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Warping Machine

8

Acceleration in Mechanisms

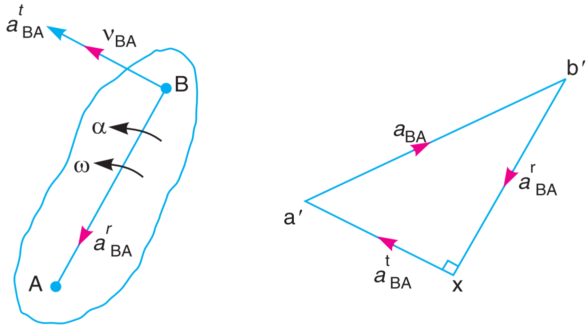
Features

8.1. Introduction

We have discussed in the previous chapter the velocities of various points in the mechanisms. Now we shall discuss the acceleration of points in the mechanisms. The acceleration analysis plays a very important role in the development of machines and mechanisms.

8.2. Acceleration Diagram for a Link

Consider two points *A* and *B* on a rigid link as shown in Fig. 8.1 (*a*). Let the point *B* moves with respect to *A*, with an angular velocity of ω rad/s and let α rad/s^2 be the angular acceleration of the link *AB*.



(*a*) Link. (*b*) Acceleration diagram.

Fig. 8.1. Acceleration for a link.

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We have already discussed that acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components :

1. The ***centripetal or radial component,*** which is perpendicular to the velocity of the particle at the given instant.

2. The ***tangential component,*** which is parallel to the velocity of the particle at the given instant.

Thus for a link *A B*, the velocity of point *B* with respect to *A* (*i.e.* v_{BA}) is perpendicular to the link *A B* as shown in Fig. 8.1 (*a*). Since the point *B* moves with respect to *A* with an angular velocity of ω rad/s, therefore centripetal or radial component of the acceleration of *B* with respect to *A*,

$$a^r_{BA} = \omega^2 \times \text{Length of link } AB = \omega^2 \times AB = v^2_{BA} / AB \qquad \ldots\left(\because \omega = \frac{v_{BA}}{AB}\right)$$

This radial component of acceleration acts perpendicular to the velocity v_{BA}, In other words, it acts ***parallel*** to the link *AB*.

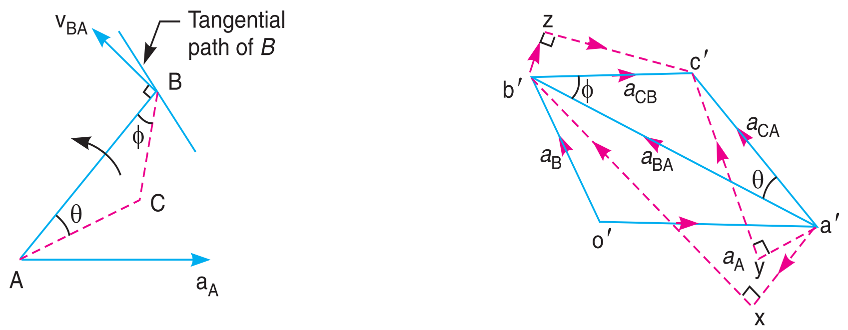
We know that tangential component of the acceleration of *B* with respect to *A*,

$$a^t_{BA} = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

This tangential component of acceleration acts parallel to the velocity v_{BA}. In other words, it acts ***perpendicular*** to the link *A B*.

In order to draw the acceleration diagram for a link *A B*, as shown in Fig. 8.1 (*b*), from any point *b'*, draw vector ***b'x parallel to BA*** to represent the radial component of acceleration of *B* with respect to *A i.e.* a^r_{BA} and from point *x* draw vector *xa'* perpendicular to *B A* to represent the tangential component of acceleration of *B* with respect to *A i.e.* a^t_{BA} . ***Join b' a'.*** The vector *b' a'* (known as ***acceleration image*** of the link *A B*) represents the total acceleration of *B* with respect to *A* (*i.e.* a_{BA}) and it is the vector sum of radial component (a^r_{BA}) and tangential component (a^t_{BA}) of acceleration.

8.3. Acceleration of a Point on a Link



(*a*) Points on a Link. (*b*) Acceleration diagram.

Fig. 8.2. Acceleration of a point on a link.

Consider two points *A* and *B* on the rigid link, as shown in Fig. 8.2 (*a*). Let the acceleration of the point *A i.e.* a_A is known in magnitude and direction and the direction of path of *B* is given. The acceleration of the point *B* is determined in magnitude and direction by drawing the acceleration diagram as discussed below.

1. From any point *o'*, draw vector *o'a'* parallel to the direction of absolute acceleration at point *A i.e.* a_A , to some suitable scale, as shown in Fig. 8.2 (*b*).

2. We know that the acceleration of B with respect to A *i.e.* a_{BA} has the following two components:

(i) Radial component of the acceleration of B with respect to A *i.e.* a^r_{BA}, and

(ii) Tangential component of the acceleration B with respect to A *i.e.* a^t_{BA}. These two components are mutually perpendicular.

3. Draw vector $a'x$ parallel to the link AB (because radial component of the acceleration of B with respect to A will pass through AB), such that

$$\text{vector } a'x = a^r_{BA} = v^2_{BA} / AB$$

where v_{BA} = Velocity of B with respect to A.

Note: The value of v_{BA} may be obtained by drawing the velocity diagram as discussed in the previous chapter.

4. From point x, draw vector xb' perpendicular to AB or vector $a'x$ (because tangential component of B with respect to A *i.e.* a^t_{BA}, is perpendicular to radial component a^r_{BA}) and through o' draw a line parallel to the path of B to represent the absolute acceleration of B *i.e.* a_B. The vectors xb' and $o'b'$ intersect at b'. Now the values of a_B and a^t_{BA} may be measured, to the scale.

A refracting telescope uses mechanisms to change directions.

Note : This picture is given as additional information and is not a direct example of the current chapter.

5. By joining the points a' and b' we may determine the total acceleration of B with respect to A *i.e.* a_{BA}. The vector $a'b'$ is known as ***acceleration image*** of the link AB.

6. For any other point C on the link, draw triangle $a'b'c'$ similar to triangle ABC. Now vector $b'c'$ represents the acceleration of C with respect to B *i.e.* a_{CB}, and vector $a'c'$ represents the acceleration of C with respect *to* A *i.e.* a_{CA}. As discussed above, a_{CB} and a_{CA} will each have two components as follows :

(i) a_{CB} has two components; a^r_{CB} and a^t_{CB} as shown by triangle $b'zc'$ in Fig. 8.2 (*b*), in which $b'z$ is parallel to BC and zc' is perpendicular to $b'z$ or BC.

(ii) a_{CA} has two components ; a^r_{CA} and a^t_{CA} as shown by triangle $a'yc'$ in Fig. 8.2 (*b*), in which $a'y$ is parallel to AC and yc' is perpendicular to $a'y$ or AC.

7. The angular acceleration of the link AB is obtained by dividing the tangential components of the acceleration of B with respect to A (a^t_{BA}) to the length of the link. Mathematically, angular acceleration of the link AB,

$$\alpha_{AB} = a^t_{BA} / AB$$

8.4. Acceleration in the Slider Crank Mechanism

A slider crank mechanism is shown in Fig. 8.3 (*a*). Let the crank OB makes an angle θ with the inner dead centre (*I.D.C*) and rotates in a clockwise direction about the fixed point O with uniform angular velocity ω_{BO} rad/s.

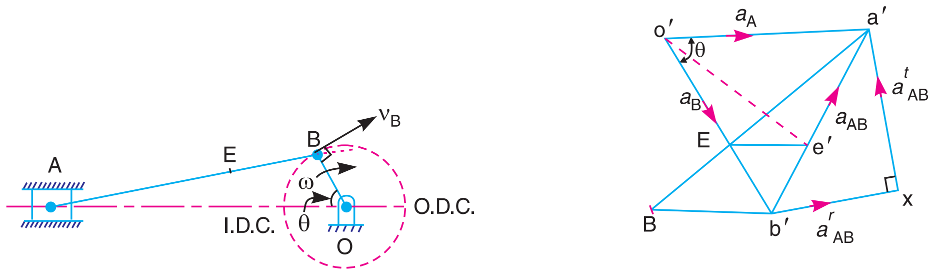
∴ Velocity of B with respect to O or velocity of B (because O is a fixed point),

$$v_{BO} = v_B = \omega_{BO} \times OB, \text{ acting tangentially at } B.$$

We know that centripetal or radial acceleration of *B* with respect to *O* or acceleration of *B* (because *O* is a fixed point),

$$a^r_{BO} = a_B = \omega^2_{BO} \times OB = \frac{v^2_{BO}}{OB}$$

Note : A point at the end of a link which moves with constant angular velocity has no tangential component of acceleration.



(*a*) Slider crank mechanism. (*b*) Acceleration diagram.

Fig. 8.3. Acceleration in the slider crank mechanism.

The acceleration diagram, as shown in Fig. 8.3 (*b*), may now be drawn as discussed below:

1. Draw vector *o′ b′* parallel to *BO* and set off equal in magnitude of $a^r_{BO} = a_B$, to some suitable scale.

2. From point *b′*, draw vector *b′x* parallel to *BA*. The vector *b′x* represents the radial component of the acceleration of *A* with respect to *B* whose magnitude is given by :

$$a^r_{AB} = v^2_{AB} / BA$$

Since the point *B* moves with constant angular velocity, therefore there will be ***no tangential*** component of the acceleration.

3. From point *x*, draw vector *xa′* perpendicular to *b′x* (or *AB*). The vector *xa′* represents the tangential component of the acceleration of *A* with respect to *B* *i.e.* a^t_{AB}.

Note: When a point moves along a straight line, it has **no centripetal or radial** component of the acceleration.

4. Since the point *A* reciprocates along *AO*, therefore the acceleration must be parallel to velocity. Therefore from *o′*, draw *o′ a′* parallel to *AO*, intersecting the vector *xa′* at *a′*.

Now the acceleration of the piston or the slider *A* (a_A) and a^t_{AB} may be measured to the scale.

5. The vector *b′ a′*, which is the sum of the vectors *b′ x* and *x a′*, represents the total acceleration of *A* with respect to *B* *i.e.* a_{AB}. The vector *b′ a′* represents the acceleration of the connecting rod *AB*.

6. The acceleration of any other point on *AB* such as *E* may be obtained by dividing the vector *b′ a′* at *e′* in the same ratio as *E* divides *AB* in Fig. 8.3 (*a*). In other words

$$a'e' / a'b' = AE / AB$$

7. The angular acceleration of the connecting rod *AB* may be obtained by dividing the tangential component of the acceleration of *A* with respect to *B* $\left(a^t_{AB}\right)$ to the length of *AB*. In other words, angular acceleration of *AB*,

$$\alpha_{AB} = a^t_{AB} / AB \text{ (Clockwise about } B)$$

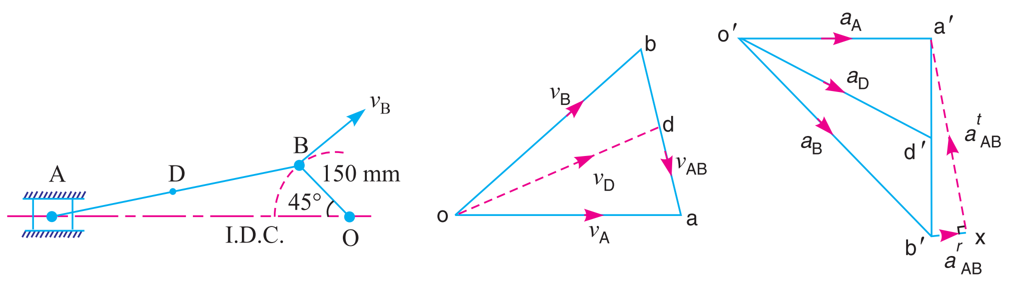
Example 8.1. *The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine :* **1.** *linear velocity and acceleration of the midpoint of the connecting rod, and* **2.** *angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.*

Solution. Given : N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s; OB = 150 mm = 0.15 m ; BA = 600 mm = 0.6 m

We know that linear velocity of B with respect to O or velocity of B,

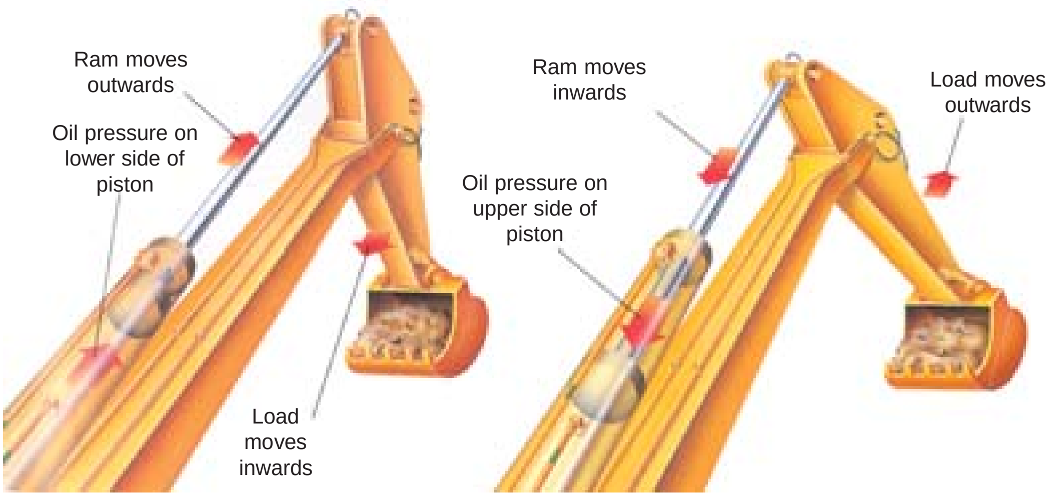
$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$

...(Perpendicular to BO)



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 8.4



Pushing with fluids

Note : This picture is given as additional information and is not a direct example of the current chapter.

1. *Linear velocity of the midpoint of the connecting rod*

First of all draw the space diagram, to some suitable scale; as shown in Fig. 8.4 (*a*). Now the velocity diagram, as shown in Fig. 8.4 (*b*), is drawn as discussed below:

1. Draw vector ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or simply velocity of B *i.e.* v_{BO} or v_B, such that

$$\text{vector } ob = v_{BO} = v_B = 4.713 \text{ m/s}$$

2. From point b, draw vector ba perpendicular to BA to represent the velocity of A with respect to B *i.e.* v_{AB}, and from point o draw vector oa parallel to the motion of A (which is along AO) to represent the velocity of A *i.e.* v_A. The vectors ba and oa intersect at a.

By measurement, we find that velocity of *A* with respect to *B*,

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

and Velocity of *A*, $v_A = \text{vector } oa = 4 \text{ m/s}$

3. In order to find the velocity of the midpoint *D* of the connecting rod *AB*, divide the vector *ba* at *d* in the same ratio as *D* divides *AB*, in the space diagram. In other words,

$$bd / ba = BD/BA$$

Note: Since *D* is the midpoint of *AB*, therefore *d* is also midpoint of vector *ba*.

4. Join *od*. Now the vector *od* represents the velocity of the midpoint *D* of the connecting rod *i.e.* v_D.

By measurement, we find that

$$v_D = \text{vector } od = 4.1 \text{ m/s} \quad \textbf{Ans.}$$

Acceleration of the midpoint of the connecting rod

We know that the radial component of the acceleration of *B* with respect to *O* or the acceleration of *B*,

$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

and the radial component of the acceleraiton of *A* with respect to *B*,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.4 (*c*) is drawn as discussed below:

1. Draw vector *o' b'* parallel to *BO*, to some suitable scale, to represent the radial component of the acceleration of *B* with respect to *O* or simply acceleration of *B i.e.* a^r_{BO} or a_B, such that

$$\text{vector } o'b' = a^r_{BO} = a_B = 148.1 \text{ m/s}^2$$

Note: Since the crank *OB* rotates at a constant speed, therefore there will be no tangential component of the acceleration of *B* with respect to *O*.

2. The acceleration of *A* with respect to *B* has the following two components:

(*a*) The radial component of the acceleration of *A* with respect to *B i.e.* a^r_{AB}, and

(*b*) The tangential component of the acceleration of *A* with respect to *B i.e.* a^t_{AB}. These two components are mutually perpendicular.

Therefore from point *b'*, draw vector *b' x* parallel to *AB* to represent $a^r_{AB} = 19.3 \text{ m/s}^2$ and from point *x* draw vector *xa'* perpendicular to vector *b' x* whose magnitude is yet unknown.

3. Now from *o'*, draw vector *o' a'* parallel to the path of motion of *A* (which is along *AO*) to represent the acceleration of *A i.e.* a_A. The vectors *xa'* and *o' a'* intersect at *a'*. Join *a' b'*.

4. In order to find the acceleration of the midpoint *D* of the connecting rod *AB*, divide the vector *a' b'* at *d'* in the same ratio as *D* divides *AB*. In other words

$$b'd'/b'a' = BD/BA$$

Note: Since *D* is the midpoint of *AB*, therefore *d'* is also midpoint of vector *b' a'*.

5. Join *o' d'*. The vector *o' d'* represents the acceleration of midpoint *D* of the connecting rod *i.e.* a_D.

By measurement, we find that

$$a_D = \text{vector } o'd' = 117 \text{ m/s}^2 \quad \textbf{Ans.}$$

2. *Angular velocity of the connecting rod*

We know that angular velocity of the connecting rod AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \text{ (Anticlockwise about } B) \textbf{ Ans.}$$

Angular acceleration of the connecting rod

From the acceleration diagram, we find that

$$a^t_{AB} = 103 \text{ m/s}^2 \qquad \text{...(By measurement)}$$

We know that angular acceleration of the connecting rod AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \text{ (Clockwise about } B) \textbf{ Ans.}$$

Example 8.2. *An engine mechanism is shown in Fig. 8.5. The crank CB = 100 mm and the connecting rod BA = 300 mm with centre of gravity G, 100 mm from B. In the position shown, the crankshaft has a speed of 75 rad/s and an angular acceleration of 1200 rad/s². Find:* ***1.*** *velocity of G and angular velocity of AB, and* ***2.*** *acceleration of G and angular acceleration of AB.*

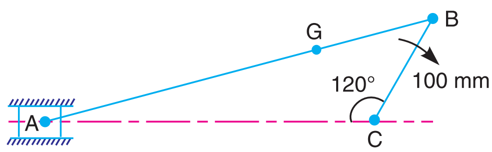


Fig. 8.5

Solution. Given : $\omega_{BC} = 75$ rad/s ; $\alpha_{BC} = 1200$ rad/s², $CB = 100$ mm = 0.1 m; $BA = 300$ mm = 0.3 m

We know that velocity of B with respect to C or velocity of B,

$$v_{BC} = v_B = \omega_{BC} \times CB = 75 \times 0.1 = 7.5 \text{ m/s} \qquad \text{...(Perpendicular to } BC)$$

Since the angular acceleration of the crankshaft, $\alpha_{BC} = 1200$ rad/s², therefore tangential component of the acceleration of B with respect to C,

$$a^t_{BC} = \alpha_{BC} \times CB = 1200 \times 0.1 = 120 \text{ m/s}^2$$

Note: When the angular acceleration is not given, then there will be no tangential component of the acceleration.

1. *Velocity of G and angular velocity of AB*

First of all, draw the space diagram, to some suitable scale, as shown in Fig. 8.6 (*a*). Now the velocity diagram, as shown in Fig. 8.6 (*b*), is drawn as discussed below:

1. Draw vector *cb* perpendicular to CB, to some suitable scale, to represent the velocity of B with respect to C or velocity of B (*i.e.* v_{BC} *or* v_B), such that

$$\text{vector } cb = v_{BC} = v_B = 7.5 \text{ m/s}$$

2. From point *b*, draw vector *ba* perpendicular to BA to represent the velocity of A with respect to B *i.e.* v_{AB}, and from point *c*, draw vector *ca* parallel to the path of motion of A (which is along AC) to represent the velocity of A *i.e.* v_A. The vectors *ba* and *ca* intersect at *a*.

3. Since the point G lies on AB, therefore divide vector *ab* at *g* in the same ratio as G divides AB in the space diagram. In other words,

$$ag / ab = AG / AB$$

The vector *cg* represents the velocity of G.

By measurement, we find that velocity of G,

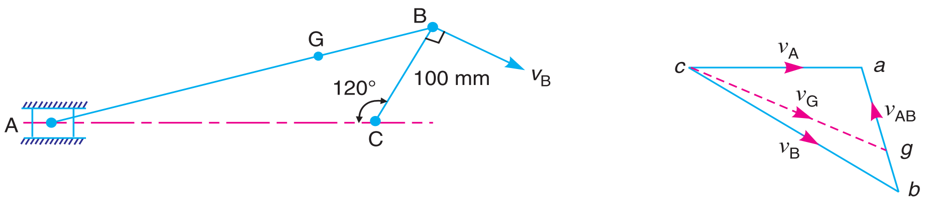
$$v_G = \text{vector } cg = 6.8 \text{ m/s} \textbf{ Ans.}$$

From velocity diagram, we find that velocity of A with respect to B,

$$v_{AB} = \text{vector } ba = 4 \text{ m/s}$$

We know that angular velocity of AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4}{0.3} = 13.3 \text{ rad/s (Clockwise)} \quad \textbf{Ans.}$$



(*a*) Space diagram. (*b*) Velocity diagram.

Fig. 8.6

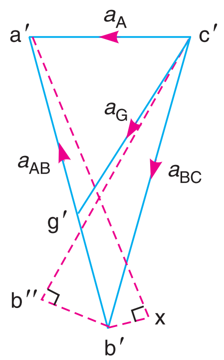
2. *Acceleration of G and angular acceleration of AB*

We know that radial component of the acceleration of B with respect to C,

$$^*\ a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(7.5)^2}{0.1} = 562.5 \text{ m/s}^2$$

and radial component of the acceleration of A with respect to B,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{4^2}{0.3} = 53.3 \text{ m/s}^2$$



(*c*) Acceleration diagram.

Fig. 8.6

Now the acceleration diagram, as shown in Fig. 8.6 (*c*), is drawn as discussed below:

1. Draw vector $c'\,b''$ parallel to CB, to some suitable scale, to represent the radial component of the acceleration of B with respect to C, *i.e.* a^r_{BC}, such that

$$\text{vector } c'b'' = a^r_{BC} = 562.5 \text{ m/s}^2$$

2. From point b'', draw vector $b''\,b'$ perpendicular to vector $c'\,b''$ or CB to represent the tangential component of the acceleration of B with respect to C *i.e.* a^t_{BC}, such that

$$\text{vector } b''b' = a^t_{BC} = 120 \text{ m/s}^2 \quad \text{... (Given)}$$

3. Join $c'\,b'$. The vector $c'\,b'$ represents the total acceleration of B with respect to C *i.e.* a_{BC}.

4. From point b', draw vector $b'\,x$ parallel to BA to represent radial component of the acceleration of A with respect to B *i.e.* a^r_{AB} such that

$$\text{vector } b'x = a^r_{AB} = 53.3 \text{ m/s}^2$$

5. From point x, draw vector xa' perpendicular to vector $b'x$ or BA to represent tangential component of the acceleration of A with respect to B *i.e.* a^t_{AB}, whose magnitude is not yet known.

6. Now draw vector $c'a'$ parallel to the path of motion of A (which is along AC) to represent the acceleration of A *i.e.* a_A. The vectors xa' and $c'a'$ intersect at a'. Join $b'\,a'$. The vector $b'\,a'$ represents the acceleration of A with respect to B *i.e.* a_{AB}.

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* When angular acceleration of the crank is not given, then there is no a^t_{BC}. In that case, $a^r_{BC} = a_{BC} = a_B$, as discussed in the previous example.

7. In order to find the acceleratio of G, divide vector $a'b'$ in g' in the same ratio as G divides BA in Fig. 8.6 (*a*). Join $c'g'$. The vector $c'g'$ represents the acceleration of G.

By measurement, we find that acceleration of G,

$$a_G = \text{vector } c'g' = 414 \text{ m/s}^2 \quad \textbf{Ans.}$$

From acceleration diagram, we find that tangential component of the acceleration of A with respect to B,

$$a^t_{AB} = \text{vector } xa' = 546 \text{ m/s}^2 \qquad \text{...(By measurement)}$$

∴ Angular acceleration of AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{546}{0.3} = 1820 \text{ rad/s}^2 \text{ (Clockwise)} \quad \textbf{Ans.}$$

Example 8.3. *In the mechanism shown in Fig. 8.7, the slider C is moving to the right with a velocity of 1 m/s and an acceleration of 2.5 m/s².*

The dimensions of various links are AB = 3 m inclined at 45° with the vertical and BC = 1.5 m inclined at 45° with the horizontal. Determine: **1.** *the magnitude of vertical and horizontal component of the acceleration of the point B, and* **2.** *the angular acceleration of the links AB and BC.*

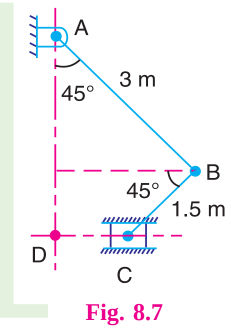


Fig. 8.7

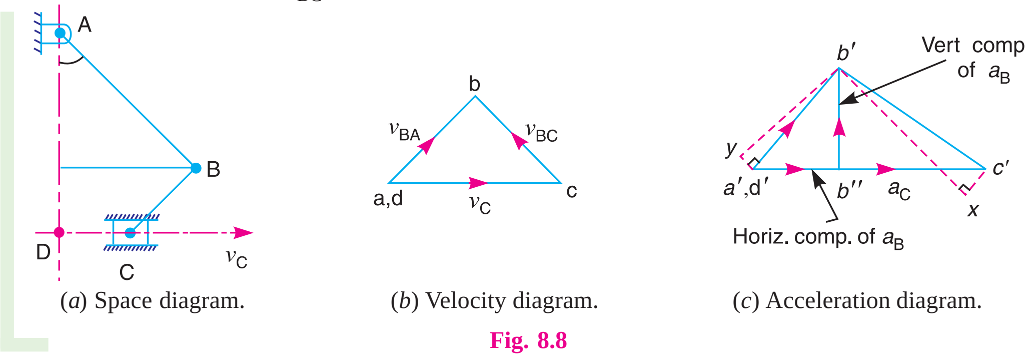
Solution. Given : $v_C = 1$ m/s ; $a_C = 2.5$ m/s²; $AB = 3$ m ; $BC = 1.5$ m

First of all, draw the space diagram, as shown in Fig. 8.8 (*a*), to some suitable scale. Now the velocity diagram, as shown in Fig. 8.8 (*b*), is drawn as discussed below:

1. Since the points A and D are fixed points, therefore they lie at one place in the velocity diagram. Draw vector dc parallel to DC, to some suitable scale, which represents the velocity of slider C with respect to D or simply velocity of C, such that

$$\text{vector } dc = v_{CD} = v_C = 1 \text{ m/s}$$

2. Since point B has two motions, one with respect to A and the other with respect to C, therefore from point a, draw vector ab perpendicular to AB to represent the velocity of B with respect to A, *i.e.* v_{BA} and from point c draw vector cb perpendicular to CB to represent the velocity of B with respect to C *i.e.* v_{BC}. The vectors ab and cb intersect at b.



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 8.8

By measurement, we find that velocity of B with respect to A,

$$v_{BA} = \text{vector } ab = 0.72 \text{ m/s}$$

and velocity of B with respect to C,

$$v_{BC} = \text{vector } cb = 0.72 \text{ m/s}$$

We know that radial component of acceleration of B with respect to C,

$$a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(0.72)^2}{1.5} = 0.346 \text{ m/s}^2$$

and radial component of acceleration of B with respect to A,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(0.72)^2}{3} = 0.173 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.8 (*c*), is drawn as discussed below:

1. *Since the points A and D are fixed points, therefore they lie at one place in the acceleration diagram. Draw vector $d'c'$ parallel to DC, to some suitable scale, to represent the acceleration of C with respect to D or simply acceleration of C *i.e.* a_{CD} or a_C such that

$$\text{vector } d'c' = a_{CD} = a_C = 2.5 \text{ m/s}^2$$

2. The acceleration of B with respect to C will have two components, *i.e.* one radial component of B with respect to C $\left(a^r_{BC}\right)$ and the other tangential component of B with respect to C $\left(a^t_{BC}\right)$. Therefore from point c', draw vector $c'x$ parallel to CB to represent a^r_{BC} such that

$$\text{vector } c'x = a^r_{BC} = 0.346 \text{ m/s}^2$$

3. Now from point x, draw vector xb' perpendicular to vector $c'x$ or CB to represent a^t_{BC} whose magnitude is yet unknown.

4. The acceleration of B with respect to A will also have two components, *i.e.* one radial component of B with respect to A (a^r_{BA}) and other tangential component of B with respect to A (a^t_{BA}). Therefore from point a' draw vector $a'y$ parallel to AB to represent a^r_{BA}, such that

$$\text{vector } a'y = a^r_{BA} = 0.173 \text{ m/s}^2$$

5. From point y, draw vector yb' perpendicular to vector $a'y$ or AB to represent a^t_{BA}. The vector yb' intersect the vector xb' at b'. Join $a'b'$ and $c'b'$. The vector $a'b'$ represents the acceleration of point B (a_B) and the vector $c'b'$ represents the acceleration of B with respect to C.

1. *Magnitude of vertical and horizontal component of the acceleration of the point B*

Draw $b'b''$ perpendicular to $a'c'$. The vector $b'b''$ is the vertical component of the acceleration of the point B and $a'b''$ is the horizontal component of the acceleration of the point B. By measurement,

vector $b'b'' = 1.13$ m/s^2 and vector $a'b'' = 0.9$ m/s^2 **Ans.**

2. *Angular acceleration of AB and BC*

By measurement from acceleration diagram, we find that tangential component of acceleration of the point B with respect to A,

$$a^t_{BA} = \text{vector } yb' = 1.41 \text{ m/s}^2$$

and tangential component of acceleration of the point B with respect to C,

$$a^t_{BC} = \text{vector } xb' = 1.94 \text{ m/s}^2$$

* If the mechanism consists of more than one fixed point, then all these points lie at the same place in the velocity and acceleration diagrams.

We know that angular acceleration of AB,

$$\alpha_{AB} = \frac{a^t_{BA}}{AB} = \frac{1.41}{3} = 0.47 \text{ rad/s}^2 \quad \textbf{Ans.}$$

and angular acceleration of BC,

$$\alpha_{BC} = \frac{a^t_{BA}}{CB} = \frac{1.94}{1.5} = 1.3 \text{ rad/s}^2 \quad \textbf{Ans.}$$

Example 8.4. *PQRS is a four bar chain with link PS fixed. The lengths of the links are PQ = 62.5 mm ; QR = 175 mm ; RS = 112.5 mm ; and PS = 200 mm. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.*

Solution. Given : ω_{QP} = 10 rad/s; PQ = 62.5 mm = 0.0625 m ; QR = 175 mm = 0.175 m ; RS = 112.5 mm = 0.1125 m ; PS = 200 mm = 0.2 m

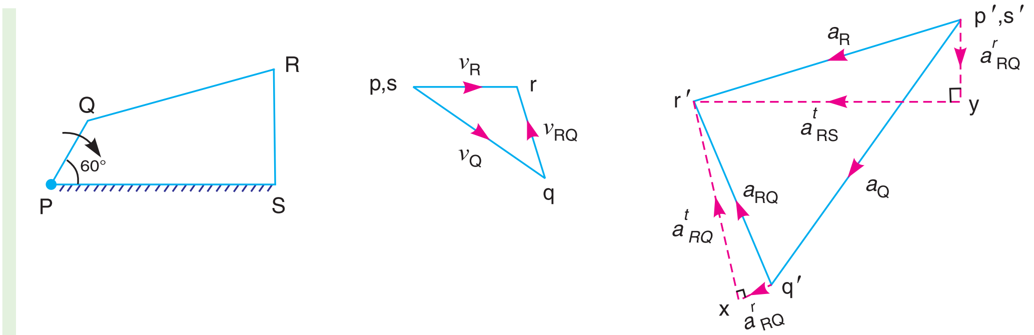
We know that velocity of Q with respect to P or velocity of Q,

$$v_{QP} = v_Q = \omega_{QP} \times PQ = 10 \times 0.0625 = 0.625 \text{ m/s}$$

...(Perpendicular to PQ)

Angular velocity of links QR and RS

First of all, draw the space diagram of a four bar chain, to some suitable scale, as shown in Fig. 8.9 (*a*). Now the velocity diagram as shown in Fig. 8.9 (*b*), is drawn as discussed below:



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 8.9

1. Since P and S are fixed points, therefore these points lie at one place in velocity diagram. Draw vector pq perpendicular to PQ, to some suitable scale, to represent the velocity of Q with respect to P or velocity of Q *i.e.* v_{QP} or v_Q such that

$$\text{vector } pq = v_{QP} = v_Q = 0.625 \text{ m/s}$$

2. From point q, draw vector qr perpendicular to QR to represent the velocity of R with respect to Q (*i.e.* v_{RQ}) and from point s, draw vector sr perpendicular to SR to represent the velocity of R with respect to S or velocity of R (*i.e.* v_{RS} or v_R). The vectors qr and sr intersect at r. By measurement, we find that

$$v_{RQ} = \text{vector } qr = 0.333 \text{ m/s, and } v_{RS} = v_R = \text{vector } sr = 0.426 \text{ m/s}$$

We know that angular velocity of link QR,

$$\omega_{QR} = \frac{v_{RQ}}{RQ} = \frac{0.333}{0.175} = 1.9 \text{ rad/s (Anticlockwise)} \quad \textbf{Ans.}$$

and angular velocity of link RS,

$$\omega_{RS} = \frac{v_{RS}}{SR} = \frac{0.426}{0.1125} = 3.78 \text{ rad/s (Clockwise)}$$ **Ans.**

Angular acceleration of links QR and RS

Since the angular acceleration of the crank PQ is not given, therefore there will be no tangential component of the acceleration of Q with respect to P.

We know that radial component of the acceleration of Q with respect to P (or the acceleration of Q),

$$a^r_{QP} = a_{QP} = a_Q = \frac{v^2_{QP}}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

Radial component of the acceleration of R with respect to Q,

$$a^r_{RQ} = \frac{v^2_{RQ}}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

and radial component of the acceleration of R with respect to S (or the acceleration of R),

$$a^r_{RS} = a_{RS} = a_R = \frac{v^2_{RS}}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$

The acceleration diagram, as shown in Fig. 8.9 (*c*) is drawn as follows :

1. Since P and S are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector $p'q'$ parallel to PQ, to some suitable scale, to represent the radial component of acceleration of Q with respect to P or acceleration of Q *i.e* a^r_{QP} or a_Q such that

$$\text{vector } p'q' = a^r_{QP} = a_Q = 6.25 \text{ m/s}^2$$

2. From point q', draw vector $q'x$ parallel to QR to represent the radial component of acceleration of R with respect to Q *i.e.* a^r_{RQ} such that

$$\text{vector } q'x = a^r_{RQ} = 0.634 \text{ m/s}^2$$

3. From point x, draw vector xr' perpendicular to QR to represent the tangential component of acceleration of R with respect to Q *i.e* a^t_{RQ} whose magnitude is not yet known.

4. Now from point s', draw vector $s'y$ parallel to SR to represent the radial component of the acceleration of R with respect to S *i.e.* a^r_{RS} such that

$$\text{vector } s'y = a^r_{RS} = 1.613 \text{ m/s}^2$$

5. From point y, draw vector yr' perpendicular to SR to represent the tangential component of acceleration of R with respect to S *i.e.* a^t_{RS}.

6. The vectors xr' and yr' intersect at r'. Join $p'r$ and $q'r'$. By measurement, we find that

$$a^t_{RQ} = \text{vector } xr' = 4.1 \text{ m/s}^2 \text{ and } a^t_{RS} = \text{vector } yr' = 5.3 \text{ m/s}^2$$

We know that angular acceleration of link QR,

$$\alpha_{QR} = \frac{a^t_{RQ}}{QR} = \frac{4.1}{0.175} = 23.43 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

and angular acceleration of link RS,

$$\alpha_{RS} = \frac{a^t_{RS}}{SR} = \frac{5.3}{0.1125} = 47.1 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

Example 8.5. *The dimensions and configuration of the four bar mechanism, shown in Fig. 8.10, are as follows :*

P_1A = 300 mm; P_2B = 360 mm; AB = 360 mm, and P_1P_2 = 600 mm.

The angle AP_1P_2 = 60°. The crank P_1A has an angular velocity of 10 rad/s and an angular acceleration of 30 rad/s², both clockwise. Determine the angular velocities and angular accelerations of P_2B, and AB and the velocity and acceleration of the joint B.

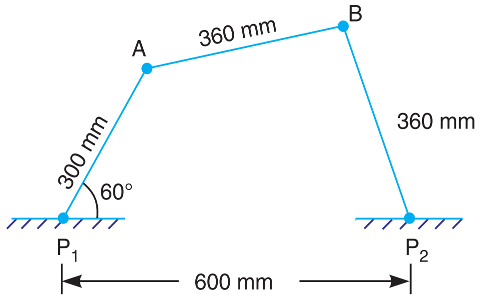


Fig. 8.10

Solution. Given : ω_{AP1} = 10 rad/s ; α_{AP1} = 30 rad/s²; P_1A = 300 mm = 0.3 m ; P_2B = AB = 360 mm = 0.36 m

We know that the velocity of A with respect to P_1 or velocity of A,

$$v_{AP1} = v_A = \omega_{AP1} \times P_1A = 10 \times 0.3 = 3 \text{ m/s}$$

Velocity of B and angular velocitites of P_2B and AB

First of all, draw the space diagram, to some suitable scale, as shown in Fig. 8.11 (*a*). Now the velocity diagram, as shown in Fig. 8.11 (*b*), is drawn as discussed below:

1. Since P_1 and P_2 are fixed points, therefore these points lie at one place in velocity diagram. Draw vector $p_1\,a$ perpendicular to P_1A, to some suitable scale, to represent the velocity of A with respect to P_1 or velocity of A *i.e.* v_{AP1} or v_A, such that

$$\text{vector } p_1a = v_{A\,P1} = v_A = 3 \text{ m/s}$$

2. From point *a*, draw vector *ab* perpendicular to *AB* to represent velocity of *B* with respect to *A* (*i.e.* v_{BA}) and from point p_2 draw vector p_2b perpendicular to P_2B to represent the velocity of *B* with respect to P_2 or velocity of *B* *i.e.* v_{BP2} or v_B. The vectors *ab* and p_2b intersect at *b*.

By measurement, we find that

$$v_{BP2} = v_B = \text{vector } p_2b = 2.2 \text{ m/s} \quad \textbf{Ans.}$$

and $$v_{BA} = \text{vector } ab = 2.05 \text{ m/s}$$

We know that angular velocity of P_2B,

$$\omega_{P2B} = \frac{v_{BP_2}}{P_2B} = \frac{2.2}{0.36} = 6.1 \text{ rad/s (Clockwise)} \quad \textbf{Ans.}$$

and angular velocity of AB,

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{2.05}{0.36} = 5.7 \text{ rad/s (Anticlockwise)} \quad \textbf{Ans.}$$

Acceleration of B and angular acceleration of P_2B and AB

We know that tangential component of the acceleration of A with respect to P_1,

$$a^t_{AP_1} = \alpha_{AP_1} \times P_1A = 30 \times 0.3 = 9 \text{ m/s}^2$$

Radial component of the acceleration of A with respect to P_1,

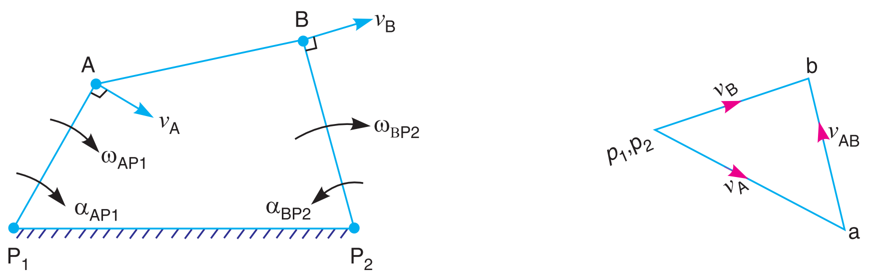
$$a^r_{AP_1} = \frac{v^2_{AP1}}{P_1A} = \omega^2_{AP_1} \times P_1A = 10^2 \times 0.3 = 30 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A.

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(2.05)^2}{0.36} = 11.67 \text{ m/s}^2$$

and radial component of the acceleration of B with respect to P_2,

$$a^r_{BP_2} = \frac{v^2_{BP_2}}{P_2B} = \frac{(2.2)^2}{0.36} = 13.44 \text{ m/s}^2$$



(*a*) Space diagram. (*b*) Velocity diagram.

Fig. 8.11

The acceleration diagram, as shown in Fig. 8.11 (*c*), is drawn as follows:

1. Since P_1 and P_2 are fixed points, therefore these points will lie at one place, in the acceleration diagram. Draw vector $p_1'\,x$ parallel to P_1A, to some suitable scale, to represent the radial component of the acceleration of A with respect to P_1, such that

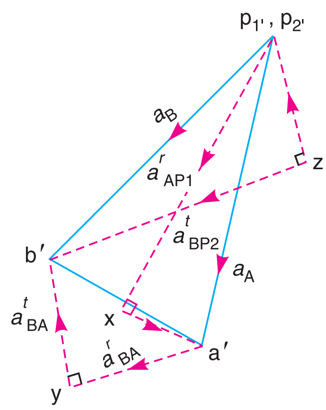
$$\text{vector } p_1'x = a^r_{AP_1} = 30 \text{ m/s}^2$$

2. From point x, draw vector xa' perpendicular to P_1A to represent the tangential component of the acceleration of A with respect to P_1, such that

$$\text{vector } xa' = a^t_{AP_1} = 9 \text{ m/s}^2$$

3. Join $p_1'a'$. The vector $p_1'a'$ represents the acceleration of A. By measurement, we find that the acceleration of A,

$$a_A = a_{AP1} = 31.6 \text{ m/s}^2$$



(*c*) Acceleration diagram

Fig. 8.11

4. From point a', draw vector $a'y$ parallel to AB to represent the radial component of the acceleration of B with respect to A, such that

$$\text{vector } a'y = a^r_{BA} = 11.67 \text{ m/s}^2$$

5. From point y, draw vector yb' perpendicular to AB to represent the tangential component of the acceleration of B with respect to A (*i.e.* a^t_{BA}) whose magnitude is yet unknown.

6. Now from point p_2', draw vector $p_2'z$ parallel to P_2B to represent the radial component of the acceleration B with respect to P_2, such that

$$\text{vector } p_2'z = a^r_{BP_2} = 13.44 \text{ m/s}^2$$

7. From point z, draw vector zb' perpendicular to P_2B to represent the tangential component of the acceleration of B with respect to P_2 *i.e.* $a^t_{BP_2}$.

8. The vectors yb' and zb' intersect at b'. Now the vector $p_2'b'$ represents the acceleration of B with respect to P_2 or the acceleration of B *i.e.* a_{BP2} or a_B. By measurement, we find that

$$a_{BP2} = a_B = \text{vector } p_2'b' = 29.6 \text{ m/s}^2 \textbf{ Ans.}$$

Also $\quad$ vector $yb' = a^t_{BA} = 13.6 \text{ m/s}^2$, and vector $zb' = a^t_{BP_2} = 26.6 \text{ m/s}^2$

We know that angular acceleration of P_2B,

$$\alpha_{P2B} = \frac{a^t_{BP_2}}{P_2B} = \frac{26.6}{0.36} = 73.8 \text{ rad/s}^2 \text{ (Anticlockwise) } \textbf{Ans.}$$

and angular acceleration of AB, $\alpha_{AB} = \dfrac{a^t_{BA}}{AB} = \dfrac{13.6}{0.36} = 37.8 \text{ rad/s}^2$ (Anticlockwise) **Ans.**

Bicycle is a common example where simple mechanisms are used.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Example 8.6. *In the mechanism, as shown in Fig. 8.12, the crank OA rotates at 20 r.p.m. anticlockwise and gives motion to the sliding blocks B and D. The dimensions of the various links are OA = 300 mm; AB = 1200 mm; BC = 450 mm and CD = 450 mm.*

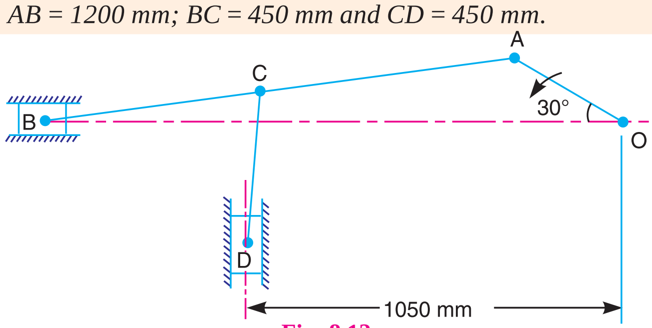


Fig. 8.12

*For the given configuration, determine : **1.** velocities of sliding at B and D, **2.** angular velocity of CD, **3.** linear acceleration of D, and **4.** angular acceleration of CD.*

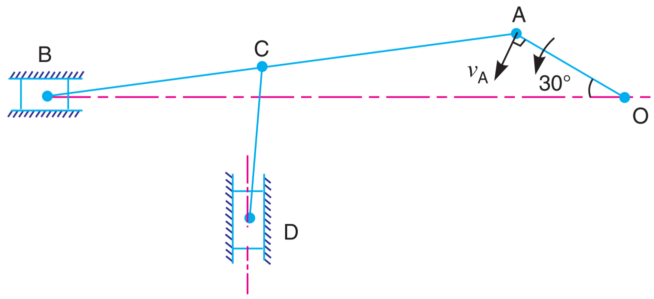
Solution. Given : $N_{AO} = 20$ r.p.m. or $\omega_{AO} = 2\,\pi \times 20/60 = 2.1$ rad/s ; $OA = 300$ mm $= 0.3$ m ; $AB = 1200$ mm $= 1.2$ m ; $BC = CD = 450$ mm $= 0.45$ m

We know that linear velocity of A with respect to O or velocity of A,

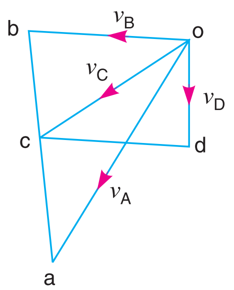
$$v_{AO} = v_A = \omega_{AO} \times OA = 2.1 \times 0.3 = 0.63 \text{ m/s} \qquad \text{...(Perpendicular to } OA\text{)}$$

1. *Velocities of sliding at B and D*

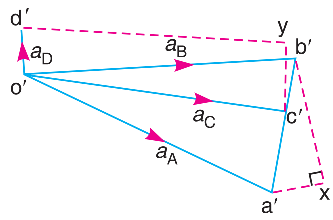
First of all, draw the space diagram, to some suitable scale, as shown in Fig. 8.13 (*a*). Now the velocity diagram, as shown in Fig. 8.13 (*b*), is drawn as discussed below:



(*a*) Space diagram.



(*b*) Velocity diagram.



(*c*) Acceleration diagram.

Fig. 8.13

1. Draw vector *oa* perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O (or simply velocity of A), such that

$$\text{vector } oa = v_{AO} = v_A = 0.63 \text{ m/s}$$

2. From point *a*, draw vector *ab* perpendicular to AB to represent the velocity of B with respect to A (*i.e.* v_{BA}) and from point *o* draw vector *ob* parallel to path of motion B (which is along BO) to represent the velocity of B with respect to O (or simply velocity of B). The vectors *ab* and *ob* intersect at *b*.

3. Divide vector *ab* at *c* in the same ratio as C divides AB in the space diagram. In other words,

$$BC/CA = bc/ca$$

4. Now from point *c*, draw vector *cd* perpendicular to CD to represent the velocity of D with respect to C (*i.e.* v_{DC}) and from point *o* draw vector *od* parallel to the path of motion of D (which along the vertical direction) to represent the velocity of D.

By measurement, we find that velocity of sliding at B,

$$v_B = \text{vector } ob = 0.4 \text{ m/s} \quad \textbf{Ans.}$$

and velocity of sliding at D, $\quad v_D = \text{vector } od = 0.24 \text{ m/s}$ **Ans.**

2. *Angular velocity of CD*

By measurement from velocity diagram, we find that velocity of D with respect to C,

$$v_{DC} = \text{vector } cd = 0.37 \text{ m/s}$$

∴ Angular velocity of CD,

$$\omega_{CD} = \frac{v_{DC}}{CD} = \frac{0.37}{0.45} = 0.82 \text{ rad/s (Anticlockwise). } \textbf{Ans.}$$

3. *Linear acceleration of D*

We know that the radial component of the acceleration of A with respect to O or acceleration of A,

$$a_{AO}^r = a_A = \frac{v_{AO}^2}{OA} = \omega_{AO}^2 \times OA = (2.1)^2 \times 0.3 = 1.323 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A,

$$a_{BA}^r = \frac{v_{BA}^2}{AB} = \frac{(0.54)^2}{1.2} = 0.243 \text{ m/s}^2$$

...(By measurement, v_{BA} = 0.54 m/s)

Radial component of the acceleration of D with respect to C,

$$a_{DC}^r = \frac{v_{DC}^2}{CD} = \frac{(0.37)^2}{0.45} = 0.304 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.13 (*c*), is drawn as discussed below:

1. Draw vector $o'a'$ parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O or simply the acceleration of A, such that

$$\text{vector } o'a' = a_{AO}^r = a_A = 1.323 \text{ m/s}^2$$

2. From point a', draw vector $a'x$ parallel to AB to represent the radial component of the acceleration of B with respect to A, such that

$$\text{vector } a'x = a_{BA}^r = 0.243 \text{ m/s}^2$$

3. From point x, draw vector xb' perpendicular to AB to represent the tangential component of the acceleration of B with respect to A (*i.e.* a_{BA}^t) whose magnitude is not yet known.

4. From point o', draw vector $o'b'$ parallel to the path of motion of B (which is along BO) to represent the acceleration of B (a_B). The vectors xb' *and* $o'b'$ intersect at b'. Join $a'b'$. The vector $a'b'$ represents the acceleration of B with respect to A.

5. Divide vector $a'b'$ at c' in the same ratio as C divides AB in the space diagram. In other words,

$$BC / BA = b'c'/b'a'$$

6. From point c', draw vector $c'y$ parallel to CD to represent the radial component of the acceleration of D with respect to C, such that

$$\text{vector } c'y = a_{DC}^r = 0.304 \text{ m/s}^2$$

7. From point y, draw yd' perpendicular to CD to represent the tangential component of acceleration of D with respect to C (*i.e.* a_{DC}^t) whose magnitude is not yet known.

8. From point o', draw vector $o'd'$ parallel to the path of motion of D (which is along the vertical direction) to represent the acceleration of D (a_D). The vectors yd' and $o'd'$ ***intersect at*** d'.

By measurement, we find that linear acceleration of D,

$$a_D = \text{vector } o'd' = 0.16 \text{ m/s}^2 \quad \textbf{Ans.}$$

4. *Angular acceleration of CD*

From the acceleration diagram, we find that the tangential component of the acceleration of D with respect to C,

$$a_{DC}^t = \text{vector } yd' = 1.28 \text{ m/s}^2$$

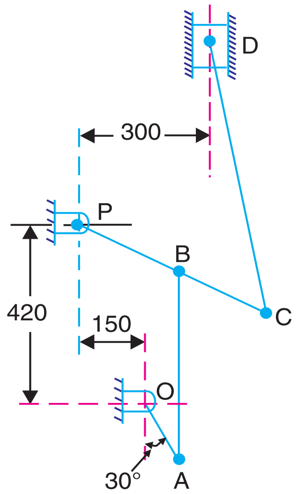
...(By measurement)

∴ Angular acceleration of *CD*,

$$\alpha_{CD} = \frac{a^t_{DC}}{CD} = \frac{1.28}{0.45} = 2.84 \text{ rad/s}^2 \text{ (Clockwise)} \quad \textbf{Ans.}$$

Example 8.7. *Find out the acceleration of the slider D and the angular acceleration of link CD for the engine mechanism shown in Fig. 8.14.*

The crank OA rotates uniformly at 180 r.p.m. in clockwise direction. The various lengths are: OA = 150 mm ; AB = 450 mm; PB = 240 mm ; BC = 210 mm ; CD = 660 mm.



All dimensions in mm.

Fig. 8.14

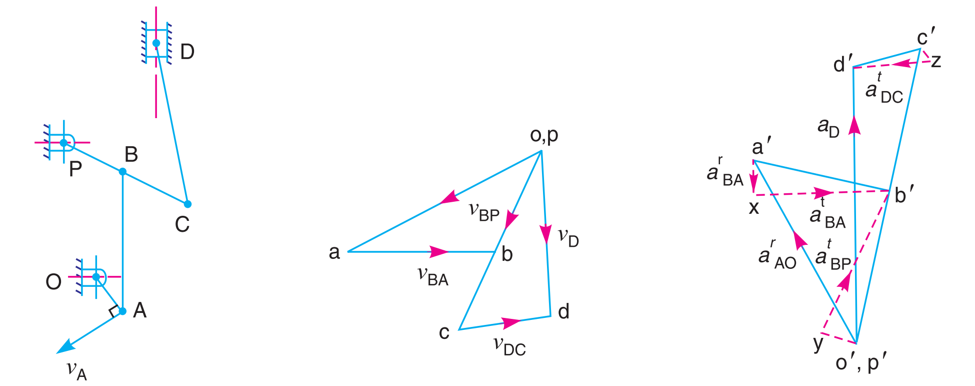
Solution. Given: N_{AO} = 180 r.p.m., or $\omega_{AO} = 2\pi \times 180/60$ = 18.85 rad/s ; *OA* = 150 mm = 0.15 m ; *AB* = 450 mm = 0.45 m ; *PB* = 240 mm = 0.24 m ; *CD* = 660 mm = 0.66 m

We know that velocity of *A* with respect to *O* or velocity of *A*,

$$v_{AO} = v_A = \omega_{AO} \times OA$$
$$= 18.85 \times 0.15 = 2.83 \text{ m/s}$$

...(Perpendicular to *OA*)

First of all draw the space diagram, to some suitable scale, as shown in Fig. 8.15 (*a*). Now the velocity diagram, as shown in Fig. 8.15 (*b*), is drawn as discussed below:



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 8.15

1. Since *O* and *P* are fixed points, therefore these points lie at one place in the velocity diagram. Draw vector *oa* perpendicular to *OA*, to some suitable scale, to represent the velocity of *A* with respect to *O* or velocity of *A* (*i.e.* v_{AO} or v_A), such that

$$\text{vector } oa = v_{AO} = v_A = 2.83 \text{ m/s}$$

2. Since the point *B* moves with respect to *A* and also with respect to *P*, therefore draw vector *ab* perpendicular to *AB* to represent the velocity of *B* with respect to *A i.e.* v_{BA} ,and from point *p* draw vector *pb* perpendicular to *PB* to represent the velocity of *B* with respect to *P* or velocity of *B* (*i.e.* v_{BP} or v_B). The vectors *ab* and *pb* intersect at *b*.

3. Since the point *C* lies on *PB* produced, therefore divide vector *pb* at *c* in the same ratio as *C* divides *PB* in the space diagram. In other words, *pb/pc* = *PB/PC*.

4. From point *c*, draw vector *cd* perpendicular to *CD* to represent the velocity of *D* with respect to *C* and from point *o* draw vector *od* parallel to the path of motion of the slider *D* (which is vertical), to represent the velocity of *D*, *i.e.* v_D.

By measurement, we find that velocity of the slider *D*,

$$v_D = \text{vector } od = 2.36 \text{ m/s}$$

Velocity of *D* with respect to *C*,

$$v_{DC} = \text{vector } cd = 1.2 \text{ m/s}$$

Velocity of *B* with respect to *A*,

$$v_{BA} = \text{vector } ab = 1.8 \text{ m/s}$$

and velocity of *B* with respect to *P*, v_{BP} = vector *pb* = 1.5 m/s

Acceleration of the slider D

We know that radial component of the acceleration of *A* with respect to *O* or acceleration of *A*,

$$a^r_{AO} = a_A = \omega^2_{AO} \times AO = (18.85)^2 \times 0.15 = 53.3 \text{ m/s}^2$$

Radial component of the acceleration of *B* with respect to *A*,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(1.8)^2}{0.45} = 7.2 \text{ m/s}^2$$

Radial component of the acceleration of *B* with respect to *P*,

$$a^r_{BP} = \frac{v^2_{BP}}{PB} = \frac{(1.5)^2}{0.24} = 9.4 \text{ m/s}^2$$

Radial component of the acceleration of *D* with respect to *C*,

$$a^r_{DC} = \frac{v^2_{DC}}{CD} = \frac{(1.2)^2}{0.66} = 2.2 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.15 (*c*), is drawn as discussed below:

1. Since *O* and *P* are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector *o' a'* parallel to *OA*, to some suitable scale, to represent the radial component of the acceleration of *A* with respect to *O* or the acceleration of *A* (*i.e.* a^r_{AO} or a_A), such that

$$\text{vector } o'a' = a^r_{AO} = a_A = 53.3 \text{ m/s}^2$$

2. From point *a*′, draw vector *a′ x* parallel to *AB* to represent the radial component of the acceleration of *B* with respect to *A* (*i.e.* a^r_{BA}), such that

$$\text{vector } a'x = a^r_{BA} = 7.2 \text{ m/s}^2$$

3. From point *x*, draw vector *xb′* perpendicular to the vector *a′x* or *AB* to represent the tangential component of the acceleration of *B* with respect to *A* *i.e.* a^t_{BA} whose magnitude is yet unknown.

4. Now from point *p*′, draw vector *p′ y* parallel to *PB* to represent the radial component of the acceleration of *B* with respect to *P* (*i.e.* a^r_{BP}), such that

$$\text{vector } p'y = a^r_{BP} = 9.4 \text{ m/s}^2$$

5. From point *y*, draw vector *yb′* perpendicular to vector *b′y* or *PB* to represent the tangential component of the acceleration of *B*, *i.e.* a^t_{BP}. The vectors *xb′* and *yb′* intersect at *b*′. Join *p′ b*′. The vector *p′ b*′ represents the acceleration of *B*, *i.e.* a_B.

6. Since the point C lies on PB produced, therefore divide vector $p'b'$ at c' in the same ratio as C divides PB in the space diagram. In other words, $p'b'/p'c' = PB/PC$

7. From point c', draw vector $c'z$ parallel to CD to represent the radial component of the acceleration of D with respect to C *i.e.* a^r_{DC}, such that

$$\text{vector } c'z = a^r_{DC} = 2.2 \text{ m/s}^2$$

8. From point z, draw vector zd' perpendicular to vector $c'z$ or CD to represent the tangential component of the acceleration of D with respect to C *i.e.* a^t_{DC}, whose magnitude is yet unknown.

9. From point o', draw vector $o'd'$ parallel to the path of motion of D (which is vertical) to represent the acceleration of D, *i.e.* a_D. The vectors zd' and $o'd'$ intersect at d'. Join $c'd'$.

By measurement, we find that acceleration of D,

$$a_D = \text{vector } o'd' = 69.6 \text{ m/s}^2 \quad \textbf{Ans.}$$

Angular acceleration of CD

From acceleration diagram, we find that tangential component of the acceleration of D with respect to C,

$$a^t_{DC} = \text{vector } zd' = 17.4 \text{ m/s}^2 \qquad \text{...(By measurement)}$$

We know that angular acceleration of CD,

$$\alpha_{CD} = \frac{a^t_{DC}}{CD} = \frac{17.4}{0.66} = 26.3 \text{ rad/s}^2 \text{ (Anticlockwise)} \quad \textbf{Ans.}$$

Example 8.8. *In the toggle mechanism shown in Fig. 8.16, the slider D is constrained to move on a horizontal path. The crank OA is rotating in the counter-clockwise direction at a speed*

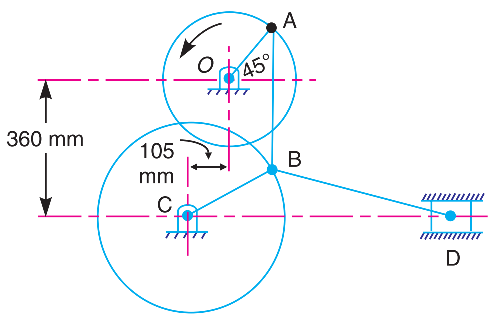


Fig. 8.16

of 180 r.p.m. increasing at the rate of 50 rad/s². The dimensions of the various links are as follows:

OA = 180 mm ; CB = 240 mm ; AB = 360 mm ; and BD = 540 mm.

For the given configuration, find **1.** *Velocity of slider D and angular velocity of BD, and* **2.** *Acceleration of slider D and angular acceleration of BD.*

Solution. Given : $N_{AO} = 180$ r.p.m. or $\omega_{AO} = 2\,\pi \times 180/60 = 18.85$ rad/s ; $OA = 180$ mm $= 0.18$ m ; $CB = 240$ mm $= 0.24$ m ; $AB = 360$ mm $= 0.36$ m ; $BD = 540$ mm $= 0.54$ m

We know that velocity of A with respect to O or velocity of A,

$$v_{AO} = v_A = \omega_{AO} \times OA = 18.85 \times 0.18 = 3.4 \text{ m/s}$$

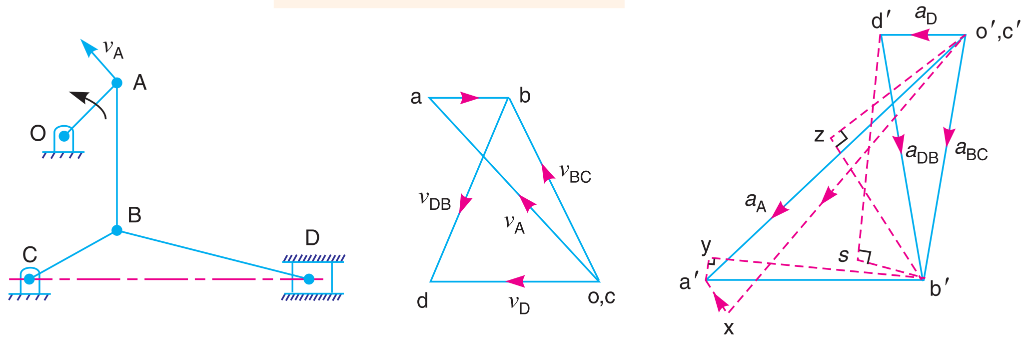
...(Perpendicular to OA)

1. *Velocity of slider D and angular velocity of BD*

First of all, draw the space diagram to some suitable scale, as shown in Fig. 8.17 (*a*). Now the velocity diagram, as shown in Fig. 8.17 (*b*), is drawn as discussed below:

1. Since *O* and *C* are fixed points, therefore these points lie at one place in the velocity diagram. Draw vector *oa* perpendicular to *OA*, to some suitable scale, to represent the velocity of *A* with respect to *O* or velocity of *A i.e.* v_{AO} or v_A, such that

$$\text{vector } oa = v_{AO} = v_A = 3.4 \text{ m/s}$$



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 8.17

2. Since *B* moves with respect to *A* and also with respect to *C*, therefore draw vector *ab* perpendicular to *AB* to represent the velocity of *B* with respect to *A i.e.* v_{BA}, and draw vector *cb* perpendicular to *CB* to represent the velocity of *B* with respect to *C ie.* v_{BC}. The vectors *ab* and *cb* intersect at *b*.

3. From point *b*, draw vector *bd* perpendicular to *BD* to represent the velocity of *D* with respect to *B i.e.* v_{DB}, and from point *c* draw vector *cd* parallel to *CD* (*i.e.*, in the direction of motion of the slider *D*) to represent the velocity of *D i.e.* v_D.

By measurement, we find that velocity of *B* with respect to *A*,

$$v_{BA} = \text{vector } ab = 0.9 \text{ m/s}$$

Velocity of *B* with respect to *C*,

$$v_{BC} = \text{vector } cb = 2.8 \text{ m/s}$$

Velocity of *D* with respect to *B*,

$$v_{DB} = \text{vector } bd = 2.4 \text{ m/s}$$

and velocity of slider *D*, $v_D = \text{vector } cd = 2.05$ m/s **Ans.**

Angular velocity of BD

We know that the angular velocity of *BD*,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{2.4}{0.54} = 4.5 \text{ rad/s} \textbf{ Ans.}$$

2. *Acceleration of slider D and angular acceleration of BD*

Since the angular acceleration of *OA* increases at the rate of 50 rad/s^2, *i.e.* $\alpha_{AO} = 50$ rad/s^2, therefore

Tangential component of the acceleration of *A* with respect to *O*,

$$a^t_{AO} = \alpha_{AO} \times OA = 50 \times 0.18 = 9 \text{ m/s}^2$$

Radial component of the acceleration of A with respect to O,

$$a^r_{AO} = \frac{v^2_{AO}}{OA} = \frac{(3.4)^2}{0.18} = 63.9 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(0.9)^2}{0.36} = 2.25 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to C,

$$a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(2.8)^2}{0.24} = 32.5 \text{ m/s}^2$$

and radial component of the acceleration of D with respect to B,

$$a^r_{DB} = \frac{v^2_{DB}}{BD} = \frac{(2.4)^2}{0.54} = 10.8 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.17 (*c*), is drawn as discussed below:

1. Since O and C are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector $o'x$ parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O *i.e.* a^r_{AO}, such that

$$\text{vector } o'x = a^r_{AO} = 63.9 \text{ m/s}^2$$

2. From point x, draw vector xa' perpendicular to vector $o'x$ or OA to represent the tangential component of the acceleration of A with respect to O *i.e.* a^t_{AO} ,such that

$$\text{vector } xa' = a^t_{AO} = 9 \text{ m/s}^2$$

3. Join $o'a'$. The vector $o'a'$ represents the total acceleration of A with respect to O or acceleration of A *i.e.* a_{AO} or a_A.

An experimental IC engine with crank shaft and cylinders.

Note : This picture is given as additional information and is not a direct example of the current chapter.

4. Now from point a', draw vector $a'y$ parallel to AB to represent the radial component of the acceleration of B with respect to A *i.e.* a^r_{BA}, such that

$$\text{vector } a'y = a^r_{BA} = 2.25 \text{ m/s}^2$$

5. From point y, draw vector yb' perpendicular to vector $a'y$ or AB to represent the tangential component of the acceleration of B with respect to A *i.e.* a^t_{BA} whose magnitude is yet unknown.

6. Now from point c', draw vector $c'z$ parallel to CB to represent the radial component of the acceleration of B with respect to C *i.e.* a^r_{BC}, such that

$$\text{vector } c'z = a^r_{BC} = 32.5 \text{ m/s}^2$$

7. From point z, draw vector zb' perpendicular to vector $c'z$ or CB to represent the tangential component of the acceleration of B with respect to C *i.e.* a^t_{BC} . The vectors yb' and zb' intersect at b'. Join $c'b'$. The vector $c'b'$ represents the acceleration of B with respect to C *i.e.* a_{BC}.

8. Now from point b', draw vector $b's$ parallel to BD to represent the radial component of the acceleration of D with respect to B *i.e.* a^r_{DB}, such that

$$\text{vector } b's = a^r_{DB} = 10.8 \text{ m/s}^2$$

9. From point s, draw vector sd' perpendicular to vector $b's$ or BD to represent the tangential component of the acceleration of D with respect to B *i.e.* a^t_{DB} whose magnitude is yet unknown.

10. From point c', draw vector $c'd'$ parallel to the path of motion of D (which is along CD) to represent the acceleration of D *i.e.* a_D. The vectors sd' and $c'd'$ intersect at d'.

By measurement, we find that acceleration of slider D,

$$a_D = \text{vector } c'd' = 13.3 \text{ m/s}^2 \textbf{ Ans.}$$

Angular acceleration of BD

By measurement, we find that tangential component of the acceleration of D with respect to B,

$$a^t_{DB} = \text{vector } sd' = 38.5 \text{ m/s}^2$$

We know that angular acceleration of BD,

$$\alpha_{BD} = \frac{a^t_{DB}}{BD} = \frac{38.5}{0.54} = 71.3 \text{ rad/s}^2 \text{ (Clockwise) } \textbf{Ans.}$$

Example 8.9. *The mechanism of a warping machine, as shown in Fig. 8.18, has the dimensions as follows:*

O_1A = 100 mm; AC = 700 mm ; BC = 200 mm ; BD = 150 mm ; O_2D = 200 mm ; O_2E = 400 mm ; O_3C = 200 mm.

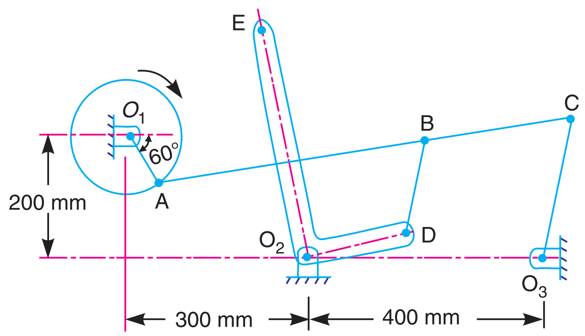


Fig. 8.18

The crank O_1A rotates at a uniform speed of 100 rad/s. For the given configuration, determine: **1.** *linear velocity of the point E on the bell crank lever,* **2.** *acceleration of the points E and B, and* **3.** *angular acceleration of the bell crank lever.*

Solution. Given : ω_{AO1} = 100 rad/s ; O_1A = 100 mm = 0.1 m

We know that linear velocity of A with respect to O_1, or velocity of A,

$$v_{AO1} = v_A = \omega_{AO1} \times O_1A = 100 \times 0.1 = 10 \text{ m/s} \quad \text{...(Perpendicular to } O_1A)$$

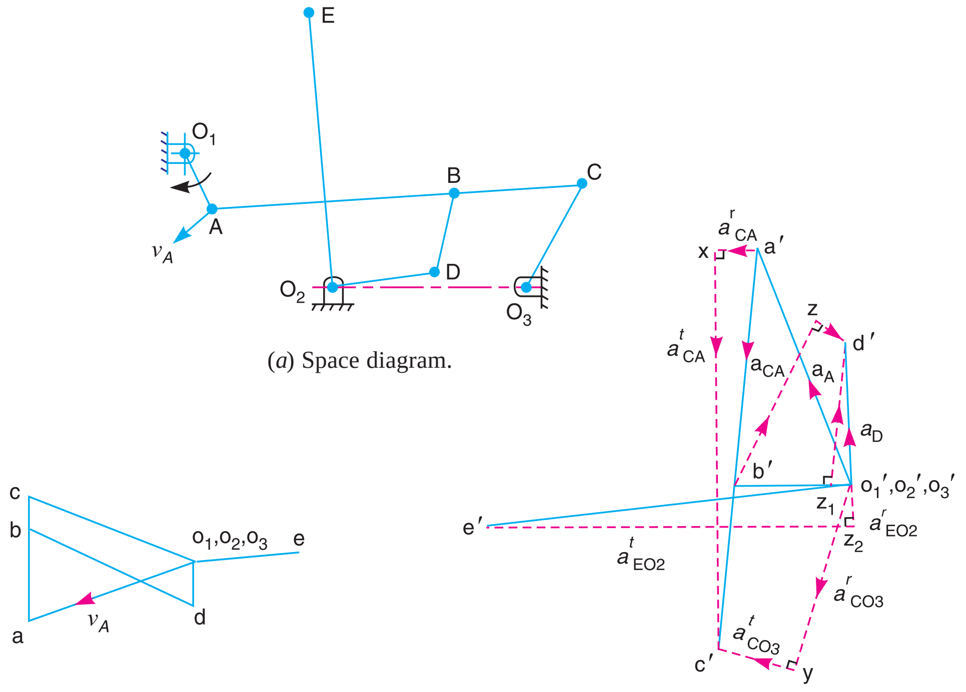
1. *Linear velocity of the point E on bell crank lever*

First of all draw the space diagram, as shown in Fig. 8.19 (*a*), to some suitable scale. Now the velocity diagram, as shown in Fig. 8.19 (*b*), is drawn as discussed below:

1. Since O_1, O_2 and O_3 are fixed points, therefore these points are marked as one point in the velocity diagram. From point o_1, draw vector o_1a perpendicular to O_1A to some suitable scale, to represent the velocity of A with respect to O or velocity of A, such that

$$\text{vector } o_1a = v_{AO1} = v_A = 10 \text{ m/s}$$

2. From point a, draw vector ac perpendicular to AC to represent the velocity of C with respect to A (*i.e.* v_{CA}) and from point o_3 draw vector o_3c perpendicular to O_3C to represent the velocity of C with respect to O_3 or simply velocity of C (*i.e.* v_C). The vectors ac and o_3c intersect at point c.



(*a*) Space diagram.

(*b*) Velocity diagram.

(*c*) Acceleration diagram.

Fig. 8.19

3. Since B lies on AC, therefore divide vector ac at b in the same ratio as B divides AC in the space diagram. In other words, $ab/ac = AB/AC$

4. From point b, draw vector bd perpendicular to BD to represent the velocity of D with respect to B (*i.e.* v_{DB}), and from point o_2 draw vector o_2d perpendicular to O_2D to represent the velocity of D with respect to O_2 or simply velocity of D (*i.e.* v_D). The vectors bd and o_2d intersect at d.

5. From point o_2, draw vector o_2e perpendicular to vector o_2d in such a way that

$$o_2e/o_2d = O_2E/O_2D$$

By measurement, we find that velocity of point C with respect to A,

$$v_{CA} = \text{vector } ac = 7 \text{ m/s}$$

Velocity of point C with respect to O_3,

$$v_{CO3} = v_C = \text{vector } o_3c = 10 \text{ m/s}$$

Velocity of point D with respect to B,

$$v_{DB} = \text{vector } bd = 10.2 \text{ m/s}$$

Warping machine uses many mechanisms.

Velocity of point D with respect to O_2,

$$v_{DO2} = v_D = \text{vector } o_2d = 2.8 \text{ m/s}$$

and velocity of the point E on the bell crank lever,

$$v_E = v_{EO2} = \text{vector } o_2e = 5.8 \text{ m/s} \quad \textbf{Ans.}$$

2. ***Acceleration of the points E and B***

Radial component of the acceleration of A with respect to O_1 (or acceleration of A),

$$a^r_{AO2} = a_{AO1} = a_A = \frac{v^2_{AO1}}{O_1A} = \frac{10^2}{0.1} = 1000 \text{ m/s}^2$$

Radial component of the acceleration of C with respect to A,

$$a^r_{CA} = \frac{v^2_{CA}}{AC} = \frac{7^2}{0.7} = 70 \text{ m/s}^2$$

Radial component of the acceleration of C with respect to O_3,

$$a^r_{CO3} = \frac{v^2_{CO3}}{O_3C} = \frac{10^2}{0.2} = 500 \text{ m/s}^2$$

Radial component of the acceleration of D with respect to B,

$$a^r_{DB} = \frac{v^2_{DB}}{BD} = \frac{(10.2)^2}{0.15} = 693.6 \text{ m/s}^2$$

Radial component of the acceleration of D with respect to O_2,

$$a^r_{DO_2} = \frac{v^2_{DO2}}{O_2D} = \frac{(2.8)^2}{0.2} = 39.2 \text{ m/s}^2$$

Radial component of the acceleration of E with respect to O_2,

$$a^r_{EO_2} = \frac{v^2_{EO2}}{O_2E} = \frac{(5.8)^2}{0.4} = 84.1 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.19 (c), is drawn as discussed below:

1. Since O_1, O_2 and O_3 are fixed points, therefore these points are marked as one point in the acceleration diagram. Draw vector $o_1'a'$ parallel to O_1A, to some suitable scale, to represent the radial component of the acceleration of A with respect to O_1 (or simply acceleration of A), such that

$$\text{vector } o_1'a' = a^r_{AO_1} = a_A = 1000 \text{ m/s}^2$$

2. From point a', draw $a'x$ parallel to AC to represent the radial component of the acceleration of C with respect to A (*i.e.* a^r_{CA}), such that

$$\text{vector } a'x = a^r_{CA} = 70 \text{ m/s}^2$$

3. From point x, draw vector xc' perpendicular to AC to represent the tangential component of the acceleration of C with respect to A (*i.e.* a^t_{CA}), the magnitude of which is yet unknown.

4. From point o_3', draw vector $o_3'y$ parallel to O_3C to represent the radial component of the acceleration of C with respect to O_3 (*i.e.* a^r_{CO3}), such that

$$\text{vector } o_3'y = a^r_{CO3} = 500 \text{ m/s}^2$$

5. From point y, draw vector yc' perpendicular to O_3C to represent the tangential component of the acceleration of C with respect to O_3 (*i.e.* a^t_{CO3}). The vectors xc' and yc' intersect at c'.

6. Join $a'c'$. The vector $a'c'$ represents the acceleration of C with respect to A (*i.e.* a_{CA}).

7. Since B lies on AC, therefore divide vector $a'c'$ at b' in the same ratio as B divides AC in the space diagram. In other words, $a'b'/a'c' = AB/AC$. Join $b'o_2'$ which represents the acceleration of point B with respect to O_2 or simply acceleration of B. By measurement, we find that

Acceleration of point B = vector $o_2'b'$ = 440 m/s^2 **Ans.**

8. Now from point b', draw vector $b'z$ parallel to BD to represent the radial component of the acceleration of D with respect to B (*i.e.* a^r_{DB}), such that

$$\text{vector } b'z = a^r_{DB} = 693.6 \text{ m/s}^2$$

9. From point z, draw vector zd' perpendicular to BD to represent the tangential component of the acceleration of D with respect to B (*i.e.* a^t_{DB}), whose magnitude is yet unknown.

10. From point o_2', draw vector $o_2'z_1$ parallel to O_2D to represent the radial component of the acceleration of D with respect to O_2 (*i.e.* $a^r_{DO_2}$), such that

$$\text{vector } o_2'z_1 = a^r_{DO_2} = 39.2 \text{ m/s}^2$$

11. From point z_1, draw vector z_1d' perpendicular to O_2D to represent the tangential component of the acceleration of D with respect to O_2 (*i.e.* $a^t_{DO_2}$). The vectors zd' and z_1d' intersect at d'.

12. Join $o_2'd'$. The vector $o_2'd'$ represents the acceleration of D with respect to O_2 or simply acceleration of D (*i.e.* a_{DO2} or a_D).

13. From point o_2', draw vector $o_2'e'$ perpendicular to $o_2'd'$ in such a way that

$$o_2'e' / o_2'd' = O_2E / O_2D$$

Note: The point e' may also be obtained drawing $a^r_{EO_2}$ and $a^t_{EO_2}$ as shown in Fig. 8.19 (*c*).

By measurement, we find that acceleration of point E,

$$a_E = a_{EO2} = \text{vector } o'_2e' = 1200 \text{ m/s}^2 \text{ **Ans.**}$$

3. *Angular acceleration of the bell crank lever*

By measurement, we find that the tangential component of the acceleration of D with respect to O_2,

$$a^t_{DO_2} = \text{vector } z_1d_1' = 610 \text{ m/s}^2$$

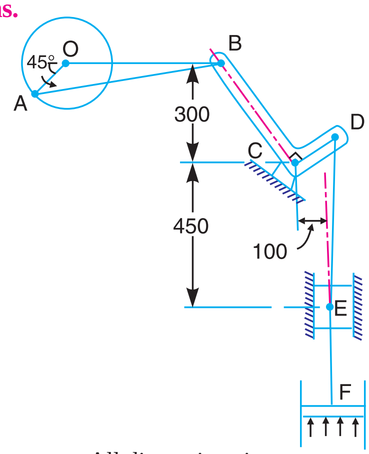
∴ Angular acceleration of the bell crank lever

$$= \frac{a^t_{DO2}}{O_2D} = \frac{610}{0.2} = 3050 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

Example 8.10. *A pump is driven from an engine crank-shaft by the mechanism as shown in Fig. 8.20. The pump piston shown at F is 250 mm in diameter and the crank speed is 100 r.p.m. The dimensions of various links are as follows:*

OA = 150 mm ; AB = 600 mm ; BC = 350 mm ; CD = 150 mm; and DE = 500 mm.

Determine for the position shown : **1**. *The velocity of the cross-head E,* **2.** *The rubbing velocity of the pins A and B which are 50 mm diameter.* **3.** *The torque required at the crank shaft to overcome a presure of 0.35 N/mm^2, and* **4.** *The acceleration of the cross-head E.*



All dimensions in mm.

Fig. 8.20

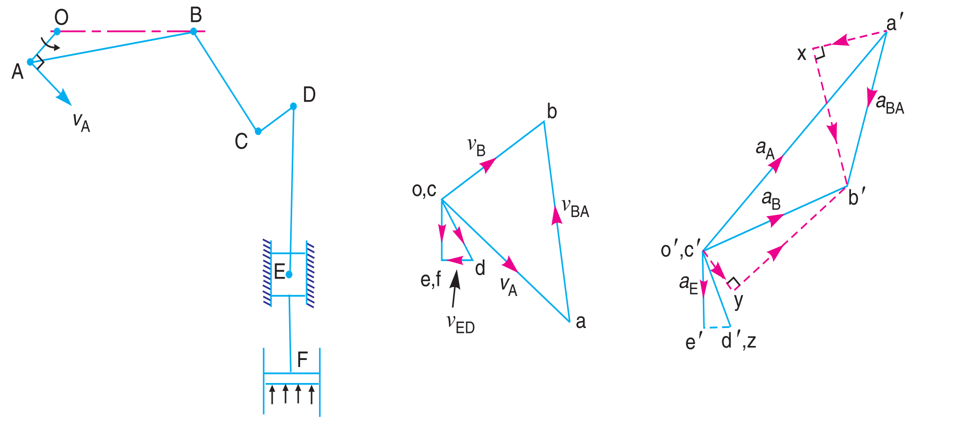
Solution. Given : N_{AO} = 100 r.p.m. or $\omega_{AO} = 2\,\pi \times 100/60 = 10.47$ rad/s; OA = 150 mm = 0.15 m ; AB = 600 mm = 0.6 m ; BC = 350 mm = 0.35 m ; CD = 150 mm = 0.15 m ; DE = 500 mm = 0.5 m

We know that velocity of A with respect to O or velocity of A,

$$v_{AO} = v_A = \omega_{AO} \times OA = 10.47 \times 0.15 = 1.57 \text{ m/s} \qquad \text{...(Perpendicular to } OA)$$

1. *Velocity of the cross-head E*

First of all, draw the space diagram, to some suitable scale, as shown in Fig. 8.21 (*a*). Now the velocity diagram, as shown in Fig. 8.21 (*b*), is drawn as discussed below:



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 8.21

1. Since O and C are fixed points, therefore these points are marked as one point in the velocity diagram. Now draw vector *oa* perpendicular to OA, to some suitable scale, to represent the velocity of A with respect ot O or the velocity of A, such that

$$\text{vector } oa = v_{AO} = v_A = 1.57 \text{ m/s}$$

2. From point *a*, draw vector *ab* perpendicular to AB to represent the velocity of B with respect to A (*i.e.* v_{BA}), and from point *c* draw vector *cb* perpendicular to CB to represent the velocity of B with respect to C (*i.e.* v_{BC}). The vectors *ab* and *cb* intersect at *b*.

By measurement, we find that

$$v_{BA} = \text{vector } ab = 1.65 \text{ m/s}$$

and $$v_{BC} = v_B = \text{vector } cb = 0.93 \text{ m/s}$$

3. From point *c*, draw vector *cd* perpendicular to *CD* or vector *cb* to represent the velocity of *D* with respect to *C* or velocity of *D*, such that

$$\text{vector } cd : \text{vector } cb = CD : CB \qquad \text{or} \qquad v_{DC} : v_{BC} = CD : CB$$

$$\therefore \qquad \frac{v_{DC}}{v_{BC}} = \frac{CD}{CB} \quad \text{or} \quad v_{DC} = v_{BC} \times \frac{CD}{CB} = 0.93 \times \frac{0.15}{0.35} = 0.4 \text{ m/s}$$

4. From point *d*, draw vector *de* perpendicular to *DE* to represent the velocity of *E* with respect to *D* (*i.e.* v_{ED}), and from point *o* draw vector *oe* parallel to the path of motion of *E* (which is vertical) to represent the velocity of *E* or *F*. The vectors *oe* and *de* intersect at *e*.

By measurement, we find that velocity of *E* with respect to *D*,

$$v_{ED} = \text{vector } de = 0.18 \text{ m/s}$$

and velocity of the cross-head *E*,

$$v_{EO} = v_E = \text{vector } oe = 0.36 \text{ m/s} \quad \textbf{Ans.}$$

2. *Rubbing velocity of the pins at A and B*

We know that angular velocity of *A* with respect to *O*,

$$\omega_{AO} = 10.47 \text{ rad/s} \qquad \text{...(Anticlockwise)}$$

Angular velocity of *B* with respect to *A*,

$$\omega_{BA} = \frac{v_{BA}}{AB} = \frac{1.65}{0.6} = 2.75 \text{ rad/s} \qquad \text{...(Anticlockwise)}$$

and angular velocity of *B* with respect to *C*,

$$\omega_{BC} = \frac{v_{BC}}{CB} = \frac{0.93}{0.35} = 2.66 \text{ rad/s} \qquad \text{...(clockwise)}$$

We know that diameter of pins at *A* and *B*,

$$d_A = d_B = 50 \text{ mm} = 0.05 \text{ m} \qquad \text{...(Given)}$$

or Radius, $r_A = r_B = 0.025$ m

∴ Rubbing velocity of pin at *A*

$$= (\omega_{AO} - \omega_{BA})\, r_A = (10.47 - 2.75)\, 0.025 = 0.193 \text{ m/s} \quad \textbf{Ans.}$$

and rubbing velocity of pin at *B*

$$= (\omega_{BA} + \omega_{BC})\, r_B = (2.75 + 2.66)\, 0.025 = 0.135 \text{ m/s} \quad \textbf{Ans.}$$

3. *Torque required at the crankshaft*

Given: Pressure to overcome by the crankshaft,

$$p_F = 0.35 \text{ N/mm}^2$$

Diameter of the pump piston

$$D_F = 250 \text{ mm}$$

∴ Force at the pump piston at *F*,

$$F_F = \text{Pressure} \times \text{Area} = p_F \times \frac{\pi}{4}(D_F)^2 = 0.35 \times \frac{\pi}{4}(250)^2 = 17\ 183 \text{ N}$$

Let F_A = Force required at the crankshaft at *A*.

Assuming transmission efficiency as 100 per cent,

Work done at *A* = Work done at *F*

$$F_A \times v_A = F_F \times v_F \quad \text{or} \quad F_A = \frac{F_F \times v_F}{v_A} = \frac{17\,183 \times 0.36}{1.57} = 3940 \text{ N}$$

$$\text{...}(\because v_F = v_E)$$

∴ Torque required at the crankshaft,

$$T_A = F_A \times OA = 3940 \times 0.15 = 591 \text{ N-m} \quad \textbf{Ans.}$$

Acceleration of the crosshead E

We know that the radial component of the acceleration of *A* with respect to *O* or the acceleration of *A*,

$$a^r_{AO} = a_A = \frac{v^2_{AO}}{OA} = \frac{(1.57)^2}{0.15} = 16.43 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A,

$$a_{BA}^{r} = \frac{v_{BA}^2}{AB} = \frac{(1.65)^2}{0.6} = 4.54 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to C.

$$a_{BC}^{r} = \frac{v_{BC}^2}{CB} = \frac{(0.93)^2}{0.35} = 2.47 \text{ m/s}^2$$

and radial component of the acceleration of E with respect to D,

$$a_{ED}^{r} = \frac{v_{ED}^2}{DE} = \frac{(0.18)^2}{0.5} = 0.065 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.21 (*c*), is drawn as discussed below:

1. Since O and C are fixed points, therefore these points are marked as one point in the acceleration diagram. Draw vector $o'a'$ parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O or the acceleration of A, such that

$$\text{vector } o'a' = a_{AO}^{r} = a_A = 16.43 \text{ m/s}^2$$

2. From point a', draw vector $a'x$ parallel to AB to represent the radial component of the acceleration of B with respect to A (*i.e.* a_{BA}^{r}), such that

$$\text{vector } a'x = a_{BA}^{r} = 4.54 \text{ m/s}^2$$

3. From point x, draw vector xb' perpendicular to AB to represent the tangential component of the acceleration of B with respect to A (*i.e.* a_{BA}^{t}) whose magnitude is yet unknown.

4. Now from point c', draw vector $c'y$ parallel to CB to represent the radial component of the acceleration of B with respect to C (*i.e.* a_{BC}^{r}), such that

$$\text{vector } c'y = a_{BC}^{r} = 2.47 \text{ m/s}^2$$

5. From point y, draw vector yb' perpendicular to CB to represent the tangential component of the acceleration of B with respect to C (*i.e.* a_{BC}^{t}). The vectors yb' and xb' intersect at b'. Join $c'b'$ and $a'b'$. The vector $c'b'$ represents the acceleration of B with respect to C (*i.e.* a_{BC}) or the acceleration of B (*i.e.* a_B) and vector $a'b'$ represents the acceleration of B with respect to A (*i.e.* a_{BA}).

By measurement, we find that

$$a_{BC} = a_B = \text{vector } c'b' = 9.2 \text{ m/s}^2$$

and $$a_{BA} = \text{vector } a'b' = 9 \text{ m/s}^2$$

6. From point c', draw vector $c'd'$ perpendicular to CD or vector $c'b'$ to represent the acceleration of D with respect to C or the acceleration of D (*i.e.* a_{DC} or a_D), such that

vector $c'd'$: vector $c'b' = CD : CB$ or $a_D : a_{BC} = CD : CB$

$$\therefore \quad \frac{a_D}{a_{BC}} = \frac{CD}{CB} \quad \text{or} \quad a_D = a_{BC} \times \frac{CD}{CB} = 9.2 \times \frac{0.15}{0.35} = 3.94 \text{ m/s}^2$$

7. Now from point d', draw vector $d'z$ parallel to DE to represent the radial component of E with respect to D (*i.e.* a_{ED}^{r}), such that

$$\text{vector } d'z = a_{ED}^{r} = 0.065 \text{ m/s}^2$$

Note: Since the magnitude of a^{r}_{ED} is very small, therefore the points d' and z coincide.

8. From point z, draw vector ze' perpendicular to DE to represent the tangential component of the acceleration of E with respect to D (*i.e.* a_{ED}^{t}) whose magnitude is yet unknown.

9. From point o', draw vector $o'e'$ parallel to the path of motion of E (which is vertical) to represent the acceleration of E. The vectors ze' and $o'e'$ intersect at e'.

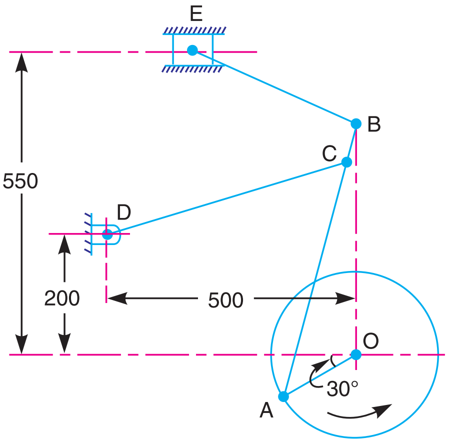
By measurement, we find that acceleration of the crosshead *E*,

$$a_E = \text{vector } o'e' = 3.8 \text{ m/s}^2 \quad \textbf{Ans.}$$

Example 8.11. *Fig. 8.22 shows the mechanism of a radial valve gear. The crank OA turns uniformly at 150 r.p.m and is pinned at A to rod AB. The point C in the rod is guided in the circular path with D as centre and DC as radius. The dimensions of various links are:*

OA = 150 mm ; AB = 550 mm ; AC = 450 mm ; DC = 500 mm ; BE = 350 mm.

Determine velocity and acceleration of the ram E for the given position of the mechanism.



All dimensions in mm.

Fig. 8.22

Solution. Given : $N_{AO} = 150$ r.p.m. or $\omega_{AO} = 2\pi \times 150/60 = 15.71$ rad/s; $OA = 150$ mm $= 0.15$ m; $AB = 550$ mm $= 0.55$ m ; $AC = 450$ mm $= 0.45$ m ; $DC = 500$ mm $= 0.5$ m ; $BE = 350$ mm $= 0.35$ m

We know that linear velocity of *A* with respect to *O* or velocity of *A*,

$$v_{AO} = v_A = \omega_{AO} \times OA = 15.71 \times 0.15 = 2.36 \text{ m/s}$$

...(Perpendicular to *OA*)

Velocity of the ram E

First of all draw the space diagram, as shown in Fig. 8.23 (*a*), to some suitable scale. Now the velocity diagram, as shown in Fig. 8.23 (*b*), is drawn as discussed below:

1. Since *O* and *D* are fixed points, therefore these points are marked as one point in the velocity diagram. Draw vector *oa* perpendicular to *OA*, to some suitable scale, to represent the velocity of *A* with respect to *O* or simply velocity of *A*, such that

$$\text{vector } oa = v_{AO} = v_A = 2.36 \text{ m/s}$$

2. From point *a*, draw vector *ac* perpendicular to *AC* to represent the velocity of *C* with respect to *A* (*i.e.* v_{CA}), and from point *d* draw vector *dc* perpendicular to *DC* to represent the velocity of *C* with respect to *D* or simply velocity of *C* (*i.e.* v_{CD} or v_C). The vectors *ac* and *dc* intersect at *c*.

3. Since the point *B* lies on *AC* produced, therefore divide vector *ac* at *b* in the same ratio as *B* divides *AC* in the space diagram. In other words *ac*:*cb* = *AC*:*CB*. Join *ob*. The vector *ob* represents the velocity of *B* (*i.e.* v_B)

4. From point *b*, draw vector *be* perpendicular to *be* to represent the velocity of *E* with respect to *B* (*i.e.* v_{EB}), and from point *o* draw vector *oe* parallel to the path of motion of the ram *E* (which is horizontal) to represent the velocity of the ram *E*. The vectors *be* and *oe* intersect at *e*.

By measurement, we find that velocity of *C* with respect to *A*,

$$v_{CA} = \text{vector } ac = 0.53 \text{ m/s}$$

Velocity of *C* with respect to *D*,

$$v_{CD} = v_C = \text{vector } dc = 1.7 \text{ m/s}$$

Velocity of E with respect to B,

$$v_{EB} = \text{vector } be = 1.93 \text{ m/s}$$

and velocity of the ram E, $v_E = \text{vector } oe = 1.05 \text{ m/s}$ **Ans.**

Acceleration of the ram E

We know that the radial component of the acceleration of A with respect to O or the acceleration of A,

$$a^r_{AO} = a_A = \frac{v^2_{AO}}{OA} = \frac{(2.36)^2}{0.15} = 37.13 \text{ m/s}^2$$

Radial component of the acceleration of C with respect to A,

$$a^r_{CA} = \frac{v^2_{CA}}{OA} = \frac{(0.53)^2}{0.45} = 0.624 \text{ m/s}^2$$

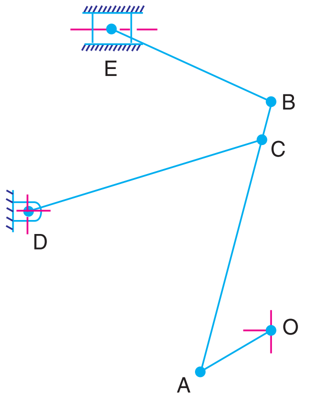
Radial component of the acceleration of C with respect to D,

$$a^r_{CD} = \frac{v^2_{CD}}{DC} = \frac{(1.7)^2}{0.5} = 5.78 \text{ m/s}^2$$

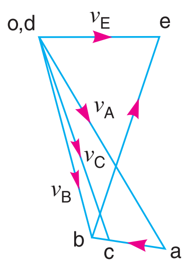
Radial component of the acceleration of E with respect to B,

$$a^r_{EB} = \frac{v^2_{EB}}{BE} = \frac{(1.93)^2}{0.35} = 10.64 \text{ m/s}^2$$

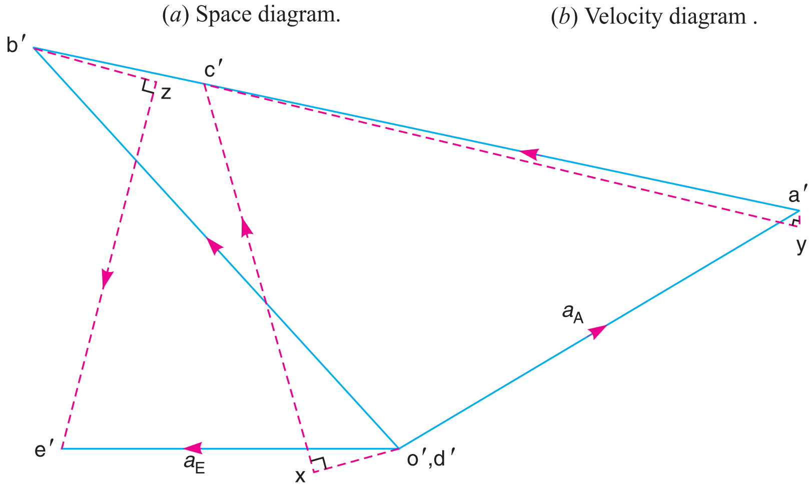
The acceleration diagram, as shown in Fig. 8.23 (c), is drawn as discussed below:



(a) Space diagram.



(b) Velocity diagram .



(c) Acceleration diagram.

Fig. 8.23

1. Since O and D are fixed points, therefore these points are marked as one point in the acceleration diagram. Draw vector $o'a'$ parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O or simply the acceleration of A, such that

$$\text{vector } o'a' = a^r_{AO} = a_A = 37.13 \text{ m/s}^2$$

2. From point d', draw vector $d'x$ parallel to DC to represent the radial component of the acceleration of C with respect to D, such that

$$\text{vector } d'x = a^r_{CD} = 5.78 \text{ m/s}^2$$

3. From point x, draw vector xc' perpendicular to DC to represent the tangential component of the acceleration of C with respect to D (*i.e.* a^t_{CD}) whose magnitude is yet unknown.

4. Now from point a', draw vector $a'y$ parallel to AC to represent the radial component of the acceleration of C with respect to A, such that

$$\text{vector } a'y = a^r_{CA} = 0.624 \text{ m/s}^2$$

5. From point y, draw vector yc' perpendicular to AC to represent the tangential component of acceleration of C with respect to A (*i.e.* a^t_{CA}). The vectors xc' and yc' intersect at c'.

6. Join $a'c'$. The vector $a'c'$ represents the acceleration of C with respect to A (*i.e.* a_{CA}).

7. Since the point B lies on AC produced, therefore divide vector $a'c'$ at b' in the same ratio as B divides AC in the space diagram. In other words, $a'c' : c'b' = AC : CB$.

8. From point b', draw vector $b'z$ parallel to BE to represent the radial component of the acceleration of E with respect to B, such that

A lathe is a machine for shaping a piece of metal, by rotating it rapidly along its axis while pressing against a fixed cutting or abrading tool.

Note : This picture is given as additional information and is not a direct example of the current chapter.

$$\text{vector } b'z = a^r_{EB} = 10.64 \text{ m/s}^2$$

9. From point z, draw vector ze' perpendicular to BE to represent the tangential component of the acceleration of E with respect to B (*i.e.* a^t_{EB}) whose magnitude is yet unknown.

10. From point o', draw vector $o'e'$ parallel to the path of motion of E (which is horizontal) to represent the acceleration of the ram E. The vectors ze' and $o'e'$ intersect at e'.

By measurement, we find that the acceleration of the ram E,

$$a_E = \text{vector } o'e' = 3.1 \text{ m/s}^2 \textbf{ Ans.}$$

Example 8.12. *The dimensions of the Andreau differential stroke engine mechanism, as shown in Fig. 8.24, are as follows:*

AB = 80 mm ; CD = 40 mm ; BE = DE = 150 mm ; and EP = 200 mm.

The links AB and CD are geared together. The speed of the smaller wheel is 1140 r.p.m. Determine the velocity and acceleration of the piston P for the given configuration.

Solution. Given: N_{DC} = 1140 r.p.m. or $\omega_{DC} = 2\pi \times 1140/60 = 119.4$ rad/s ; AB = 80 mm = 0.08 m ; CD = 40 mm = 0.04 m ; $BE = DE$ = 150 mm = 0.15 m ; EP = 200 mm = 0.2 m

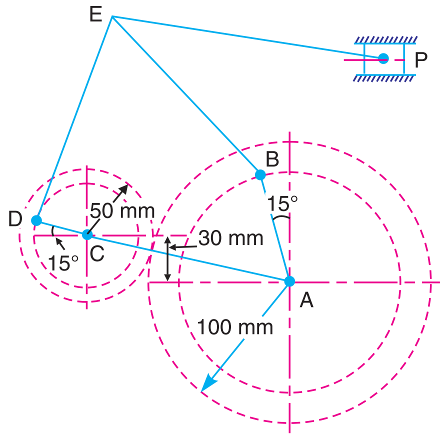


Fig. 8.24

We know that velocity of D with respect to C or velocity of D,

$$v_{DC} = v_D = \omega_{DC} \times CD = 119.4 \times 0.04 = 4.77 \text{ m/s} \qquad \text{...(Perpendicular to } CD)$$

Since the speeds of the gear wheels are inversely proportional to their diameters, therefore

$$\frac{\text{Angular speed of larger wheel}}{\text{Angular speed of smaller wheel}} = \frac{\omega_{BA}}{\omega_{DC}} = \frac{2CD}{2AB}$$

∴ Angular speed of larger wheel,

$$\omega_{BA} = \omega_{DC} \times \frac{CD}{AB} = 119.4 \times \frac{0.04}{0.08} = 59.7 \text{ rad/s}$$

and velocity of B with respect to A or velocity of B,

$$v_{BA} = v_B = \omega_{BA} \times AB = 59.7 \times 0.08 = 4.77 \text{ m/s}$$

...(Perpendicular to AB)

Velocity of the piston P

First of all draw the space diagram, to some suitable scale, as shown in Fig. 8.25 (*a*). Now the velocity diagram, as shown in Fig. 8.25 (*b*), is drawn as discussed below:

1. Since A and C are fixed points, therefore these points are marked as one point in the velocity diagram. Draw vector cd perpendicular to CD, to some suitable scale, to represent the velocity of D with respect to C or velocity of D (*i.e.* v_{DC} or v_D), such that

$$\text{vector } cd = v_{DC} = v_D = 4.77 \text{ m/s}$$

2. Draw vector ab perpendicular to AB to represent the velocity of B with respect to A or velocity of B (*i.e.* v_{BA} or v_B), such that

$$\text{vector } ab = v_{BA} = v_B = 4.77 \text{ m/s}$$

3. Now from point b, draw vector be perpendicular to BE to represent the velocity of E with respect to B (*i.e.* v_{EB}), and from point d draw vector de perpendicular to DE to represent the velocity of E with respect to D (*i.e.* v_{ED}). The vectors be and de intersect at e.

4. From point e, draw vector ep perpendicular to EP to represent the velocity of P with respect to E (*i.e.* v_{PE}), and from point a draw vector ap parallel to the path of motion of P (which is horizontal) to represent the velocity of P. The vectors ep and ap intersect at p.

By measurement, we find that velocity of E with respect to B,

$$v_{EB} = \text{vector } be = 8.1 \text{ m/s}$$

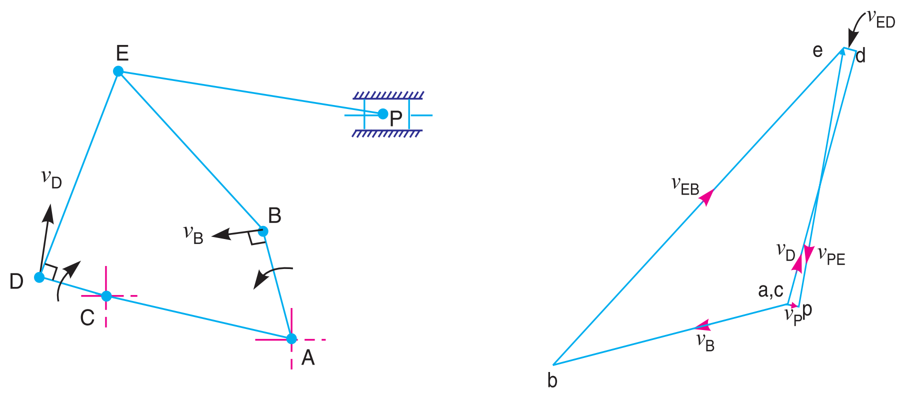
Velocity of E with respect to D,

$$v_{ED} = \text{vector } de = 0.15 \text{ m/s}$$

Velocity of P with respect to E,

$$v_{PE} = \text{vector } ep = 4.7 \text{ m/s}$$

and velocity of P, v_P = vector ap = 0.35 m/s **Ans.**



(*a*) Space diagram. (*b*) Velocity diagram.

Fig. 8.25

Acceleration of the piston P

We know that the radial component of the acceleration of B with respect A (or the acceleration of B),

$$a^r_{BA} = a_B = \frac{v^2_{BA}}{AB} = \frac{(4.77)^2}{0.08} = 284.4 \text{ m/s}^2$$

Radial component of the acceleration of D with respect to C (or the acceleration of D),

$$a^r_{DC} = a_D = \frac{v^2_{DC}}{CD} = \frac{(4.77)^2}{0.04} = 568.8 \text{ m/s}^2$$

Radial component of the acceleration of E with respect to B,

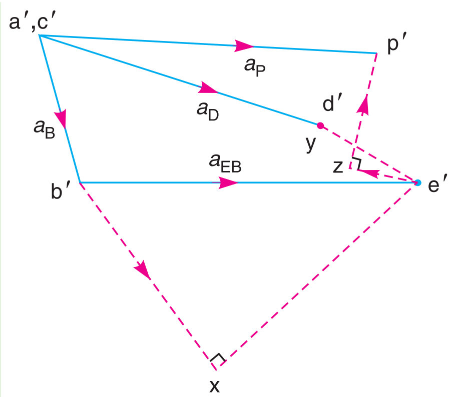
$$a^r_{EB} = \frac{v^2_{EB}}{BE} = \frac{(8.1)^2}{0.15} = 437.4 \text{ m/s}^2$$

Radial component of the acceleration of E with respect to D,

$$a^r_{ED} = \frac{v^2_{ED}}{DE} = \frac{(0.15)^2}{0.15} = 0.15 \text{ m/s}^2$$

and radial component of the acceleration of P with respect to E,

$$a^r_{PE} = \frac{v^2_{PE}}{EP} = \frac{(4.7)^2}{0.2} = 110.45 \text{ m/s}^2$$



(*c*) Acceleration diagram.

Fig. 8.25

Now the acceleration diagram, as shown in Fig. 8.25 (*c*), is drawn as discussed below:

1. Since *A* and *C* are fixed points, therefore these points are marked as one point in the acceleration diagram. Draw vector *a'b'* parallel to *AB*, to some suitable scale, to represent the radial component of the acceleration of *B* with respect to *A* or the acceleration of *B*, such that

$$\text{vector } a'b' = a^r_{BA} = a_B = 284.4 \text{ m/s}^2$$

2. Draw vector *c'd'* parallel to *CD* to represent the radial component of the acceleration of *D* with respect to *C* or the acceleration of *D*, such that

$$\text{vector } c'd' = a^r_{DC} = a_D = 568.8 \text{ m/s}^2$$

3. Now from point *b'*, draw vector *b'x* parallel to *BE* to represent the radial component of the acceleration of *E* with respect to *B*, such that

$$\text{vector } b'x' = a^r_{EB} = 437.4 \text{ m/s}^2$$

4. From point *x*, draw vector *xe'* perpendicular to *BE* to represent the tangential component of acceleration of *E* with respect to *B* (*i.e.* a^t_{EB}) whose magnitude is yet unknown.

5. From point *d'*, draw vector *d'y* parallel to *DE* to represent the radial component of the acceleration of *E* with respect to *D*, such that

$$\text{vector } d'y = a^r_{ED} = 0.15 \text{ m/s}^2$$

Note: Since the magnitude of a^r_{ED} is very small (*i.e.* 0.15 m/s^2), therefore the points *d'* and *y* coincide.

6. From point *y*, draw vector *ye'* perpendicular to *DE* to represent the tangential component of the acceleration of *E* with respect to *D* (*i.e.* a^t_{ED}). The vectors *xe'* and *ye'* intersect at *e'*.

7. From point *e'*, draw vector *e'z* parallel to *EP* to represent the radial component of the acceleration of *P* with respect to *E*, such that

$$\text{vector } e'z = a^r_{PE} = 110.45 \text{ m/s}^2$$

8. From point *z*, draw vector *zp'* perpendicular to *EP* to represent the tangential component of the acceleration of *P* with respect to *E* (*i.e.* a^t_{PE}) whose magnitude is yet unknown.

9. From point *a'*, draw vector *a'p'* parallel to the path of motion of *P* (which is horizontal) to represent the acceleration of *P*. The vectors *zp'* and *a'p'* intersect at *p'*.

By measurement, we find that acceleration of the piston *P*,

$$a_P = \text{vector } a'p' = 655 \text{ m/s}^2 \quad \textbf{Ans.}$$

8.5. Coriolis Component of Acceleration

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of the acceleration must be calculated.

Consider a link *OA* and a slider *B* as shown in Fig. 8.26 (*a*). The slider *B* moves along the link *OA*. The point *C* is the coincident point on the link *OA*.

Let ω = Angular velocity of the link *OA* at time *t* seconds.

v = Velocity of the slider *B* along the link *OA* at time *t* seconds.

$\omega.r$ = Velocity of the slider *B* with respect to *O* (perpendicular to the link *OA*) at time *t* seconds, and

$(\omega + \delta\omega)$, $(v + \delta v)$ and $(\omega + \delta\omega)\,(r + \delta r)$

= Corresponding values at time $(t + \delta t)$ seconds.

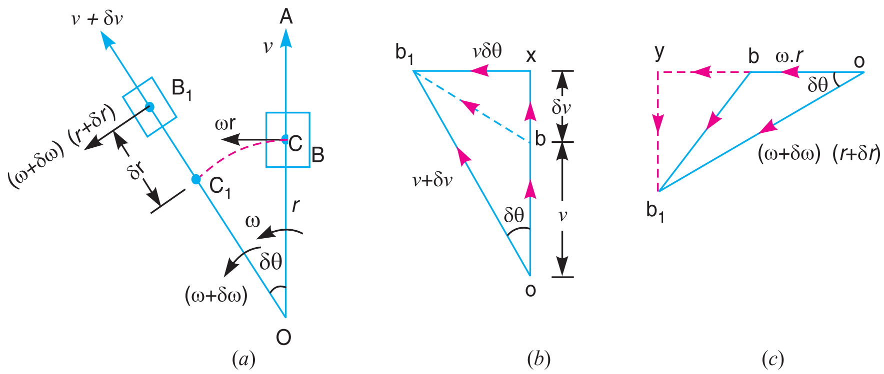


Fig. 8.26. Coriolis component of acceleration.

Let us now find out the acceleration of the slider *B* with respect to *O* and with respect to its coincident point *C* lying on the link *OA*.

Fig. 8.26 (*b*) shows the velocity diagram when their velocities *v* and $(v + \delta v)$ are considered. In this diagram, the vector bb_1 represents the change in velocity in time δt sec ; the vector *bx* represents the component of change of velocity bb_1 along *OA* (*i.e.* along radial direction) and vector xb_1 represents the component of change of velocity bb_1 in a direction perpendicular to *OA* (*i.e.* in tangential direction). Therefore

$$bx = ox - ob = (v + \delta v)\cos\delta\theta - v \uparrow$$

Since $\delta\theta$ is very small, therefore substituting $\cos\delta\theta = 1$, we have

$$bx = (v + \delta v - v) \uparrow = \delta v \uparrow$$

...(Acting radially outwards)

and $$xb_1 = (v + \delta v)\sin\delta\theta$$

Since $\delta\theta$ is very small, therefore substituting $\sin\delta\theta = \delta\theta$, we have

$$xb_1 = (v + \delta v)\,\delta\theta = v.\delta\theta + \delta v.\delta\theta$$

Neglecting $\delta v.\delta\theta$ being very small, therefore

$$xb_1 = \overleftarrow{v.\delta\theta}$$ ...(Perpendicular to *OA* and towards left)

A drill press has a pointed tool which is used for boring holes in hard materials usually by rotating abrasion or repeated bolows.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Fig. 8.26 (*c*) shows the velocity diagram when the velocities $\omega.r$ and $(\omega + \delta\omega)\,(r + \delta r)$ are considered. In this diagram, vector bb_1 represents the change in velocity ; vector yb_1 represents the component of change of velocity bb_1 along *OA* (*i.e.* along radial direction) and vector *by* represents the component of change of velocity bb_1 in a direction perpendicular to *OA* (*i.e.* in a tangential direction). Therefore

$$yb_1 = (\omega + \delta\omega)\,(r + \delta r)\sin\delta\theta \downarrow$$

$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r)\sin\delta\theta$$

Since $\delta\theta$ is very small, therefore substituting $\sin \delta\theta = \delta\theta$ in the above expression, we have

$$yb_1 = \omega.r.\delta\theta + \omega.\delta r.\delta\theta + \delta\omega.r.\delta\theta + \delta\omega.\delta r.\delta\theta$$

$$= \omega.r.\delta\theta \downarrow \text{, acting radially inwards} \qquad \text{...(Neglecting all other quantities)}$$

and $\qquad by = oy - ob = (\omega + \delta\omega)(r + \delta r)\cos\delta\theta - \omega.r$

$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r)\cos\delta\theta - \omega.r$$

Since $\delta\theta$ is small, therefore substituting $\cos\delta\theta = 1$, we have

$$by = \omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r - \omega.r = \omega.\delta r + r.\delta\omega \qquad \text{...(Neglecting } \delta\omega.\delta r)$$

...(Perpendicular to OA and towards left)

Therefore, total component of change of velocity along radial direction

$$= bx - yb_1 = (\delta v - \omega.r.\delta\theta) \uparrow \qquad \text{...(Acting radially outwards from } O \text{ to } A)$$

$\therefore$ Radial component of the acceleration of the slider B with respect to O on the link OA, acting radially outwards from O to A,

$$a_{BO}^r = \text{Lt}\,\frac{\delta v - \omega.r.\delta\theta}{\delta t} = \frac{dv}{dt} - \omega.r \times \frac{d\theta}{dt} = \frac{dv}{dt} - \omega^2.r \uparrow \qquad \text{...}(i)$$

$$\text{...}(\because d\theta/dt = \omega)$$

Also, the total component of change of velocity along tangential direction,

$$= xb_1 + by = v.\overleftarrow{\delta\theta} + (\omega.\overleftarrow{\delta r} + r.\delta\omega)$$

...(Perpendicular to OA and towards left)

$\therefore$ Tangential component of acceleration of the slider B with respect to O on the link OA, acting perpendicular to OA and towards left,

$$a_{BO}^t = \text{Lt}\,\frac{v.\delta\theta + (\omega.\delta r + r.\delta\omega)}{\delta t} = v\frac{d\theta}{dt} + \omega\frac{dr}{dt} + r\frac{d\omega}{dt}$$

$$= v.\omega + \omega.v + r.\alpha = (2\,\overleftarrow{v.\omega} + r.\alpha) \qquad \text{...}(ii)$$

$$\text{...}(\because dr/dt = v, \text{ and } d\omega/dt = \alpha)$$

Now radial component of acceleration of the coincident point C with respect to O, acting in a direction from C to O,

$$a_{CO}^r = \omega^2.r \uparrow \qquad \text{...}(iii)$$

and tangential component of acceleraiton of the coincident point C with respect to O, acting in a direction perpendicular to CO and towards left,

$$a_{CO}^t = \overleftarrow{\alpha.r} \uparrow \qquad \text{...}(iv)$$

Radial component of the slider B with respect to the coincident point C on the link OA, acting radially outwards,

$$a_{BC}^r = a_{BO}^r - a_{CO}^r = \left(\frac{dv}{dt} - \omega^2.r\right) - \left(-\omega^2.r\right) = \frac{dv}{dt} \uparrow$$

and tangential component of the slider B with respect to the coincident point C on the link OA acting in a direction perpendicular to OA and towards left,

$$a_{BC}^t = a_{BO}^t - a_{CO}^t = (2\,\omega.v + \alpha.r) - \alpha.r = 2\,\overleftarrow{\omega.v}$$

This tangential component of acceleration of the slider B with respect to the coincident point C on the link is known as ***coriolis component of acceleration*** and is always perpendicualr to the link.

∴ Coriolis component of the acceleration of B with respect of C,

$$a_{BC}^{c} = a_{BC}^{t} = 2\,\omega.v$$

where ω = Angular velocity of the link OA, and

v = Velocity of slider B with respect to coincident point C.

In the above discussion, the anticlockwise direction for ω and the radially outward direction for v are taken as ***positive.*** It may be noted that the direction of coriolis component of acceleration changes sign, if either ω or v is reversed in direction. But the direction of coriolis component of acceleration will not be changed in sign if both ω and v are reversed in direction. It is concluded that the direction of coriolis component of acceleration is obtained by rotating v, at 90°, about its origin in the same direction as that of ω.

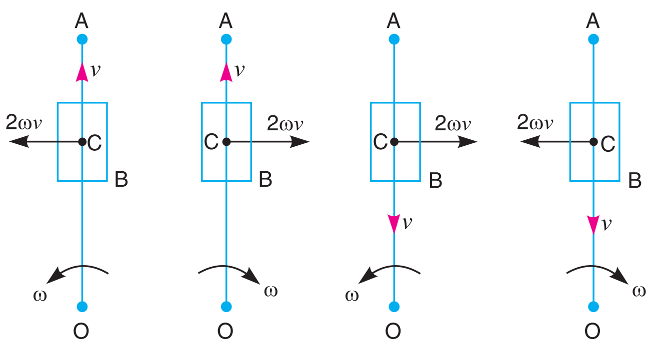


Fig. 8.27. Direction of coriolis component of acceleration.

The direction of coriolis component of acceleration (2 ω.v) for all four possible cases, is shown in Fig. 8.27. The directions of ω and v are given.

Example 8.13. *A mechanism of a crank and slotted lever quick return motion is shown in Fig. 8.28. If the crank rotates counter clockwise at 120 r.p.m., determine for the configuration shown, the velocity and acceleration of the ram D. Also determine the angular acceleration of the slotted lever.*

Crank, AB = 150 mm ; Slotted arm, OC = 700 mm and link CD = 200 mm.

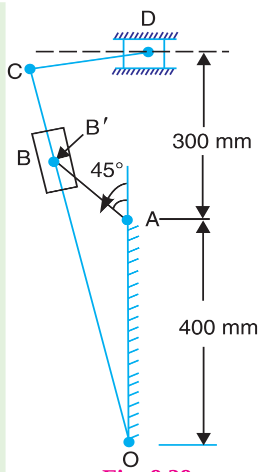


Fig. 8.28

Solution. Given : N_{BA} = 120 r.p.m or $\omega_{BA} = 2\,\pi \times 120/60 = 12.57$ rad/s ; AB = 150 mm = 0.15 m; OC = 700 mm = 0.7 m; CD = 200 mm = 0.2 m

We know that velocity of B with respect to A,

$$v_{BA} = \omega_{BA} \times AB$$

$$= 12.57 \times 0.15 = 1.9 \text{ m/s}$$

...(Perpendicular to AB)

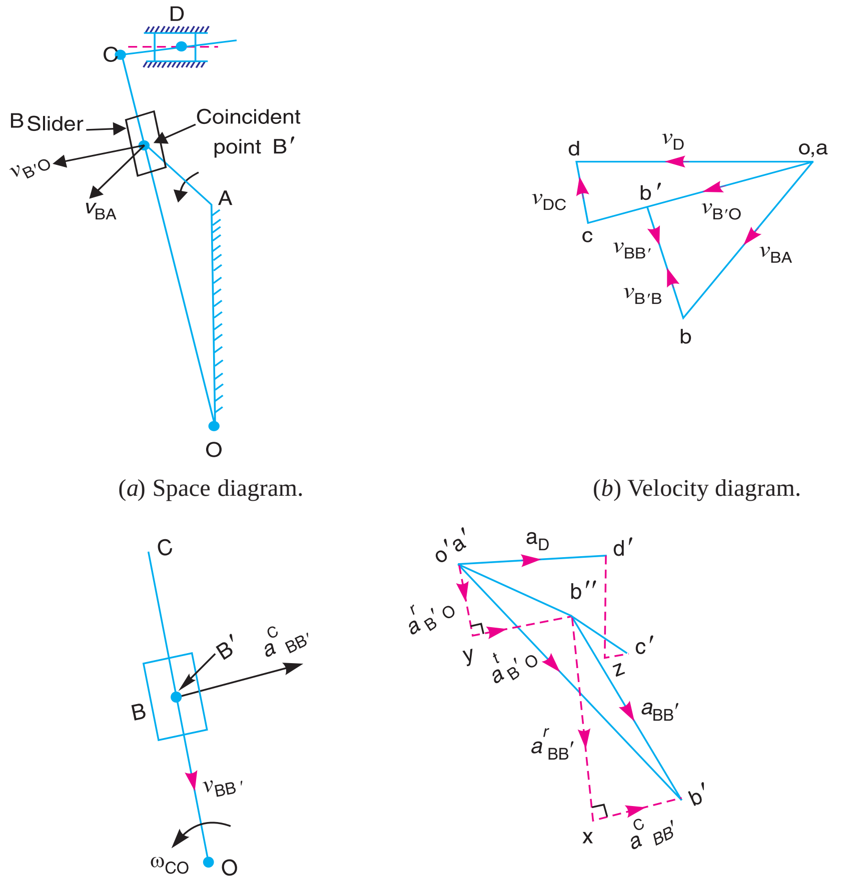
Velocity of the ram D

First of all draw the space diagram, to some suitable scale, as shown in Fig. 8.29 (*a*). Now the velocity diagram, as shown in Fig. 8.29

(*b*), is drawn as discussed below:

1. Since *O* and *A* are fixed points, therefore these points are marked as one point in velocity diagram. Now draw vector *ab* in a direction perpendicular to *AB*, to some suitable scale, to represent the velocity of slider *B* with respect to *A* *i.e.* v_{BA}, such that

$$\text{vector } ab = v_{BA} = 1.9 \text{ m/s}$$



(*a*) Space diagram. (*b*) Velocity diagram.

(*c*) Direction of coriolis component. (*d*) Acceleration diagram.

Fig. 8.29

2. From point *o*, draw vector *ob′* perpendicular to *OB′* to represent the velocity of coincident point *B′* (on the link *OC*) with respect to *O* *i.e.* $v_{B'O}$ and from point *b* draw vector *bb′* parallel to the path of motion of *B′* (which is along the link *OC*) to represent the velocity of coincident point *B′* with respect to the slider *B* *i.e.* $v_{B'B}$. The vectors *ob′* and *bb′* intersect at *b′*.

Note: Since we have to find the coriolis component of acceleration of the slider *B* with respect to the coincident point *B′*, therefore we require the velocity of *B* with respect to *B′* *i.e.* $v_{BB'}$. The vector *b′b* will represent $v_{BB'}$ as shown in Fig. 8.29 (*b*).

3. Since the point *C* lies on *OB′* produced, therefore, divide vector *ob′* at *c* in the same ratio as *C* divides *OB′* in the space diagram. In other words,

$$ob'/oc = OB'/OC$$

The vector *oc* represents the velocity of *C* with respect to *O* *i.e.* v_{CO}.

4. Now from point *c*, draw vector *cd* perpendicular to *CD* to represent the velocity of *D* with respect to *C i.e.* v_{DC} ,and from point *o* draw vector *od* parallel to the path of motion of *D* (which is along the horizontal) to represent the velocity of *D i.e.* v_D.The vectors *cd* and *od* intersect at *d*.

By measurement, we find that velocity of the ram *D*,

$$v_D = \text{vector } od = 2.15 \text{ m/s} \quad \textbf{Ans.}$$

From velocity diagram, we also find that

Velocity of *B* with respect to *B'*,

$$v_{BB'} = \text{vector } b'b = 1.05 \text{ m/s}$$

Velocity of *D* with respect to *C*,

$$v_{DC} = \text{vector } cd = 0.45 \text{ m/s}$$

Velocity of *B'* with respect to *O*

$$v_{B'O} = \text{vector } ob' = 1.55 \text{ m/s}$$

Velocity of *C* with respect to *O*,

$$v_{CO} = \text{vector } oc = 2.15 \text{ m/s}$$

∴ Angular velocity of the link *OC* or *OB'*,

$$\omega_{CO} = \omega_{B'O} = \frac{v_{CO}}{OC} = \frac{2.15}{0.7} = 3.07 \text{ rad/s (Anticlockwise)}$$

Acceleration of the ram D

We know that radial component of the acceleration of *B* with respect to *A*,

$$a^r_{BA} = \omega^2_{BA} \times AB = (12.57)^2 \times 0.15 = 23.7 \text{ m/s}^2$$

Coriolis component of the acceleration of slider *B* with respect to the coincident point *B'*,

$$a^c_{BB'} = 2\omega.v = 2\omega_{CO}.v_{BB'} = 2 \times 3.07 \times 1.05 = 6.45 \text{ m/s}^2$$

...($\because$ $\omega = \omega_{CO}$ and $v = v_{BB'}$)

Radial component of the acceleration of *D* with respect to *C*,

$$a^r_{DC} = \frac{v^2_{DC}}{CD} = \frac{(0.45)^2}{0.2} = 1.01 \text{ m/s}^2$$

Radial component of the acceleration of the coincident point *B'* with respect to *O*,

$$a^r_{B'O} = \frac{v^2_{B'O}}{B'O} = \frac{(1.55)^2}{0.52} = 4.62 \text{ m/s}^2$$

...(By measurement *B'O* = 0.52 m)

Now the acceleration diagram, as shown in Fig. 8.29 (*d*), is drawn as discussed below:

1. Since *O* and *A* are fixed points, therefore these points are marked as one point in the acceleration diagram. Draw vector *a'b'* parallel to *A B*, to some suitable scale, to represent the radial component of the acceleration of *B* with respect to *A i.e.* a^r_{BA} or a_B, such that

$$\text{vector } a'b' = a^r_{BA} = a_B = 23.7 \text{ m/s}^2$$

2. The acceleration of the slider *B* with respect to the coincident point *B'* has the following two components :

(*i*) Coriolis component of the acceleration of *B* with respect to *B' i.e.* $a^c_{BB'}$, and

(*ii*) Radial component of the acceleration of *B* with respect to *B' i.e.* $a^r_{BB'}$.

These two components are mutually perpendicular. Therefore from point *b'* draw vector *b'x* perpendicular to *B'O i.e.* in a direction as shown in Fig. 8.29 (*c*) to represent $a^c_{BB'} = 6.45 \text{ m/s}^2$. The direction of $a^c_{BB'}$ is obtained by rotating $v_{BB'}$ (represented by vector *b'b* in velocity diagram) through 90° in the same sense as that of link *OC* which rotates in the counter clockwise direction. Now from point *x*, draw vector *xb''* perpendicular to vector *b'x* (or parallel to *B'O*) to represent $a^r_{BB'}$ whose magnitude is yet unknown.

3. The acceleration of the coincident point *B'* with respect to *O* has also the following two components:

(*i*) Radial component of the acceleration of coincident point *B'* with respect to *O i.e.* $a^r_{B'O}$, and

(*ii*) Tangential component of the acceleration of coincident point *B'* with respect to *O*, *i.e.* $a^t_{B'O}$.

These two components are mutually perpendicular. Therefore from point *o'*, draw vector *o'y* parallel to *B'O* to represent $a^r_{B'O} = 4.62$ m/s^2 and from point *y* draw vector *yb''* perpendicular to vector *o'y* to represent $a^t_{B'O}$. The vectors *xb''* and *yb''* intersect at *b''*. Join *o'b''*. The vector *o'b''* represents the acceleration of *B'* with respect to *O*, *i.e.* $a_{B'O}$.

4. Since the point *C* lies on *OB'* produced, therefore divide vector *o'b''* at *c'* in the same ratio as *C* divides *OB'* in the space diagram. In other words,

$$o'b''/o'c' = OB'/OC$$

5. The acceleration of the ram *D* with respect to *C* has also the following two components:

(*i*) Radial component of the acceleration of *D* with respect to *C i.e.* a^r_{DC}, and

(*ii*) Tangential component of the acceleration of *D* with respect to *C*, *i.e.* a^t_{DC}.

The two components are mutually perpendicular. Therefore draw vector *c'z* parallel to *CD* to represent $a^r_{DC} = 1.01$ m/s^2 and from *z* draw *zd'* perpendicular to vector *zc'* to represent a^t_{DC}, whose magnitude is yet unknown.

6. From point *o'*, draw vector *o'd'* in the direction of motion of the ram *D* which is along the horizontal. The vectors *zd'* and *o'd'* intersect at *d'*. The vector *o'd'* represents the acceleration of ram *D i.e.* a_D.

By measurement, we find that acceleration of the ram *D*,

$$a_D = \text{vector } o'd' = 8.4 \text{ m/s}^2 \quad \textbf{Ans.}$$

Angular acceleration of the slotted lever

By measurement from acceleration diagram, we find that tangential component of the coincident point *B'* with respect to *O*,

$$a^t_{B'O} = \text{vector } yb'' = 6.4 \text{ m/s}^2$$

We know that angular acceleration of the slotted lever,

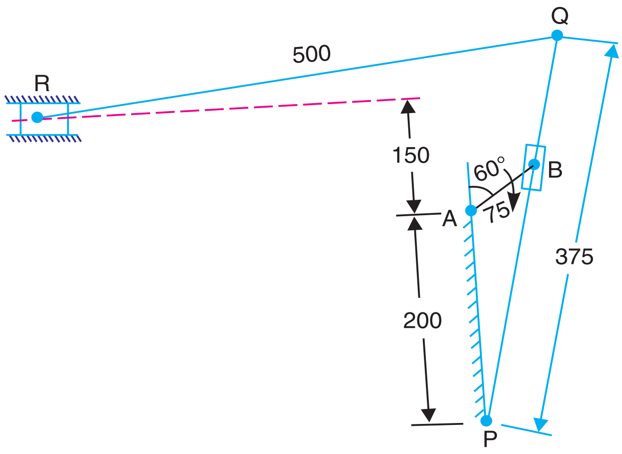
$$= \frac{a^t_{B'O}}{OB'} = \frac{6.4}{0.52} = 12.3 \text{ rad/s}^2 \text{ (Anticlockwise)} \quad \textbf{Ans.}$$

Example 8.14. *The driving crank AB of the quick-return mechanism, as shown in Fig. 8.30, revolves at a uniform speed of 200 r.p.m. Find the velocity and acceleration of the tool-box R, in the position shown, when the crank makes an angle of 60° with the vertical line of centres PA. What is the acceleration of sliding of the block at B along the slotted lever PQ ?*

Solution. Given : $N_{BA} = 200$ r.p.m. or $\omega_{BA} = 2\pi \times 200/60 = 20.95$ rad/s ; $AB = 75$ mm $= 0.075$ m

We know that velocity of B with respect to A,

$$v_{BA} = \omega_{BA} \times AB = 20.95 \times 0.075 = 1.57 \text{ m/s} \quad \text{...(Perpendicular to } AB)$$



All dimensions in mm.

Fig. 8.30

Velocity of the tool-box R

First of all draw the space diagram, to some suitable scale, as shown in Fig. 8.31 (*a*). Now the velocity diagram, as shown in Fig. 8.31 (*b*), is drawn as discussed below:

1. Since A and P are fixed points, therefore these points are marked as one point in the velocity diagram. Now draw vector ab in a direction perpendicular to AB, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B (*i.e.* v_{BA} or v_B), such that

$$\text{vector } ab = v_{BA} = v_B = 1.57 \text{ m/s}$$

2. From point p, draw vector pb' perpendicular to PB' to represent the velocity of coincident point B' with respect to P (*i.e.* $v_{B'P}$ or $v_{B'}$) and from point b, draw vector bb' parallel to the path of motion of B' (which is along PQ) to represent the velocity of coincident point B' with respect to the slider B *i.e.* $v_{B'B}$. The vectors pb' and bb' intersect at b'.

Note. The vector $b'b$ will represent the velocity of the slider B with respect to the coincident point B' *i.e.* $v_{BB'}$.

3. Since the point Q lies on PB' produced, therefore divide vector pb' at q in the same ratio as Q divides PB'. In other words,

$$pb'/pq = PB'/PQ$$

The vector pq represents the velocity of Q with respect to P *i.e.* v_{QP}.

4. Now from point q, draw vector qr perpendicular to QR to represent the velocity of R with respect to Q *i.e.* v_{RQ}, and from point a draw vector ar parallel to the path of motion of the tool-box R (which is along the horizontal), to represent the velocity of R *i.e.* v_R. The vectors qr and ar intersect at r.

By measurement, we find that velocity of the tool-box R,

$$v_R = \text{vector } ar = 1.6 \text{ m/s} \quad \textbf{Ans.}$$

We also find that velocity of B' with respect to B,

$$v_{B'B} = \text{vector } bb' = 1.06 \text{ m/s}$$

Velocity of B' with respect to P,

$$v_{B'P} = \text{vector } pb' = 1.13 \text{ m/s}$$

Velocity of R with respect to Q,

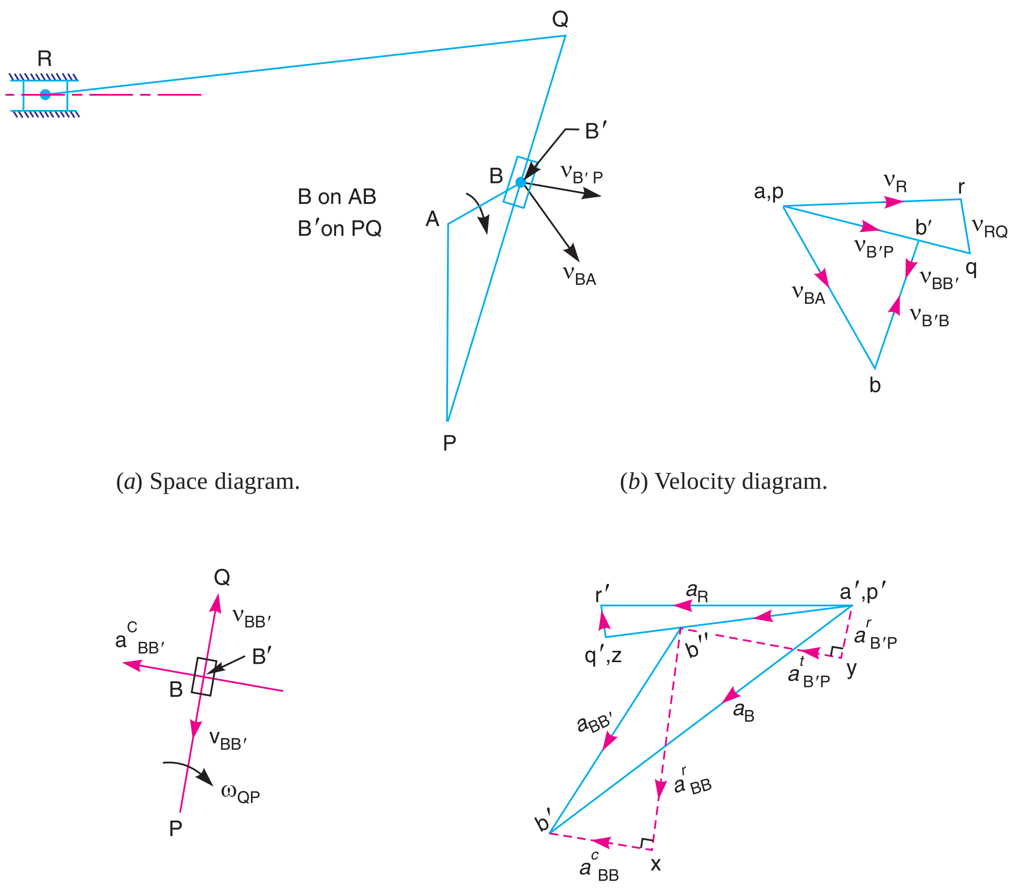
$$v_{RQ} = \text{vector } qr = 0.4 \text{ m/s}$$

Velocity of Q with respect to P,

$$v_{QP} = \text{vector } pq = 1.7 \text{ m/s}$$

$\therefore$ Angular velocity of the link PQ,

$$\omega_{PQ} = \frac{v_{QP}}{QP} = \frac{1.7}{0.375} = 4.53 \text{ rad/s} \qquad ...(\because PQ = 0.375 \text{ m})$$



(*a*) Space diagram. (*b*) Velocity diagram.

(*c*) Direction of coriolis component. (*d*) Acceleration diagram.

Fig. 8.31

Acceleration of the tool box R

We know that the radial component of the acceleration of B with respect to A,

$$a^r_{BA} = \omega^2_{BA} \times AB = (20.95)^2 \times 0.075 = 32.9 \text{ m/s}^2$$

Coriolis component of the acceleration of the slider B with respect to coincident point B'.

$$a^c_{BB'} = 2\omega.v = 2\omega_{QP} \times v_{BB'} = 2 \times 4.53 \times 1.06 = 9.6 \text{ m/s}^2$$

$$...(\because \omega = \omega_{QP}, \text{ and } v = v_{BB'})$$

Radial component of the acceleration of R with respect to Q,

$$a^r_{RQ} = \frac{v^2_{RQ}}{QR} = \frac{(0.4)^2}{0.5} = 0.32 \text{ m/s}^2$$

Radial component of the acceleration of *B'* with respect to *P*,

$$a^r_{B'P} = \frac{v^2_{B'P}}{PB'} = \frac{(1.13)^2}{0.248} = 5.15 \text{ m/s}^2$$

...(By measurement, PB' = 248 mm = 0.248 m)

Now the acceleration diagram, as shown in Fig. 8.31 (*d*), is drawn as discussed below:

1. Since *A* and *P* are fixed points, therefore these points are marked as one point in the acceleration diagram. Draw vector *a'b'* parallel to *A B*, to some suitable scale, to represent the radial component of the acceleration of *B* with respect to *A i.e.* a^r_{BA}, or a_B such that

$$\text{vector } a'b' = a^r_{BA} = a_B = 32.9 \text{ m/s}^2$$

2. The acceleration of the slider *B* with respect to the coincident point *B'* has the following two components:

(*i*) Coriolis component of the acceleration of *B* with respect to *B' i.e.* $a^c_{BB'}$, and

(*ii*) Radial component of the acceleration of *B* with respect to *B' i.e.* $a^r_{BB'}$.

These two components are mutually perpendicular. Therefore from point *b'*, draw vector *b'x* perpendicular to *BP* [*i.e.* in a direction as shown in Fig. 8.31 (*c*)] to represent $a^c_{BB'} = 9.6 \text{ m/s}^2$. The direction of $a^c_{BB'}$ is obtained by rotating $v_{BB'}$ (represented by vector *b'b* in the velocity diagram) through 90° in the same sense as that of link *PQ* which rotates in the clockwise direction. Now from point *x*, draw vector *xb''* perpendicular to vector *b'x* (or parallel to *B'P*) to represent $a^r_{BB'}$ whose magnitude is yet unknown.

3. The acceleration of the coincident point *B'* with respect to *P* has also the following two components:

(*i*) Radial component of the acceleration of *B'* with respect to *P i.e.* $a^r_{B'P}$, and

(*ii*) Tangential component of the acceleration of *B'* with respect to *P i.e.* $a^t_{B'P}$.

These two components are mutually perpendicular. Therefore from point *p'* draw vector *p'y* parallel to *B'P* to represent $a^r_{B'P} = 5.15 \text{ m/s}^2$, and from point *y* draw vector *yb''* perpendicular to vector *p'y* to represent $a^t_{B'P}$. The vectors *xb''* and *yb''* intersect at *b''*, join *p'b''*. The vector *p'b''* represents the acceleration of *B'* with respect to *P i.e.* $a_{B'P}$ and the vector *b''b'* represents the acceleration of *B* with respect to *B' i.e.* $a_{BB'}$.

4. Since the point *Q* lies on *PB'* produced, therefore divide vector *p'b''* at *q'* in the same ratio as *Q* divides *PB* in the space diagram. In other words,

$$p'b''/p'q' = PB'/PQ$$

5. The acceleration of the tool-box *R* with respect to *Q* has the following two components:

(*i*) Radial component of the acceleration of *R* with respect to *Q i.e.* a^r_{RQ}, and

(*ii*) Tangential component of the acceleration of *R* with respect to *Q i.e.* a^t_{RQ}.

These two components are mutually perpendicular. Therefore from point *q'*, draw vector *a'z* parallel to *QR* to represent $a^r_{RQ} = 0.32 \text{ m/s}^2$. Since the magnitude of this component is very small, therefore the points *q'* and *z* coincide as shown in Fig. 8.31 (*d*). Now from point *z* (same as *q'*), draw vector *zr'* perpendicular to vector *q'z* (or *QR*) to represent a^t_{RQ} whose magnitude is yet unknown.

6. From point *a'* draw vector *a'r'* parallel to the path of motion of the tool-box *R* (*i.e.* along the horizontal) which intersects the vector *zr'* at *r'*. The vector *a'r'* represents the acceleration of the tool-box *R i.e.* a_R.

By measurement, we find that

$$a_R = \text{vector } a'r' = 22 \text{ m/s}^2 \textbf{ Ans.}$$

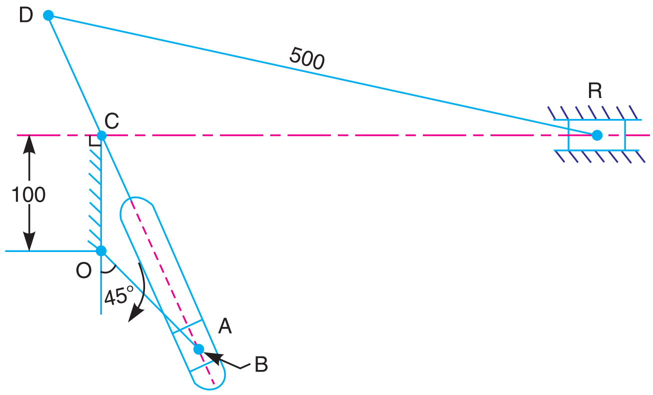
Acceleration of sliding of the block B along the slotted lever PQ

By measurement, we find that the acceleration of sliding of the block *B* along the slotted lever *PQ*

$$= a_{BB'} = \text{vector } b''x = 18 \text{ m/s}^2 \textbf{ Ans.}$$

Example 8.15. *In a Whitworth quick return motion, as shown in Fig. 8.32. OA is a crank rotating at 30 r.p.m. in a clockwise direction. The dimensions of various links are : OA = 150 mm; OC = 100 mm; CD = 125 mm; and DR = 500 mm.*

Determine the acceleraion of the sliding block R and the angular acceleration of the slotted lever CA.



All dimensions in mm.

Fig. 8.32

Solution. Given : N_{AO} = 30 r.p.m. or ω_{AO} = 2π × 30/60 = 3.142 rad/s ; *OA* = 150 mm = 0.15 m; *OC* = 100 mm = 0.1 m ; *CD* = 125 mm = 0.125 m ; *DR* = 500 mm = 0.5 m

We know that velocity of *A* with respect to *O* or velocity of *A*,

$$v_{AO} = v_A = \omega_{AO} \times OA = 3.142 \times 0.15 = 0.47 \text{ m/s}$$

...(Perpendicular to *OA*)

First of all draw the space diagram, to some suitable scale, as shown in Fig. 8.33 (*a*). Now the velocity diagram, as shown in Fig. 8.33 (*b*), is drawn as discussed below:

1. Since *O* and *C* are fixed points, therefore these are marked at the same place in velocity diagram. Now draw vector *ca* perpendicular to *OA*, to some suitable scale, to represent the velocity of *A* with respect to *O* or simply velocity of *A* *i.e.* v_{AO} or v_A, such that

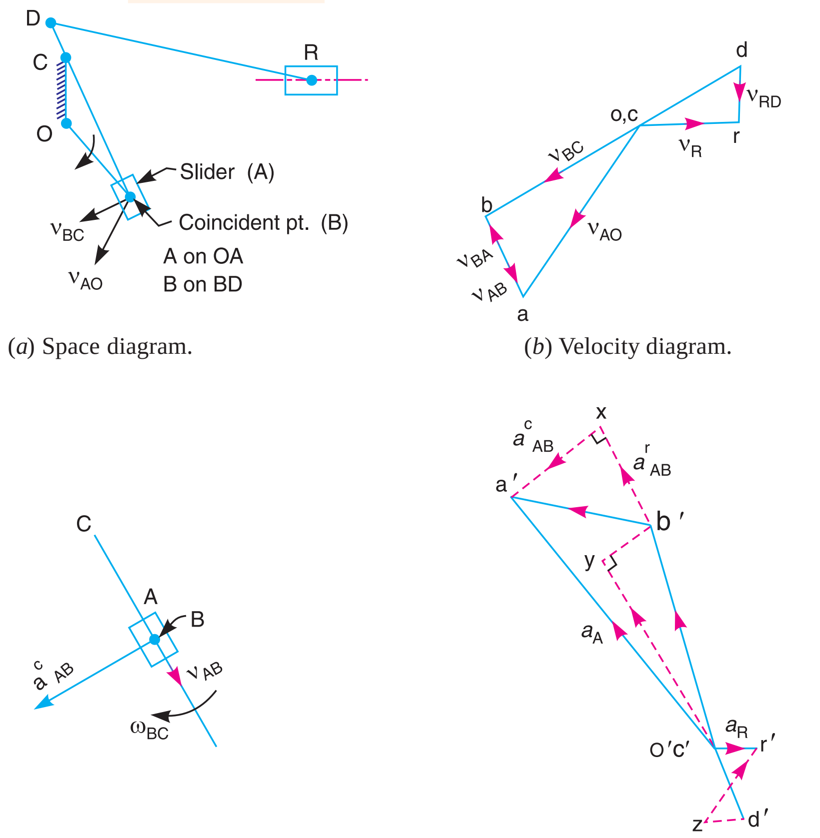
$$\text{vector } oa = v_{AO} = v_A = 0.47 \text{ m/s}$$

2. From point *c*, draw vector *cb* perpendicular to *BC* to represent the velocity of the coincident point *B* with respect to *C* *i.e.* v_{BC} or v_B and from point *a* draw vector *ab* parallel to the path of motion of *B* (which is along *BC*) to represent the velocity of coincident point *B* with respect to *A* *i.e.* v_{BA}. The vectors *cb* and *ab* intersect at *b*.

Note: Since we have to find the coriolis component of acceleration of slider *A* with respect to coincident point *B*, therefore we require the velocity of *A* with respect to *B* *i.e.* v_{AB}. The vector *ba* will represent v_{AB} as shown in Fig. 8.33 (*b*).

3. Since D lies on BC produced, therefore divide vector bc at d in the same ratio as D divides BC in the space diagram. In other words,

$$bd/bc = BD/BC$$



(*a*) Space diagram. (*b*) Velocity diagram.

(*c*) Direction of coriolis component. (*d*) Acceleration diagram.

Fig. 8.33

4. Now from point d, draw vector dr perpendicular to DR to represent the velocity of R with respect to D *i.e.* v_{RD}, and from point c draw vector cr parallel to the path of motion of R (which is horizontal) to represent the velocity of R *i.e.* v_R.

By measurement, we find that velocity of B with respect to C,

$$v_{BC} = \text{vector } cb = 0.46 \text{ m/s}$$

Velocity of A with respect to B,

$$v_{AB} = \text{vector } ba = 0.15 \text{ m/s}$$

and velocity of R with respect to D,

$$v_{RD} = \text{vector } dr = 0.12 \text{ m/s}$$

We know that angular velocity of the link BC,

$$\omega_{BC} = \frac{v_{BC}}{CB} = \frac{0.46}{0.24} = 1.92 \text{ rad/s (Clockwise)}$$

...(By measurement, CB = 0.24 m)

Acceleration of the sliding block R

We know that the radial component of the acceleration of A with respect to O,

$$a_{AO}^{r} = \frac{v_{AO}^{2}}{OA} = \frac{(0.47)^2}{0.15} = 1.47 \text{ m/s}^2$$

Coriolis component of the acceleration of slider A with respect to coincident point B,

$$a_{AB}^{c} = 2\,\omega_{BC} \times v_{AB} = 2 \times 1.92 \times 0.15 = 0.576 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to C,

$$a_{BC}^{r} = \frac{v_{BC}^{2}}{CB} = \frac{(0.46)^2}{0.24} = 0.88 \text{ m/s}^2$$

Radial component of the acceleration of R with respect to D,

$$a_{RD}^{r} = \frac{v_{RD}^{2}}{DR} = \frac{(0.12)^2}{0.5} = 0.029 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.33 (*d*), is drawn as discussed below:

1. Since O and C are fixed points, therefore these are marked at the same place in the acceleration diagram. Draw vector *o'a'* parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O *i.e.* a_{AO}^{r}, or a_A such that

$$\text{vector } o'a' = a_{AO}^{r} = a_A = 1.47 \text{ m/s}^2$$

2. The acceleration of the slider A with respect to coincident point B has the following two components:

(*i*) Coriolis component of the acceleration of A with respect to B *i.e.* a_{AB}^{c}, and

(*ii*) Radial component of the acceleration of A with respect to B *i.e.* a_{AB}^{r}.

These two components are mutually perpendicular. Therefore from point *a'* draw vector *a'x* perpendicular to *BC* to represent $a_{AB}^{c} = 0.576$ m/s^2 in a direction as shown in Fig. 8.33 (*c*), and draw vector *xb'* perpendicular to vector *a'x* (or parallel to *BC*) to represent a_{AB}^{r} whose magnitude is yet unknown.

Note: The direction of a_{AB}^{c} is obtained by rotating v_{AB} (represented by vector *ba* in velocity diagram) through 90° in the same sense as that of ω_{BC} which rotates in clockwise direction.

3. The acceleration of B with respect to C has the following two components:

(*i*) Radial component of B with respect to C *i.e.* a_{BC}^{r}, and

(*ii*) Tangential component of B with respect to C *i.e.* a_{BC}^{t}.

These two components are mutually perpendicular. Therefore, draw vector *c'y* parallel to *BC* to represent $a_{BC}^{r} = 0.88$ m/s^2 and from point *y* draw vector *yb'* perpendicular to *c'y* to represent a_{BC}^{t}. The vectors *xb'* and *yb'* intersect at *b'*. Join *b'c'*.

4. Since the point *D* lies on *BC* produced, therefore divide vector *b'c'* at *d'* in the same ratio as *D* divides *BC* in the space diagram. In other words,

$$b'd'/b'c' = BD/BC.$$

5. The acceleration of the sliding block R with respect to D has also the following two components:

(*i*) Radial component of R with respect to D *i.e.* a_{RD}^{r}, and

(*ii*) Tangential component of R with respect to D *i.e.* a_{RD}^{t}.

These two components are mutually perpendicular. Therefore from point *d*′, draw vector $d'z$ parallel to *DR* to represent $a^r_{RD} = 0.029$ m/s^2 and from *z* draw *zr*′ perpendicular to *d*′*z* to represent a^t_{RD} whose magnitude is yet unknown.

6. From point *c*′, draw vector *c*′*r*′ parallel to the path of motion of *R* (which is horizontal). The vector *c*′*r*′ intersects the vector *zr*′ at *r*′. The vector *c*′*r*′ represents the acceleration of the sliding block *R*.

By measurement, we find that acceleration of the sliding block *R*,

$$a_R = \text{vector } c'r' = 0.18 \text{ m/s}^2 \textbf{ Ans.}$$

Angular acceleration of the slotted lever CA

By measurement from acceleration diagram, we find that tangential component of *B* with respect to *C*,

$$a^t_{BC} = \text{vector } yb' = 0.14 \text{ m/s}^2$$

We know that angular acceleration of the slotted lever *CA*,

$$\alpha_{CA} = \alpha_{BC} = \frac{a^t_{CB}}{BC} = \frac{0.14}{0.24} = 0.583 \text{ rad/s}^2 \text{ (Anticlockwise) } \textbf{Ans.}$$

Example 8.16. *The kinematic diagram of one of the cylinders of a rotary engine is shown in Fig. 8.34. The crank OA which is vertical and fixed , is 50 mm long. The length of the connecting rod AB is 125 mm. The line of the stroke OB is inclined at 50° to the vertical.*

The cylinders are rotating at a uniform speed of 300 r.p.m., in a clockwise direction, about the fixed centre O. Determine: **1.** *acceleration of the piston inside the cylinder, and* **2.** *angular acceleration of the connecting rod.*

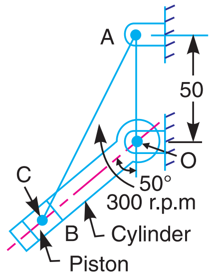


Fig. 8.34

Solution. Given: $AB = 125$ mm $= 0.125$ m ; $N_{CO} = 300$ r.p.m. or $\omega_{CO} = 2\pi \times 300/60 = 31.4$ rad/s

First of all draw the space diagram, as shown in Fig. 8.35 (*a*), to some suitable scale. By measurement from the space diagram, we find that

$$OC = 85 \text{ mm} = 0.085 \text{ m}$$

∴ Velocity of *C* with respect to *O*,

$$v_{CO} = \omega_{CO} \times OC = 31.4 \times 0.85 = 2.7 \text{ m/s}$$

...(Perpendicular to *CO*)

Now the velocity diagram, as shown in Fig. 8.35 (*b*), is drawn as discussed below:

1. Since *O* and *A* are fixed points, therefore these are marked at the same place in the velocity diagram. Draw vector *oc* perpendicular to *OC* to represent the velocity of *C* with respect to *O* *i.e.* v_{CO}, such that

$$\text{vector } oc = v_{CO} = v_C = 2.7 \text{ m/s.}$$

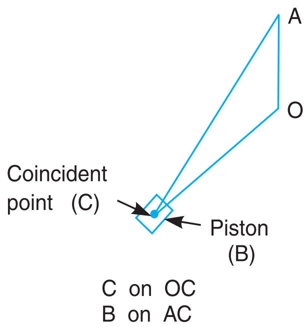
2. From point *c*, draw vector *cb* parallel to the path of motion of the piston *B* (which is along *CO*) to represent the velocity of *B* with respect to *C* *i.e.* v_{BC} , and from point *a* draw vector *ab* perpendicular to *AB* to represent the velocity of *B* with respect to *A* *i.e.* v_{BA} or v_B.

By measurement, we find that velocity of piston *B* with respect to coincident point *C*,

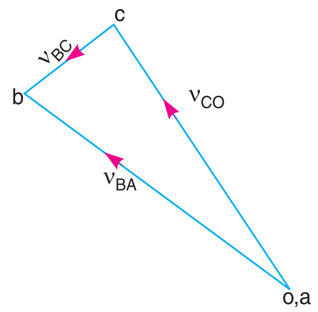
$$v_{BC} = \text{vector } cb = 0.85 \text{ m/s}$$

and velocity of piston *B* with respect to *A*,

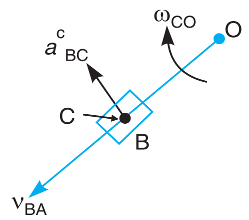
$$v_{BA} = v_B = \text{vector } ab = 2.85 \text{ m/s}$$



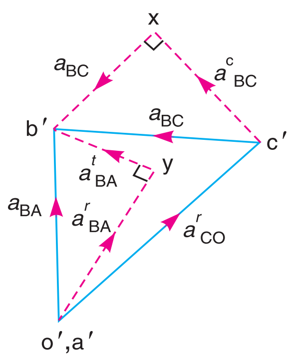
(*a*) Space diagram.



(*b*) Velocity diagram.



(*c*) Direction of coriolis component.



(*d*) Acceleration diagram.

Fig. 8.35

1. *Acceleration of the piston inside the cylinder*

We know that the radial component of the acceleration of the coincident point *C* with respect to *O*,

$$a^r_{CO} = \frac{v^2_{CO}}{OC} = \frac{(2.7)^2}{0.085} = 85.76 \text{ m/s}^2$$

Coriolis component of acceleration of the piston *B* with respect to the cylinder or coincident point *C*,

$$a^c_{BC} = 2\,\omega_{CO} \times v_{BC} = 2 \times 31.4 \times 0.85 = 53.4 \text{ m/s}^2$$

Radial component of acceleration of *B* with respect to *A*,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(2.85)^2}{0.125} = 65 \text{ m/s}^2$$

The acceleration diagram, as shown in Fig. 8.35 (*d*), is drawn as discussed below:

1. Since O and A are fixed points, therefore these are marked as one point in the acceleration diagram. Draw vector $o'c'$ parallel to OC, to some suitable scale, to represent the radial component of the acceleration of C with respect to O *i.e.*, a^r_{CO}, such that

$$\text{vector } o'c' = a^r_{CO} = 85.76 \text{ m/s}^2$$

2. The acceleration of piston B with respect to coincident point C has the following two components:

(*i*) Coriolis component of the acceleration of B with respect to C *i.e.* a^c_{BC}, and

(*ii*) Radial component of the acceleration of B with respect to C *i.e.* a^r_{BC}.

These two components are mutually perpendicular. Therefore from point c', draw vector $c'x$ perpendicular to CO to represent $a^c_{BC} = 53.4$ m/s^2 in a direction as shown in Fig. 8.35 (*c*). The direction of a^c_{BC} is obtained by rotating v_{BC} (represented by vector cb in velocity diagram) through 90° in the same sense as that of ω_{CO} which rotates in the clockwise direction. Now from point x, draw vector xb' perpendicular to vector $c'x$ (or parallel to OC) to represent a^r_{BC} whose magnitude is yet unknown.

3. The acceleration of B with respect to A has also the following two components:

(*i*) Radial component of the acceleration of B with respect to A *i.e.* a^r_{BA}, and

(*ii*) Tangential component of the acceleration of B with respect to A *i.e.* a^t_{BA}.

These two components are mutually perpendicular. Therefore from point a', draw vector $a'y$ parallel to AB to represent $a^r_{BA} = 65$ m/s^2, and from point y draw vector yb' perpendicular to vector $a'y$ to represent a^t_{BA}. The vectors xb' and yb' intersect at b'.

4. Join $c'b'$ and $a'b'$. The vector $c'b'$ represents the acceleration of B with respect to C (*i.e.* acceleration of the piston inside the cylinder).

By measurement, we find that acceleration of the piston inside the cylinder,

$$a_{BC} = \text{vector } c'b' = 73.2 \text{ m/s}^2 \quad \textbf{Ans.}$$

2. *Angular acceleration of the connecting rod*

By measurement from acceleration diagram, we find that the tangential component of the acceleration of B with respect to A,

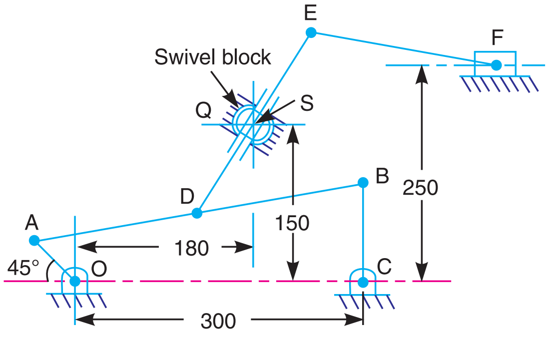
$$a^t_{BA} = \text{vector } yb' = 37.6 \text{ m/s}^2$$

∴ Angular acceleration of the connecting rod AB,

$$\alpha_{AB} = \frac{a^t_{BA}}{AB} = \frac{37.6}{0.125} = 301 \text{ rad/s}^2 \text{ (Clockwise)} \quad \textbf{Ans.}$$

Example 8.17. *In a swivelling joint mechanism, as shown in Fig. 8.36, the driving crank OA is rotating clockwise at 100 r.p.m. The lengths of various links are : OA = 50 mm ; AB = 350 mm; AD = DB ; DE = EF = 250 mm and CB = 125 mm. The horizontal distance between the fixed points O and C is 300 mm and the vertical distance between F and C is 250 mm.*

For the given configuration, determine: **1.** *Velocity of the slider block F,* **2.** *Angular velocity of the link DE,* **3.** *Velocity of sliding of the link DE in the swivel block,and* **4.** *Acceleration of sliding of the link DE in the trunnion.*



All dimensions in mm.

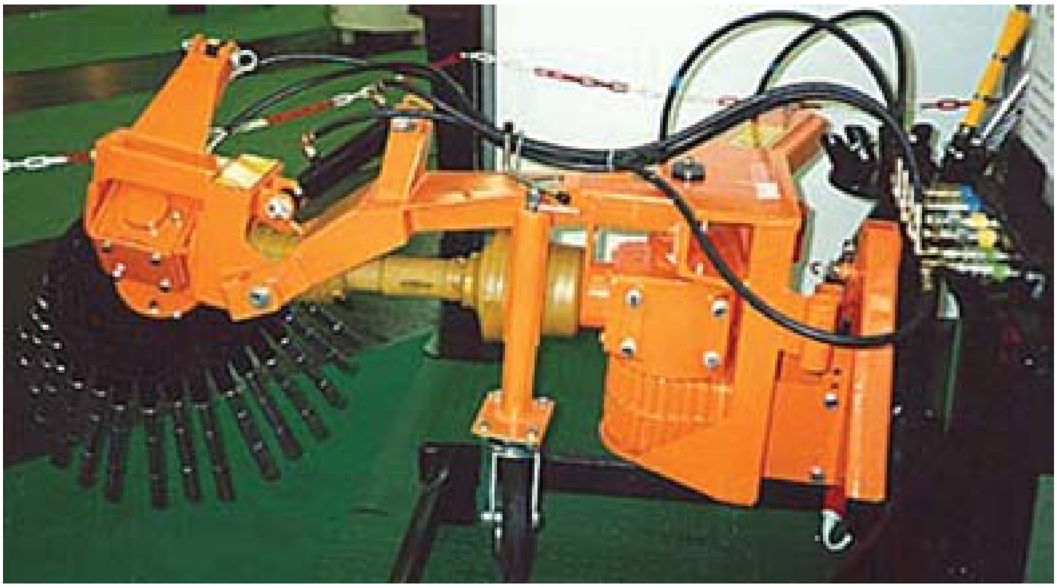
Fig. 8.36

Solution. Given: N_{AO} = 100 r.p.m. or $\omega_{AO} = 2\pi \times 100/60 = 10.47$ rad/s ; OA = 50 mm = 0.05 m; AB = 350 mm = 0.35 m ; CB = 125 mm = 0.125 m ; $DE = EF$ = 250 mm = 0.25 m

We know that velocity of A with respect to O or velocity of A,

$$v_{AO} = v_A = \omega_{AO} \times OA = 10.47 \times 0.05 = 0.523 \text{ m/s}$$

...(Perpendicular to OA)



This machine uses swivelling joint.

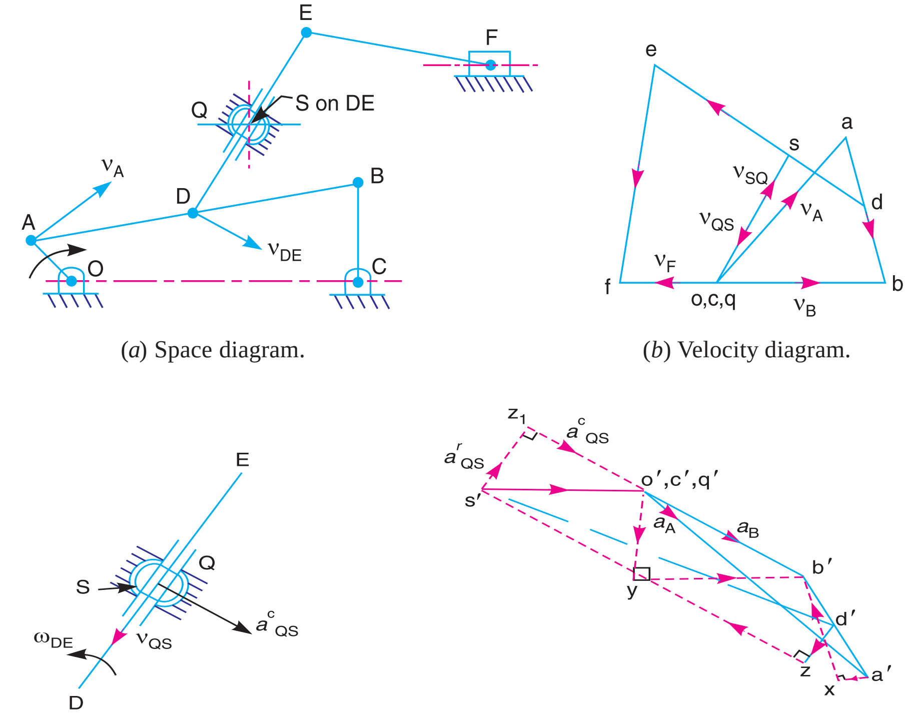
1. *Velocity of slider block F*

First of all draw the space diagram, to some suitable scale, as shown in Fig. 8.37 (*a*). Now the velocity diagram, as shown in Fig. 8.37 (*b*), is drawn as discussed below:

1. Since O, C and Q are fixed points, therefore these points are marked at one place in the velocity diagram. Draw vector oa perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O or simply velocity of A, *i.e.* v_{AO} or v_A, such that

vector $oa = v_{AO} = v_A = 0.523$ m/s

2. From point *a*, draw vector *ab* perpendicular to *AB* to represent the velocity of *B* with respect to *A* *i.e.* v_{BA},and from point *c* draw vector *cb* perpendicular to *CB* to represent the velocity of *B* with respect to *C* or simply velocity of *B* *i.e.* v_{BC} or v_B. The vectors *ab* and *cb* intersect at *b*.



(*a*) Space diagram. (*b*) Velocity diagram.

(*c*) Direction of coriolis component. (*d*) Acceleration diagram.

Fig. 8.37

3. Since point *D* lies on *AB*, therefore divide vector *ab* at *d* in the same ratio as *D* divides *AB* in the space diagram. In other words,

$$ad/ab = AD/AB$$

Note: Since point *D* is mid-point of *AB*, therefore *d* is also mid-point of *ab*.

4. Now from point *d*, draw vector *ds* perpendicular to *DS* to represent the velocity of *S* with respect to *D* *i.e.* v_{SD}, and from point *q* draw vector *qs* parallel to the path of motion of swivel block *Q* (which is along *DE*) to represent the velocity of *S* with respect to *Q* *i.e.* v_{SQ}. The vectors *ds* and *qs* intersect at *s*.

Note: The vector *sq* will represent the velocity of swivel block *Q* with respect to *S* *i.e.* v_{QS}.

5. Since point *E* lies on *DS* produced, therefore divide vector *ds* at *e* in the same ratio as *E* divides *DS* in the space diagram. In other words,

$$de/ds = DE/DS$$

6. From point *e*, draw vector *ef* perpendicular to *EF* to represent the velocity of *F* with respect to *E* *i.e.* v_{FE} , and from point *o* draw vector *of* parallel to the path of motion of *F* (which is along the horizontal direction) to represent the velocity of *F* *i.e.* v_F..The vectors *ef* and *of* intersect at *f*.

By measurement, we find that velocity of *B* with respect to *A*,

$$v_{BA} = \text{vector } ab = 0.4 \text{ m/s}$$

Velocity of B with respect to C,

$$v_{BC} = v_B = \text{vector } cb = 0.485 \text{ m/s}$$

Velocity of S with respect to D,

$$v_{SD} = \text{vector } ds = 0.265 \text{ m/s}$$

Velocity of Q with respect to S,

$$v_{QS} = \text{vector } sq = 0.4 \text{ m/s}$$

Velocity of E with respect to D,

$$v_{ED} = \text{vector } de = 0.73 \text{ m/s}$$

Velocity of F with respect to E,

$$v_{FE} = \text{vector } ef = 0.6 \text{ m/s}$$

and velocity of the slider block F, v_F = vector of = 0.27 m/s **Ans.**

2. Angular velocity of the link DE

We know that angular velocity of the link DE,

$$\omega_{DE} = \frac{v_{ED}}{DE} = \frac{0.73}{0.25} = 2.92 \text{ rad/s (Anticlockwise)} \quad \textbf{Ans.}$$

3. Velocity of sliding of the link DE in the swivel block

The velocity of sliding of the link DE in the swivel block Q will be same as that of velocity of S *i.e.* v_S.

∴ Velocity of sliding of the link DE in the swivel block,

$$v_S = v_{SQ} = 0.4 \text{ m/s} \quad \textbf{Ans.}$$

4. Acceleration of sliding of the link DE in the trunnion

We know that the radial component of the acceleration of A with respect to O or the acceleration of A,

$$a^r_{AO} = a_A = \frac{v^2_{AO}}{OA} = \frac{(0.523)^2}{0.05} = 5.47 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(0.4)^2}{0.35} = 0.457 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to C,

$$a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(0.485)^2}{0.125} = 1.88 \text{ m/s}^2$$

Radial component of the acceleration of S with respect to D,

$$a^r_{SD} = \frac{v^2_{SD}}{DS} = \frac{(0.265)^2}{0.085} = 0.826 \text{ m/s}^2$$

...(By measurement DS = 85 mm = 0.085 m)

Coriolis component of the acceleration of Q with respect to S,

$$a^c_{QS} = 2\,\omega_{DE} \times v_{QS} = 2 \times 2.92 \times 0.4 = 2.336 \text{ m/s}^2$$

and radial component of the acceleration of F with respect to E,

$$a^r_{FE} = \frac{v^2_{FE}}{EF} = \frac{(0.6)^2}{0.25} = 1.44 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.37 (*d*), is drawn as discussed below:

1. Since O, C and Q are fixed points, therefore these points are marked at one place in the acceleration diagram. Now draw vector $o'a'$ parallel to OA, to some suitable scale, to represent a^r_{AO}, or a_A such that

$$\text{vector } o'a' = a^r_{AO} = a_A = 5.47 \text{ m/s}^2$$

Note : Since OA rotates with uniform speed, therefore there will be no tangential component of the acceleration.

2. The acceleration of B with respect to A has the following two components:

(*i*) Radial component of the acceleration of B with respect to A *i.e.* a^r_{BA}, and

(*ii*) Tangential component of the acceleration of B with respect to A *i.e.* a^t_{BA}.

These two components are mutually perpendicular. Therefore from point a', draw vector $a'x$ parallel to AB to represent $a^r_{BA} = 0.457 \text{ m/s}^2$, and from point x draw vector xb' perpendicular to vector $a'x$ to represent a^t_{BA} whose magnitude is yet unknown.

3. The acceleration of B with respect to C has the following two components:

(*i*) Radial component of the acceleration of B with respect to C *i.e.* a^r_{BC}, and

(*ii*) Tangential component of the acceleration of B with respect to C *i.e.* a^t_{BC}.

These two components are mutually perpendicular. Therefore from point c', draw vector $c'y$ parallel to CB to represent $a^r_{BC} = 1.88 \text{ m/s}^2$ and from point y draw vector yb' perpendicular to vector $c'y$ to represent a^t_{BC}. The vectors xb' and yb' intersect at b'.

4. Join $a'b'$ and $c'b'$. The vector $a'b'$ represents the acceleration of B with respect to A *i.e.* a_{BA} and the vector $c'b'$ represents the acceleration of B with respect to C or simply the acceleration of B *i.e.* a_{BC} or a_B, because C is a fixed point.

5. Since the point D lies on AB, therefore divide vector $a'b'$ at d' in the same ratio as D divides AB in the space diagram. In other words,

$$a'd'/a'b' = AD/AB$$

Note: Since D is the mid-point of AB, therefore d' is also mid-point of vector $a'd'$.

6. The acceleration of S with respect to D has the following two components:

(*i*) Radial component of the acceleration of S with respect to D *i.e.* a^r_{SD}, and

(*ii*) Tangential component of the acceleration of S with respect to D *i.e.* a^t_{SD}.

These two components are mutually perpendicular. Therefore from point d', draw vector $d'z$ parallel to DS to represent $a^r_{SD} = 0.826 \text{ m/s}^2$, and from point z draw vector zs' perpendicular to vector $d'z$ to represent a^t_{SD} whose magnitude is yet unknown.

7. The acceleration of Q (swivel block) with respect to S (point on link DE *i.e.* coincident point) has the following two components:

(*i*) Coriolis component of acceleration of Q with respect to S *i.e.* a^c_{QS}, and

(*ii*) Radial component of acceleration of Q with respect to S, *i.e.* a^r_{QS}.

These two components are mutually perpendicular. Therefore from point q', draw vector $q'z_1$, perpendicular to DS to represent $a^c_{QS} = 2.336$ m/s^2 in a direction as shown in Fig. 8.37 (*c*). The direction of a^c_{QS} is obtained by rotating v_{QS} (represented by vector sq in velocity diagram) through 90° in the same sense as that of ω_{DE} which rotates in the anticlockwise direction. Now from z_1, draw vector z_1s' perpendicular to vector $q'z_1$ (or parallel to DS) to represent a^r_{QS}. The vectors zs' and z_1s' intersect at s'.

8. Join $s'q'$ and $d's'$. The vector $s'q'$ represents the acceleration of Q with respect to S *i.e.* a_{QS} and vector $d's'$ represents the acceleration of S with respect to D *i.e.* a_{SD}.

By measurement, we find that the acceleration of sliding the link DE in the trunnion,

$$= a^r_{QS} = \text{vector } z_1s' = 1.55 \text{ m/s}^2 \quad \textbf{Ans.}$$

EXERCISES

1. The engine mechanism shown in Fig. 8.38 has crank OB = 50 mm and length of connecting rod AB = 225 mm. The centre of gravity of the rod is at G which is 75 mm from B. The engine speed is 200 r.p.m.

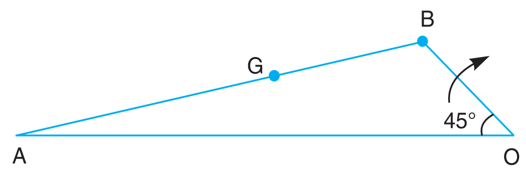


Fig. 8.38

For the position shown, in which OB is turned 45° from OA, Find 1. the velocity of G and the angular velocity of AB, and 2. the acceleration of G and angular acceleration of AB.

[Ans. 6.3 m/s ; 22.6 rad/s ; 750 m/s^2 ; 6.5 rad/s^2]

2. In a pin jointed four bar mechanism $ABCD$, the lengths of various links are as follows:

AB = 25 mm ; BC = 87.5 mm ; CD = 50 mm and AD = 80 mm.

The link AD is fixed and the angle BAD = 135°. If the velocity of B is 1.8 m/s in the clockwise direction, find 1. velocity and acceleration of the mid point of BC, and 2. angular velocity and angular acceleration of link CB and CD.

[Ans. 1.67 m/s, 110 m/s^2 ; 8.9 rad/s, 870 rad/s^2 ; 32.4 rad/s, 1040 rad/s^2]

3. In a four bar chain $ABCD$, link AD is fixed and the crank AB rotates at 10 radians per second clockwise. Lengths of the links are AB = 60 mm ; BC = CD = 70 mm ; DA = 120 mm. When angle DAB = 60° and both B and C lie on the same side of AD, find 1. angular velocities (magnitude and direction) of BC and CD ; and 2. angular acceleration of BC and CD.

[Ans. 6.43 rad/s (anticlockwise), 6.43 rad/s (clockwise) ; 10 rad/s^2 105 rad/s^2]

4. In a mechanism as shown in Fig. 8.39, the link AB rotates with a uniform angular velocity of 30 rad/s. The lengths of various links are :

AB = 100 mm ; BC = 300 mm ; BD = 150 mm ; DE = 250 mm ; EF = 200 mm ; DG = 165 mm.

Determine the velocity and acceleration of G for the given configuration.

[Ans. 0.6 m/s ; 66 m/s^2]

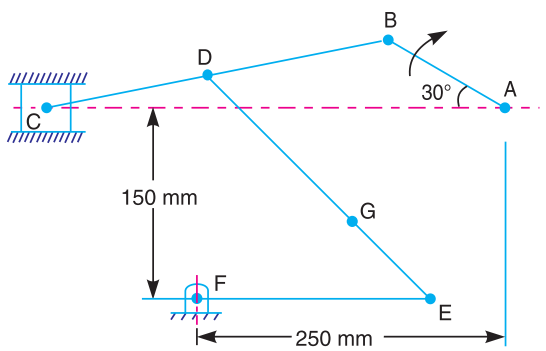


Fig. 8.39

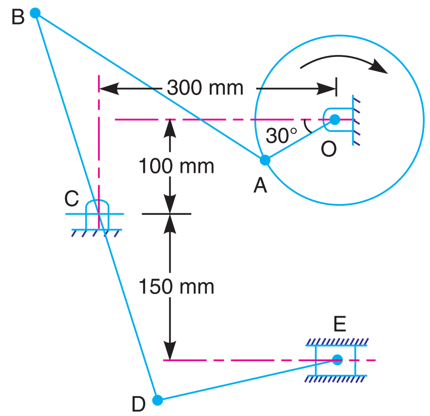


Fig. 8.40

5. In a mechanism as shown in Fig. 8.40, the crank *OA* is 100 mm long and rotates in a clockwise direction at a speed of 100 r.p.m. The straight rod *BCD* rocks on a fixed point at *C*. The links *BC* and *CD* are each 200 mm long and the link *AB* is 300 mm long. The slider *E*, which is driven by the rod *DE* is 250 mm long. Find the velocity and acceleration of *E*.

[Ans. 1.26 m/s; 10.5 m/s²]

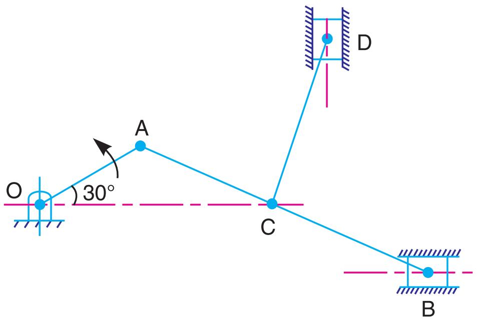


Fig. 8.41

6. The dimensions of the various links of a mechanism, as shown in Fig. 8.41, are as follows:

OA = 80 mm ; *AC* = *CB* = *CD* = 120 mm

If the crank *OA* rotates at 150 r.p.m. in the anti-clockwise direction, find, for the given configuration: 1. velocity and acceleration of *B* and *D* ; 2. rubbing velocity on the pin at *C*, if its diameter is 20 mm ; and 3. angular acceleration of the links *AB* and *CD*.

[Ans. 1.1 m/s ; 0.37 m/s ; 20.2 m/s², 16.3 m/s² ; 0.15 m/s ; 34.6 rad/s²; 172.5 rad/s²]

7. In the toggle mechanism, as shown in Fig. 8.42, *D* is constrained to move on a horizontal path. The dimensions of various links are : *AB* = 200 mm; *BC* = 300 mm ; *OC* = 150 mm; and *BD* = 450 mm.

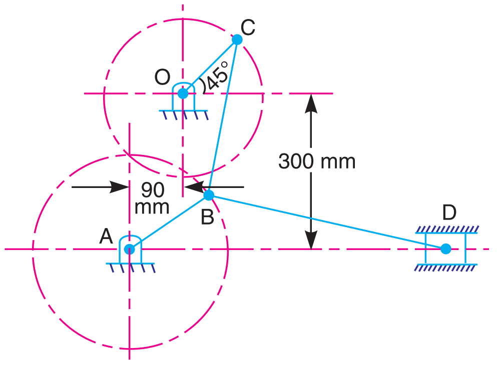


Fig. 8.42

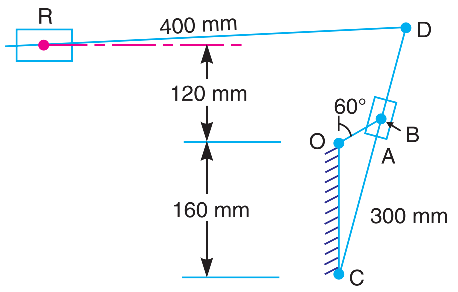


Fig. 8.43

The crank *OC* is rotating in a counter clockwise direction at a speed of 180 r.p.m., increasing at the rate of 50 rad/s². Find, for the given configuration 1. velocity and acceleration of *D*, and 2. angular velocity and angular acceleration of *BD*.

8. In a quick return mechanism, as shown in Fig. 8.43, the driving crank OA is 60 mm long and rotates at a uniform speed of 200 r.p.m. in a clockwise direction. For the position shown, find 1. velocity of the ram R ; 2. acceleration of the ram R, and 3. acceleration of the sliding block A along the slotted bar CD.
[Ans. 1.3 m/s ; 9 m/s^2 ; 15 m/s^2]

9. Fig. 8.44 shows a quick return motion mechanism in which the driving crank OA rotates at 120 r.p.m. in a clockwise direction. For the position shown, determine the magnitude and direction of 1, the acceleration of the block D ; and 2. the angular acceleration of the slotted bar QB.
[Ans. 7.7 m/s^2 ; 17 rad/s^2]

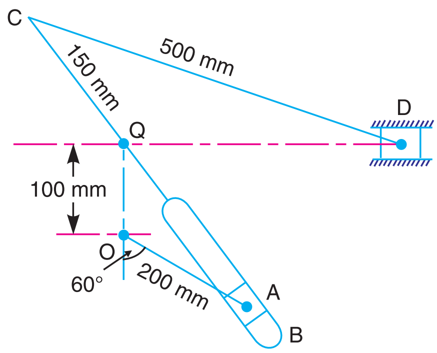


Fig. 8.44

10. In the oscillating cylinder mechanism as shown in Fig. 8.45, the crank OA is 50 mm long while the piston rod AB is 150 mm long. The crank OA rotates uniformly about O at 300 r.p.m.

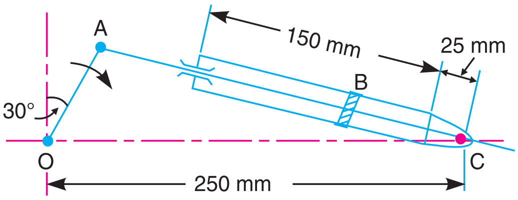


Fig. 8.45

Determine, for the position shown : 1. velocity of the piston B relative to the cylinder walls, 2. angular velocity of the piston rod AB, 3. sliding acceleration of the piston B relative to the cylinder walls, and 4. angular acceleration of the piston rod AB.

[Ans. 1.5 m/s ; 2.2 rad/s (anticlockwise) ; 16.75 m/s^2 ; 234 rad/s^2]

11. The mechanism as shown in Fig 8.46 is a marine steering gear, called Rapson's slide. O_2B is the tiller and AC is the actuating rod. If the velocity of AC is 25 mm/min to the left, find the angular velocity and angular acceleration of the tiller. Either graphical or analytical technique may be used.
[Ans. 0.125 rad/s; 0.018 rad/s^2]

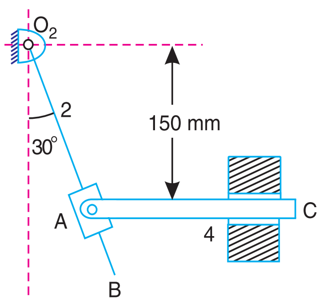


Fig. 8.46

DO YOU KNOW ?

1. Explain how the acceleration of a point on a link (whose direction is known) is obtained when the acceleration of some other point on the same link is given in magnitude and direction.
2. Draw the acceleration diagram of a slider crank mechanism.
3. Explain how the coriolis component of acceleration arises when a point is rotating about some other fixed point and at the same time its distance from the fixed point varies.
4. Derive an expression for the magnitude and direction of coriolis component of acceleration.
5. Sketch a quick return motion of the crank and slotted lever type and explain the procedure of drawing the velocity and acceleration diagram, for any given configuration of the mechanism.

OBJECTIVE TYPE QUESTIONS

1. The component of the acceleration, parallel to the velocity of the particle, at the given instant is called

   (*a*) radial component (*b*) tangential component

   (*c*) coriolis component (*d*) none of these

2. A point B on a rigid link AB moves with respect to A with angular velocity ω rad/s. The radial component of the acceleration of B with respect to A,

   (*a*) $v_{BA} \times AB$ (*b*) $v^2_{BA} \times AB$ (*c*) $\frac{v_{BA}}{AB}$ (*d*) $\frac{v^2_{BA}}{AB}$

   where v_{BA} = Linear velocity of B with respect to $A = \omega \times AB$

3. A point B on a rigid link AB moves with respect to A with angular velocity ω rad/s. The angular acceleration of the link AB is

   (*a*) $\frac{a^r_{BA}}{AB}$ (*b*) $\frac{a^t_{BA}}{AB}$ (*c*) $v_{BA} \times AB$ (*d*) $\frac{v^2_{BA}}{AB}$

4. A point B on a rigid link AB moves with respect to A with angular velocity ω rad/s. The total acceleration of B with respect to A will be equal to

   (*a*) vector sum of radial component and coriolis component

   (*b*) vector sum of tangential component and coriolis component

   (*c*) vector sum of radial component and tangential component

   (*d*) vector difference of radial component and tangential component

5. The coriolis component of acceleration is taken into account for

   (*a*) slider crank mechanism (*b*) four bar chain mechanism

   (*c*) quick return motion mechanism (*d*) none of these

ANSWERS

1. (*b*)	**2.** (*d*)	**3.** (*b*)	**4.** (*c*)	**5.** (*c*)

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