explain the reason for the shape.

5.6 **DESCRIBE** the shape of a typical integral control rod worth curve and explain the reason for the shape.

5.7 Given an integral or differential control rod worth curve, **CALCULATE** the reactivity change due to a control rod movement between two positions.

5.8 Given differential control rod worth data, **PLOT** differential and integral control rod worth curves.

NEUTRON LIFE CYCLE

Some number of the fast neutrons produced by fission in one generation will eventually cause fission in the next generation. The series of steps that fission neutrons go through as they slow to thermal energies and are absorbed in the reactor is referred to as the neutron life cycle. The neutron life cycle is markedly different between fast reactors and thermal reactors. This chapter presents the neutron life cycle for thermal reactors.

EO 1.1 DEFINE the following terms:

a. Infinite multiplication factor, k_∞
b. Effective multiplication factor, k_{eff}
c. Subcritical
d. Critical
e. Supercritical

EO 1.2 DEFINE each term in the six factor formula using the ratio of the number of neutrons present at different points in the neutron life cycle.

EO 1.3 Given the macroscopic cross sections for various materials, CALCULATE the thermal utilization factor.

EO 1.4 Given microscopic cross sections for absorption and fission, atom density, and ν, CALCULATE the reproduction factor.

EO 1.5 Given the numbers of neutrons present at the start of a generation and values for each factor in the six factor formula, CALCULATE the number of neutrons that will be present at any point in the life cycle.

EO 1.6 LIST physical changes in the reactor core that will have an effect on the thermal utilization factor, reproduction factor, or resonance escape probability.

EO 1.7 EXPLAIN the effect that temperature changes will have on the following factors:

a. Thermal utilization factor
b. Resonance escape probability
c. Fast non-leakage probability
d. Thermal non-leakage probability

Infinite Multiplication Factor, k_∞

Not all of the neutrons produced by fission will have the opportunity to cause new fissions because some neutrons will be absorbed by non-fissionable material. Some will be absorbed parasitically in fissionable material and will not cause fission, and others will leak out of the reactor. For the maintenance of a self-sustaining chain reaction, however, it is not necessary that every neutron produced in fission initiate another fission. The minimum condition is for each nucleus undergoing fission to produce, on the average, at least one neutron that causes fission of another nucleus. This condition is conveniently expressed in terms of a multiplication factor.

The number of neutrons absorbed or leaking out of the reactor will determine the value of this multiplication factor, and will also determine whether a new generation of neutrons is larger, smaller, or the same size as the preceding generation. Any reactor of a finite size will have neutrons leak out of it. Generally, the larger the reactor, the lower the fraction of neutron leakage. For simplicity, we will first consider a reactor that is infinitely large, and therefore has no neutron leakage. A measure of the increase or decrease in neutron flux in an infinite reactor is the infinite multiplication factor, k_∞. The *infinite multiplication factor* is the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption in the preceding generation. This can be expressed mathematically as shown below.

$$k_\infty = \frac{\text{neutron production from fission in one generation}}{\text{neutron absorption in the preceding generation}}$$

Four Factor Formula

A group of fast neutrons produced by fission can enter into several reactions. Some of these reactions reduce the size of the neutron group while other reactions allow the group to increase in size or produce a second generation. There are four factors that are completely independent of the size and shape of the reactor that give the inherent multiplication ability of the fuel and moderator materials without regard to leakage. This *four factor formula* accurately represents the infinite multiplication factor as shown in the equation below.

$$k_\infty = \varepsilon\ p\ f\ \eta$$

where:

ε = fast fission factor
p = resonance escape probability
f = thermal utilization factor
η = reproduction factor

Each of these four factors, which are explained in the following subsections, represents a process that adds to or subtracts from the initial neutron group produced in a generation by fission.

Fast Fission Factor, (ε)

The first process that the neutrons of one generation may undergo is fast fission. Fast fission is fission caused by neutrons that are in the fast energy range. Fast fission results in the net increase in the fast neutron population of the reactor core. The cross section for fast fission in uranium-235 or uranium-238 is small; therefore, only a small number of fast neutrons cause fission. The fast neutron population in one generation is therefore increased by a factor called the fast fission factor. The *fast fission factor* (ε) is defined as the ratio of the net number of fast neutrons produced by all fissions to the number of fast neutrons produced by thermal fissions. The mathematical expression of this ratio is shown below.

$$\varepsilon = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}}$$

In order for a neutron to be absorbed by a fuel nucleus as a fast neutron, it must pass close enough to a fuel nucleus while it is a fast neutron. The value of ε will be affected by the arrangement and concentrations of the fuel and the moderator. The value of ε is essentially 1.00 for a homogenous reactor where the fuel atoms are surrounded by moderator atoms. However, in a heterogeneous reactor, all the fuel atoms are packed closely together in elements such as pins, rods, or pellets. Neutrons emitted from the fission of one fuel atom have a very good chance of passing near another fuel atom before slowing down significantly. The arrangement of the core elements results in a value of about 1.03 for ε in most heterogeneous reactors. The value of ε is not significantly affected by variables such as temperature, pressure, enrichment, or neutron poison concentrations. Poisons are non-fuel materials that easily absorb neutrons and will be discussed in more detail later.

Resonance Escape Probability, (p)

After increasing in number as a result of some fast fissions, the neutrons continue to diffuse through the reactor. As the neutrons move they collide with nuclei of fuel and non-fuel material and moderator in the reactor losing part of their energy in each collision and slowing down. While they are slowing down through the resonance region of uranium-238, which extends from about 6 eV to 200 eV, there is a chance that some neutrons will be captured. The probability that a neutron will not be absorbed by a resonance peak is called the resonance escape probability. The *resonance escape probability* (p) is defined as the ratio of the number of neutrons that reach thermal energies to the number of fast neutrons that start to slow down. This ratio is shown below.

$$p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}$$

The value of the resonance escape probability is determined largely by the fuel-moderator arrangement and the amount of enrichment of uranium-235 (if any is used). To undergo resonance absorption, a neutron must pass close enough to a uranium-238 nucleus to be absorbed while slowing down. In a homogeneous reactor the neutron does its slowing down in the region of the fuel nuclei, and this condition is easily met. This means that a neutron has a high probability of being absorbed by uranium-238 while slowing down; therefore, its escape probability is lower. In a heterogeneous reactor, however, the neutron slows down in the moderator where there are no atoms of uranium-238 present. Therefore, it has a low probability of undergoing resonance absorption, and its escape probability is higher.

The value of the resonance escape probability is not significantly affected by pressure or poison concentration. In water moderated, low uranium-235 enrichment reactors, raising the temperature of the fuel will raise the resonance absorption in uranium-238 due to the doppler effect (an apparent broadening of the normally narrow resonance peaks due to thermal motion of nuclei). The increase in resonance absorption lowers the resonance escape probability, and the fuel temperature coefficient for resonance escape is negative (explained in detail later). The temperature coefficient of resonance escape probability for the moderator temperature is also negative. As water temperature increases, water density decreases. The decrease in water density allows more resonance energy neutrons to enter the fuel and be absorbed. The value of the resonance escape probability is always slightly less than one (normally 0.95 to 0.99).

The product of the fast fission factor and the resonance escape probability (ε p) is the ratio of the number of fast neutrons that survive slowing down (thermalization) compared to the number of fast neutrons originally starting the generation.

Thermal Utilization Factor, (f)

Once thermalized, the neutrons continue to diffuse throughout the reactor and are subject to absorption by other materials in the reactor as well as the fuel. The thermal utilization factor describes how effectively thermal neutrons are absorbed by the fuel, or how well they are utilized within the reactor. The *thermal utilization factor* (f) is defined as the ratio of the number of thermal neutrons absorbed in the fuel to the number of thermal neutrons absorbed in any reactor material. This ratio is shown below.

$$f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in all reactor materials}}$$

The thermal utilization factor will always be less than one because some of the thermal neutrons absorbed within the reactor will be absorbed by atoms of non-fuel materials.

An equation can be developed for the thermal utilization factor in terms of reaction rates as follows.

$$f = \frac{\text{rate of absorption of thermal neutrons by the fuel}}{\text{rate of absorption of thermal neutrons by all reactor materials}}$$

$$f = \frac{\Sigma_a^U \phi^U V^U}{\Sigma_a^U \phi^U V^U + \Sigma_a^m \phi^m V^m + \Sigma_a^p \phi^p V^p}$$

The superscripts U, m, and p refer to uranium, moderator, and poison, respectively. In a heterogeneous reactor, the flux will be different in the fuel region than in the moderator region due to the high absorption rate by the fuel. Also, the volumes of fuel, moderator, and poisons will be different. Although not shown in the above equation, other non-fuel materials, such as core construction materials, may absorb neutrons in a heterogeneous reactor. These other materials are often lumped together with the superscript designation OS, for "other stuff." To be completely accurate, the above equation for the thermal utilization factor should include all neutron-absorbing reactor materials when dealing with heterogeneous reactors. However, for the purposes of this text, the above equation is satisfactory.

In a homogeneous reactor the neutron flux seen by the fuel, moderator, and poisons will be the same. Also, since they are spread throughout the reactor, they all occupy the same volume. This allows the previous equation to be rewritten as shown below.

$$f = \frac{\Sigma_a^U}{\Sigma_a^U + \Sigma_a^m + \Sigma_a^p} \qquad (3\text{-}1)$$

Equation (3-1) gives an approximation for a heterogeneous reactor if the fuel and moderator are composed of small elements distributed uniformly throughout the reactor.

Since absorption cross sections vary with temperature, it would appear that the thermal utilization factor would vary with a temperature change. But, substitution of the temperature correction formulas (see Module 2) in the above equation will reveal that all terms change by the same amount, and the ratio remains the same. In heterogeneous water-moderated reactors, there is another important factor. When the temperature rises, the water moderator expands, and a significant amount of it will be forced out of the reactor core. This means that N^m, the number of moderator atoms per cm^3, will be reduced, making it less likely for a neutron to be absorbed by a moderator atom. This reduction in N^m results in an increase in thermal utilization as moderator temperature increases because a neutron now has a better chance of hitting a fuel atom. Because of this effect, the temperature coefficient for the thermal utilization factor is positive. The amount of enrichment of uranium-235 and the poison concentration will affect the thermal utilization factor in a similar manner as can be seen from the equation above.

Example:

Calculate the thermal utilization factor for a homogeneous reactor. The macroscopic absorption cross section of the fuel is 0.3020 cm^{-1}, the macroscopic absorption cross section of the moderator is 0.0104 cm^{-1}, and the macroscopic absorption cross section of the poison is 0.0118 cm^{-1}.

Solution:

$$f = \frac{\Sigma_a^U}{\Sigma_a^U + \Sigma_a^m + \Sigma_a^p}$$

$$= \frac{0.3020\ cm^{-1}}{0.3020\ cm^{-1} + 0.0104 cm^{-1} + 0.0118 cm^{-1}}$$

$$= 0.932$$

Reproduction Factor, (η)

Most of the neutrons absorbed in the fuel cause fission, but some do not. The *reproduction factor* (η) is defined as the ratio of the number of fast neurtons produces by thermal fission to the number of themal neutrons absorbed in the fuel. The reproduction factor is shown below.

$$\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}$$

The reproduction factor can also be stated as a ratio of rates as shown below.

$$\eta = \frac{\text{rate of production of fast neutrons by thermal fission}}{\text{rate of absorption of thermal neutrons by the fuel}}$$

The rate of production of fast neutrons by thermal fission can be determined by the product of the fission reaction rate ($\Sigma_f{}^u\phi^u$) and the average number of neutrons produced per fission (ν). The average number of neutrons released in thermal fission of uranium-235 is 2.42. The rate of absorption of thermal neutrons by the fuel is $\Sigma_a{}^u\phi^u$. Substituting these terms into the equation above results in the following equation.

$$\eta = \frac{\Sigma_f^U\ \phi^U\ \nu}{\Sigma_a^U\ \phi^U}$$

Table 1 lists values of ν and η for fission of several different materials by thermal neutrons and fast neutrons.

TABLE 1
Average Number of Neutrons Liberated in Fission

Fissile Nucleus	Thermal Neutrons		Fast Neutrons	
	ν	η	ν	η
Uranium-233	2.49	2.29	2.58	2.40
Uranium-235	2.42	2.07	2.51	2.35
Plutonium-239	2.93	2.15	3.04	2.90

In the case where the fuel contains several fissionable materials, it is necessary to account for each material. In the case of a reactor core containing both uranium-235 and uranium-238, the reproduction factor would be calculated as shown below.

$$\eta = \frac{N^{U-235}\,\sigma_f^{U-235}\,\nu^{U-235}}{N^{U-235}\,\sigma_a^{U-235} + N^{U-238}\,\sigma_a^{U-238}} \qquad (3\text{-}2)$$

Example:

Calculate the reproduction factor for a reactor that uses 10% enriched uranium fuel. The microscopic absorption cross section for uranium-235 is 694 barns. The cross section for uranium-238 is 2.71 barns. The microscopic fission cross section for uranium-235 is 582 barns. The atom density of uranium-235 is 4.83×10^{21} atoms/cm^3. The atom density of uranium-238 is 4.35×10^{22} atoms/cm^3. ν is 2.42.

Solution:

Use Equation (3-2) to calculate the reproduction factor.

$$\eta = \frac{N^{U-235}\,\sigma_f^{U-235}\,\nu^{U-235}}{N^{U-235}\,\sigma_a^{U-235} + N^{U-238}\,\sigma_a^{U-238}}$$

$$= \frac{\left(4.83 \times 10^{21}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(582 \times 10^{-24}\ \text{cm}^2\right)\left(2.42\right)}{\left(4.83 \times 10^{21}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(694 \times 10^{-24}\ \text{cm}^2\right) + \left(4.35 \times 10^{22}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(2.71 \times 10^{-24}\ \text{cm}^2\right)}$$

$$= 1.96$$

As temperature varies, each absorption and fission microscopic cross section varies according to the 1/v relationship (see Module 2). Since both the numerator and the denominator change equally, the net change in η is zero. Therefore, η changes only as uranium-235 enrichment changes. η increases with enrichment because there is less uranium-238 in the reactor making it more likely that a neutron absorbed in the fuel will be absorbed by uranium-235 and cause fission.

To determine the reproduction factor for a single nuclide rather than for a mixture, the calculation may be further simplified to the one shown below.

$$\eta = \frac{\sigma_f \nu}{\sigma_a}$$

Effective Multiplication Factor

The infinite multiplication factor can fully represent only a reactor that is infinitely large, because it assumes that no neutrons leak out of the reactor. To completely describe the neutron life cycle in a real, finite reactor, it is necessary to account for neutrons that leak out. The multiplication factor that takes leakage into account is the *effective multiplication factor* (k_{eff}), which is defined as the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption and leakage in the preceding generation.

The effective multiplication factor may be expressed mathematically as shown below.

$$k_{eff} = \frac{\text{neutron production from fission in one generation}}{\text{neutron absorption in the preceding generation} + \text{neutron leakage in the preceding generation}}$$

So, the value of k_{eff} for a self-sustaining chain reaction of fissions, where the neutron population is neither increasing nor decreasing, is one. The condition where the neutron chain reaction is self-sustaining and the neutron population is neither increasing nor decreasing is referred to as the *critical* condition and can be expressed by the simple equation $k_{eff} = 1$.

If the neutron production is greater than the absorption and leakage, the reactor is called supercritical. In a *supercritical* reactor, k_{eff} is greater than one, and the neutron flux increases each generation. If, on the other hand, the neutron production is less than the absorption and leakage, the reactor is called subcritical. In a *subcritical* reactor, k_{eff} is less than one, and the flux decreases each generation.

When the multiplication factor of a reactor is not equal to exactly one, the neutron flux will change and cause a change in the power level. Therefore, it is essential to know more about how this factor depends upon the contents and construction of the reactor. The balance between production of neutrons and their absorption in the core and leakage out of the core determines the value of the multiplication factor. If the leakage is small enough to be neglected, the multiplication factor depends upon only the balance between production and absorption, and is called the infinite multiplication factor (k_∞) since an infinitely large core can have no leakage. When the leakage is included, the factor is called the effective multiplication factor (k_{eff}).

The effective multiplication factor (k_{eff}) for a finite reactor may be expressed mathematically in terms of the infinite multiplication factor and two additional factors which account for neutron leakage as shown below.

$$k_{eff} = k_\infty \mathcal{L}_f \mathcal{L}_t$$

Fast Non-Leakage Probability ($\mathcal{L}_f$)

In a realistic reactor of finite size, some of the fast neutrons leak out of the boundaries of the reactor core before they begin the slowing down process. The *fast non-leakage probability* ($\mathcal{L}_f$) is defined as the ratio of the number of fast neutrons that do not leak from the reactor core to the number of fast neutrons produced by all fissions. This ratio is stated as follows.

$$\mathcal{L}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

Thermal Non-Leakage Probability ($\mathcal{L}_t$)

Neutrons can also leak out of a finite reactor core after they reach thermal energies. The *thermal non-leakage probability* ($\mathcal{L}_t$) is defined as the ratio of the number of thermal neutrons that do not leak from the reactor core to the number of neutrons that reach thermal energies. The thermal non-leakage probability is represented by the following.

$$\mathcal{L}_t = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

The fast non-leakage probability ($\mathcal{L}_f$) and the thermal non-leakage probability ($\mathcal{L}_t$) may be combined into one term that gives the fraction of all neutrons that do not leak out of the reactor core. This term is called the *total non-leakage probability* and is given the symbol $\mathcal{L}_T$, where $\mathcal{L}_T = \mathcal{L}_f\mathcal{L}_t$. $\mathcal{L}_f$ and $\mathcal{L}_t$ are both effected by a change in coolant temperature in a heterogeneous water-cooled, water-moderated reactor. As coolant temperature rises, the coolant expands. The density of the moderator is lower; therefore, neutrons must travel farther while slowing down. This effect increases the probability of leakage and thus decreases the non-leakage probability. Consequently, the temperature coefficient (defined later) for the non-leakage probabilities is negative, because as temperature rises, $\mathcal{L}_f$ and $\mathcal{L}_t$ decrease.

Six Factor Formula

With the inclusion of these last two factors it is possible to determine the fraction of neutrons that remain after every possible process in a nuclear reactor. The effective multiplication factor (k_{eff}) can then be determined by the product of six terms.

$$k_{eff} = \varepsilon \, \mathcal{L}_f \, p \, \mathcal{L}_t \, f \, \eta \qquad (3\text{-}3)$$

Equation (3-3) is called the *six factor formula*. Using this six factor formula, it is possible to trace the entire neutron life cycle from production by fission to the initiation of subsequent fissions. Figure 1 illustrates a neutron life cycle with nominal values provided for each of the six factors. Refer to Figure 1 for the remainder of the discussion on the neutron life cycle and sample calculations. The generation begins with 1000 neutrons. This initial number is represented by N_o. The first process is fast fission and the population has been increased by the neutrons from this fast fission process. From the definition of the fast fission factor it is possible to calculate its value based on the number of neutrons before and after fast fission occur.

$$\varepsilon = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}}$$

$$= \frac{1040}{1000}$$

$$= 1.04$$

The total number of fast neutrons produced by thermal and fast fission is represented by the quantity $N_o\varepsilon$.

Next, it can be seen that 140 neutrons leak from the core before reaching the thermal energy range. The fast non-leakage probability is calculated from its definition, as shown below.

$$\mathcal{L}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

$$= \frac{1040 - 140}{1040}$$

$$= 0.865$$

The number of neutrons that remain in the core during the slowing down process is represented by the quantity $N_o \, \varepsilon \, \mathcal{L}_f$.

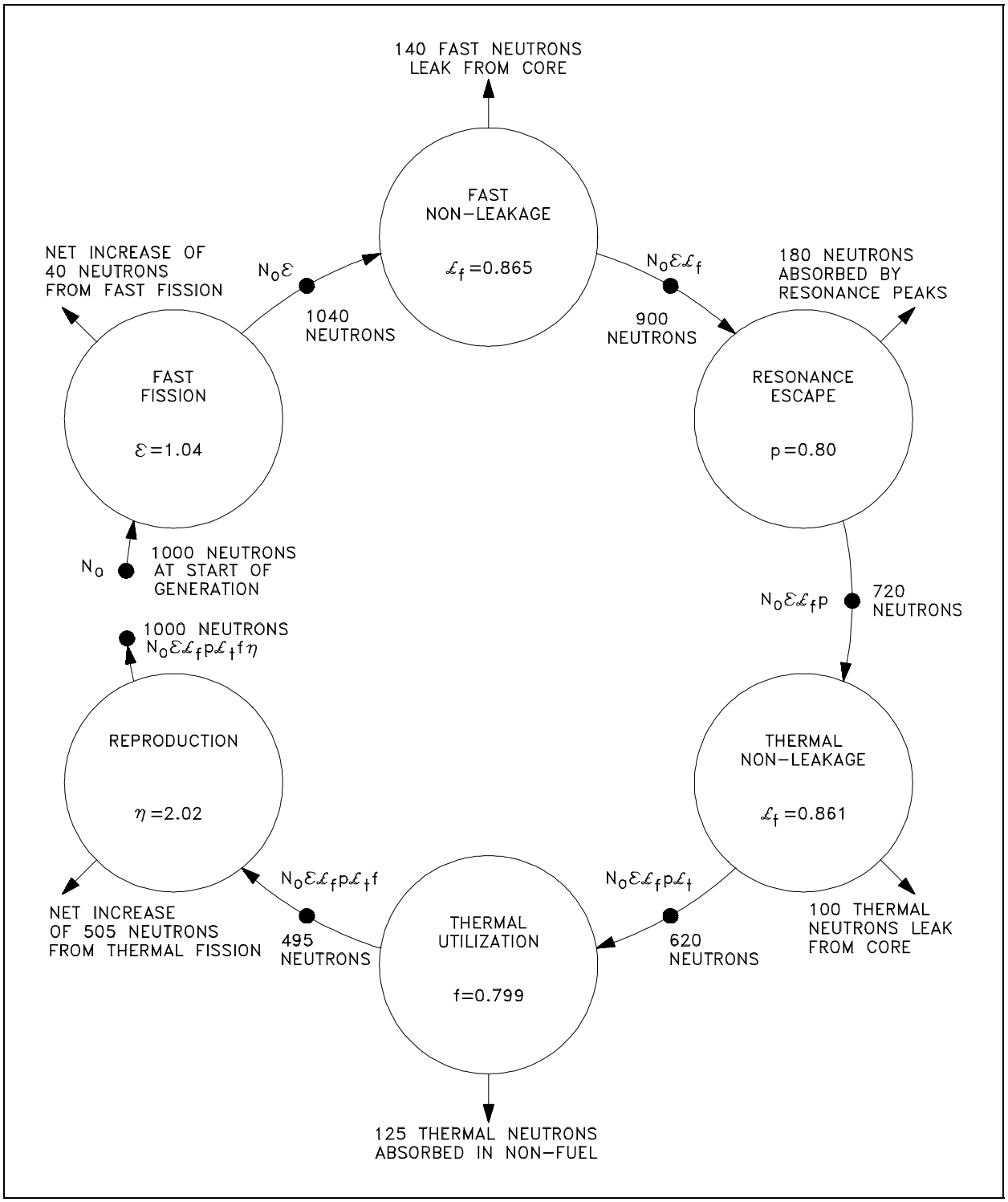

Figure 1 Neutron Life Cycle with k_{eff} = 1

The next step in the analysis is to consider the number of neutrons that are absorbed in the intermediate energy level. The probability of escaping this resonance absorption (p) is stated as follows.

$$p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}$$

$$= \frac{720}{900}$$

$$= 0.80$$

The number of neutrons entering the thermal energy range is now represented by the quantity $N_o \varepsilon \mathcal{L}_f p$.

After reaching thermal energies, 100 neutrons leak from the core. The value for $\mathcal{L}_t$ can be calculated by substitution of the known values in the definition as shown below.

$$\mathcal{L}_t = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

$$= \frac{620}{720}$$

$$= 0.861$$

The number of thermal neutrons available for absorption anywhere in the core is represented by the quantity $N_o \varepsilon \mathcal{L}_f p \mathcal{L}_t$.

Figure 1 indicates that 125 neutrons were absorbed in non-fuel materials. Since a total of 620 thermal neutrons were absorbed, the number absorbed by the fuel equals 620 - 125 = 495. Therefore, the thermal utilization factor can be calculated as follows.

$$f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in any reactor material}}$$

$$= \frac{495}{620}$$

$$= 0.799$$

The final factor numerically describes the production of fission neutrons resulting from thermal neutrons being absorbed in the fuel. This factor is called the reproduction factor (η). The value for the reproduction factor can be determined as shown below.

$$\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}$$

$$= \frac{1000}{495}$$

$$= 2.02$$

The number of fission neutrons that exist at the end of the life cycle which are available to start a new generation and cycle is represented by the quantity $N_o\ \varepsilon\ \mathcal{L}_f\ p\ \mathcal{L}_t\ f\ \eta$.

In the example illustrated in Figure 1, k_{eff} is equal to one. Therefore, 1000 neutrons are available to start the next generation.

Example:

10,000 neutrons exist at the beginning of a generation. The values for each factor of the six factor formula are listed below. Calculate the number of neutrons that exist at the points in the neutron life cycle listed below.

1) Number of neutrons that exist after fast fission.
2) Number of neutrons that start to slow down in the reactor.
3) Number of neutrons that reach thermal energies.
4) Number of thermal neutrons that are absorbed in the reactor.
5) Number of thermal neutrons absorbed in the fuel.
6) Number of neutrons produced from thermal fission.

$\varepsilon = 1.031$ $\quad \mathcal{L}_f = 0.889$ $\quad f = 0.751$
$p = 0.803$ $\quad \mathcal{L}_t = 0.905$ $\quad \eta = 2.012$

Solution:

1) $N = N_o\ \varepsilon = 10{,}310$
2) $N = N_o\ \varepsilon\ \mathcal{L}_f = 9{,}166$
3) $N = N_o\ \varepsilon\ \mathcal{L}_f\ p = 7{,}360$
4) $N = N_o\ \varepsilon\ \mathcal{L}_f\ p\ \mathcal{L}_t = 6{,}661$
5) $N = N_o\ \varepsilon\ \mathcal{L}_f\ p\ \mathcal{L}_t\ f = 5{,}002$
6) $N = N_o\ \varepsilon\ \mathcal{L}_f\ p\ \mathcal{L}_t\ f\ \eta = 10{,}065$

Neutron Life Cycle of a Fast Reactor

The neutron life cycle in a fast reactor is markedly different than that for a thermal reactor. In a fast reactor, care is taken during the reactor design to minimize thermalization of neutrons. Virtually all fissions taking place in a fast reactor are caused by fast neutrons. Due to this, many factors that are taken into account by the thermal reactor neutron life cycle are irrelevant to the fast reactor neutron life cycle. The resonance escape probability is not significant because very few neutrons exist at energies where resonance absorption is significant. The thermal non-leakage probability does not exist because the reactor is designed to avoid the thermalization of neutrons. A separate term to deal with fast fission is not necessary because all fission is fast fission and is handled by the reproduction factor.

The thermal utilization factor is modified to describe the utilization of fast neutrons instead of thermal neutrons. The reproduction factor is similarly modified to account for fast fission instead of thermal fission.

Summary

The important information in this chapter is summarized on the following pages.

Neutron Life Cycle Summary

- The infinite multiplication factor, k_{∞}, is the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption in the preceding generation.
- The effective multiplication factor, k_{eff}, is the ratio of the number of neutrons produced by fission in one generation to the number of neutrons lost through absorption and leakage in the preceding generation.
- Critical is the condition where the neutron chain reaction is self-sustaining and the neutron population is neither increasing nor decreasing.
- Subcritical is the condition in which the neutron population is decreasing each generation.
- Supercritical is the condition in which the neutron population is increasing each generation.
- The six factor formula is stated as $k_{eff} = \varepsilon \, \mathcal{L}_f \, p \, \mathcal{L}_t \, f \, \eta$. Each of the six factors is defined below.

$$\varepsilon = \frac{\text{number of fast neutrons produced by all fissions}}{\text{number of fast neutrons produced by thermal fissions}}$$

$$\mathcal{L}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

$$p = \frac{\text{number of neutrons that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}$$

$$\mathcal{L}_t = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

$$f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in all reactor materials}}$$

$$\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}$$

Neutron Life Cycle Summary (Cont.)

- The thermal utilization factor can be calculated from the macroscopic cross section for absorption of reactor materials using Equation (3-1).

$$f = \frac{\Sigma_a^U}{\Sigma_a^U + \Sigma_a^m + \Sigma_a^p}$$

- The reproduction factor can be calculated based on the characteristics of the reactor fuel using Equation (3-2).

$$\eta = \frac{N^{U-235}\ \sigma_f^{U-235}\ \nu^{U-235}}{N^{U-235}\ \sigma_a^{U-235} + N^{U-238}\ \sigma_a^{U-238}}$$

- The number of neutrons present at any point in the neutron life cycle can be calculated as the product of the number of neutrons present at the start of the generation and all the factors preceding that point in the life cycle.
- The thermal utilization factor is effected by the enrichment of uranium-235, the amount of neutron poisons, and the moderator-to-fuel ratio.
- The reproduction factor is effected by the enrichment of uranium-235.
- The resonance escape probability is effected by the enrichment of uranium-235, the temperature of the fuel, and the temperature of the moderator.
- An increase in moderator temperature will have the following effects.

  Increase the thermal utilization factor
  Decrease resonance escape probability
  Decrease fast non-leakage probability
  Decrease thermal non-leakage probability

REACTIVITY

Reactivity is a measure of the departure of a reactor from criticality. The reactivity is related to the value of k_{eff}. Reactivity is a useful concept to predict how the neutron population of a reactor will change over time.

EO 1.8 **Given the number of neutrons in a reactor core and the effective multiplication factor, CALCULATE the number of neutrons present after any number of generations.**

EO 1.9 **DEFINE the term reactivity.**

EO 1.10 **CONVERT between reactivity and the associated value of k_{eff}.**

EO 1.11 **CONVERT measures of reactivity between the following units:**

a. **$\Delta k/k$**
b. **$\% \Delta k/k$**
c. **$10^{-4}\ \Delta k/k$**
d. **Percent millirho (pcm)**

EO 1.12 **EXPLAIN the relationship between reactivity coefficients and reactivity defects.**

Application of the Effective Multiplication Factor

When k_{eff} remains constant from generation to generation, it is possible to determine the number of neutrons beginning any particular generation by knowing only the value of k_{eff} and the number of neutrons starting the first generation. If N_o neutrons start the first generation, then $N_o(k_{eff})$ neutrons start the second generation. Equation (3-4) can be used to calculate the number of neutrons after the completion of "n" generations.

$$N_n = N_o \left(k_{eff}\right)^n \qquad (3\text{-}4)$$

Example:

The number of neutrons in the core at time zero is 1000 and k_{eff} = 1.002. Calculate the number of neutrons after 50 generations.

Solution:

Use Equation (3-4) to calculate the number of neutrons.

$$N_n = N_o \left(k_{eff}\right)^n$$

$$N_{50} = 1000 \text{ neutrons } \left(1.002\right)^{50}$$

$$= 1105 \text{ neutrons}$$

Reactivity

If there are N_o neutrons in the preceding generation, then there are $N_o(k_{eff})$ neutrons in the present generation. The numerical change in neutron population is $(N_o k_{eff} - N_o)$. The gain or loss in neutron population $(N_o k_{eff} - N_o)$, expressed as a fraction of the present generation $(N_o k_{eff})$, is shown below.

$$\frac{N_o\ k_{eff} - N_o}{N_o\ k_{eff}}$$

This relationship represents the fractional change in neutron population per generation and is referred to as *reactivity* (ρ). Cancelling out the term N_o from the numerator and denominator, the reactivity is determined as shown in the equation below.

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \tag{3-5}$$

From Equation (3-5) it may be seen that ρ may be positive, zero, or negative, depending upon the value of k_{eff}. The larger the absolute value of reactivity in the reactor core, the further the reactor is from criticality. It may be convenient to think of reactivity as a measure of a reactor's departure from criticality.

Example:

Calculate the reactivity in the reactor core when k_{eff} is equal to 1.002 and 0.998.

Solution:

The reactivity for each case is determined by substituting the value of k_{eff} into Equation (3-5).

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \qquad \rho = \frac{k_{eff} - 1}{k_{eff}}$$

$$= \frac{1.002 - 1}{1.002} \qquad = \frac{0.998 - 1}{0.998}$$

$$= 0.001996 \qquad = -0.0020$$

Units of Reactivity

Reactivity is a dimensionless number. It does not have dimensions of time, length, mass, or any combination of these dimensions. It is simply a ratio of two quantities that are dimensionless. As shown in the calculation in the previous example, the value of reactivity is often a small decimal value. In order to make this value easier to express, artificial units are defined.

By definition, the value for reactivity that results directly from the calculation of Equation (3-5) is in units of $\Delta k/k$. Alternative units for reactivity are $\%\Delta k/k$ and pcm (percent millirho). The conversions between these units of reactivity are shown below.

$$1\% \ \frac{\Delta k}{k} = 0.01 \ \frac{\Delta k}{k}$$

$$1 \text{ pcm} = 0.00001 \ \frac{\Delta k}{k}$$

Another unit of reactivity that is used at some reactors is equivalent to 10^{-4} $\Delta k/k$. This unit of reactivity does not have a unique name. Special units for reactivity that do have unique names are dollars and cents. These units and their applications will be described in a later chapter.

Example:

Convert the values of reactivity listed below to the indicated units.

a. 0.000421 $\Delta k/k$ = _____ pcm
b. 0.0085 $\Delta k/k$ = _____ % $\Delta k/k$
c. 16×10^{-4} $\Delta k/k$ = _____ $\Delta k/k$

Solution:

a. 42.1 pcm
b. 0.85% $\Delta k/k$
c. 0.0016 $\Delta k/k$

If the reactivity is known, the effective multiplication factor can be determined by solving Equation (3-5) for k_{eff} in terms of the reactivity. This results in the following relationship.

$$k_{eff} = \frac{1}{1 - \rho} \tag{3-6}$$

Reactivity must be in units of $\Delta k/k$ for use in Equation (3-6).

Example:

Given a reactivity of -20.0×10^{-4} $\Delta k/k$, calculate k_{eff}.

Solution:

$$k_{eff} = \frac{1}{1 - \rho}$$

$$= \frac{1}{1 - (-20.0 \times 10^{-4})}$$

$$= 0.998$$

Reactivity Coefficients and Reactivity Defects

The amount of reactivity (ρ) in a reactor core determines what the neutron population, and consequently the reactor power, are doing at any given time. The reactivity can be effected by many factors (for example, fuel depletion, temperature, pressure, or poisons). The next several chapters discuss the factors affecting reactivity and how they are used to control or predict reactor behavior.

To quantify the effect that a variation in parameter (that is, increase in temperature, control rod insertion, increase in neutron poison) will have on the reactivity of the core, *reactivity coefficients* are used. Reactivity coefficients are the amount that the reactivity will change for a given change in the parameter. For instance, an increase in moderator temperature will cause a decrease in the reactivity of the core. The amount of reactivity change per degree change in the moderator temperature is the moderator temperature coefficient. Typical units for the moderator temperature coefficient are pcm/^{o}F. Reactivity coefficients are generally symbolized by α_x, where x represents some variable reactor parameter that affects reactivity. The definition of a reactivity coefficient in equation format is shown below.

$$\alpha_x = \frac{\Delta \rho}{\Delta x}$$

If the parameter x increases and positive reactivity is added, then α_x is positive. If the parameter x increases and negative reactivity is added, then α_x is negative.

Reactivity defects ($\Delta\rho$) are the total reactivity change caused by a variation in a parameter. Reactivity defects can be determined by multiplying the change in the parameter by the average value of the reactivity coefficient for that parameter. The equation below shows the general method for relating reactivity coefficients to reactivity defects.

$$\Delta\rho = \alpha_x \, \Delta x$$

Example:

The moderator temperature coefficient for a reactor is -8.2 pcm/^{o}F. Calculate the reactivity defect that results from a temperature decrease of 5^{o}F.

Solution:

$$\Delta \rho = \alpha_T \, \Delta T$$

$$= \left(-8.2 \ \frac{\text{pcm}}{^{\circ}\text{F}} \right) \left(-5 \ ^{\circ}\text{F} \right)$$

$$= 41 \ \text{pcm}$$

The reactivity addition due to the temperature decrease was positive because of the negative temperature coefficient.

Summary

The important information in this chapter is summarized below.

Reactivity Summary

- The number of neutrons present in the core after a given number of generations is calculated using Equation (3-4).

$$N_n = N_o \left(k_{eff}\right)^n$$

- Reactivity is the fractional change in neutron population per generation.
- Reactivity and k_{eff} are represented in Equation (3-5) and Equation (3-6), respectively.

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \qquad k_{eff} = \frac{1}{1 - \rho}$$

- The relationship between units of reactivity are listed below.

$$1\% \ \frac{\Delta k}{k} = 0.01 \ \frac{\Delta k}{k}$$

$$1 \text{ pcm} = 0.00001 \ \frac{\Delta k}{k}$$

- A reactivity coefficient is the amount of change in reactivity per unit change in the parameter. A reactivity defect is the total reactivity change caused by a change in the parameter. The reactivity defect is the product of the reactivity coefficient and the magnitude of the parameter change.

REACTIVITY COEFFICIENTS

Changes in the physical properties of the materials in the reactor will result in changes in the reactivity. Reactivity coefficients are useful in quantifying the reactivity change that will occur due to the change in a physical property such as the temperature of the moderator or fuel.

EO 2.1 **EXPLAIN the conditions of over moderation and under moderation.**

EO 2.2 **EXPLAIN why many reactors are designed to be operated in an under moderated condition.**

EO 2.3 **STATE the effect that a change in moderator temperature will have on the moderator to fuel ratio.**

EO 2.4 **DEFINE the temperature coefficient of reactivity.**

EO 2.5 **EXPLAIN why a negative temperature coefficient of reactivity is desirable.**

EO 2.6 **EXPLAIN why the fuel temperature coefficient is more effective than the moderator temperature coefficient in terminating a rapid power rise.**

EO 2.7 **EXPLAIN the concept of Doppler broadening of resonance absorption peaks.**

EO 2.8 **LIST two nuclides that are present in some types of reactor fuel assemblies that have significant resonance absorption peaks.**

EO 2.9 **DEFINE the pressure coefficient of reactivity.**

EO 2.10 **EXPLAIN why the pressure coefficient of reactivity is usually negligible in a reactor cooled and moderated by a subcooled liquid.**

EO 2.11 **DEFINE the void coefficient of reactivity.**

EO 2.12 **IDENTIFY the moderator conditions under which the void coefficient of reactivity becomes significant.**

Moderator Effects

As discussed in the previous module, a moderator possesses specific desirable characteristics.

(a) large neutron scattering cross section
(b) low neutron absorption cross section
(c) large neutron energy loss per collision

With the exception of the Liquid Metal Fast Breeder Reactor (LMFBR), the remaining major reactor types that are currently employed use moderating materials to reduce fission neutron energies to the thermal range. Light moderators (composed of light nuclei) are found to be more effective than heavy moderators because the light moderator removes more energy per collision than a heavy moderator. Therefore, the neutrons reach thermal energy more rapidly and they are less likely to be lost through resonance absorption.

As discussed in a previous module, the ability of a given material to slow down neutrons is referred to as the macroscopic slowing down power (MSDP) and is defined as the product of the logarithmic energy decrement per collision (ξ) times the macroscopic scattering cross section for neutrons as follows.

$$MSDP = \xi \Sigma_s$$

Macroscopic slowing down power indicates how rapidly slowing down occurs in the material in question, but it does not completely define the effectiveness of the material as a moderator. An element such as boron has a high logarithmic energy decrement and a good slowing down power, but is a poor moderator. It is a poor moderator because of its high probability of absorbing neutrons, and may be accounted for by dividing the macroscopic slowing down power by the macroscopic absorption cross section. This relationship is called the moderating ratio (MR).

$$MR = \frac{\xi \Sigma_s}{\Sigma_a}$$

The moderating ratio is merely the ratio of slowing down power to the macroscopic absorption cross section. The higher the moderating ratio, the more effectively the material performs as a moderator.

Another ratio, the *moderator-to-fuel ratio* (N^m/N^u), is very important in the discussion of moderators. As the reactor designer increases the amount of moderator in the core (that is, N^m/N^u increases), neutron leakage decreases. Neutron absorption in the moderator (Σ_a^m) increases and causes a decrease in the thermal utilization factor. Having insufficient moderator in the core (that is, N^m/N^u decreases) causes an increase in slowing down time and results in a greater loss of neutrons by resonance absorption. This also causes an increase in neutron leakage. The effects of varying the moderator-to-fuel ratio on the thermal utilization factor and the resonance probability are shown in Figure 2.

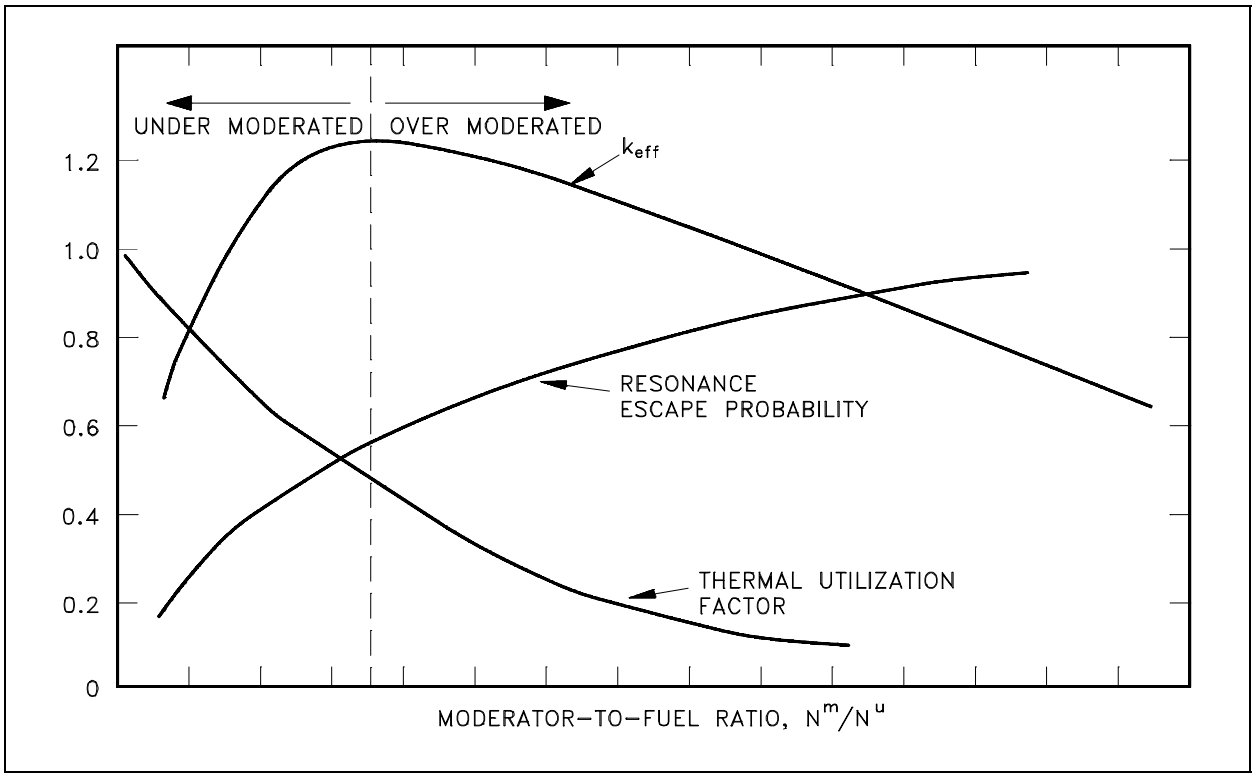

Figure 2 Effects of Over and Under Moderation on k_{eff}

Because the moderator-to-fuel ratio affects the thermal utilization factor and the resonance escape probability, it also affects k_{eff}. The remaining factors in the six factor formula are also affected by the moderator-to-fuel ratio, but to a lesser extent than f and p. As illustrated in Figure 2, which is applicable to a large core fueled with low-enriched fuel, there is an optimum point above which increasing the moderator-to-fuel ratio decreases k_{eff} due to the dominance of the decreasing thermal utilization factor. Below this point, a decrease in the moderator-to-fuel ratio decreases k_{eff} due to the dominance of the increased resonance absorption in the fuel. If the ratio is above this point, the core is said to be over moderated, and if the ratio is below this point, the core is said to be under moderated.

In practice, water-moderated reactors are designed with a moderator-to-fuel ratio so that the reactor is operated in an under moderated condition. The reason that some reactors are designed to be under moderated is if the reactor were over moderated, an increase in temperature would decrease the N^m/N^u due to the expansion of the water as its density became lower. This decrease in N^m/N^u would be a positive reactivity addition, increasing k_{eff} and further raising power and temperature in a dangerous cycle. If the reactor is under moderated, the same increase in temperature results in the addition of negative reactivity, and the reactor becomes more self-regulating.

Moderator Temperature Coefficient

The change in reactivity per degree change in temperature is called the *temperature coefficient of reactivity*. Because different materials in the reactor have different reactivity changes with temperature and the various materials are at different temperatures during reactor operation, several different temperature coefficients are used. Usually, the two dominant temperature coefficients are the moderator temperature coefficient and the fuel temperature coefficient.

The change in reactivity per degree change in moderator temperature is called the *moderator temperature coefficient* of reactivity. The magnitude and sign (+ or -) of the moderator temperature coefficient is primarily a function of the moderator-to-fuel ratio. If a reactor is under moderated, it will have a negative moderator temperature coefficient. If a reactor is over moderated, it will have a positive moderator temperature coefficient. A negative moderator temperature coefficient is desirable because of its self-regulating effect. For example, an increase in reactivity causes the reactor to produce more power. This raises the temperature of the core and adds negative reactivity, which slows down, or turns, the power rise.

Fuel Temperature Coefficient

Another temperature coefficient of reactivity, the fuel temperature coefficient, has a greater effect than the moderator temperature coefficient for some reactors. The *fuel temperature coefficient* is the change in reactivity per degree change in fuel temperature. This coefficient is also called the "prompt" temperature coefficient because an increase in reactor power causes an immediate change in fuel temperature. A negative fuel temperature coefficient is generally considered to be even more important than a negative moderator temperature coefficient because fuel temperature immediately increases following an increase in reactor power. The time for heat to be transferred to the moderator is measured in seconds. In the event of a large positive reactivity insertion, the moderator temperature cannot turn the power rise for several seconds, whereas the fuel temperature coefficient starts adding negative reactivity immediately.

Another name applied to the fuel temperature coefficient of reactivity is the fuel doppler reactivity coefficient. This name is applied because in typical low enrichment, light water-moderated, thermal reactors the fuel temperature coefficient of reactivity is negative and is the result of the doppler effect, also called doppler broadening. The phenomenon of the doppler effect is caused by an apparent broadening of the resonances due to thermal motion of nuclei as illustrated in Figure 3. Stationary nuclei absorb only neutrons of energy E_o. If the nucleus is moving away from the neutron, the velocity (and energy) of the neutron must be greater than E_o to undergo resonance absorption. Likewise, if the nucleus is moving toward the neutron, the neutron needs less energy than E_o to be absorbed. Raising the temperature causes the nuclei to vibrate more rapidly within their lattice structures, effectively broadening the energy range of neutrons that may be resonantly absorbed in the fuel. Two nuclides present in large amounts in the fuel of some reactors with large resonant peaks that dominate the doppler effect are uranium-238 and plutonium-240.

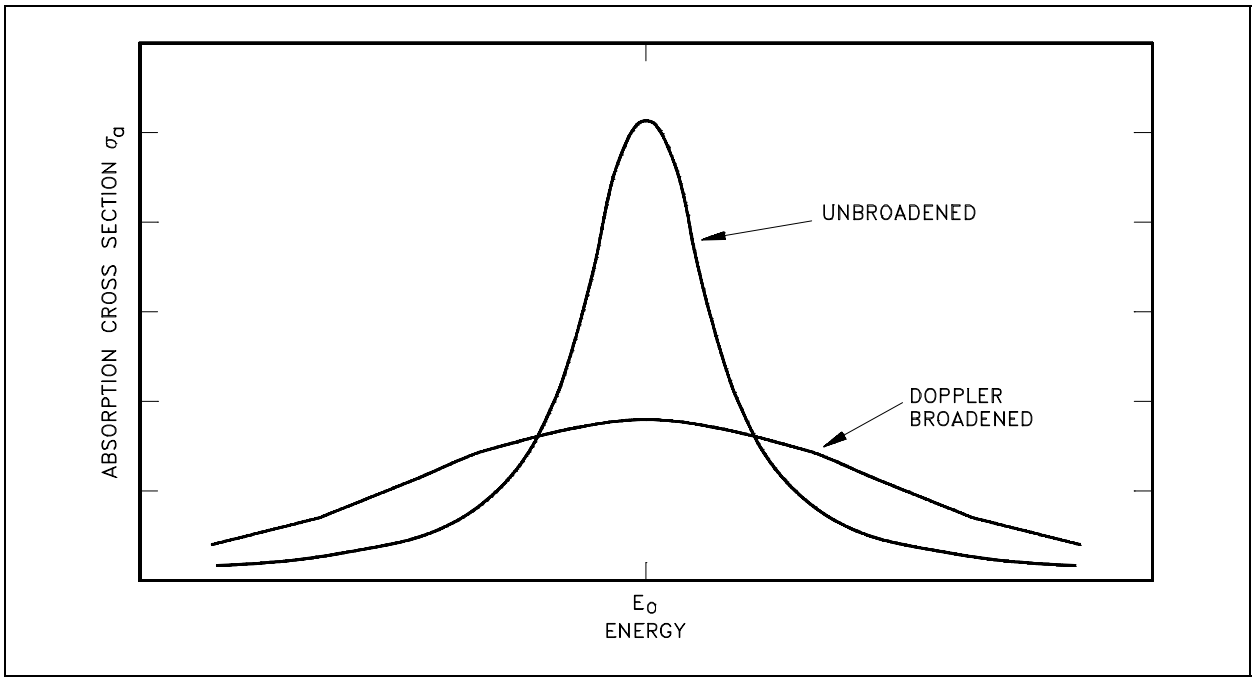

Figure 3 Effect of Fuel Temperature on Resonance Absorption Peaks

Pressure Coefficient

The reactivity in a reactor core can be affected by the system pressure. The *pressure coefficient* of reactivity is defined as the change in reactivity per unit change in pressure. The pressure coefficient of reactivity for the reactor is the result of the effect of pressure on the density of the moderator. For this reason, it is sometimes referred to as the moderator density reactivity coefficient. As pressure increases, density correspondingly increases, which increases the moderator-to-fuel ratio in the core. In the typical under moderated core the increase in the moderator-to-fuel ratio will result in a positive reactivity addition. In reactors that use water as a moderator, the absolute value of the pressure reactivity coefficient is seldom a major factor because it is very small compared to the moderator temperature coefficient of reactivity.

Void Coefficient

In systems with boiling conditions, such as boiling water reactors (BWR), the pressure coefficient becomes an important factor due to the larger density changes that occur when the vapor phase of water undergoes a pressure change. Of prime importance during operation of a BWR, and a factor in some other water-moderated reactors, is the void coefficient. The void coefficient is caused by the formation of steam voids in the moderator. The *void coefficient* of reactivity is defined as the change in reactivity per percent change in void volume. As the reactor power is raised to the point where the steam voids start to form, voids displace moderator from the coolant channels within the core. This displacement reduces the moderator-to-fuel ratio, and in an under moderated core, results in a negative reactivity addition, thereby limiting reactor power rise. The void coefficient is significant in water-moderated reactors that operate at or near saturated conditions.

Summary

The important information in this chapter is summarized below.

Reactivity Coefficients Summary

- The temperature coefficient of reactivity is the change in reactivity per degree change in temperature.
- A reactor is under moderated when a decrease in the moderator-to-fuel ratio decreases k_{eff} due to the increased resonance absorption. A reactor is over moderated when an increase in the moderator-to-fuel ratio decreases k_{eff} due to the decrease in the thermal utilization factor.
- Reactors are usually designed to operate in an under moderated condition so that the moderator temperature coefficient of reactivity is negative.
- Increasing the moderator temperature will decrease the moderator-to-fuel ratio. Decreasing the moderator temperature will increase the moderator-to-fuel ratio.
- A negative temperature coefficient of reactivity is desirable because it makes the reactor more self-regulating. An increase in power, resulting in an increase in temperature, results in negative reactivity addition due to the temperature coefficient. The negative reactivity addition due to the temperature increase will slow or stop the power increase.
- The fuel temperature coefficient is more effective than the moderator temperature coefficient in terminating a rapid power rise because the fuel temperature immediately increases following a power increase, while the moderator temperature does not increase for several seconds.
- The Doppler broadening of resonance peaks occurs because the nuclei may be moving either toward or away from the neutron at the time of interaction. Therefore, the neutron may actually have either slightly more or slightly less than the resonant energy, but still appear to be at resonant energy relative to the nucleus.
- Uranium-238 and plutonium-240 are two nuclides present in some reactor fuels that have large resonance absorption peaks.

Reactivity Coefficients Summary (Cont.)

- The pressure coefficient of reactivity is the change in reactivity per unit change in pressure.
- The pressure coefficient of reactivity is usually negligible in reactors moderated by subcooled liquids because the density of the liquid does not change significantly within the operating pressure range.
- The void coefficient of reactivity is the change in reactivity per unit change in void volume.
- The void coefficient of reactivity becomes significant in a reactor in which the moderator is at or near saturated conditions.

NEUTRON POISONS

In some reactors, neutron-absorbing materials called poisons are intentionally designed into the reactor for specific purposes. Some of these poisons deplete as they absorb neutrons during reactor operation, and others remain relatively constant.

EO 3.1 DEFINE the following terms:

a. Burnable poison
b. Non-burnable poison
c. Chemical shim

EO 3.2 EXPLAIN the use of burnable neutron poisons in a reactor core.

EO 3.3 LIST the advantages and disadvantages of chemical shim over fixed burnable poisons.

EO 3.4 STATE two reasons why fixed non-burnable neutron poisons are used in reactor cores.

EO 3.5 STATE an example of a material used as a fixed non-burnable neutron poison.

Fixed Burnable Poisons

During operation of a reactor the amount of fuel contained in the core constantly decreases. If the reactor is to operate for a long period of time, fuel in excess of that needed for exact criticality must be added when the reactor is built. The positive reactivity due to the excess fuel must be balanced with negative reactivity from neutron-absorbing material. Moveable control rods containing neutron-absorbing material are one method used to offset the excess fuel. Control rods will be discussed in detail in a later chapter. Using control rods alone to balance the excess reactivity may be undesirable or impractical for several reasons. One reason for a particular core design may be that there is physically insufficient room for the control rods and their large mechanisms.

To control large amounts of excess fuel without adding additional control rods, burnable poisons are loaded into the core. *Burnable poisons* are materials that have a high neutron absorption cross section that are converted into materials of relatively low absorption cross section as the result of neutron absorption. Due to the burnup of the poison material, the negative reactivity of the burnable poison decreases over core life. Ideally, these poisons should decrease their negative reactivity at the same rate the fuel's excess positive reactivity is depleted. Fixed burnable poisons are generally used in the form of compounds of boron or gadolinium that are shaped into separate lattice pins or plates, or introduced as additives to the fuel. Since they can usually be distributed more uniformly than control rods, these poisons are less disruptive to the core power distribution.

Soluble Poisons

Soluble poisons, also called *chemical shim*, produce a spatially uniform neutron absorption when dissolved in the water coolant. The most common soluble poison in commercial pressurized water reactors (PWR) is boric acid, which is often referred to as "soluble boron," or simply "solbor." The boric acid in the coolant decreases the thermal utilization factor, causing a decrease in reactivity. By varying the concentration of boric acid in the coolant (a process referred to as boration and dilution), the reactivity of the core can be easily varied. If the boron concentration is increased, the coolant/moderator absorbs more neutrons, adding negative reactivity. If the boron concentration is reduced (dilution), positive reactivity is added. The changing of boron concentration in a PWR is a slow process and is used primarily to compensate for fuel burnout or poison buildup. The variation in boron concentration allows control rod use to be minimized, which results in a flatter flux profile over the core than can be produced by rod insertion. The flatter flux profile is due to the fact that there are no regions of depressed flux like those that would be produced in the vicinity of inserted control rods.

DOE reactors typically do not use soluble neutron poisons during normal operation. Some DOE reactors do, however, include emergency shutdown systems that inject solutions containing neutron poisons into the system that circulates reactor coolant. Various solutions, including sodium polyborate and gadolinium nitrate, are used.

Fixed burnable poisons possess some advantages over chemical shim. Fixed burnable poisons may be discretely loaded in specific locations in order to shape or control flux profiles in the core. Also, fixed burnable poisons do not make the moderator temperature reactivity coefficient less negative as chemical shim does. With chemical shim, as temperature rises and the moderator expands, some moderator is pushed out of the active core area. Boron is also moved out, and this has a positive effect on reactivity. This property of chemical shim limits the allowable boron concentration because any greater concentration makes the moderator temperature coefficient of reactivity positive.

Non-Burnable Poisons

A *non-burnable poison* is one that maintains a constant negative reactivity worth over the life of the core. While no neutron poison is strictly non-burnable, certain materials can be treated as non-burnable poisons under certain conditions. One example is hafnium. The removal (by absorption of neutrons) of one isotope of hafnium leads to the production of another neutron absorber, and continues through a chain of five absorbers. This absorption chain results in a long-lived burnable poison which approximates non-burnable characteristics. Absorbers with low neutron absorption cross sections can also be treated as non-burnable under most conditions.

It is possible to make the reactivity of a poison material that is usually a burnable poison more uniform over core life through the use of self-shielding. In self-shielding, the poison material is thick enough that only the outer layer of the poison is exposed to the neutron flux. The absorptions that take place in the outer layers reduce the number of neutrons that penetrate to the inner material. As the outer layers of poison absorb neutrons and are converted to non-poison materials, the inner layers begin absorbing more neutrons, and the negative reactivity of the poison is fairly uniform.

The normal use of fixed non-burnable poisons is in power shaping, or to prevent excessive flux and power peaking near moderator regions of the reactor.

Summary

The important information in this chapter is summarized below.

Neutron Poisons Summary

- A burnable neutron poison is a material that has a high neutron absorption cross section that is converted into a material of relatively low absorption cross section as the result of neutron absorption.
- A non-burnable neutron poison is a material that has relatively constant neutron absorption characteristics over core life. The absorption of a neutron by one isotope in the material produces another isotope that also has a high absorption cross section.
- Chemical shim is a soluble neutron poison that is circulated in the coolant during normal operation.
- Burnable neutron poisons are used in reactor cores to compensate for the excess positive reactivity of the fuel when the reactor is initially started up.
- Chemical shim has several advantages over fixed burnable poisons.
  Has a spatially uniform effect
  Possible to increase or decrease amount of poison in the core during reactor operation
- Fixed burnable poisons have several advantages over chemical shim.
  Can be used to shape flux profiles
  Do not have an adverse effect on moderator temperature coefficient
- Two reasons for using non-burnable neutron poisons in reactor cores are to shape power and to prevent excessive flux and power peaking near moderator regions.
- An example of a material that is used as a fixed non-burnable neutron poison is hafnium.

XENON

Xenon-135 has a tremendous impact on the operation of a nuclear reactor. It is important to understand the mechanisms that produce and remove xenon from the reactor to predict how the reactor will respond following changes in power level.

EO 4.1 LIST two methods of production and two methods of removal for xenon-135 during reactor operation.

EO 4.2 STATE the equation for equilibrium xenon-135 concentration.

EO 4.3 DESCRIBE how equilibrium xenon-135 concentration varies with reactor power level.

EO 4.4 DESCRIBE the causes and effects of a xenon oscillation.

EO 4.5 DESCRIBE how xenon-135 concentration changes following a reactor shutdown from steady-state conditions.

EO 4.6 EXPLAIN the effect that pre-shutdown power levels have on the xenon-135 concentration after shutdown.

EO 4.7 STATE the approximate time following a reactor shutdown at which the reactor can be considered "xenon free."

EO 4.8 EXPLAIN what is meant by the following terms:

a. Xenon precluded startup
b. Xenon dead time

EO 4.9 DESCRIBE how xenon-135 concentration changes following an increase or a decrease in the power level of a reactor.

Fission Product Poisons

Fission fragments generated at the time of fission decay to produce a variety of fission products. Fission products are of concern in reactors primarily because they become parasitic absorbers of neutrons and result in long term sources of heat. Although several fission products have significant neutron absorption cross sections, xenon-135 and samarium-149 have the most substantial impact on reactor design and operation. Because these two fission product poisons remove neutrons from the reactor, they will have an impact on the thermal utilization factor and thus k_{eff} and reactivity.

Production and Removal of Xenon-135

Xenon-135 has a 2.6 x 10^6 barns neutron absorption cross section. It is produced directly by some fissions, but is more commonly a product of the tellurium-135 decay chain shown below. The fission yield (γ) for xenon-135 is about 0.3%, while γ for tellurium-135 is about 6%.

$$^{135}_{52}\text{Te} \xrightarrow[19.0\ \text{sec}]{\beta^-} {}^{135}_{53}\text{I} \xrightarrow[6.57\ \text{hr}]{\beta^-} {}^{135}_{54}\text{Xe} \xrightarrow[9.10\ \text{hr}]{\beta^-} {}^{135}_{55}\text{Cs} \xrightarrow[2.3\text{x}10^6\ \text{yr}]{\beta^-} {}^{135}_{56}\text{Ba} \quad \text{(stable)}$$

The half-life for tellurium-135 is so short compared to the other half-lives that it can be assumed that iodine-135 is produced directly from fission. Iodine-135 is not a strong neutron absorber, but decays to form the neutron poison xenon-135. Ninety-five percent of all the xenon-135 produced comes from the decay of iodine-135. Therefore, the half-life of iodine-135 plays an important role in the amount of xenon-135 present.

The rate of change of iodine concentration is equal to the rate of production minus the rate of removal. This can be expressed in the equation below.

rate of change of iodine concentration = yield from fission - decay rate - burnup rate

or

$$\frac{dN_I}{dt} = \gamma_I \Sigma_f^{fuel} \phi - \lambda_I N_I - \sigma_a^I N_I \phi$$

where:

N_I = ^{135}I concentration

γ_I = fission yield of ^{135}I

Σ_f^{fuel} = macroscopic fission cross section fuel

ϕ = thermal neutron flux

λ_I = decay constant for ^{135}I

σ_a^I = microscopic absorption cross section ^{135}I

Since the σ_a^I is very small, the burn up rate term may be ignored, and the expression for the rate of change of iodine concentration is modified as shown below.

$$\frac{dN_I}{dt} = \gamma_I \Sigma_f^{fuel} \phi - \lambda_I N_I$$

When the rate of production of iodine equals the rate of removal of iodine, equilibrium exists. The iodine concentration remains constant and is designated $N_I(eq)$. The following equation for the equilibrium concentration of iodine can be determined from the preceding equation by setting the two terms equal to each other and solving for $N_I(eq)$.

$$N_I(eq) = \frac{\gamma_I \Sigma_f^{fuel} \phi}{\lambda_I}$$

Since the equilibrium iodine concentration is proportional to the fission reaction rate, it is also proportional to reactor power level.

The rate of change of the xenon concentration is equal to the rate of production minus the rate of removal. Recall that 5% of xenon comes directly from fission and 95% comes from the decay of iodine. The rate of change of xenon concentration is expressed by the following equations.

$$\begin{pmatrix}\text{rate of change of}\\ \text{xenon-135 concentration}\end{pmatrix} = \begin{pmatrix}\text{xenon-135 yield}\\ \text{from fission}\end{pmatrix} + \begin{pmatrix}\text{iodine-135}\\ \text{decay}\end{pmatrix} - \begin{pmatrix}\text{xenon-135}\\ \text{decay}\end{pmatrix} - \begin{pmatrix}\text{xenon-135}\\ \text{burnup}\end{pmatrix}$$

$$\frac{dN_{Xe}}{dt} = \gamma_{Xe} \Sigma_f^{fuel} \phi + \lambda_I N_I - \lambda_{Xe} N_{Xe} - \sigma_a^{Xe} N_{Xe} \phi$$

where:

N_{Xe} = ^{135}Xe concentration

γ_{Xe} = fission yield of ^{135}Xe

Σ_f^{fuel} = macroscopic fission cross section of the fuel

ϕ = thermal neutron flux

λ_I = decay constant for ^{135}I

N_I = ^{135}I concentration

λ_{Xe} = decay constant for ^{135}Xe

σ_a^{Xe} = microscopic absorption cross section ^{135}Xe

The xenon burnup term above refers to neutron absorption by xenon-135 by the following reaction.

$$^{135}_{54}Xe + ^{1}_{0}n \rightarrow ^{136}_{54}Xe + \gamma$$

Xenon-136 is not a significant neutron absorber; therefore, the neutron absorption by xenon-135 constitutes removal of poison from the reactor. The burnup rate of xenon-135 is dependent upon the neutron flux and the xenon-135 concentration.

The equilibrium concentration of xenon-135 is designated $N_{Xe}(eq)$, and is represented as shown below.

$$N_{Xe}(eq) = \frac{\gamma_{Xe} \Sigma_f^{fuel} \phi + \lambda_I N_I}{\lambda_{Xe} + \sigma_a^{Xe} \phi}$$

For xenon-135 to be in equilibrium, iodine-135 must also be in equilibrium. Substituting the expression for equilibrium iodine-135 concentration into the equation for equilibrium xenon results in the following.

$$N_{Xe}\ (eq) = \frac{\left(\gamma_{Xe} + \gamma_I\right) \Sigma_f^{fuel}\ \phi}{\lambda_{Xe} + \sigma_a^{Xe}\ \phi}$$

From this equation it can be seen that the equilibrium value for xenon-135 increases as power increases, because the numerator is proportional to the fission reaction rate. Thermal flux is also in the denominator; therefore, as the thermal flux exceeds 10^{12} neutrons/cm^2-sec, the $\sigma_a^{Xe}\phi$ term begins to dominate, and at approximately 10^{15} neutrons/cm^2-sec, the xenon-135 concentration approaches a limiting value. The equilibrium iodine-135 and xenon-135 concentrations as a function of neutron flux are illustrated in Figure 4.

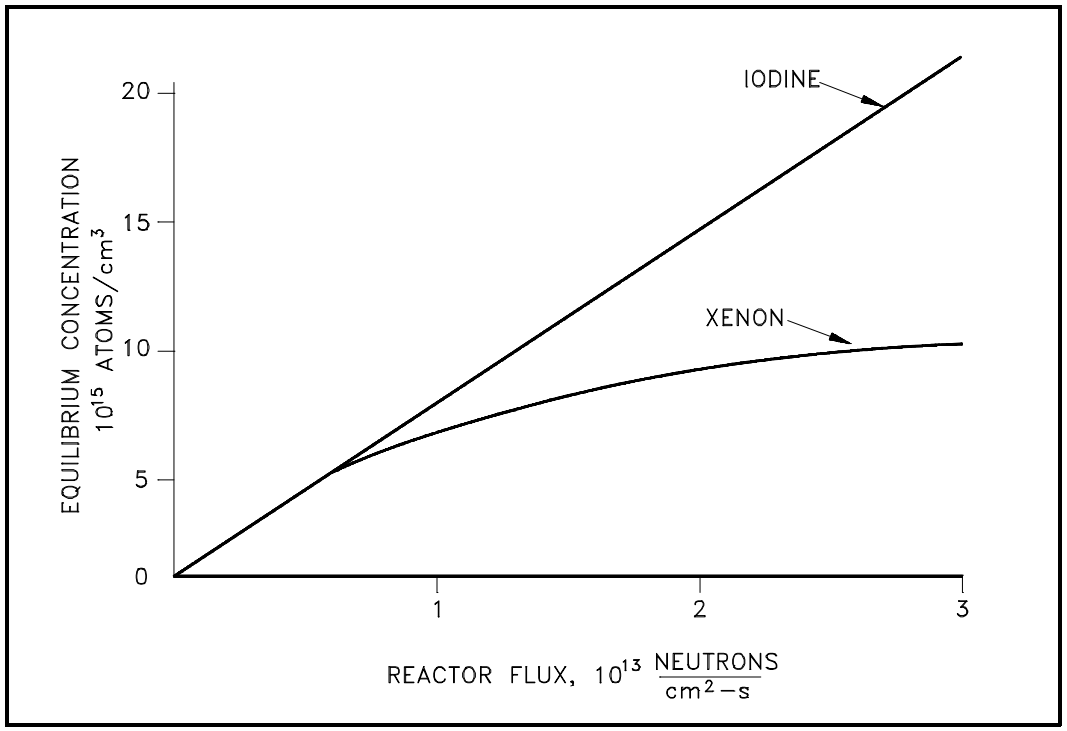

Figure 4 Equilibrium Iodine-135 and Xenon-135 Concentrations Versus Neutron Flux

The higher the power level, or flux, the higher the equilibrium xenon-135 concentration, but equilibrium xenon-135 is not directly proportional to power level. For example, equilibrium xenon-135 at 25% power is more than half the value for equilibrium xenon-135 at 100% power for many reactors. Because the xenon-135 concentration directly affects the reactivity level in the reactor core, the negative reactivity due to the xenon concentrations for different power levels or conditions are frequently plotted instead of the xenon concentration.

Xenon-135 Response to Reactor Shutdown

When a reactor is shutdown, the neutron flux is reduced essentially to zero. Therefore, after shutdown, xenon-135 is no longer produced by fission and is no longer removed by burnup. The only remaining production mechanism is the decay of the iodine-135 which was in the core at the time of shutdown. The only removal mechanism for xenon-135 is decay.

$$\frac{dN_{Xe}}{dt} = \lambda_I N_I - \lambda_{Xe} N_{Xe}$$

Because the decay rate of iodine-135 is faster than the decay rate of xenon-135, the xenon concentration builds to a peak. The peak is reached when the product of the terms $\lambda_I N_I$ is equal to $\lambda_{Xe} N_{Xe}$ (in about 10 to 11 hours). Subsequently, the production from iodine decay is less than the removal of xenon by decay, and the concentration of xenon-135 decreases. The greater the flux level prior to shutdown, the greater the concentration of iodine-135 at shutdown; therefore, the greater the peak in xenon-135 concentration after shutdown. This phenomenon can be seen in Figure 5, which illustrates the negative reactivity value of xenon-135 following shutdown from various neutron flux levels.

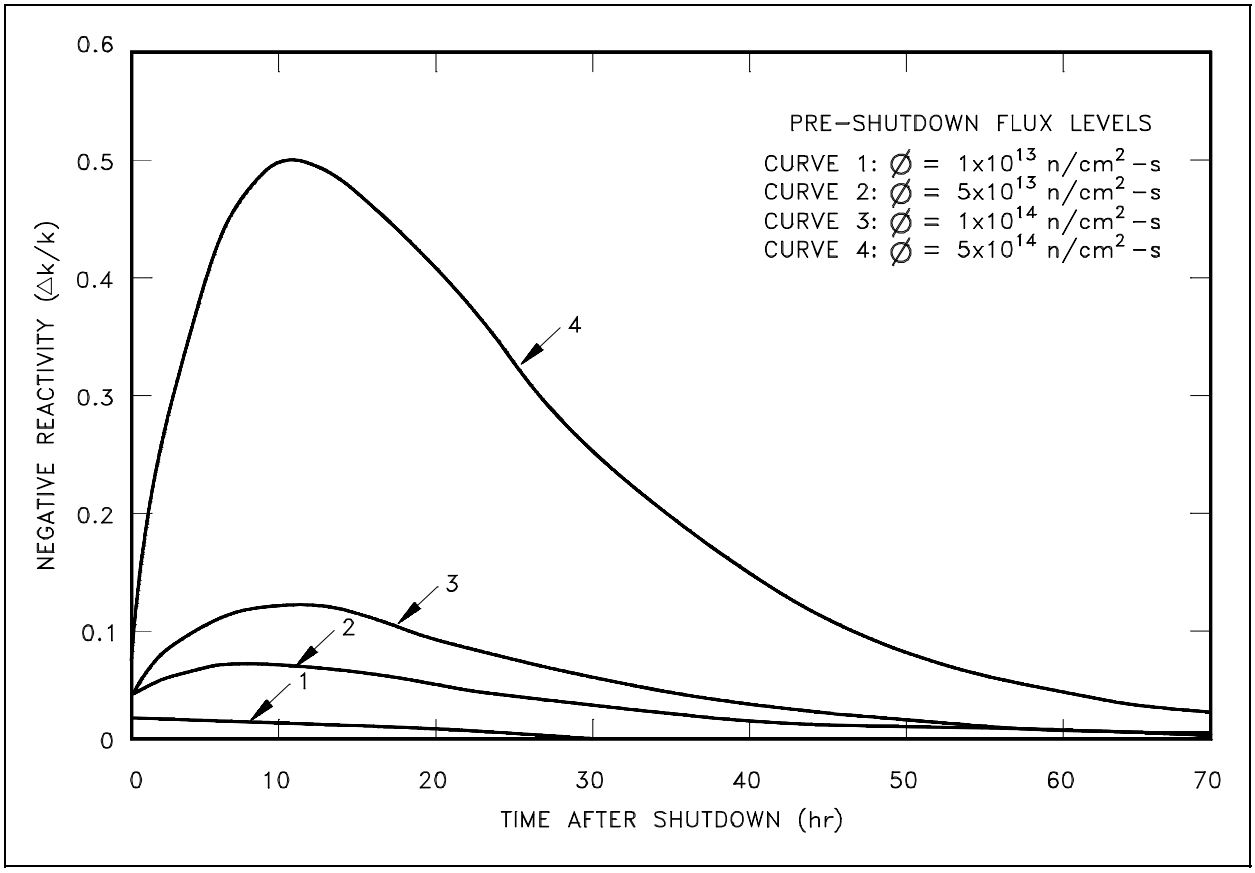

Figure 5 Xenon-135 Reactivity After Reactor Shutdown

Negative xenon reactivity, also called xenon poisoning, may provide sufficient negative reactivity to make the reactor inoperable because there is insufficient positive reactivity available from control rod removal or chemical shim dilution (if used) to counteract it. The inability of the reactor to be started due to the effects of xenon is sometimes referred to as a *xenon precluded startup*. The period of time where the reactor is unable to "override" the effects of xenon is called *xenon dead time*. Because the amount of excess core reactivity available to override the negative reactivity of the xenon is usually less than 10% $\Delta k/k$, thermal power reactors are normally limited to flux levels of about 5×10^{13} neutrons/cm^2-sec so that timely restart can be ensured after shutdown. For reactors with very low thermal flux levels ($\sim 5 \times 10^{12}$ neutrons/cm^2-sec or less), most xenon is removed by decay as opposed to neutron absorption. For these cases, reactor shutdown does not cause any xenon-135 peaking effect.

Following the peak in xenon-135 concentration about 10 hours after shutdown, the xenon-135 concentration will decrease at a rate controlled by the decay of iodine-135 into xenon-135 and the decay rate of xenon-135. For some reactors, the xenon-135 concentration about 20 hours after shutdown from full power will be the same as the equilibrium xenon-135 concentration at full power. About 3 days after shutdown, the xenon-135 concentration will have decreased to a small percentage of its pre-shutdown level, and the reactor can be assumed to be xenon free without a significant error introduced into reactivity calculations.

Xenon-135 Oscillations

Large thermal reactors with little flux coupling between regions may experience spatial power oscillations because of the non-uniform presence of xenon-135. The mechanism is described in the following four steps.

(1) An initial lack of symmetry in the core power distribution (for example, individual control rod movement or misalignment) causes an imbalance in fission rates within the reactor core, and therefore, in the iodine-135 buildup and the xenon-135 absorption.

(2) In the high-flux region, xenon-135 burnout allows the flux to increase further, while in the low-flux region, the increase in xenon-135 causes a further reduction in flux. The iodine concentration increases where the flux is high and decreases where the flux is low.

(3) As soon as the iodine-135 levels build up sufficiently, decay to xenon reverses the initial situation. Flux decreases in this area, and the former low-flux region increases in power.

(4) Repetition of these patterns can lead to xenon oscillations moving about the core with periods on the order of about 15 hours.

With little change in overall power level, these oscillations can change the local power levels by a factor of three or more. In a reactor system with strongly negative temperature coefficients, the xenon-135 oscillations are damped quite readily. This is one reason for designing reactors to have negative moderator-temperature coefficients.

Xenon-135 Response to Reactor Power Changes

During periods of steady state operation, at a constant neutron flux level, the xenon-135 concentration builds up to its equilibrium value for that reactor power in about 40 to 50 hours. Figure 6 illustrates a typical xenon transient that occurs as a result of a change in reactor power level. At time zero, reactor power is raised from 50% power to 100% power. When the reactor power is increased, xenon concentration initially decreases because the burnup is increased at the new higher power level. Because 95% of the xenon production is from iodine-135 decay, which has a 6 to 7 hour half-life, the production of xenon remains constant for several hours. After a few hours (roughly 4 to 6 hours depending on power levels) the rate of production of xenon from iodine and fission equals the rate of removal of xenon by burnup and decay. At this point, the xenon concentration reaches a minimum. The xenon concentration then increases to the new equilibrium level for the new power level in roughly 40 to 50 hours. It should be noted that the magnitude and the rate of change of xenon concentration during the initial 4 to 6 hours following the power change is dependent upon the initial power level and on the amount of change in power level. The xenon concentration change is greater for a larger change in power level.

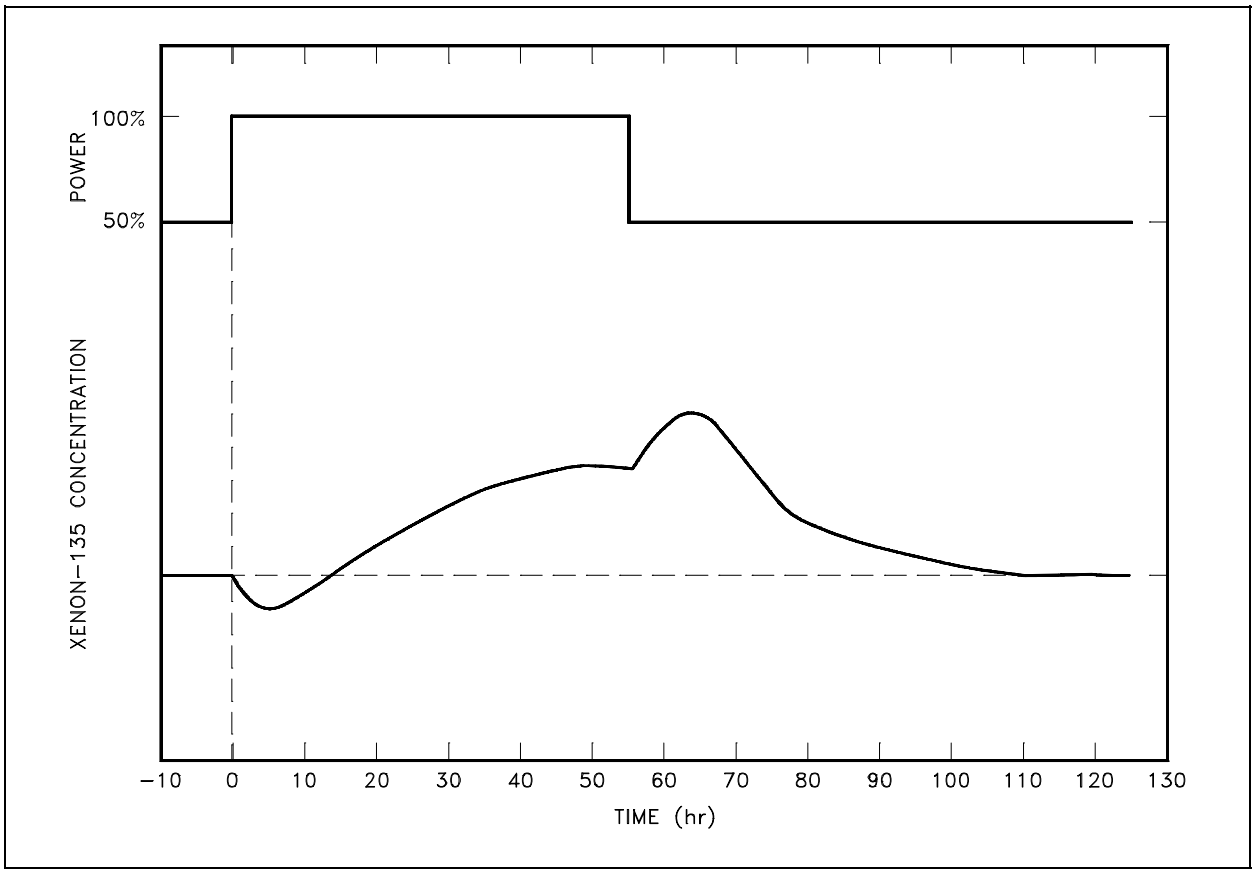

Figure 6 Xenon-135 Variations During Power Changes

When reactor power is decreased from 100% to 50% power (t = 55 hours), the process is reversed. There is an immediate decrease in xenon burnup, which results in an increase in xenon-135 concentration. The iodine-135 concentration is still at the higher equilibrium level for 100% power and is therefore still producing xenon-135 at the higher rate. The xenon-135 concentration continues to rise until the rate of production of xenon-135 becomes equal to the rate of removal (roughly 7 to 8 hours after the initial reduction in power level). The xenon-135 concentration then gradually decreases to the new equilibrium level in about 50 to 60 hours. The magnitude of the xenon peak is greatest if the initial power level is very high.

Maximum peak xenon occurs when a reactor that is operating at 100% equilibrium xenon concentration is suddenly shut down. The most rapid possible burnout of xenon occurs when a reactor is started up and operated at full power while this maximum peak xenon condition exists.

Summary

The important information in this chapter is summarized below.

Xenon Summary

- Xenon-135 is produced directly as a fission product and by the decay of iodine-135 during reactor operation. Xenon-135 is removed from the core by radioactive decay and by neutron absorption during reactor operation.
- The equilibrium concentration for xenon-135 is determined by the following equation.

$$N_{Xe}\ (eq) = \frac{\gamma_{Xe}\ \Sigma_f^{fuel}\ \phi + \lambda_I\ N_I}{\lambda_{Xe} + \sigma_a^{Xe}\ \phi} \quad \text{or} \quad N_{Xe}\ (eq) = \frac{(\gamma_{Xe} + \gamma_I)\ \Sigma_f^{fuel}\ \phi}{\lambda_{Xe} + \sigma_a^{Xe}\ \phi}$$

- The xenon-135 concentration increases with increasing power level in a non-linear manner. Equilibrium xenon-135 concentration reaches a maximum at a flux of about 10^{15} neutrons/cm^2-sec.
- After a power increase, xenon-135 concentration will initially decrease due to the increased removal by burnout. Xenon-135 will reach a minimum about 5 hours after the power increase and then increase to a new, higher equilibrium value as the production from iodine decay increases.

Xenon Summary (Cont.)

- A xenon-135 oscillation may be caused by a rapid perturbation in the core power distribution. The xenon-135 oscillation can change local power levels in the core by a factor of three or more.
- Following a reactor shutdown, xenon-135 concentration will increase due to the decay of the iodine inventory of the core. Xenon-135 will peak approximately 10 hours after the shutdown (from 100%) and then decrease as xenon-135 decay becomes greater than the iodine-135 decay.
- The greater the pre-shutdown power level, the greater the peak value of xenon.
- The core can be considered xenon-free about 3 days after shutdown.
- A xenon precluded startup occurs when there is insufficient reactivity in the control rods to overcome the negative reactivity of xenon-135.
- Xenon dead time is the period of time where the reactor is unable to override the effects of xenon.
- After a power decrease, xenon-135 concentration will initially increase due to production by iodine decay being greater than the burnout. Xenon-135 will reach a maximum about 8 hours after the power decrease and then decrease to a new, lower equilibrium value.

SAMARIUM AND OTHER FISSION PRODUCT POISONS

The fission product poison that has the most significant effect on reactor operations other than xenon-135 is samarium-149. Samarium-149 behaves significantly different from xenon-135 due to its different nuclear properties.

EO 4.10 DESCRIBE how samarium-149 is produced and removed from the reactor core during reactor operation.

EO 4.11 STATE the equation for equilibrium samarium-149 concentration.

EO 4.12 DESCRIBE how equilibrium samarium-149 concentration varies with reactor power level.

EO 4.13 DESCRIBE how samarium-149 concentration changes following a reactor shutdown from steady-state conditions.

EO 4.14 DESCRIBE how samarium-149 concentration changes following a reactor startup.

EO 4.15 STATE the conditions under which helium-3 will have a significant effect on the reactivity of a reactor.

Production and Removal of Samarium-149

Samarium-149 is the second most important fission-product poison because of its high thermal neutron absorption cross section of 4.1×10^4 barns. Samarium-149 is produced from the decay of the neodymium-149 fission fragment as shown in the decay chain below.

$$^{149}_{60}\text{Nd} \xrightarrow[1.72\ \text{hr}]{\beta^-} {}^{149}_{61}\text{Pm} \xrightarrow[53.1\ \text{hr}]{\beta^-} {}^{149}_{62}\text{Sm} \quad \text{(stable)}$$

For the purpose of examining the behavior of samarium-149, the 1.73 hour half-life of neodymium-149 is sufficiently shorter than the 53.1 hour value for promethium-149 that the promethium-149 may be considered as if it were formed directly from fission. This assumption, and neglecting the small amount of promethium burnup, allows the situation to be described as follows.

Rate of change of ^{149}Pm = yield from fission - decay ^{149}Pm concentration

therefore:

$$\frac{dN_{Pm}}{dt} = \gamma_{Pm} \Sigma_f^{fuel} \phi - \lambda_{Pm} N_{Pm}$$

where:

N_{Pm} = ^{149}Pm concentration

γ_{Pm} = ^{149}Pm fission yield

λ_{Pm} = decay constant for ^{149}Pm

Solving for the equilibrium value of promethium-149 gives the following.

$$N_{Pm}(eq) = \frac{\gamma_{Pm} \Sigma_f^{fuel} \phi}{\lambda_{Pm}}$$

The rate of samarium-149 formation is described as follows.

Rate of change of ^{149}Sm = yield from fission + ^{149}Pm decay - ^{149}Sm burnup

therefore:

$$\frac{dN_{Sm}}{dt} = \gamma_{Sm} \Sigma_f^{fuel} \phi + \lambda_{Pm} N_{Pm} - N_{Sm} \sigma_a^{Sm} \phi$$

where:

N_{Sm} = ^{149}Sm concentration

γ_{Sm} = ^{149}Sm fission yield

σ_a^{Sm} = microscopic absorption cross section of ^{149}Sm

The fission yield of samarium-149, however, is nearly zero; therefore, the equation becomes the following.

$$\frac{dN_{Sm}}{dt} = \lambda_{Pm} N_{Pm} - N_{Sm} \sigma_a^{Sm} \phi$$

Solving this equation for the equilibrium concentration of samarium-149 and substituting $\gamma_{Pm}\Sigma_f^{fuel} \phi / \lambda_{Pm}$ for $N_{Pm}(eq)$ yields the following.

$$N_{Sm}(eq) = \frac{\gamma_{Pm} \Sigma_f^{fuel}}{\sigma_a^{Sm}}$$

This expression for equilibrium samarium-149 concentration during reactor operation illustrates that equilibrium samarium-149 concentration is independent of neutron flux and power level. The samarium concentration will undergo a transient following a power level change, but it will return to its original value.

Samarium-149 Response to Reactor Shutdown

Since the neutron flux drops to essentially zero after reactor shutdown, the rate of samarium-149 production becomes the following.

$$\frac{dN_{Sm}}{dt} = \lambda_{Pm} N_{Pm}$$

Because samarium-149 is not radioactive and is not removed by decay, it presents problems somewhat different from those encountered with xenon-135, as illustrated in Figure 7. The equilibrium concentration and the poisoning effect build to an equilibrium value during reactor operation. This equilibrium is reached in approximately 20 days (500 hours), and since samarium-149 is stable, the concentration remains essentially constant during reactor operation. When the reactor is shutdown, the samarium-149 concentration builds up as a result of the decay of the accumulated promethium-149. The buildup of samarium-149 after shutdown depends upon the power level before shutdown. Samarium-149 does not peak as xenon-135 does, but increases slowly to a maximum value as shown in Figure 7. After shutdown, if the reactor is then operated at power, samarium-149 is burned up and its concentration returns to the equilibrium value. Samarium poisoning is minor when compared to xenon poisoning. Although samarium-149 has a constant poisoning effect during long-term sustained operation, its behavior during initial startup and during post-shutdown and restart periods requires special considerations in reactor design.

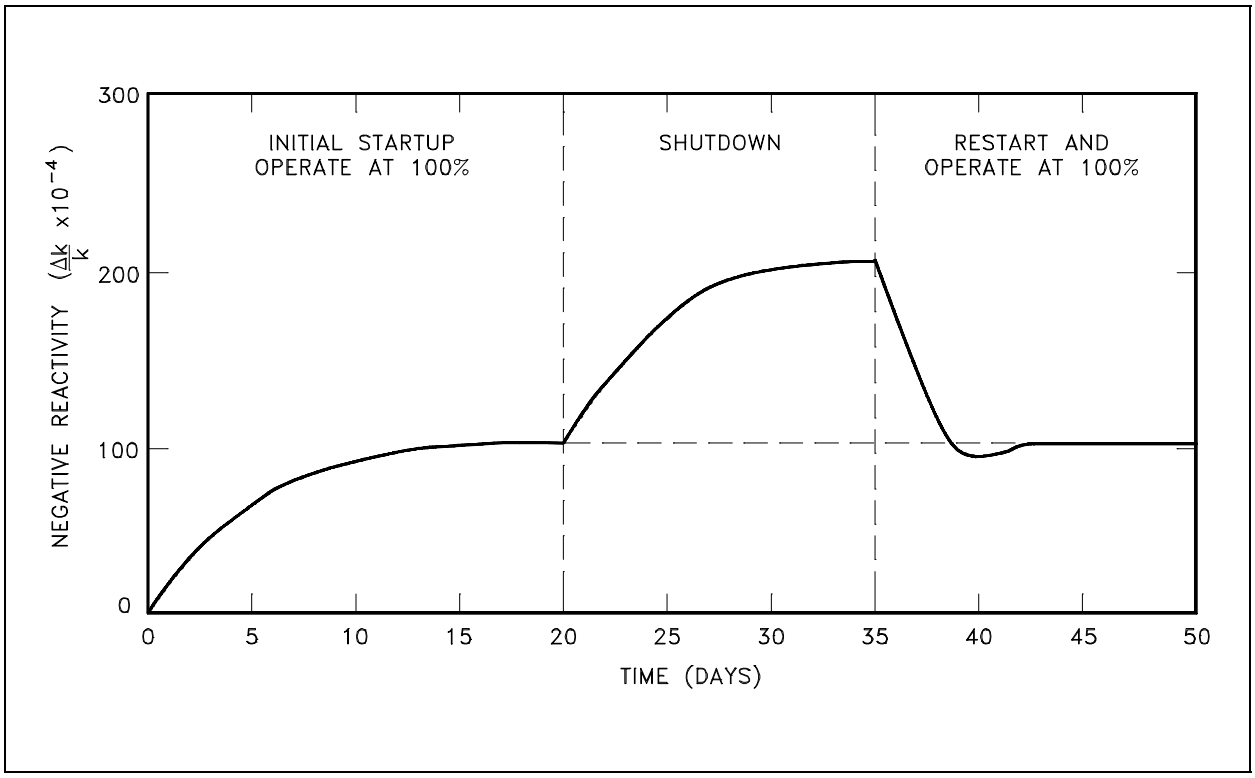

Figure 7 Behavior of Samarium-149 in a Typical Light Water Reactor

The xenon-135 and samarium-149 mechanisms are dependent on their very large thermal neutron cross sections and only affect thermal reactor systems. In fast reactors, neither these nor any other fission products have a major poisoning influence.

Other Neutron Poisons

There are numerous other fission products that, as a result of their concentration and thermal neutron absorption cross section, have a poisoning effect on reactor operation. Individually, they are of little consequence, but "lumped" together they have a significant impact. These are often characterized as "lumped fission product poisons" and accumulate at an average rate of 50 barns per fission event in the reactor.

In addition to fission product poisons, other materials in the reactor decay to materials that act as neutron poisons. An example of this is the decay of tritium to helium-3. Since tritium has a half-life of 12.3 years, normally this decay does not significantly affect reactor operations because the rate of decay of tritium is so slow. However, if tritium is produced in a reactor and then allowed to remain in the reactor during a prolonged shutdown of several months, a

sufficient amount of tritium may decay to helium-3 to add a significant amount of negative reactivity. Any helium-3 produced in the reactor during a shutdown period will be removed during subsequent operation by a neutron-proton reaction.

Summary

The important information in this chapter is summarized below.

Samarium and Other Fission Product Poisons Summary

- Samarium-149 is produced directly from fission and from the decay of promethium-149 during reactor operation. Samarium-149 is removed from the core by neutron absorption.
- The equation for equilibrium samarium-149 concentration is stated below.

$$N_{Sm}(eq) = \frac{\gamma_{Pm}\,\Sigma_f^{fuel}}{\sigma_a^{Sm}}$$

- The equilibrium samarium-149 concentration is independent of power level.
- Following a reactor shutdown, the samarium-149 concentration increases due to the decay of the promethium-149 inventory of the core and the loss of the burnup factor.
- If the reactor is restarted following a shutdown, the samarium-149 concentration decreases as samarium is burned up and returns to its equilibrium operating value.
- Helium-3 will become a significant neutron poison if significant amounts of tritium are left in a reactor during a shutdown period that lasts longer than several months.

CONTROL RODS

Most reactors contain control rods made of neutron absorbing materials that are used to adjust the reactivity of the core. Control rods can be designed and used for coarse control, fine control, or fast shutdowns.

EO 5.1 DESCRIBE the difference between a "grey" neutron absorbing material and a "black" neutron absorbing material.

EO 5.2 EXPLAIN why a "grey" neutron absorbing material may be preferable to a "black" neutron absorbing material for use in control rods.

EO 5.3 EXPLAIN why resonance absorbers are sometimes preferred over thermal absorbers as a control rod material.

EO 5.4 DEFINE the following terms:

a. Integral control rod worth
b. Differential control rod worth

EO 5.5 DESCRIBE the shape of a typical differential control rod worth curve and explain the reason for the shape.

EO 5.6 DESCRIBE the shape of a typical integral control rod worth curve and explain the reason for the shape.

EO 5.7 Given an integral or differential control rod worth curve, CALCULATE the reactivity change due to a control rod movement between two positions.

EO 5.8 Given differential control rod worth data, PLOT differential and integral control rod worth curves.

Selection of Control Rod Materials

Rods of neutron-absorbing material are installed in most reactors to provide precise, adjustable control of reactivity. These rods are able to be moved into or out of the reactor core and typically contain elements such as silver, indium, cadmium, boron, or hafnium.

The material used for the control rods varies depending on reactor design. Generally, the material selected should have a good absorption cross section for neutrons and have a long lifetime as an absorber (not burn out rapidly). The ability of a control rod to absorb neutrons can be adjusted during manufacture. A control rod that is referred to as a "black" absorber absorbs essentially all incident neutrons. A "grey" absorber absorbs only a part of them. While it takes more grey rods than black rods for a given reactivity effect, the grey rods are often preferred because they cause smaller depressions in the neutron flux and power in the vicinity of the rod. This leads to a flatter neutron flux profile and more even power distribution in the core.

If grey rods are desired, the amount of material with a high absorption cross section that is loaded in the rod is limited. Material with a very high absorption cross section may not be desired for use in a control rod, because it will burn out rapidly due to its high absorption cross section. The same amount of reactivity worth can be achieved by manufacturing the control rod from material with a slightly lower cross section and by loading more of the material. This also results in a rod that does not burn out as rapidly.

Another factor in control rod material selection is that materials that resonantly absorb neutrons are often preferred to those that merely have high thermal neutron absorption cross sections. Resonance neutron absorbers absorb neutrons in the epithermal energy range. The path length traveled by the epithermal neutrons in a reactor is greater than the path length traveled by thermal neutrons. Therefore, a resonance absorber absorbs neutrons that have their last collision farther (on the average) from the control rod than a thermal absorber. This has the effect of making the area of influence around a resonance absorber larger than around a thermal absorber and is useful in maintaining a flatter flux profile.

Types of Control Rods

There are several ways to classify the types of control rods. One classification method is by the purpose of the control rods. Three purposes of control rods are listed below.

Shim rods - used for coarse control and/or to remove reactivity in relatively large amounts.

Regulating rods - used for fine adjustments and to maintain desired power or temperature.

Safety rods - provide a means for very fast shutdown in the event of an unsafe condition. Addition of a large amount of negative reactivity by rapidly inserting the safety rods is referred to as a "scram" or "trip."

Not all reactors have different control rods to serve the purposes mentioned above. Depending upon the type of reactor and the controls necessary, it is possible to use dual-purpose or even triple-purpose rods. For example, consider a set of control rods that can insert enough reactivity to be used as shim rods. If the same rods can be operated at slow speeds, they will function as regulating rods. Additionally, these same rods can be designed for rapid insertion, or scram. These rods serve a triple function yet meet other specifications such as precise control, range of control, and efficiency.

Control Rod Effectiveness

The effectiveness of a control rod depends largely upon the value of the ratio of the neutron flux at the location of the rod to the average neutron flux in the reactor. The control rod has maximum effect (inserts the most negative reactivity) if it is placed in the reactor where the flux is maximum. If a reactor has only one control rod, the rod should be placed in the center of the reactor core. The effect of such a rod on the flux is illustrated in Figure 8.

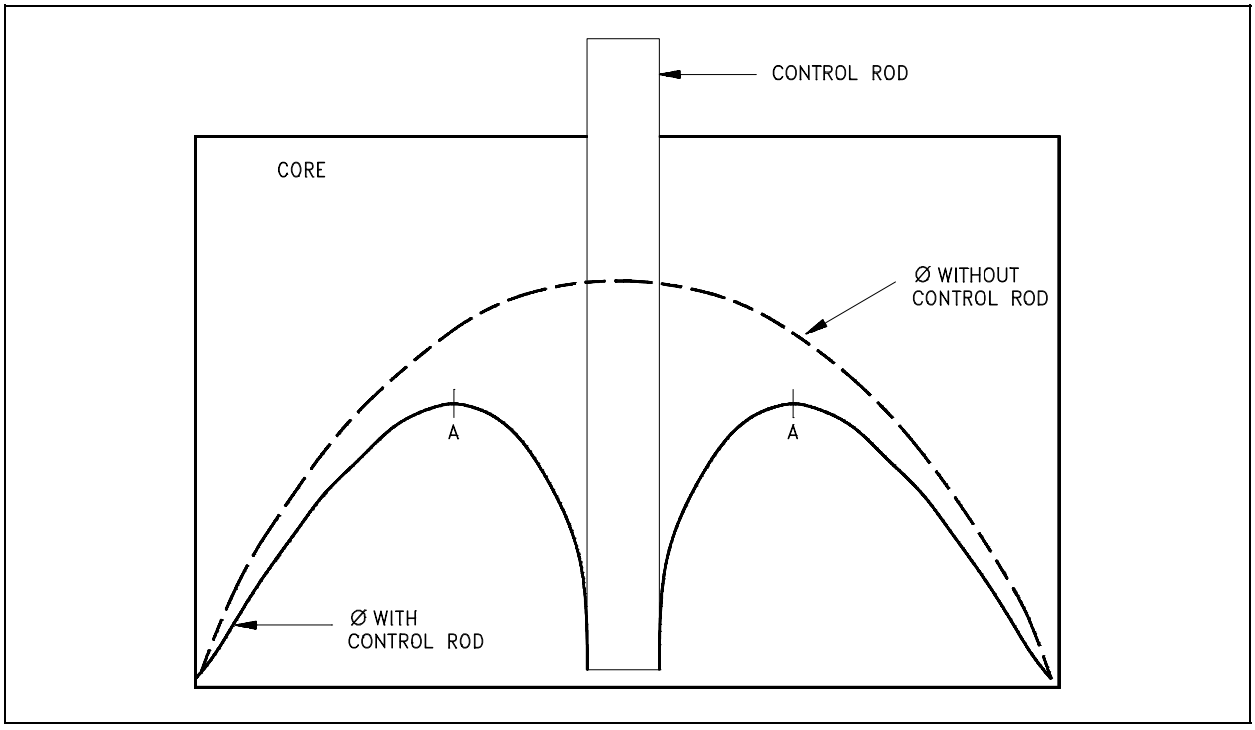

Figure 8 Effect of Control Rod on Radial Flux Distribution

If additional rods are added to this simple reactor, the most effective location is where the flux is maximum, that is, at point A. Numerous control rods are required for a reactor that has a large amount of excess reactivity (that amount of reactivity in excess of that needed to be critical). The exact amount of reactivity that each control rod inserts depends upon the reactor design. The change in reactivity caused by control rod motion is referred to as control rod worth.

Integral and Differential Control Rod Worth

The exact effect of control rods on reactivity can be determined experimentally. For example, a control rod can be withdrawn in small increments, such as 0.5 inch, and the change in reactivity can be determined following each increment of withdrawal. By plotting the resulting reactivity versus the rod position, a graph similar to Figure 9 is obtained. The graph depicts integral control rod worth over the full range of withdrawal. The *integral control rod worth* is the total reactivity worth of the rod at that particular degree of withdrawal and is usually defined to be the greatest when the rod is fully withdrawn.

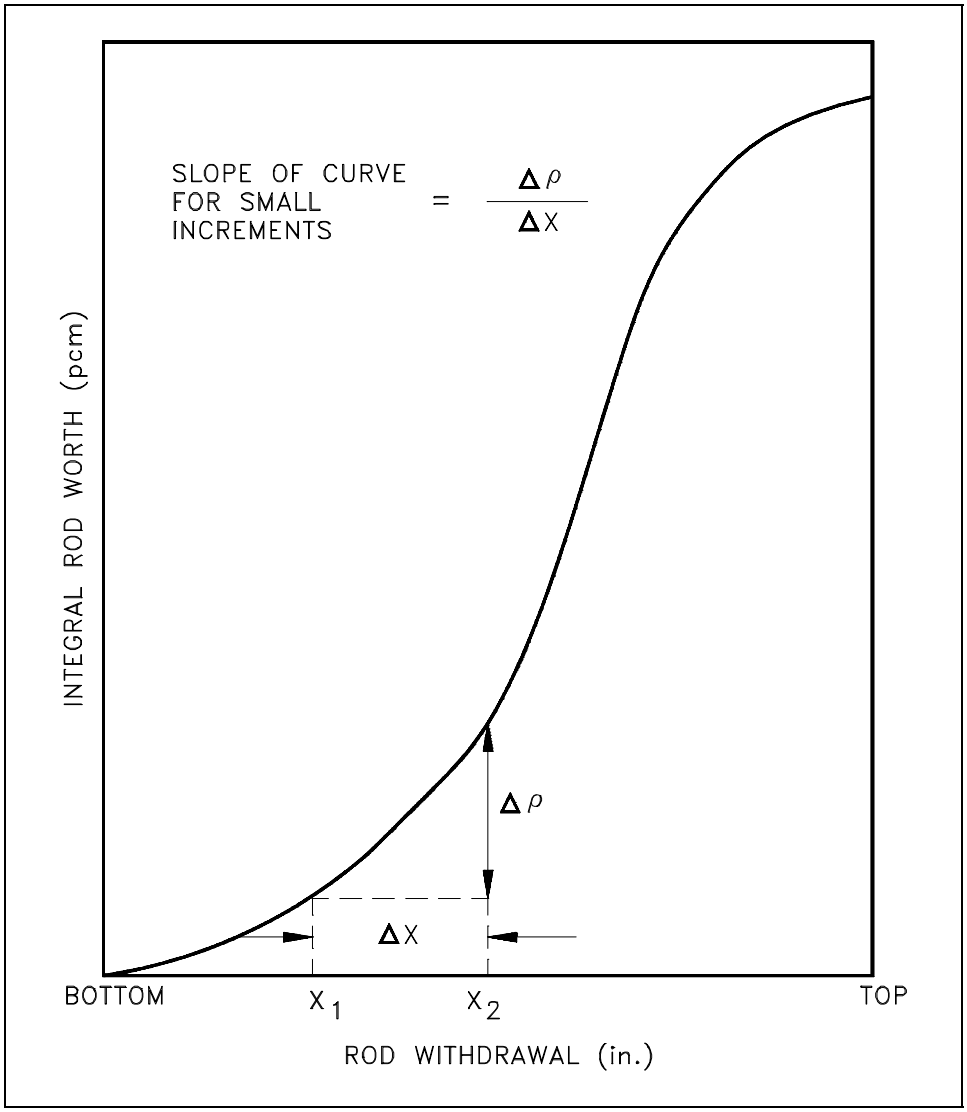

Figure 9 Integral Control Rod Worth

The slope of the curve ($\Delta\rho/\Delta x$), and therefore the amount of reactivity inserted per unit of withdrawal, is greatest when the control rod is midway out of the core. This occurs because the area of greatest neutron flux is near the center of the core; therefore, the amount of change in neutron absorption is greatest in this area. If the slope of the curve for integral rod worth in Figure 9 is taken, the result is a value for rate of change of control rod worth as a function of control rod position. A plot of the slope of the integral rod worth curve, also called the differential control rod worth, is shown in Figure 10. At the bottom of the core, where there are few neutrons, rod movement has little effect so the change in rod worth per inch varies little. As the rod approaches the center of the core its effect becomes greater, and the change in rod worth per inch is greater. At the center of the core the differential rod worth is greatest and varies little with rod motion. From the center of the core to the top, the rod worth per inch is basically the inverse of the rod worth per inch from the center to the bottom.

Differential control rod worth is the reactivity change per unit movement of a rod and is normally expressed as ρ/inch, $\Delta k/k$ per inch, or pcm/inch. The integral rod worth at a given withdrawal is merely the summation of all the differential rod worths up to that point of withdrawal. It is also the area under the differential rod worth curve at any given withdrawal position.

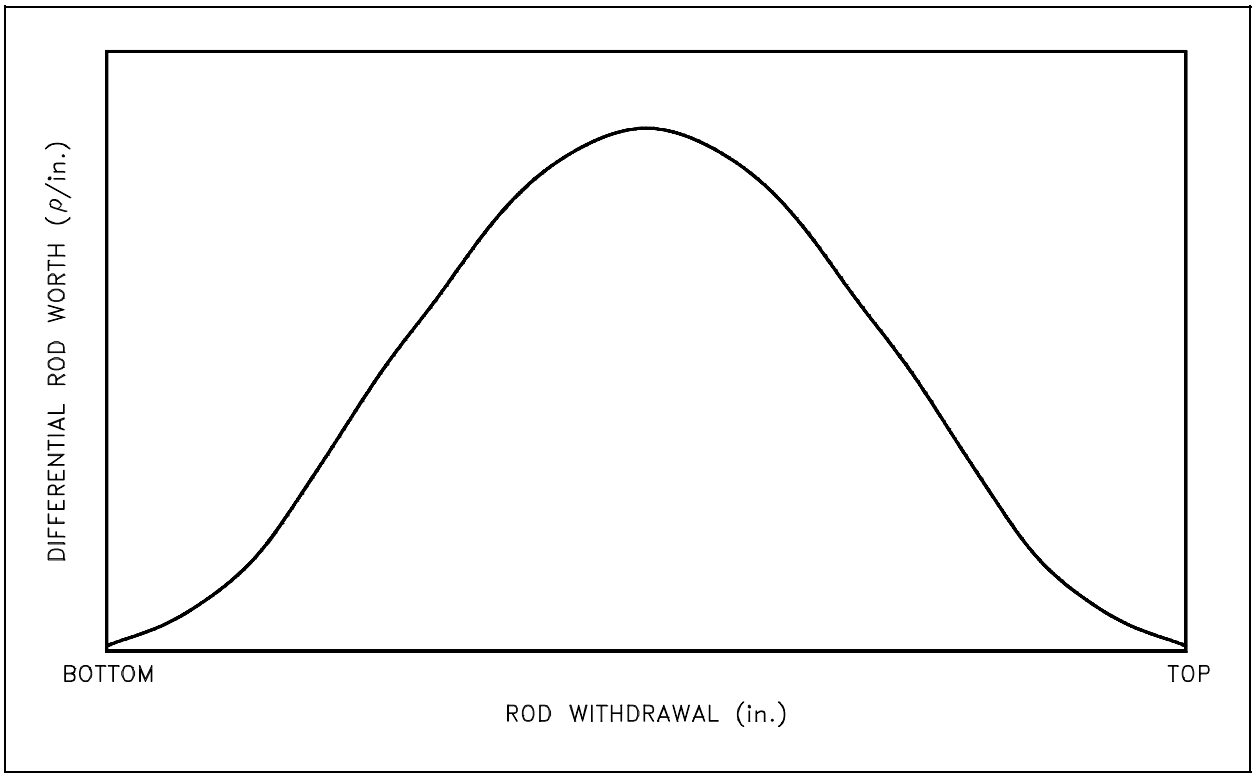

Figure 10 Differential Control Rod Worth

The following exercises are intended to reinforce an understanding of the concepts of integral and differential rod worth.

Example 1:

Using the integral rod worth curve provided in Figure 11, find the reactivity inserted by moving the rod from 12 inches withdrawn out to 18 inches withdrawn.

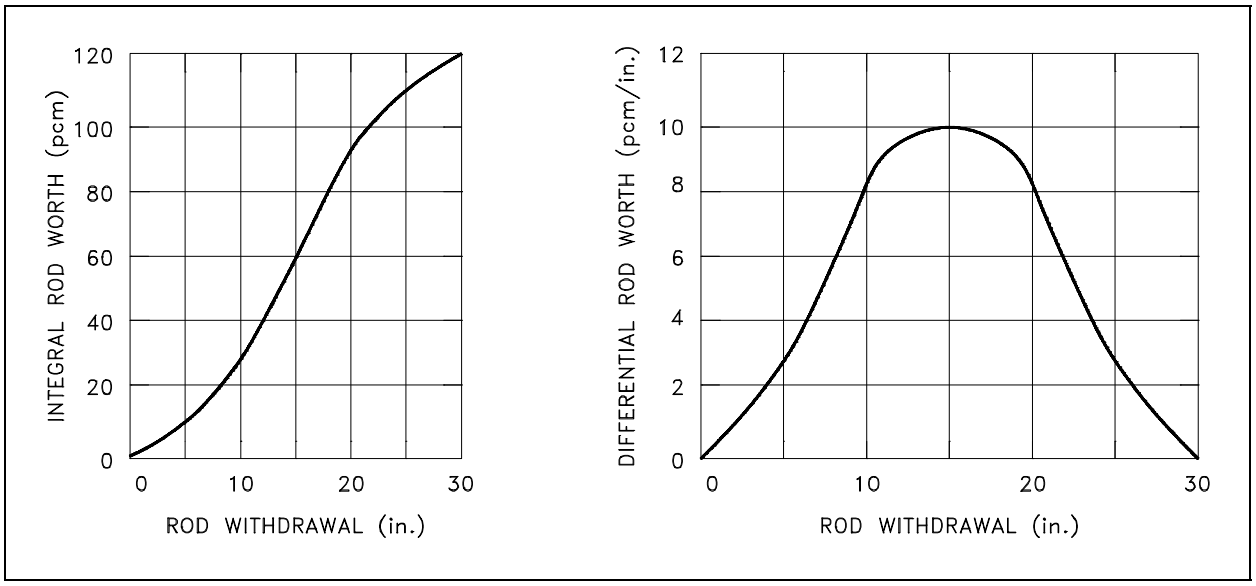

Figure 11 Rod Worth Curves for Example Problems

Solution:

The integral rod worth at 12 inches is 40 pcm and the integral rod worth at 18 inches is 80 pcm.

$$\Delta\rho = \rho_{final} - \rho_{initial}$$

$$= \rho_{18} - \rho_{12}$$

$$= 80 \text{ pcm} - 40 \text{ pcm}$$

$$= 40 \text{ pcm}$$

Example 2:

Using the differential rod worth curve provided in Figure 11, calculate the reactivity inserted by moving the rod from 10 inches withdrawn to 6 inches withdrawn.

Solution:

The solution is basically given by the area under the curve for the interval. The answers obtained in the following approximation may vary slightly depending upon the degree of approximation.

Method 1. Treating the range from 10 inches to 6 inches as a trapezoid, that is, taking the end values of pcm/inch and multiplying their average by the 4 inches moved yields the following.

$$\left(\frac{8\ \frac{\text{pcm}}{\text{inch}} + 3\ \frac{\text{pcm}}{\text{inch}}}{2}\right)(4\ \text{inches}) = -22\ \text{pcm}$$

This is negative because the rod was inserted.

Method 2. Using the central value of rod position at 8 inches yields an average rod worth of 5.5 pcm/inch. Multiplying by the 4 inches of rod travel yields the answer.

(5.5 pcm/in.)(4 in.) = -22 pcm

Method 3. Breaking the rod travel total into two parts (10 inches to 8 inches and 8 inches to 6 inches) yields:

$$\left(\frac{8\ \frac{\text{pcm}}{\text{inch}} + 5.5\ \frac{\text{pcm}}{\text{inch}}}{2}\right)(-2\ \text{inches}) = -13.5\ \text{pcm}$$

$$\left(\frac{5.5\ \frac{\text{pcm}}{\text{inch}} + 3\ \frac{\text{pcm}}{\text{inch}}}{2}\right)(-2\ \text{inches}) = -8.5\ \text{pcm}$$

(- 13.5 pcm) + (- 8.5 pcm) = -22 pcm

In this example the various approximations used did not cause any difference because the problem deals with a section of the curve with an approximately constant slope. To obtain the value over the interval between 8 inches and 20 inches, however, would require the use of several subintervals (as in the last approximation) to obtain an accurate answer.

Example 3:

For the differential rod worth data given below, construct differential and integral rod worth curves.

Interval (inches)	Reactivity Inserted (pcm)
0 to 2	10
2 to 4	20
4 to 6	40
6 to 8	60
8 to 10	60
10 to 12	40
12 to 14	20
14 to 16	10

Solution:

Differential rod worth:

For each interval, the number of pcm/inch must be determined. For example, in the first interval (0 inches to 2 inches), 10 pcm is added. Therefore, the differential rod worth equals an average 5 pcm/inch. This value of differential rod worth is plotted at the center of each interval. The center of the interval 0 inches to 2 inches is 1 inch. The values of pcm/inch for each interval are then listed as shown below and plotted on Figure 12.

Interval Center	pcm/inch
1	5
3	10
5	20
7	30
9	30
11	20
13	10
15	5

Integral rod worth:

To plot the integral rod worth, merely develop a cumulative total of the reactivity added after each interval and plot the summed reactivity insertion vs. rod position as shown in Figure 12.

Interval Endpoint	Summed Reactivity
2	10
4	30
6	70
8	130
10	190
12	230
14	250
16	260

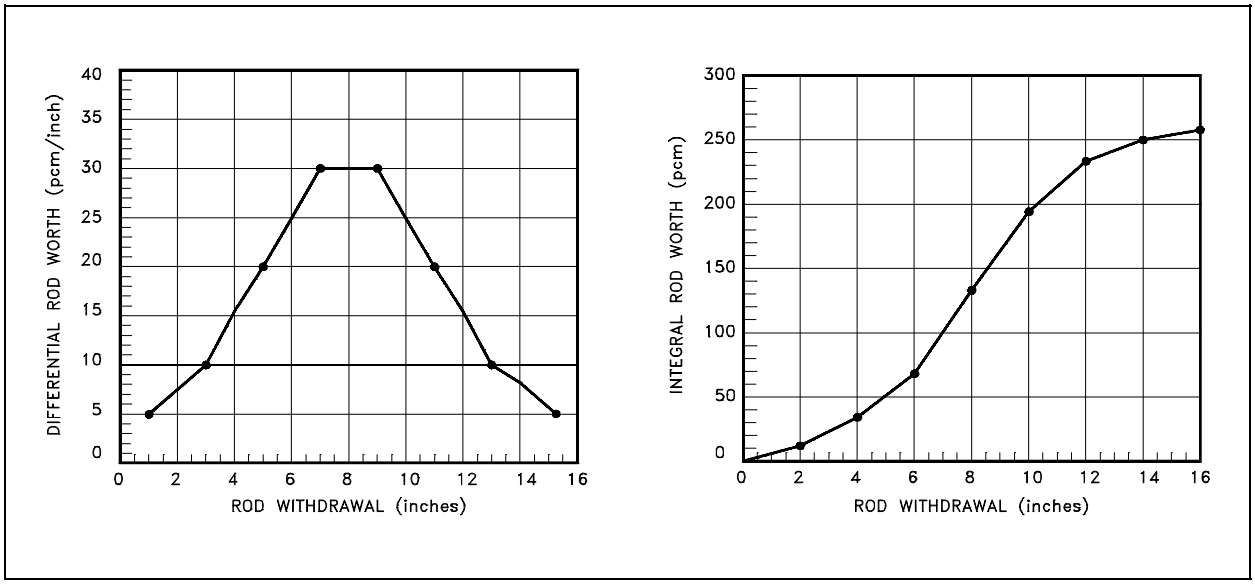

Figure 12 Rod Worth Curves From Example 3

If an integral rod worth curve is supplied, a differential rod worth curve can be generated from the integral rod worth data. Merely select a convenient interval of rod withdrawal, such as 1 inch or 2 inches. Then, determine from the curve the amount of reactivity added for each constant interval of rod withdrawal. A plot of this reactivity addition versus rod withdrawal represents differential rod worth.

Rod Control Mechanisms

The control rod insertion rates on a scram are designed to be sufficient to protect the reactor against damage in all transients that are expected to occur during the life of the reactor.

During normal rod motion, the control rods must be able to move rapidly enough to compensate for the most rapid rate at which positive reactivity is expected to build within the reactor in order to provide positive control. The transient that is normally considered when setting this minimum rod speed is the burnout of maximum peak xenon while at full power. Xenon burnout is usually the most rapid, non-accident transient expected. The maximum rod speed is normally limited in order to reduce the severity of an accident involving the continuous withdrawal of control rods.

Summary

The important information in this chapter is summarized on the following page.

Control Rods Summary

- A black neutron-absorbing material absorbs essentially all incident neutrons. A grey neutron-absorbing material absorbs only part of the incident neutrons.
- A grey neutron-absorbing material may be preferable to a black neutron-absorbing material in the construction of control rods because the grey absorber causes smaller depressions in neutron flux and power in the vicinity of the rod.
- Resonance absorbers are sometimes preferred to thermal absorbers as control rod materials because they have a larger area of influence and result in a flatter flux profile.
- Integral control rod worth is the total reactivity worth of the control rod at a particular degree of withdrawal from the core.
- Differential control rod worth is the reactivity change per unit movement of a control rod.
- The typical differential control rod worth curve has a bell shape. It has very low values at the top and bottom of the core and a maximum value at the center of the core. The curve has this shape because rod worth is related to neutron flux, and flux is highest in the center of the core.
- The typical integral control rod worth curve has an "S" shape. It has a relatively flat slope at the top and bottom of the core and a maximum slope at the center of the core.
- Integral or differential control rod worth curves can be used to determine the reactivity change due to a control rod movement between two positions.
- Integral or differential control rod worth curves can be plotted based on measured control rod worth data.

Department of Energy
Fundamentals Handbook

NUCLEAR PHYSICS AND REACTOR THEORY

Module 4

Reactor Theory (Reactor Operations)

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TERMINAL OBJECTIVE

1.0 Given the necessary information and equations, **EXPLAIN** how subcritical multiplication occurs.

ENABLING OBJECTIVES

1.1 **DEFINE** the following terms:

a. Subcritical multiplication
b. Subcritical multiplication factor

1.2 Given a neutron source strength and a subcritical system of known k_{eff}, **CALCULATE** the steady-state neutron level.

1.3 Given an initial count rate and k_{eff}, **CALCULATE** the final count rate that will result from the addition of a known amount of reactivity.

1.4 Given count rates vs. the parameter being adjusted, **ESTIMATE** the value of the parameter at which the reactor will become critical through the use of a 1/M plot.

TERMINAL OBJECTIVE

2.0 Given the necessary information and equations, **DESCRIBE** how power changes in a reactor that is near criticality.

ENABLING OBJECTIVES

2.1 **DEFINE** the following terms:

a. Reactor period
b. Doubling time
c. Reactor startup rate

2.2 **DESCRIBE** the relationship between the delayed neutron fraction, average delayed neutron fraction, and effective delayed neutron fraction.

2.3 **WRITE** the period equation and **IDENTIFY** each symbol.

2.4 Given the reactivity of the core and values for the effective average delayed neutron fraction and decay constant, **CALCULATE** the reactor period and the startup rate.

2.5 Given the initial power level and either the doubling or halving time, **CALCULATE** the power at any later time.

2.6 Given the initial power level and the reactor period, **CALCULATE** the power at any later time.

2.7 **EXPLAIN** what is meant by the terms prompt drop and prompt jump.

2.8 **DEFINE** the term prompt critical.

2.9 **DESCRIBE** reactor behavior during the prompt critical condition.

2.10 **EXPLAIN** the use of measuring reactivity in units of dollars.

TERMINAL OBJECTIVE

3.0 Without references, **EXPLAIN** the concepts concerning reactor startup, operation, and shutdown.

ENABLING OBJECTIVES

3.1 **EXPLAIN** why a startup neutron source may be required for a reactor.

3.2 **LIST** four variables typically involved in a reactivity balance.

3.3 **EXPLAIN** how a reactivity balance may be used to predict the conditions under which the reactor will become critical.

3.4 **LIST** three methods used to shape or flatten the core power distribution.

3.5 **DESCRIBE** the concept of power tilt.

3.6 **DEFINE** the term shutdown margin.

3.7 **EXPLAIN** the rationale behind the one stuck rod criterion.

3.8 **IDENTIFY** five changes that will occur during and after a reactor shutdown that will affect the reactivity of the core.

3.9 **EXPLAIN** why decay heat is present following reactor operation.

3.10 **LIST** three variables that will affect the amount of decay heat present following reactor shutdown.

3.11 **ESTIMATE** the approximate amount of decay heat that will exist one hour after a shutdown from steady state conditions.

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SUBCRITICAL MULTIPLICATION

Subcritical multiplication is the phenomenon that accounts for the changes in neutron flux that takes place in a subcritical reactor due to reactivity changes. It is important to understand subcritical multiplication in order to understand reactor response to changes in conditions.

EO 1.1 DEFINE the following terms:

a. Subcritical multiplication
b. Subcritical multiplication factor

EO 1.2 Given a neutron source strength and a subcritical system of known k_{eff}, CALCULATE the steady-state neutron level.

EO 1.3 Given an initial count rate and k_{eff}, CALCULATE the final count rate that will result from the addition of a known amount of reactivity.

EO 1.4 Given count rates vs. the parameter being adjusted, ESTIMATE the value of the parameter at which the reactor will become critical through the use of a 1/M plot.

Subcritical Multiplication Factor

When a reactor is in a shutdown condition, neutrons are still present to interact with the fuel. These source neutrons are produced by a variety of methods that were discussed in Module 2. If neutrons and fissionable material are present in the reactor, fission will take place. Therefore, a reactor will always be producing a small number of fissions even when it is shutdown.

Consider a reactor in which k_{eff} is 0.6. If 100 neutrons are suddenly introduced into the reactor, these 100 neutrons that start the current generation will produce 60 neutrons (100 x 0.6) from fission to start the next generation. The 60 neutrons that start the second generation will produce 36 neutrons (60 x 0.6) to start the third generation. The number of neutrons produced by fission in subsequent generations due to the introduction of 100 source neutrons into the reactor is shown below.

Generation	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th
Neutrons	100	60	36	22	13	8	5	3	2	1	0	0

Because the reactor is subcritical, neutrons introduced in the reactor will have a decreasing effect on each subsequent generation. The addition of source neutrons to the reactor containing fissionable material has the effect of maintaining a much higher stable neutron level due to the fissions occurring than the neutron level that would result from the source neutrons alone. The effects of adding source neutrons at a rate of 100 neutrons per generation to a reactor with a k_{eff} of 0.6 are shown below.

Generation	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th
	100	60	36	22	13	8	5	3	2	1	0	0
		100	60	36	22	13	8	5	3	2	1	0
			100	60	36	22	13	8	5	3	2	1
				100	60	36	22	13	8	5	3	2
					100	60	36	22	13	8	5	3
						100	60	36	22	13	8	5
							100	60	36	22	13	8
								100	60	36	22	13
									100	60	36	22
										100	60	36
											100	60
												100
Total n	100	160	196	218	231	239	244	247	249	250	250	...

A neutron source strength of 100 neutrons per generation will result in 250 neutrons per generation being produced from a combination of sources and fission in a shutdown reactor with a k_{eff} of 0.6. If the value of k_{eff} were higher, the source neutrons would produce a greater number of fission neutrons and their effects would be felt for a larger number of subsequent generations after their addition to the reactor.

The effect of fissions in the fuel increasing the effective source strength of a reactor with a k_{eff} of less than one is *subcritical multiplication*. For a given value of k_{eff} there exists a *subcritical multiplication factor* (M) that relates the source level to the steady-state neutron level of the core. If the value of k_{eff} is known, the amount that the neutron source strength will be multiplied (M) can easily be determined by Equation (4-1).

$$M = \frac{1}{1 - k_{eff}} \qquad (4\text{-}1)$$

Example:

Calculate the subcritical multiplication factors for the following values of k_{eff}.

1) $k_{eff} = 0.6$
2) $k_{eff} = 0.986$

Solution:

1)

$$M = \frac{1}{1 - k_{eff}}$$

$$= \frac{1}{1 - 0.6}$$

$$= 2.5$$

2)

$$M = \frac{1}{1 - k_{eff}}$$

$$= \frac{1}{1 - 0.986}$$

$$= 71.4$$

The example above illustrates that the subcritical multiplication factor will increase as positive reactivity is added to a shutdown reactor, increasing the value of k_{eff}. If the source strength of this reactor were 1000 neutrons/sec, the neutron level would increase from 2500 neutrons/second at a k_{eff} of 0.6 to a neutron level of 71,400 neutrons/sec at a k_{eff} of 0.986.

Effect of Reactivity Changes on Subcritical Multiplication

In a subcritical reactor, the neutron level is related to the source strength by Equation (4-2).

$$N = (S)\ (M) \qquad (4\text{-}2)$$

where:

N = neutron level
S = neutron source strength
M = subcritical multiplication factor

If the term M in Equation (4-2) is replaced by the expression $1/1-k_{eff}$ from Equation (4-1), the following expression results.

$$N = S\left(\frac{1}{1 - k_{eff}}\right) \qquad (4\text{-}3)$$

Example:

A reactor contains a neutron source that produces 110,000 neutrons per second. The reactor has a k_{eff} of 0.986. Calculate the stable total neutron production rate in the reactor.

Solution:

The neutron production rate is calculated using Equation (4-3).

$$N = S\left(\frac{1}{1 - k_{eff}}\right)$$

$$= 110{,}000\ \frac{\text{neutrons}}{\text{second}}\left(\frac{1}{1 - 0.986}\right)$$

$$= 7.86 \times 10^{6}\ \frac{\text{neutrons}}{\text{second}}$$

To this point it has been necessary to know the neutron source strength of the reactor in order to use the concept of subcritical multiplication. In most reactors the actual strength of the neutron sources is difficult, if not impossible, to determine. Even though the actual source strength may not be known, it is still possible to relate the change in reactivity to a change in neutron level.

Consider a reactor at two different times when k_{eff} is two different values, k_1 and k_2. The neutron level at each time can be determined based on the neutron source strength and the subcritical multiplication factor using Equation (4-3).

$$N_1 = S\left(\frac{1}{1 - k_1}\right) \qquad N_2 = S\left(\frac{1}{1 - k_2}\right)$$

The equation for N_1 can be divided by the equation for N_2.

$$\frac{N_1}{N_2} = \frac{S\left(\frac{1}{1 - k_1}\right)}{S\left(\frac{1}{1 - k_2}\right)}$$

$$\frac{N_1}{N_2} = \frac{1 - k_2}{1 - k_1}$$

Because the source strength appears in both the numerator and denominator, it cancels out of the equation. Therefore, the neutron level at any time can be determined based on the neutron level present at any other time provided the values of k_{eff} or reactivity for both times are known.

The neutron level in a shutdown reactor is typically monitored using instruments that measure the neutron leakage out of the reactor. The neutron leakage is proportional to the neutron level in the reactor. Typical units for displaying the instrument reading are counts per second (cps). Because the instrument count rate is proportional to the neutron level, the above equation can be restated as shown in Equation (4-4).

$$\frac{CR_1}{CR_2} = \frac{1 - k_2}{1 - k_1} \qquad (4\text{-}4)$$

where:

CR_1 = count rate at time 1
CR_2 = count rate at time 2
k_1 = k_{eff} at time 1
k_2 = k_{eff} at time 2

Equation (4-4) is very useful during the shutdown operation of a reactor. Before adding positive reactivity to a reactor, it is possible to predict the effect the reactivity addition will have on the neutron level.

Example:

A reactor that has a reactivity of -1000 pcm has a count rate of 42 counts per second (cps) on the neutron monitoring instrumentation. Calculate what the neutron level should be after a positive reactivity insertion of 500 pcm from the withdrawal of control rods.

Solution:

Step 1: Determine the initial value of k_{eff} for the core.

$$k_1 = \frac{1}{1 - \rho_1}$$

$$= \frac{1}{1 - (-0.01000)}$$

$$= 0.9901$$

Step 2: Determine the final value of k_{eff} for the core. The final value of reactivity will be -500 pcm (-1000 + 500).

$$k_2 = \frac{1}{1 - \rho_2}$$

$$= \frac{1}{1 - (-0.00500)}$$

$$= 0.9950$$

Step 3: Use Equation (4-4) to determine the final count rate.

$$\frac{CR_1}{CR_2} = \frac{1 - k_2}{1 - k_1}$$

$$CR_2 = CR_1 \left(\frac{1 - k_1}{1 - k_2} \right)$$

$$= 42 \text{ cps} \left(\frac{1 - 0.9901}{1 - 0.9950} \right)$$

$$= 83 \text{ cps}$$

Notice from this example that the count rate doubled as the reactivity was halved (e.g., reactivity was changed from -1000 pcm to -500 pcm).

Use of 1/M Plots

Because the subcritical multiplication factor is related to the value of k_{eff}, it is possible to monitor the approach to criticality through the use of the subcritical multiplication factor. As positive reactivity is added to a subcritical reactor, k_{eff} will get nearer to one. As k_{eff} gets nearer to one, the subcritical multiplication factor (M) gets larger. The closer the reactor is to criticality, the faster M will increase for equal step insertions of positive reactivity. When the reactor becomes critical, M will be infinitely large. For this reason, monitoring and plotting M during an approach to criticality is impractical because there is no value of M at which the reactor clearly becomes critical.

Instead of plotting M directly, its inverse (1/M) is plotted on a graph of 1/M versus rod height.

$$M = \frac{1}{1 - k_{eff}}$$

$$\frac{1}{M} = 1 - k_{eff}$$

As control rods are withdrawn and k_{eff} approaches one and M approaches infinity, 1/M approaches zero. For a critical reactor, 1/M is equal to zero. A true 1/M plot requires knowledge of the neutron source strength. Because the actual source strength is usually unknown, a reference count rate is substituted, and the calculation of the factor 1/M is through the use of Equation (4-5).

$$\frac{1}{M} = \frac{CR_o}{CR} \qquad (4\text{-}5)$$

where:

1/M = inverse multiplication factor
CR_o = reference count rate
CR = current count rate

In practice, the reference count rate used is the count rate prior to the beginning of the reactivity change. The startup procedures for many reactors include instructions to insert positive reactivity in incremental steps with delays between the reactivity insertions to allow time for subcritical multiplication to increase the steady-state neutron population to a new, higher level and allow more accurate plotting of 1/M. The neutron population will typically reach its new steady-state value within 1-2 minutes, but the closer the reactor is to criticality, the longer the time will be to stabilize the neutron population.

Example:

Given the following rod withdrawal data, construct a 1/M plot and estimate the rod position when criticality would occur. The initial count rate on the nuclear instrumentation prior to rod withdrawal is 50 cps.

Rod Withdrawal (inches)	Count Rate (cps)
2	55
4	67
6	86
8	120
10	192
12	500

Solution:

Step 1: Calculate 1/M for each of the rod positions using equation (4-5). The reference count rate is 50 cps at a rod position of zero.

Rod Withdrawal (inches)	Count Rate (cps)	CR_o/CR
0	50	1
2	55	0.909
4	67	0.746
6	86	0.581
8	120	0.417
10	192	0.260
12	500	0.100

Step 2: Plotting these values, as shown in Figure 1, and extrapolating to a 1/M value of 0 reveals that the reactor will go critical at approximately 13 inches of rod withdrawal.

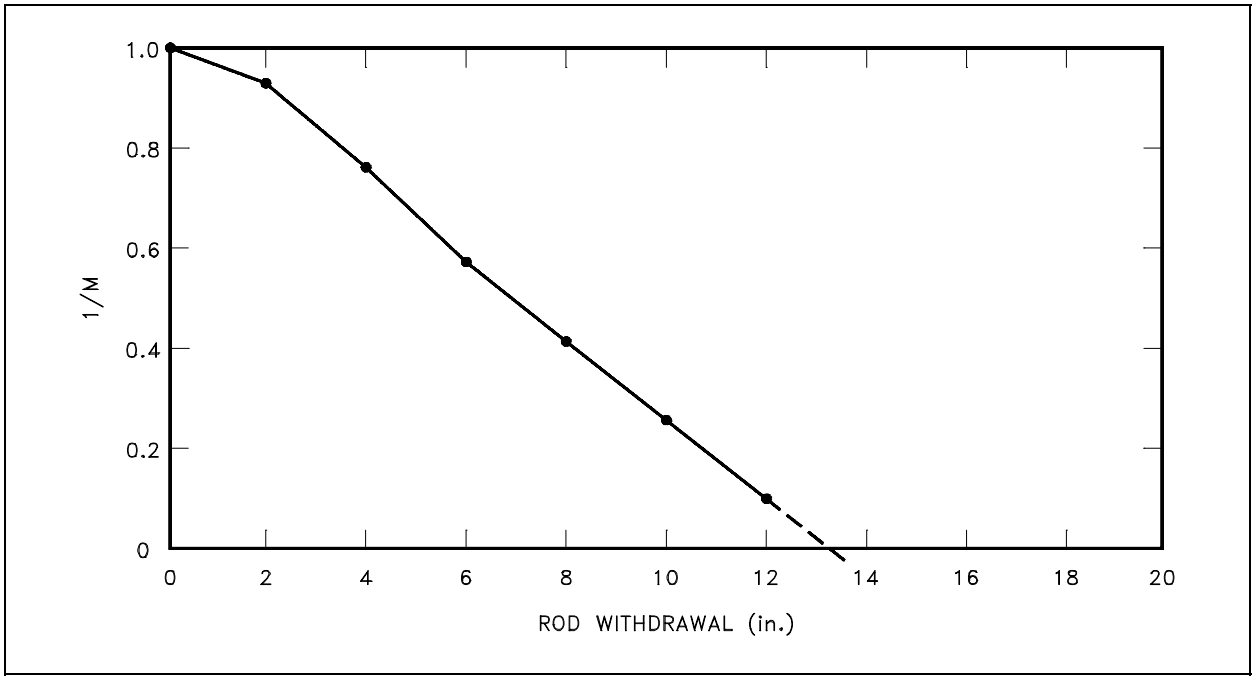

Figure 1 1/M Plot vs. Rod Withdrawal

Summary

The important information in this chapter is summarized below.

Subcritical Multiplication Summary

- Subcritical multiplication is the effect of fissions in the fuel increasing the effective source strength of a reactor with a k_{eff} less than one.
- Subcritical multiplication factor is the factor that relates the source level to the steady-state neutron level of the core.
- The steady-state neutron level of a subcritical reactor can be calculated based on the source strength and k_{eff} using Equation (4-3).

$$N = S\left(\frac{1}{1 - k_{eff}}\right)$$

- The count rate expected in a subcritical reactor following a change in reactivity can be calculated based on the initial count rate, initial k_{eff}, and amount of reactivity addition using Equation (4-4).

$$\frac{CR_1}{CR_2} = \frac{1 - k_2}{1 - k_1}$$

- 1/M plots can be used to predict the point of criticality.

REACTOR KINETICS

The response of neutron flux and reactor power to changes in reactivity is much different in a critical reactor than in a subcritical reactor. The reliance of the chain reaction on delayed neutrons makes the rate of change of reactor power controllable.

EO 2.1 DEFINE the following terms:

a. Reactor period
b. Doubling time
c. Reactor startup rate

EO 2.2 DESCRIBE the relationship between the delayed neutron fraction, average delayed neutron fraction, and effective delayed neutron fraction.

EO 2.3 WRITE the period equation and IDENTIFY each symbol.

EO 2.4 Given the reactivity of the core and values for the effective average delayed neutron fraction and decay constant, CALCULATE the reactor period and the startup rate.

EO 2.5 Given the initial power level and either the doubling or halving time, CALCULATE the power at any later time.

EO 2.6 Given the initial power level and the reactor period, CALCULATE the power at any later time.

EO 2.7 EXPLAIN what is meant by the terms prompt drop and prompt jump.

EO 2.8 DEFINE the term prompt critical.

EO 2.9 DESCRIBE reactor behavior during the prompt critical condition.

EO 2.10 EXPLAIN the use of measuring reactivity in units of dollars.

Reactor Period (τ)

The *reactor period* is defined as the time required for reactor power to change by a factor of "e," where "e" is the base of the natural logarithm and is equal to about 2.718. The reactor period is usually expressed in units of seconds. From the definition of reactor period, it is possible to develop the relationship between reactor power and reactor period that is expressed by Equation (4-6).

$$P = P_o \, e^{t/\tau} \tag{4-6}$$

where:

P = transient reactor power
P_o = initial reactor power
τ = reactor period (seconds)
t = time during the reactor transient (seconds)

The smaller the value of τ, the more rapid the change in reactor power. If the reactor period is positive, reactor power is increasing. If the reactor period is negative, reactor power is decreasing.

There are numerous equations used to express reactor period, but Equation (4-7) shown below, or portions of it, will be useful in most situations. The first term in Equation (4-7) is the prompt term and the second term is the delayed term.

$$\tau = \frac{\ell^*}{\rho} + \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff} \, \rho + \dot{\rho}} \tag{4-7}$$

where:

ℓ^* = prompt generation lifetime
$\bar{\beta}_{eff}$ = effective delayed neutron fraction
ρ = reactivity
λ_{eff} = effective delayed neutron precursor decay constant
$\dot{\rho}$ = rate of change of reactivity

Effective Delayed Neutron Fraction

Recall that β, the *delayed neutron fraction*, is the fraction of all fission neutrons that are born as delayed neutrons. The value of β depends upon the actual nuclear fuel used. As discussed in Module 1, the delayed neutron precursors for a given type of fuel are grouped on the basis of half-life. The following table lists the fractional neutron yields for each delayed neutron group of three common types of fuel.

TABLE 1
Delayed Neutron Fractions for Various Fuels

Group	Half-Life (sec)	Uranium-235	Uranium-238	Plutonium-239
1	55.6	0.00021	0.0002	0.00021
2	22.7	0.00141	0.0022	0.00182
3	6.22	0.00127	0.0025	0.00129
4	2.30	0.00255	0.0061	0.00199
5	0.61	0.00074	0.0035	0.00052
6	0.23	0.00027	0.0012	0.00027
TOTAL	-	0.00650	0.0157	0.00200

The term $\bar{\beta}$ (pronounced beta-bar) is the *average delayed neutron fraction*. The value of $\bar{\beta}$ is the weighted average of the total delayed neutron fractions of the individual types of fuel. Each total delayed neutron fraction value for each type of fuel is weighted by the percent of total neutrons that the fuel contributes through fission. If the percentage of fissions occurring in the different types of fuel in a reactor changes over the life of the core, the average delayed neutron fraction will also change. For a light water reactor using low enriched fuel, the average delayed neutron fraction can change from 0.0070 to 0.0055 as uranium-235 is burned out and plutonium-239 is produced from uranium-238.

Delayed neutrons do not have the same properties as prompt neutrons released directly from fission. The average energy of prompt neutrons is about 2 MeV. This is much greater than the average energy of delayed neutrons (about 0.5 MeV). The fact that delayed neutrons are born at lower energies has two significant impacts on the way they proceed through the neutron life cycle. First, delayed neutrons have a much lower probability of causing fast fissions than prompt neutrons because their average energy is less than the minimum required for fast fission to occur. Second, delayed neutrons have a lower probability of leaking out of the core while they are at fast energies, because they are born at lower energies and subsequently travel a shorter distance as fast neutrons. These two considerations (lower fast fission factor and higher fast non-leakage probability for delayed neutrons) are taken into account by a term called the *importance factor* (I). The importance factor relates the average delayed neutron fraction to the effective delayed neutron fraction.

The *effective delayed neutron fraction* $\left(\bar{\beta}_{eff}\right)$ is defined as the fraction of neutrons at thermal energies which were born delayed. The effective delayed neutron fraction is the product of the average delayed neutron fraction and the importance factor.

$$\bar{\beta}_{eff} = \bar{\beta}\ I$$

where:

$\bar{\beta}_{eff}$ = effective delayed neutron fraction
$\bar{\beta}$ = average delayed neutron fraction
I = importance factor

In a small reactor with highly enriched fuel, the increase in fast non-leakage probability will dominate the decrease in the fast fission factor, and the importance factor will be greater than one. In a large reactor with low enriched fuel, the decrease in the fast fission factor will dominate the increase in the fast non-leakage probability and the importance factor will be less than one (about 0.97 for a commercial PWR).

Effective Delayed Neutron Precursor Decay Constant

Another new term has been introduced in the reactor period (τ) equation. That term is λ_{eff} (pronounced lambda effective), the *effective delayed neutron precursor decay constant*. The decay rate for a given delayed neutron precursor can be expressed as the product of precursor concentration and the decay constant (λ) of that precursor. The decay constant of a precursor is simply the fraction of an initial number of the precursor atoms that decays in a given unit time. A decay constant of 0.1 sec^{-1}, for example, implies that one-tenth, or ten percent, of a sample of precursor atoms decays within one second. The value for the effective delayed neutron precursor decay constant, λ_{eff}, varies depending upon the balance existing between the concentrations of the precursor groups and the nuclide(s) being used as the fuel.

If the reactor is operating at a constant power, all the precursor groups reach an equilibrium value. During an up-power transient, however, the shorter-lived precursors decaying at any given instant were born at a higher power level (or flux level) than the longer-lived precursors decaying at the same instant. There is, therefore, proportionately more of the shorter-lived and fewer of the longer-lived precursors decaying at that given instant than there are at constant power. The value of λ_{eff} is closer to that of the shorter-lived precursors.

During a down-power transient the longer-lived precursors become more significant. The longer-lived precursors decaying at a given instant were born at a higher power level (or flux level) than the shorter-lived precursors decaying at that instant. Therefore, proportionately more of the longer-lived precursors are decaying at that instant, and the value of λ_{eff} approaches the values of the longer-lived precursors.

Approximate values for λ_{eff} are 0.08 sec^{-1} for steady-state operation, 0.1 sec^{-1} for a power increase, and 0.05 sec^{-1} for a power decrease. The exact values will depend upon the materials used for fuel and the value of the reactivity of the reactor core.

Returning now to Equation (4-7) for reactor period.

$$\tau = \underbrace{\frac{\ell^*}{\rho}}_{\left(\begin{array}{c}\text{prompt}\\ \text{term}\end{array}\right)} + \underbrace{\frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho + \dot{\rho}}}_{\left(\begin{array}{c}\text{delayed}\\ \text{term}\end{array}\right)}$$

If the positive reactivity added is less than the value of $\bar{\beta}_{eff}$, the emission of prompt fission neutrons alone is not sufficient to overcome losses to non-fission absorption and leakage. If delayed neutrons were not being produced, the neutron population would decrease as long as the reactivity of the core has a value less than the effective delayed neutron fraction. The positive reactivity insertion is followed immediately by a small immediate power increase called the *prompt jump*. This power increase occurs because the rate of production of prompt neutrons changes abruptly as the reactivity is added. Recall from an earlier module that the generation time for prompt neutrons is on the order of 10^{-13} seconds. The effect can be seen in Figure 2. After the prompt jump, the rate of change of power cannot increase any more rapidly than the built-in time delay the precursor half-lives allow. Therefore, the power rise is controllable, and the reactor can be operated safely.

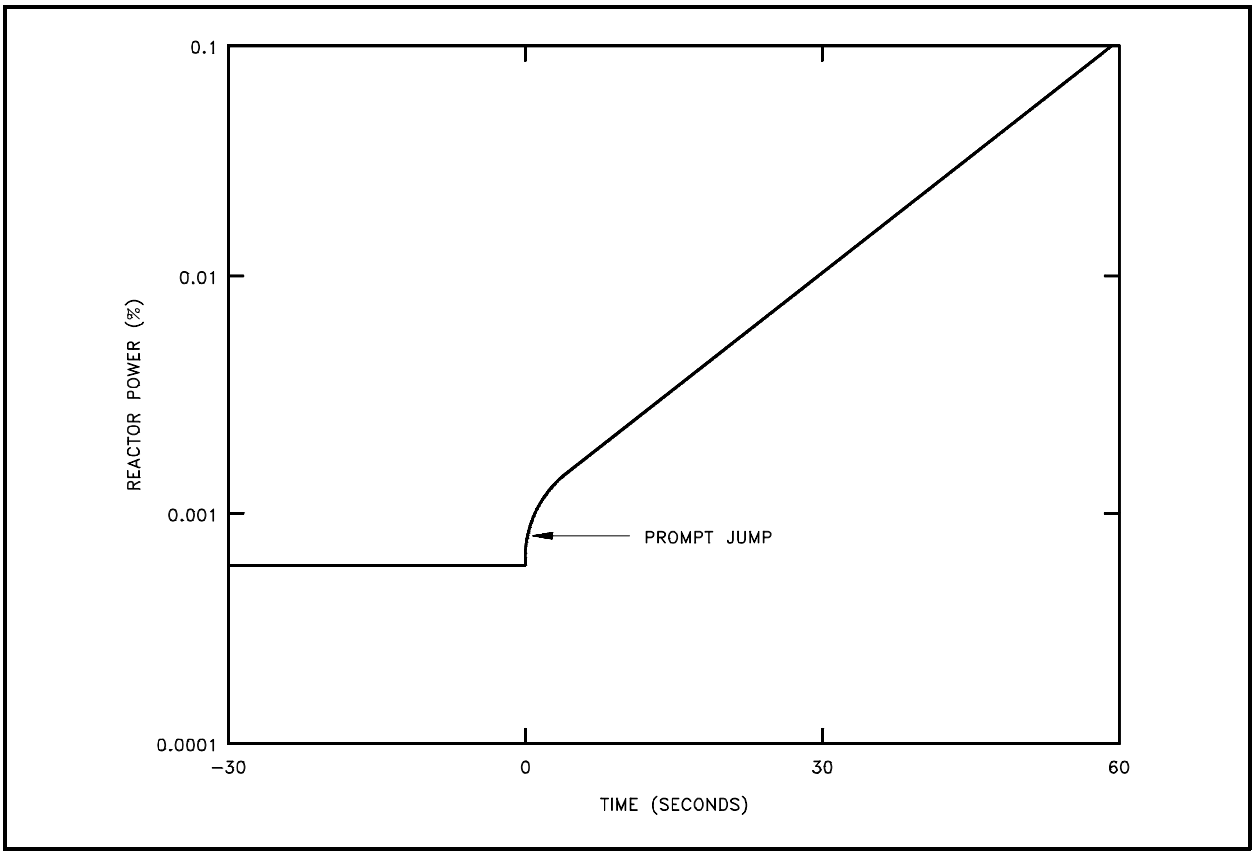

Figure 2 Reactor Power Response to Positive Reactivity Addition

Conversely, in the case where negative reactivity is added to the core there will be a prompt drop in reactor power. The *prompt drop* is the small immediate decrease in reactor power caused by the negative reactivity addition. The prompt drop is illustrated in Figure 3. After the prompt drop, the rate of change of power slows and approaches the rate determined by the delayed term of Equation (4-7).

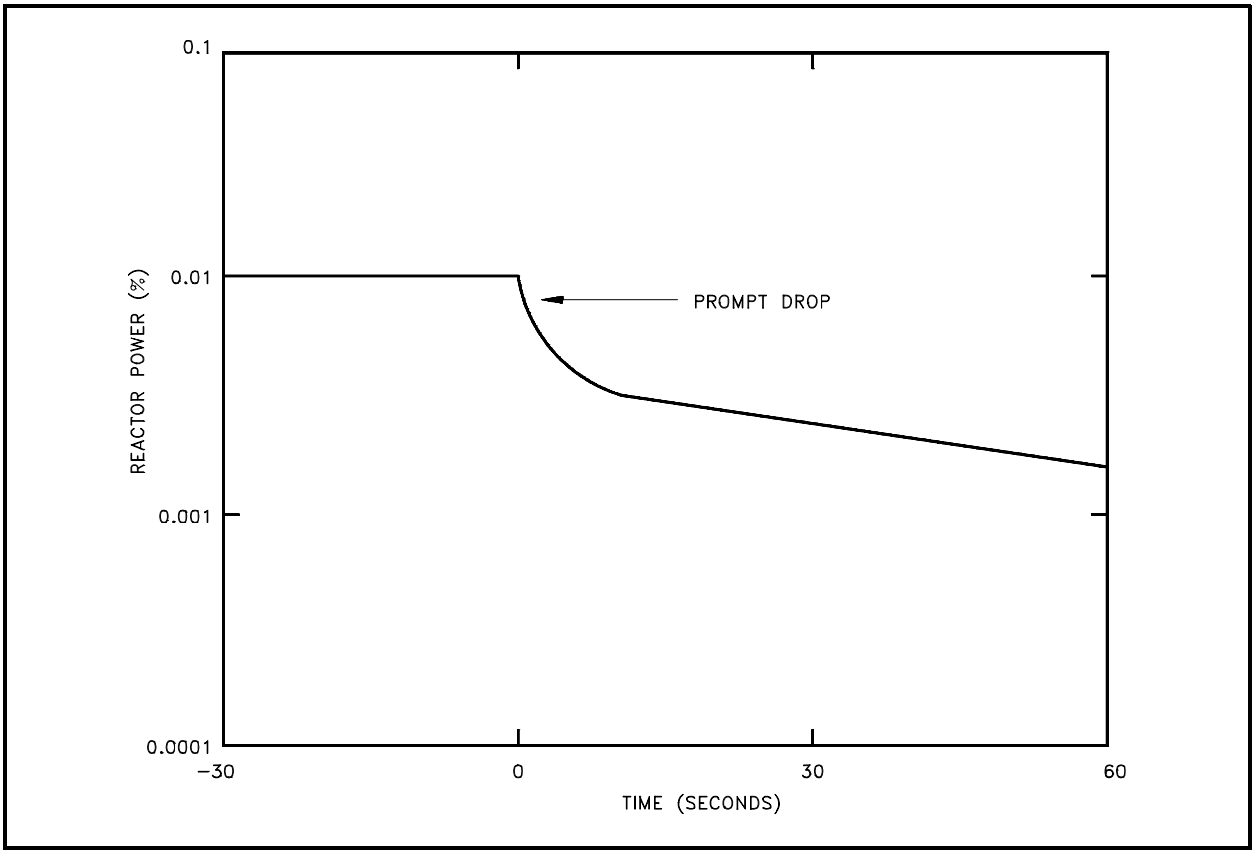

Figure 3 Reactor Power Response to Negative Reactivity Addition

Prompt Criticality

It can be readily seen from Equation (4-7) that if the amount of positive reactivity added equals the value of $\bar{\beta}_{eff}$, the reactor period equation becomes the following.

$$\tau = \frac{\ell^*}{\rho}$$

In this case, the production of prompt neutrons alone is enough to balance neutron losses and increase the neutron population. The condition where the reactor is critical on prompt neutrons, and the neutron population increases as rapidly as the prompt neutron generation lifetime allows is known as *prompt critical*. The prompt critical condition does not signal a dramatic change in neutron behavior. The reactor period changes in a regular manner between reactivities above and below this reference. Prompt critical is, however, a convenient condition for marking the transition from delayed neutron to prompt neutron time scales. A reactor whose reactivity even approaches prompt critical is likely to suffer damage due to the rapid rise in power to a very high level. For example, a reactor which has gone prompt critical could experience a several thousand percent power increase in less than one second.

Because the prompt critical condition is so important, a specific unit of reactivity has been defined that relates to it. The unit of reactivity is the dollar ($), where one dollar of reactivity is equivalent to the effective delayed neutron fraction $(\bar{\beta}_{eff})$. A reactivity unit related to the dollar is the cent, where one cent is one-hundredth of a dollar. If the reactivity of the core is one dollar, the reactor is prompt critical. Because the effective delayed neutron fraction is dependent upon the nuclides used as fuel, the value of the dollar is also dependent on the nuclides used as fuel.

Stable Period Equation

For normal reactor operating conditions, the value of positive reactivity in the reactor is never permitted to approach the effective delayed neutron fraction, and the reactor period equation is normally written as follows.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho + \dot{\rho}} \qquad (4\text{-}8)$$

Equation (4-8) is referred to as the *transient period equation* since it incorporates the $\dot{\rho}$ term to account for the changing amount of reactivity in the core. The ℓ^*/ρ term (prompt period) is normally negligible with respect to the remainder of the equation and is often not included.

For conditions when the amount of reactivity in the core is constant ($\dot{\rho} = 0$), and the reactor period is unchanging, Equation (4-8) can be simplified further to Equation (4-9) which is known as the stable period equation.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho} \qquad (4\text{-}9)$$

Reactor Startup Rate (SUR)

The *reactor startup rate* (SUR) is defined as the number of factors of ten that power changes in one minute. The units of SUR are powers of ten per minute, or decades per minute (DPM). Equation (4-10) shows the relationship between reactor power and startup rate.

$$P = P_o \, 10^{SUR\,(t)} \tag{4-10}$$

where:

SUR = reactor startup rate (DPM)
t = time during reactor transient (minutes)

The relationship between reactor period and startup rate can be developed by considering Equations (4-6) and (4-10).

$$P = P_o \, e^{t/\tau} \quad \text{and} \quad P = P_o \, 10^{SUR\,(t)}$$

$$\frac{P}{P_o} = e^{t/\tau} = 10^{SUR\,(t)}$$

Changing the base of the exponential term on the right side to "e" ($10 = e^{2.303}$) and solving the result yields the following.

$$e^{t\,(sec)/\tau} = e^{2.303\ SUR\ (t\ (min))}$$

$$\frac{t\ (sec)}{\tau} = 2.303\ SUR(t\ (min))$$

$$\frac{60}{\tau} = 2.303\ SUR$$

$$SUR = \frac{26.06}{\tau} \tag{4-11}$$

Doubling Time

Sometimes it is useful to discuss the rate of change of reactor power in terms similar to those used in radioactive decay calculations. *Doubling or halving time* are terms that relate to the amount of time it takes reactor power to double or be reduced to one-half the initial power level. If the stable reactor period is known, doubling time can be determined as follows.

Doubling time (DT) = τ (ln 2)

where:

τ = stable reactor period
ln 2 = natural logarithm of 2

When the doubling time is known, the power level change from P_o is given by the following equation.

$$P = P_o \, 2^{t/DT} \qquad (4\text{-}12)$$

where:

t = time interval of transient
DT = doubling time

The following example problems reinforce the concepts of period and startup rate.

Example 1:

A reactor has a λ_{eff} of 0.10 sec^{-1} and an effective delayed neutron fraction of 0.0070. If k_{eff} is equal to 1.0025, what is the stable reactor period and the SUR?

Solution:

Step 1: First solve for reactivity using Equation (3-5).

$$\rho = \frac{k_{eff} - 1}{k_{eff}}$$

$$= \frac{1.0025 - 1}{1.0025}$$

$$= 0.00249 \ \Delta k/k$$

Step 2: Use this value of reactivity in Equation (4-9) to calculate reactor period.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff} \, \rho}$$

$$= \frac{0.0070 - 0.00249}{(0.10 \ sec^{-1}) \, 0.00249}$$

$$= 18.1 \ sec$$

Step 3: The startup rate can be calculated from the reactor period using Equation (4-11).

$$SUR = \frac{26.06}{\tau}$$

$$= \frac{26.06}{18.1 \text{ sec}}$$

$$= 1.44 \text{ DPM}$$

Example 2:

130 pcm of negative reactivity is added to a reactor that is initially critical at a power of 100 watts. λ_{eff} for the reactor is 0.05 sec^{-1} and the effective delayed neutron fraction is 0.0068. Calculate the steady state period and startup rate. Also calculate the power level 2 minutes after the reactivity insertion.

Solution:

Step 1: Use Equation (4-9) to calculate the reactor period.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff} \rho}$$

$$= \frac{0.0068 - (-0.00130)}{(0.05 \text{ sec}^{-1})(-0.00130)}$$

$$= -124.6 \text{ sec}$$

Step 2: The startup rate can be calculated from the reactor period using Equation (4-11).

$$SUR = \frac{26.06}{\tau}$$

$$= \frac{26.06}{-124.6 \text{ sec}}$$

$$= -0.2091 \text{ DPM}$$

Step 3: Use either Equation (4-1) or Equation (4-10) to calculate the reactor power two minutes after the reactivity insertion.

$$P = P_o e^{t/\tau}$$

$$= (100 \text{ W}) e^{(120 \text{ s}/-124.6 \text{ s})}$$

$$= 38.2 \text{ W}$$

$$P = P_o 10^{SUR (t)}$$

$$= (100 \text{ W}) 10^{(-0.2091 \text{ DPM}) (2 \text{ min})}$$

$$= 38.2 \text{ W}$$

Example 3:

A reactor has a power level of 1000 watts and a doubling time of 2 minutes. What is the reactor power level 10 minutes later?

Solution:

Use Equation (4-12) to calculate the final power level.

$$P = P_o\ (2)^{t/DT}$$

$$= (1{,}000\ W)\ (2)^{10\ min/2\ min}$$

$$= 32{,}000\ W$$

Summary

The important information in this chapter is summarized below.

Reactor Kinetics Summary

- Reactor period is the time required for reactor power to change by a factor of e (2.718).
- Doubling time is the time required for reactor power to double.
- Reactor startup rate is the number of factors of ten that reactor power changes in one minute.
- The delayed neutron fraction (β) is the fraction of all fission neutrons that are born as delayed neutrons for a particular type of fuel (that is, uranium-235 and plutonium-239).
- The average delayed neutron fraction ($\bar{\beta}$) is the weighted average of the total delayed neutron fractions of the different types of fuel used in a particular reactor.
- The effective delayed neutron fraction ($\bar{\beta}_{eff}$) is the average delayed neutron fraction multiplied by an Importance Factor which accounts for the fact that delayed neutrons are born at lower average energies than fast neutrons.
- The reactor period equation is stated below.

$$\tau = \underbrace{\frac{\ell^*}{\rho}}_{\left(\begin{smallmatrix}\text{prompt}\\ \text{term}\end{smallmatrix}\right)} + \underbrace{\frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho + \dot{\rho}}}_{\left(\begin{smallmatrix}\text{delayed}\\ \text{term}\end{smallmatrix}\right)}$$

where:

τ = reactor period
ℓ^* = prompt generation lifetime
$\bar{\beta}_{eff}$ = effective delayed neutron fraction
ρ = reactivity
λ_{eff} = effective delayed neutron precursor decay constant
$\dot{\rho}$ = rate of change of reactivity

Reactor Kinetics Summary (Cont.)

- Equations (4-9) and (4-11) can be used to calculate the stable reactor period and startup rate.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\ \rho} \qquad SUR = \frac{26.06}{\tau}$$

- The concept of doubling time can be used in a similar manner to reactor period to calculate changes in reactor power using Equation (4-12).

$$P = P_o\ 2^{t/DT}$$

- The reactor period or the startup rate can be used to determine the reactor power using Equations (4-6) and (4-10).

$$P = P_o\ e^{t/\tau} \qquad P = P_o\ 10^{SUR\ (t)}$$

- Prompt jump is the small, immediate power increase that follows a positive reactivity insertion related to an increase in the prompt neutron population.
- Prompt drop is the small, immediate power decrease that follows a negative reactivity insertion related to a decrease in the prompt neutron population.
- Prompt critical is the condition when the reactor is critical on prompt neutrons alone.
- When a reactor is prompt critical, the neutron population, and hence power, can increase as quickly as the prompt neutron generation time allows.
- Measuring reactivity in units of dollars is useful when determining if a reactor is prompt critical. A reactor that contains one dollar of positive reactivity is prompt critical since one dollar of reactivity is equivalent to λ_{eff}.

REACTOR OPERATION

It is important to understand the principles that determine how a reactor responds during all modes of operation. Special measures must be taken during the startup of a reactor to ensure that expected responses are occurring. During power operation, control of the flux shape is necessary to ensure operation within limits and maximum core performance. Even when a reactor is shut down, the fact that the fission products created by the fission process continue to generate heat results in a need to monitor support systems to ensure adequate cooling of the core.

EO 3.1 **EXPLAIN why a startup neutron source may be required for a reactor.**

EO 3.2 **LIST four variables typically involved in a reactivity balance.**

EO 3.3 **EXPLAIN how a reactivity balance may be used to predict the conditions under which the reactor will become critical.**

EO 3.4 **LIST three methods used to shape or flatten the core power distribution.**

EO 3.5 **DESCRIBE the concept of power tilt.**

EO 3.6 **DEFINE the term shutdown margin.**

EO 3.7 **EXPLAIN the rationale behind the one stuck rod criterion.**

EO 3.8 **IDENTIFY five changes that will occur during and after a reactor shutdown that will affect the reactivity of the core.**

EO 3.9 **EXPLAIN why decay heat is present following reactor operation.**

EO 3.10 **LIST three variables that will affect the amount of decay heat present following reactor shutdown.**

EO 3.11 **ESTIMATE the approximate amount of decay heat that will exist one hour after a shutdown from steady state conditions.**

Startup

When a reactor is started up with unirradiated fuel, or on those occasions when the reactor is restarted following a long shutdown period, the source neutron population will be very low. In some reactors, the neutron population is frequently low enough that it cannot be detected by the nuclear instrumentation during the approach to criticality. Installed neutron sources, such as those discussed in Module 2, are frequently used to provide a safe, easily monitored reactor startup. The neutron source, together with the subcritical multiplication process, provides a sufficiently large neutron population to allow monitoring by the nuclear instruments throughout the startup procedure. Without the installed source, it may be possible to withdraw the control rods to the point of criticality, and then continue withdrawal without detecting criticality because the reactor goes critical below the indicating range. Continued withdrawal of control rods at this point could cause reactor power to rise at an uncontrollable rate before neutron level first becomes visible on the nuclear instruments.

An alternative to using a startup source is to limit the rate of rod withdrawal, or require waiting periods between rod withdrawal increments. By waiting between rod withdrawal increments, the neutron population is allowed to increase through subcritical multiplication. Subcritical multiplication is the process where source neutrons are used to sustain the chain reaction in a reactor with a multiplication factor (k_{eff}) of less than one. The chain reaction is not "self-sustaining," but if the neutron source is of sufficient magnitude, it compensates for the neutrons lost through absorption and leakage. This process can result in a constant, or increasing, neutron population even though k_{eff} is less than one.

Estimated Critical Position

In the first chapter of this module, 1/M plots were discussed. These plots were useful for monitoring the approach to criticality and predicting when criticality will occur based on indications received while the startup is actually in progress. Before the reactor startup is initiated, the operator calculates an estimate of the amount of rod withdrawal that will be necessary to achieve criticality. This process provides an added margin of safety because a large discrepancy between actual and estimated critical rod positions would indicate that the core was not performing as designed. Depending upon a reactor's design or age, the buildup of xenon within the first several hours following a reactor shutdown may introduce enough negative reactivity to cause the reactor to remain shutdown even with the control rods fully withdrawn. In this situation it is important to be able to predict whether criticality can be achieved, and if criticality cannot be achieved, the startup should not be attempted.

For a given set of conditions (such as time since shutdown, temperature, pressure, fuel burnup, samarium and xenon poisoning) there is only one position of the control rods (and boron concentrations for a reactor with chemical shim) that results in criticality, using the normal rod withdrawal sequence. Identification of these conditions allows accurate calculation of control rod position at criticality. The calculation of an *estimated critical position* (ECP) is simply a mathematical procedure that takes into account all of the changes in factors that significantly affect reactivity that have occurred between the time of reactor shutdown and the time that the reactor is brought critical again.

For most reactor designs, the only factors that change significantly after the reactor is shut down are the average reactor temperature and the concentration of fission product poisons. The reactivities normally considered when calculating an ECP include the following.

Basic Reactivity of the Core- The reactivity associated with the critical control rod position for a xenon-free core at normal operating temperature. This reactivity varies with the age of the core (amount of fuel burnup).

Direct Xenon Reactivity - The reactivity related to the xenon that was actually present in the core at the time it was shutdown. This reactivity is corrected to allow for xenon decay.

Indirect Xenon Reactivity - The reactivity related to the xenon produced by the decay of iodine that was present in the core at the time of shutdown.

Temperature Reactivity - The reactivity related to the difference between the actual reactor temperature during startup and the normal operating temperature.

To arrive at an ECP of the control rods, the basic reactivity, direct and indirect xenon reactivity, and temperature reactivity are combined algebraically to determine the amount of positive control rod reactivity that must be added by withdrawing control rods to attain criticality. A graph of control rod worth versus rod position is used to determine the estimated critical position.

Core Power Distribution

In order to ensure predictable temperatures and uniform depletion of the fuel installed in a reactor, numerous measures are taken to provide an even distribution of flux throughout the power producing section of the reactor. This shaping, or flattening, of the neutron flux is normally achieved through the use of *reflectors* that affect the flux profile across the core, or by the installation of poisons to suppress the neutron flux where desired. The last method, although effective at shaping the flux, is the least desirable since it reduces neutron economy by absorbing the neutrons.

A reactor core is frequently surrounded by a "reflecting" material to reduce the ratio of peak flux to the flux at the edge of the core fuel area. Reflector materials are normally not fissionable, have a high scattering cross section, and have a low absorption cross section. Essentially, for thermal reactors a good moderator is a good reflector. Water, heavy water, beryllium, zirconium, or graphite are commonly used as reflectors. In fast reactor systems, reflectors are not composed of moderating materials because it is desired to keep neutron energy high. The reflector functions by scattering some of the neutrons, which would have leaked from a bare (unreflected) core, back into the fuel to produce additional fissions.

Figure 4 shows the general effect of reflection in the thermal reactor system where core power is proportional to the thermal flux. Notice that a reflector can raise the power density of the core periphery and thus increase the core average power level without changing the peak power. As illustrated in Figure 4, the thermal flux in the reflector may actually be higher than that in the outermost fuel since there are very few absorptions in the reflector.

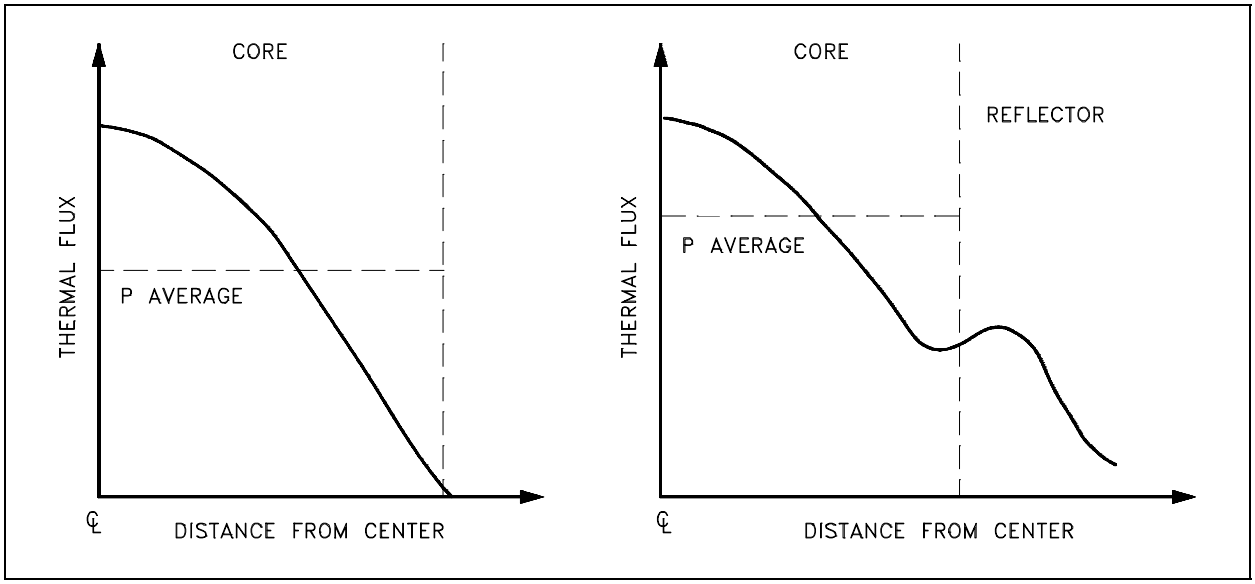

Figure 4 Neutron Radial Flux Shapes for Bare and Reflected Cores

Varying the fuel enrichment or fuel concentrations in the core radially, axially, or both, can readily be used to control power distribution. The simplified example illustrated in Figure 5 shows the effect of using a higher enrichment in the outer regions of the core. Varying fuel concentrations or poison loading for flux shaping is frequently referred to as zoning. In the example illustrated the large central peak is reduced, but the average power level remains the same.

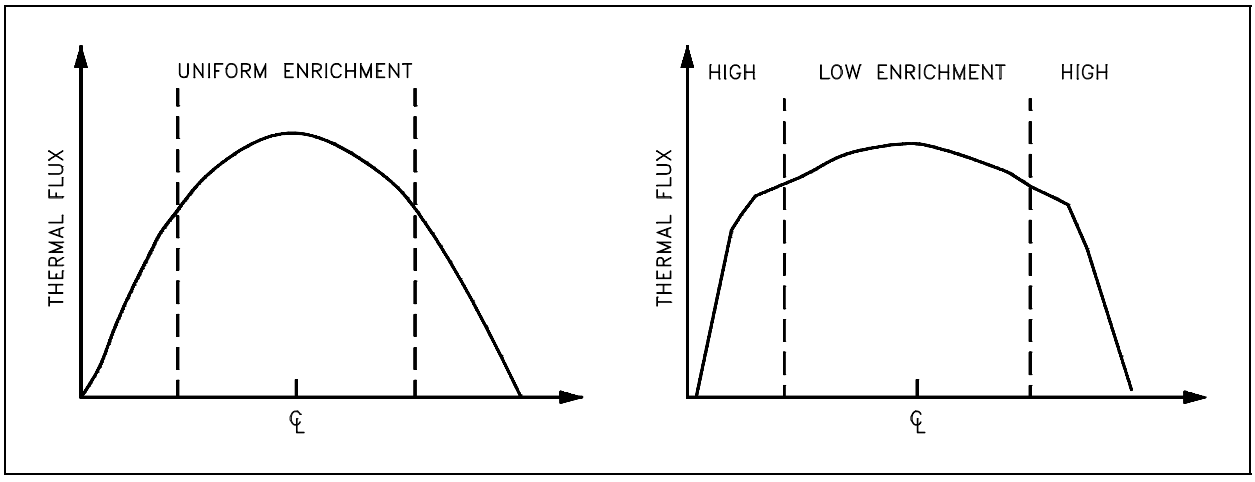

Figure 5 Effect of Non-Uniform Enrichment on Radial Flux Shape

The previous examples discuss changes in radial power distribution. Large variations also exist in axial power distribution. Figure 6(A) illustrates the power distribution that may exist for a reactor with a cylindrical geometry. The control rods in this reactor are inserted from the top, and the effect of inserting control rods further is shown in Figure 6(B). The thermal flux is largely suppressed in the vicinity of the control rods, and the majority of the power is generated low in the core. This flux profile can be flattened by the use of axial fuel and/or poison zoning.

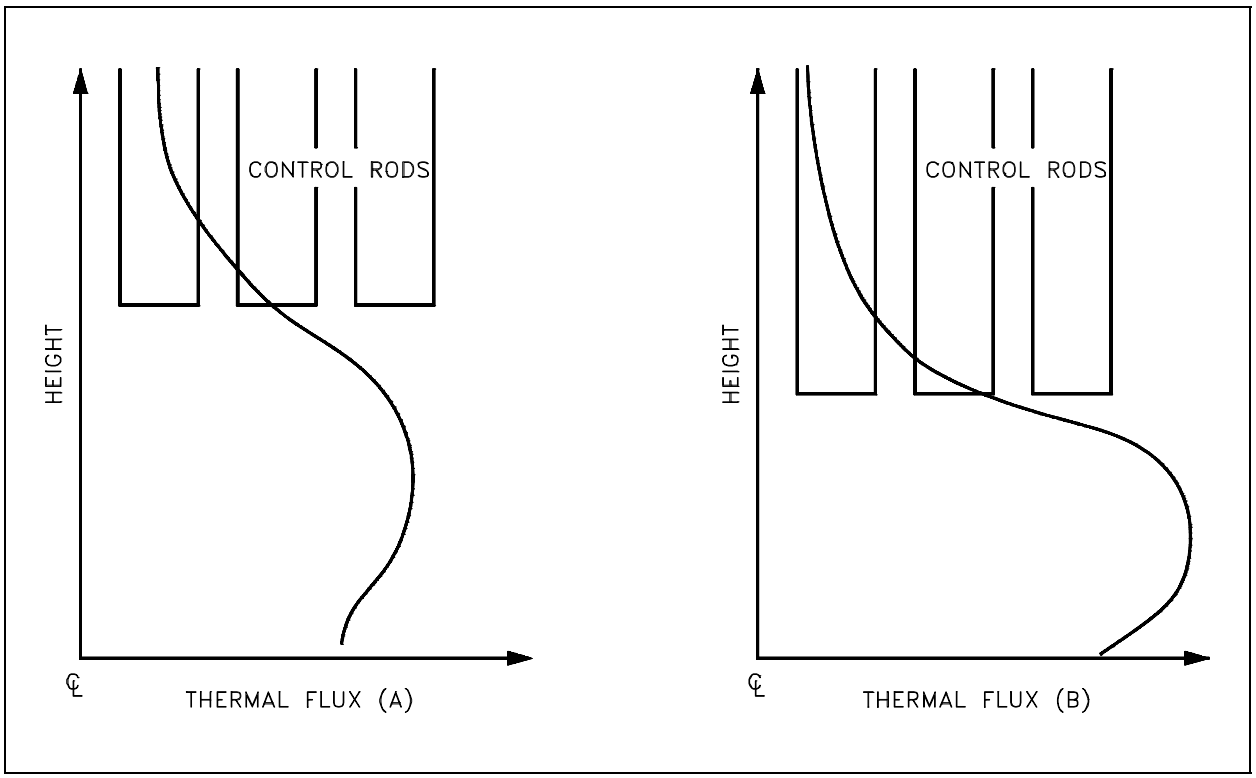

Figure 6 Effect of Control Rod Position on Axial Flux Distribution

Power Tilt

A *power tilt*, or flux tilt, is a specific type of core power distribution problem. It is a non-symmetrical variation of core power in one quadrant of the core relative to the others. The power in one portion might be suppressed by over-insertion of control rods in that portion of the core, which, for a constant overall power level, results in a relatively higher flux in the remainder of the core. This situation can lead to xenon oscillations, which were previously discussed.

Shutdown Margin

Shutdown margin is the instantaneous amount of reactivity by which a reactor is subcritical or would be subcritical from its present condition assuming all control rods are fully inserted except for the single rod with the highest integral worth, which is assumed to be fully withdrawn. Shutdown margin is required to exist at all times, even when the reactor is critical. It is important that there be enough negative reactivity capable of being inserted by the control rods to ensure complete shutdown at all times during the core lifetime. A shutdown margin in the range of one to five percent reactivity is typically required.

The stuck rod criterion refers to the fact that the shutdown margin does not take credit for the insertion of the highest worth control rod. The application of the stuck rod criterion ensures that the failure of a single control rod will not prevent the control rod system from shutting down the reactor.

Operation

During reactor operation, numerous parameters such as temperature, pressure, power level, and flow are continuously monitored and controlled to ensure safe and stable operation of the reactor. The specific effects of variations in these parameters vary greatly depending upon reactor design, but generally the effects for thermal reactors are as follows.

Temperature

The most significant effect of a variation in temperature upon reactor operation is the addition of positive or negative reactivity. As previously discussed, reactors are generally designed with negative temperature coefficients of reactivity (moderator and fuel temperature coefficients) as a self-limiting safety feature. A rise in reactor temperature results in the addition of negative reactivity. If the rise in temperature is caused by an increase in reactor power, the negative reactivity addition slows, and eventually turns the increase in reactor power. This is a highly desirable effect because it provides a negative feedback in the event of an undesired power excursion.

Negative temperature coefficients can also be utilized in water cooled and moderated power reactors to allow reactor power to automatically follow energy demands that are placed upon the system. For example, consider a reactor operating at a stable power level with the heat produced being transferred to a heat exchanger for use in an external closed cycle system. If the energy demand in the external system increases, more energy is removed from reactor system causing the temperature of the reactor coolant to decrease. As the reactor temperature decreases, positive reactivity is added and a corresponding increase in reactor power level results.

As reactor power increases to a level above the level of the new energy demand, the temperature of the moderator and fuel increases, adding negative reactivity and decreasing reactor power level to near the new level required to maintain system temperature. Some slight oscillations above and below the new power level occur before steady state conditions are achieved. The final result is that the average temperature of the reactor system is essentially the same as the initial temperature, and the reactor is operating at the new higher required power level. The same inherent stability can be observed as the energy demand on the system is decreased.

If the secondary system providing cooling to the reactor heat exchanger is operated as an open system with once-through cooling, the above discussion is not applicable. In these reactors, the temperature of the reactor is proportional to the power level, and it is impossible for the reactor to be at a higher power level and the same temperature.

Pressure

The pressure applied to the reactor system can also affect reactor operation by causing changes in reactivity. The reactivity changes result from changes in the density of the moderator in response to the pressure changes. For example, as the system pressure rises, the moderator density increases and results in greater moderation, less neutron leakage, and therefore the insertion of positive reactivity. A reduction in system pressure results in the addition of negative reactivity. Typically, in pressurized water reactors (PWR), the magnitude of this effect is considerably less than that of a change in temperature. In two-phase systems such as boiling water reactors (BWR), however, the effects of pressure changes are more noticeable because there is a greater change in moderator density for a given change in system pressure.

Power Level

A change in reactor power level can result in a change in reactivity if the power level change results in a change in system temperature.

The power level at which the reactor is producing enough energy to make up for the energy lost to ambient is commonly referred to as the *point of adding heat*. If a reactor is operating well below the point of adding heat, then variations in power level produce no measurable variations in temperature. At power levels above the point of adding heat, temperature varies with power level, and the reactivity changes will follow the convention previously described for temperature variations.

The inherent stability and power turning ability of a negative temperature coefficient are ineffective below the point of adding heat. If a power excursion is initiated from a very low power level, power will continue to rise unchecked until the point of adding heat is reached, and the subsequent temperature rise adds negative reactivity to slow, and turn, the rise of reactor power. In this region, reactor safety is provided by automatic reactor shutdown systems and operator action.

Flow

At low reactor power levels, changing the flow rate of the coolant through the reactor does not result in a measurable reactivity change because fuel and moderator temperatures and the fraction of steam voids occurring in the core are not changed appreciably.

When the flow rate is varied, however, the change in temperature that occurs across the core (outlet versus inlet temperature) will vary inversely with the flow rate. At higher power levels, on liquid cooled systems, increasing flow will lower fuel and coolant temperatures slightly, resulting in a small positive reactivity insertion. A positive reactivity addition also occurs when flow is increased in a two-phase (steam-water) cooled system. Increasing the flow rate decreases the fraction of steam voids in the coolant and results in a positive reactivity addition. This property of the moderator in a two-phase system is used extensively in commercial BWRs. Normal power variations required to follow load changes on BWRs are achieved by varying the coolant/moderator flow rate.

Core Burnup

As a reactor is operated, atoms of fuel are constantly consumed, resulting in the slow depletion of the fuel frequently referred to as core burnup. There are several major effects of this fuel depletion. The first, and most obvious, effect of the fuel burnup is that the control rods must be withdrawn or chemical shim concentration reduced to compensate for the negative reactivity effect of this burnup.

Some reactor designs incorporate the use of supplemental burnable poisons in addition to the control rods to compensate for the reactivity associated with excess fuel in a new core. These fixed burnable poisons burn out at a rate that approximates the burnout of the fuel and they reduce the amount of control rod movement necessary to compensate for fuel depletion early in core life.

As control rods are withdrawn to compensate for fuel depletion, the effective size of the reactor is increased. By increasing the effective size of the reactor, the probability that a neutron slows down and is absorbed while it is still in the reactor is also increased. Therefore, neutron leakage decreases as the effective reactor size is increased. The magnitude of the moderator negative temperature coefficient is determined in part by the change in neutron leakage that occurs as the result of a change in moderator temperature. Since the fraction of neutrons leaking out is less with the larger core, a given temperature change will have less of an effect on the leakage. Therefore, the magnitude of the moderator negative temperature coefficient decreases with fuel burnup.

There is also another effect that is a consideration only on reactors that use dissolved boron in the moderator (chemical shim). As the fuel is burned up, the dissolved boron in the moderator is slowly removed (concentration diluted) to compensate for the negative reactivity effects of fuel burnup. This action results in a larger (more negative) moderator temperature coefficient of reactivity in a reactor using chemical shim. This is due to the fact that when water density is decreased by rising moderator temperature in a reactor with a negative temperature coefficient, it results in a negative reactivity addition because some moderator is forced out of the core. With a coolant containing dissolved poison, this density decrease also results in some poison being forced out of the core, which is a positive reactivity addition, thereby reducing the magnitude of the negative reactivity added by the temperature increase. Because as fuel burnup increases the concentration of boron is slowly lowered, the positive reactivity added by the above poison removal process is lessened, and this results in a larger negative temperature coefficient of reactivity.

The following effect of fuel burnup is most predominant in a reactor with a large concentration of uranium-238. As the fission process occurs in a thermal reactor with low or medium enrichment, there is some conversion of uranium-238 into plutonium-239. Near the end of core life in certain reactors, the power contribution from the fission of plutonium-239 may be comparable to that from the fission of uranium-235. The value of the delayed neutron fraction (β) for uranium-235 is 0.0064 and for plutonium-239 is 0.0021. Consequently, as core burnup progresses, the effective delayed neutron fraction for the fuel decreases appreciably. It follows then that the amount of reactivity insertion needed to produce a given reactor period decreases with burnup of the fuel.

Shutdown

A reactor is considered to be shut down when it is subcritical and sufficient shutdown reactivity exists so there is no immediate probability of regaining criticality. Shutdown is normally accomplished by insertion of some (or all) of the control rods, or by introduction of soluble neutron poison into the reactor coolant.

The rate at which the reactor fission rate decays immediately following shutdown is similar for all reactors provided a large amount of negative reactivity is inserted. After a large negative reactivity addition the neutron level undergoes a rapid decrease of about two decades (prompt drop) until it is at the level of production of delayed neutrons. Then the neutron level slowly drops off as the delayed neutron precursors decay, and in a short while only the longest-lived precursor remains in any significant amount. This precursor determines the final rate of decrease in reactor power until the neutron flux reaches the steady state level corresponding to the subcritical multiplication of the neutron source.

The half-life of the longest lived delayed neutron precursor results in a reactor period of around -80 seconds or a startup rate of -1/3 DPM for most reactors after a reactor shutdown. One noticeable exception to this is a heavy water reactor. In a heavy water reactor, the photo-neutron source is extremely large after shutdown due to the amount of deuterium in the moderator and the large number of high energy gammas from short-lived fission product decay. The photo-neutron source is large enough to have a significant impact on neutron population immediately after shutdown. The photo-neutron source has the result of flux levels decreasing more slowly so that a heavy water reactor will have a significantly larger negative reactor period after a shutdown.

Throughout the process of reactor shutdown the nuclear instrumentation is closely monitored to observe that reactor neutron population is decreasing as expected, and that the instrumentation is functioning properly to provide continuous indication of neutron population. Instrumentation is observed for proper overlap between ranges, comparable indication between multiple instrument channels, and proper decay rate of neutron population.

A distinction should be made between indicated reactor power level after shutdown and the actual thermal power level. The indicated reactor power level is the power produced directly from fission in the reactor core, but the actual thermal power drops more slowly due to decay heat production as previously discussed. Decay heat, although approximately 5 to 6% of the steady state reactor power prior to shutdown, diminishes to less than 1% of the pre-shutdown power level after about one hour.

After a reactor is shutdown, provisions are provided for the removal of decay heat. If the reactor is to be shut down for only a short time, operating temperature is normally maintained. If the shutdown period will be lengthy or involves functions requiring cooldown of the reactor, the reactor temperature can be lowered by a number of methods. The methods for actually conducting cooldown of the reactor vary depending on plant design, but in all cases limitations are imposed on the maximum rate at which the reactor systems may be cooled. These limits are provided to reduce the stress applied to system materials, thereby reducing the possibility of stress induced failure.

Although a reactor is shut down, it must be continuously monitored to ensure the safety of the reactor. Automatic monitoring systems are employed to continuously collect and assess the data provided by remote sensors. It is ultimately the operator who must ensure the safety of the reactor.

Decay Heat

About 7 percent of the 200 MeV produced by an average fission is released at some time after the instant of fission. This energy comes from the decay of the fission products. When a reactor is shut down, fission essentially ceases, but decay energy is still being produced. The energy produced after shutdown is referred to as decay heat. The amount of decay heat production after shutdown is directly influenced by the power history of the reactor prior to shutdown. A reactor operated at full power for 3 to 4 days prior to shutdown has much higher decay heat generation than a reactor operated at low power for the same period. The decay heat produced by a reactor shutdown from full power is initially equivalent to about 5 to 6% of the thermal rating of the reactor. This decay heat generation rate diminishes to less than 1% approximately one hour after shutdown. However, even at these low levels, the amount of heat generated requires the continued removal of heat for an appreciable time after shutdown. Decay heat is a long-term consideration and impacts spent fuel handling, reprocessing, waste management, and reactor safety.

Summary

The important information in this chapter is summarized below.

Reactor Operation Summary

- An installed neutron source, together with the subcritical multiplication process, may be needed to increase the neutron population to a level where it can be monitored throughout the startup procedure.
- Reactivity balances, such as Estimated Critical Position calculations, typically consider the basic reactivity of the core and the reactivity effects of temperature, direct xenon, and indirect xenon.
- A reactivity balance called an Estimated Critical Position is used to predict the position of the control rods at which criticality will be achieved during a startup. To arrive at an ECP of the control rods, the basic reactivity, direct and indirect xenon reactivity, and temperature reactivity are added together to determine the amount of positive reactivity that must be added by withdrawing control rods to attain criticality. A graph of control rod worth versus rod position is used to determine the estimated critical position.

Reactor Operation Summary (Cont.)

- Three methods are used to shape or flatten the core power distribution.
  - Use of reflectors
  - Installation of neutron poisons
  - Axial or radial variation of fuel enrichment

- Power tilt is a non-symmetrical variation of core power in one quadrant of the core relative to the other quadrants.

- Shutdown margin is the instantaneous amount of reactivity by which a reactor is subcritical or would be subcritical from its present condition assuming all control rods are fully inserted except for the single rod with the highest integral worth, which is assumed to be fully withdrawn.

- The stuck rod criterion is applied to the shutdown margin to ensure that the failure of a single control rod will not prevent the control rod system from shutting down the reactor.

- Several factors may change during and after the shutdown of the reactor that affect the reactivity of the core.
  - Control rod position
  - Soluble neutron poison concentration
  - Temperature of the fuel and coolant
  - Xenon
  - Samarium

- Decay heat is always present following reactor operation due to energy resulting from the decay of fission products.

- The amount of decay heat present in the reactor is dependent on three factors.
  - The pre-shutdown power level
  - How long the reactor operated
  - The amount of time since reactor shutdown

- Decay heat immediately after shutdown is approximately 5-6% of the pre-shutdown power level. Decay heat will decrease to approximately 1% of the pre-shutdown power level within one hour of reactor shutdown.

end of text.

CONCLUDING MATERIAL

Review activities:

DOE - ANL-W, BNL, EG&G Idaho, EG&G Mound, EG&G Rocky Flats, LLNL, LANL, MMES, ORAU, REECo, WHC, WINCO, WEMCO, and WSRC.

Preparing activity:

DOE - NE-73
Project Number 6910-0025

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