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25

Computer Aided Analysis and Synthesis of Mechanisms

Features (Main)

25.1. Introduction

We have already discussed in chapters 7 and 8, the graphical methods to determine velocity and acceleration analysis of a mechanism. It may be noted that graphical method is only suitable for determining the velocity and acceleration of the links in a mechanism for a single position of the crank. In order to determine the velocity and acceleration of the links in a mechanism for different positions of the crank, we have to draw the velocity and acceleration diagrams for each position of the crank which is inconvenient. In this chapter, we shall discuss the analytical expressions for the displacement, velocity and acceleration in terms of general parameters of a mechanism and calculations may be performed either by a desk calculator or digital computer.

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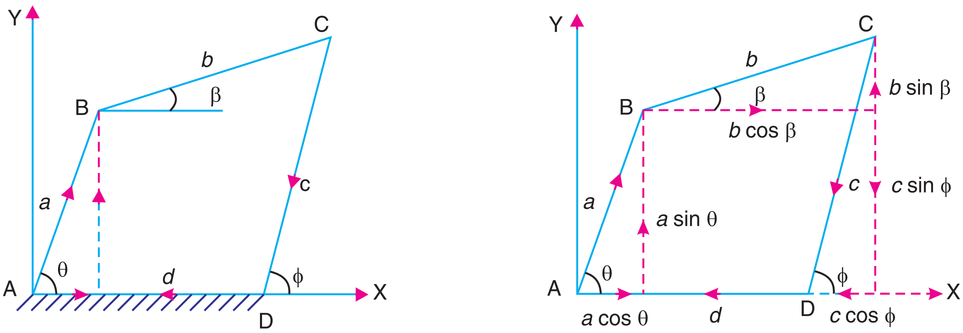
25.2. Computer Aided Analysis for Four Bar Mechanism (Freudenstein's Equation)

Consider a four bar mechanism *ABCD*, as shown in Fig. 25.1 (*a*), in which *AB* = *a*, *BC* = *b*, *CD* = *c*, and *DA* = *d*. The link *AD* is fixed and lies along *X*-axis. Let the links *AB* (input link), *BC* (coupler) and *DC* (output link) make angles θ, β and ϕ respectively along the *X*-axis or fixed link *AD*.



(*a*) Four bar mechanism. (*b*) Components along *X*-axis and *Y*-axis.

Fig. 25.1

The relation between the angles and link lengths may be developed by considering the links as vectors. The expressions for displacement, velocity and acceleration analysis are derived as discussed below :

1. *Displacement analysis*

For equilibrium of the mechanism, the sum of the components along *X*-axis and along *Y*-axis must be equal to zero. First of all, taking the sum of the components along *X*-axis as shown in Fig. 25.1 (*b*), we have

$$a\cos\theta + b\cos\beta - c\cos\phi - d = 0 \quad \ldots (i)$$

or

$$b\cos\beta = c\cos\phi + d - a\cos\theta$$

Squaring both sides

$$b^2\cos^2\beta = (c\cos\phi + d - a\cos\theta)^2$$

$$= c^2\cos^2\phi + d^2 + 2c\,d\cos\phi + a^2\cos^2\theta$$

$$-2a\,c\cos\phi\cos\theta - 2a\,d\cos\theta \quad \ldots (ii)$$

Now taking the sum of the components along *Y*-axis, we have

$$a\sin\theta + b\sin\beta - c\sin\phi = 0 \quad \ldots (iii)$$

or

$$b\sin\beta = c\sin\phi - a\sin\theta$$

Squaring both sides,

$$b^2\sin^2\beta = (c\sin\phi - a\sin\theta)^2$$

$$= c^2\sin^2\phi + a^2\sin^2\theta - 2a\,c\sin\phi\sin\theta \quad \ldots (iv)$$

Adding equations (*ii*) and (*iv*),

$$b^2(\cos^2\beta + \sin^2\beta) = c^2(\cos^2\phi + \sin^2\phi) + d^2 + 2\,c\,d\cos\phi + a^2(\cos^2\theta + \sin^2\theta)$$

$$-2\,a\,c(\cos\phi\cos\theta + \sin\phi\sin\theta) - 2\,a\,d\cos\theta$$

or $$b^2 = c^2 + d^2 + 2\,c\,d\cos\phi + a^2 - 2\,a\,c(\cos\phi\cos\theta + \sin\phi\sin\theta) - 2\,a\,d\cos\theta$$

or $$2\,a\,c(\cos\phi\cos\theta + \sin\phi\sin\theta) = a^2 - b^2 + c^2 + d^2 + 2\,c\,d\cos\phi - 2a\,d\cos\theta$$

$$\cos\phi\cos\theta + \sin\phi\sin\theta = \frac{a^2 - b^2 + c^2 + d^2}{2\,a\,c} + \frac{d}{a}\cos\phi - \frac{d}{c}\cos\theta \qquad \text{. . . }(v)$$

Let $$\frac{d}{a} = k_1;\ \frac{d}{c} = k_2; \text{ and } \frac{a^2 - b^2 + c^2 + d^2}{2\,a\,c} = k_3 \qquad \text{. . . }(vi)$$

Equation **(*v*)** may be written as

$$\cos\phi\cos\theta + \sin\phi\sin\theta = k_1\cos\phi - k_2\cos\theta + k_3 \qquad \text{. . . }(vii)$$

or cos (ϕ – θ) or $\cos(\theta - \phi) = k_1\cos\phi - k_2\cos\theta + k_3$

The equation **(*vii*)** is known as ***Freudenstein's equation.***

Since it is very difficult to determine the value of ϕ for the given value of θ, from equation **(*vii*)**, therefore it is necessary to simplify this equation.

From trigonometrical ratios, we know that

$$\sin\phi = \frac{2\tan(\phi/2)}{1 + \tan^2(\phi/2)} \quad \text{and} \quad \cos\phi = \frac{1 - \tan^2(\phi/2)}{1 + \tan^2(\phi/2)}$$

Substituting these values of $\sin\phi$ and $\cos\phi$ in equation **(*vii*)**,

$$\frac{1 - \tan^2(\phi/2)}{1 + \tan^2(\phi/2)} \times \cos\theta + \frac{2\tan(\phi/2)}{1 + \tan^2(\phi/2)} \times \sin\theta$$

$$= k_1 \times \frac{1 - \tan^2(\phi/2)}{1 + \tan^2(\phi/2)} - k_2\cos\theta + k_3$$

$$\cos\theta\,[1 - \tan^2(\phi/2)] + 2\sin\theta\tan(\phi/2)$$

$$= k_1\,[1 - \tan^2(\phi/2)] - k_2\cos\theta\,[1 + \tan^2(\phi/2)] + k_3[1 + \tan^2(\phi/2)]$$

$$\cos\theta - \cos\theta\tan^2(\phi/2) + 2\sin\theta\tan(\phi/2)$$

$$= k_1 - k_1\tan^2(\phi/2) - k_2\cos\theta - k_2\cos\theta\tan^2(\phi/2) + k_3 + k_3\tan^2(\phi/2)$$

Rearranging this equation,

$$-\cos\theta\tan^2(\phi/2) + k_1\tan^2(\phi/2) + k_2\cos\theta\tan^2(\phi/2) - k_3\tan^2(\phi/2) + 2\sin\theta\tan(\phi/2)$$

$$= -\cos\theta + k_1 - k_2\cos\theta + k_3$$

$$-\tan^2(\phi/2)\,[\cos\theta - k_1 - k_2\cos\theta + k_3] + 2\sin\theta\tan(\phi/2) - k_1 - k_3 + \cos\theta(1 + k_2) = 0$$

$$[\,(1 - k_2)\cos\theta + k_3 - k_1]\tan^2\phi/2 + (-2\sin\theta)\tan\phi/2 + [k_1 + k_3 - (1 + k_2)\cos\theta] = 0$$

(By changing the sign)

or $$A\tan^2(\phi/2) + B\tan(\phi/2) + C = 0 \qquad \text{. . . }(viii)$$

where
$$\left.\begin{aligned} A &= (1-k_2)\cos\theta + k_3 + k_1, \\ B &= -2\sin\theta, \text{ and} \\ C &= k_1 + k_3 - (1+k_2)\cos\theta \end{aligned}\right] \quad \ldots (ix)$$

Inner view of an aircraft engine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

The equation ***(viii)*** is a quadratic equation in $\tan(\phi/2)$. Its two roots are

$$\tan(\phi/2) = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

or
$$\phi = 2\tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right] \quad \ldots (x)$$

From this equation ***(x)***, we can find the position of output link *CD* (*i.e.* angle ϕ) if the length of the links (*i.e. a*, *b*, *c* and *d*) and position of the input link *AB* (*i.e.* angle θ) is known.

If the relation between the position of input link *AB* (*i.e.* angle θ) and the position of coupler link *BC* (*i.e.* angle β) is required, then eliminate angle ϕ from the equations ***(i)*** and ***(iii)***.

The equation ***(i)*** may be written as

$$c\cos\phi = a\cos\theta + b\cos\beta - d \quad \ldots (xi)$$

Squaring both sides,

$$c^2\cos^2\phi = a^2\cos^2\theta + b^2\cos^2\beta + 2\,a\,b\cos\theta\cos\beta$$
$$+\ d^2 - 2\,a\,d\cos\theta - 2\,b\,d\cos\beta \quad \ldots (xii)$$

Now equation ***(iii)*** may be written as

$$c\sin\phi = a\sin\theta + b\sin\beta \quad \ldots (xiii)$$

Squaring both sides,

$$c^2 \sin^2 \phi = a^2 \sin^2 \theta + b^2 \sin^2 \beta + 2\, a\, b \sin\theta \sin\beta \qquad \text{... } (xiv)$$

Adding equations (*xii*) and (*xiv*),

$$c^2(\cos^2\phi + \sin^2\theta) = a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\beta + \sin^2\beta)$$

$$+ 2ab(\cos\theta\cos\beta + \sin\theta\sin\beta) + d^2 - 2\, a\, d\, \cos\theta - 2\, b\, d\, \cos\beta$$

or

$$c^2 = a^2 + b^2 + 2\, a\, b(\cos\theta\cos\beta + \sin\theta\sin\beta)$$

$$+ d^2 - 2ad\cos\theta - 2\, b\, d\cos\beta$$

or

$$2ab(\cos\theta\cos\beta + \sin\theta\sin\beta) = c^2 - a^2 - b^2 - d^2 + 2a\, d\cos\theta + 2\, b\, d\cos\beta$$

$$\cos\theta\cos\beta + \sin\theta\sin\beta = \frac{c^2 - a^2 - b^2 - d^2}{2ab} + \frac{d}{b}\cos\theta + \frac{d}{a}\cos\beta \qquad \text{. . . } (xv)$$

Let

$$\frac{d}{a} = k_1; \frac{d}{b} = k_4; \quad \text{and} \quad \frac{c^2 - a^2 - b^2 - d^2}{2a\, b} = k_5 \qquad \text{. . . } (xvi)$$

∴ Equation (*xvi*) may be written as

$$\cos\theta\cos\beta + \sin\theta\sin\beta = k_1\cos\beta + k_4\cos\theta + k_5 \qquad \text{. . . } (xvii)$$

From trigonometrical ratios, we know that

$$\sin\beta = \frac{2\tan(\beta/2)}{1 + \tan^2(\beta/2)}, \quad \text{and} \quad \cos\beta = \frac{1 - \tan^2(\beta/2)}{1 + \tan^2(\beta/2)}$$

Substituting these values of $\sin\beta$ and $\cos\beta$ in equation (*xvii*),

$$\cos\theta\left[\frac{1 - \tan^2(\beta/2)}{1 + \tan^2(\beta/2)}\right] + \sin\theta\left[\frac{2\tan(\beta/2)}{1 + \tan^2(\beta/2)}\right]$$

$$= k_1\left[\frac{1 - \tan^2(\beta/2)}{1 + \tan^2(\beta/2)}\right] + k_4\cos\theta + k_5$$

$$\cos\theta[1 - \tan^2(\beta/2)] + 2\sin\theta\tan(\beta/2)$$

$$= k_1\left[1 - \tan^2(\beta/2)\right] + k_4\cos\theta\left[1 + \tan^2(\beta/2)\right] \; + k_5\left[1 + \tan^2(\beta/2)\right]$$

$$\cos\theta - \cos\theta\tan^2(\beta/2) + 2\sin\theta\tan(\beta/2)$$

$$= k_1 - k_1\tan^2(\beta/2) + k_4\cos\theta + k_4\cos\theta\tan^2(\beta/2)$$

$$+ k_5 + k_5\tan^2(\beta/2)$$

$$-\cos\theta\tan^2(\beta/2) + k_1\tan^2(\beta/2) - k_4\cos\theta\tan^2(\beta/2) - k_5\tan^2(\beta/2)$$

$$+ 2\sin\theta\tan(\beta/2) - k_1 - k_4\cos\theta - k_5 + \cos\theta = 0$$

$$-\tan^2(\beta/2)[(k_4 + 1)\cos\theta + k_5 - k_1] + 2\sin\theta\tan(\beta/2) - [(k_4 - 1)\cos\theta + k_5 + k_1] = 0$$

or $$[(k_4+1)\cos\theta+k_5-k_1]\tan^2(\beta/2)+(-2\sin\theta)\tan(\beta/2)+[(k_4-1)\cos\theta+k_5+k_1]=0$$

(By changing the sign)

or $$D\tan^2(\beta/2)+E\tan(\beta/2)+F=0 \quad \text{. . . (xviii)}$$

where $$\left.\begin{aligned} D &= (k_4+1)\cos\theta+k_5-k_1, \\ E &= -2\sin\theta, \quad \text{and} \\ F &= [(k_4-1)\cos\theta+k_5+k_1] \end{aligned}\right] \quad \text{... (xix)}$$

The equation ***(xviii)*** is a quadratic equation in $\tan(\beta/2)$. Its two roots are

$$\tan(\beta/2)=\frac{-E\pm\sqrt{E^2-4D\,F}}{2D}$$

or $$\beta=2\tan^{-1}\left[\frac{-E\pm\sqrt{E^2-4DF}}{2\,D}\right] \quad \text{. . . (xx)}$$

From this equation ***(xx)***, we can find the position of coupler link *BC* (*i.e.* angle β).

Note: The angle α may be obtained directly from equation ***(i)*** or ***(iii)*** after determining the angle ϕ.

2. *Velocity analysis*

Let ω_1 = Angular velocity of the link $AB = d\theta/dt$,

ω_2 = Angular velocity of the link $BC = d\beta/dt$, and

ω_3 = Angular velocity of the link $CD = d\phi/dt$.

Differentiating equation ***(i)*** with respect to time,

$$-a\sin\theta\times\frac{d\theta}{dt}-b\sin\beta\times\frac{d\beta}{dt}+c\sin\phi\times\frac{d\phi}{dt}=0$$

or $$-a\,\omega_1\sin\theta-b\,\omega_2\sin\beta+c\,\omega_3\sin\phi=0 \quad \text{... (xxi)}$$

Again, differentiating equation ***(iii)*** with respect to time,

$$a\cos\theta\times\frac{d\theta}{dt}+b\cos\beta\times\frac{d\beta}{dt}-c\cos\phi\times\frac{d\phi}{dt}=0$$

or $$a\,\omega_1\cos\theta+b\,\omega_2\cos\beta-c\,\omega_3\cos\phi=0 \quad \text{... (xxii)}$$

Multiplying the equation ***(xxi)*** by $\cos\beta$ and equation ***(xxii)*** by $\sin\beta$,

$$-a\,\omega_1\sin\theta\cos\beta-b\,\omega_2\sin\beta\cos\beta+c\,\omega_3\sin\phi\cos\beta=0 \quad \text{... (xxiii)}$$

and $$a\,\omega_1\cos\theta\sin\beta+b\,\omega_2\cos\beta\sin\beta-c\,\omega_3\cos\phi\sin\beta=0 \quad \text{... (xxiv)}$$

Adding equations ***(xxiii)*** and ***(xxiv)***,

$$a\,\omega_1\sin(\beta-\theta)+c\,\omega_3\sin(\phi-\beta)=0$$

$$\therefore \quad \omega_3=\frac{-a\,\omega_1\sin(\beta-\theta)}{c\sin(\phi-\beta)} \quad \text{... (xxv)}$$

Again, multiplying the equation **(*xxi*)** by $\cos\phi$ and equation **(*xxii*)** by $\sin\phi$,

$$-a\,\omega_1 \sin\theta\cos\phi - b\,\omega_2 \sin\beta\cos\phi + c\,\omega_3 \sin\phi\cos\phi = 0 \quad \ldots (xxvi)$$

and
$$a\,\omega_1 \cos\theta\sin\phi + b\,\omega_2 \cos\beta\sin\phi - c\,\omega_3 \cos\phi\sin\phi = 0 \quad \ldots (xxvii)$$

Adding equations **(*xxvi*)** and **(*xxvii*)**,

$$a\,\omega_1 \sin(\phi-\theta) + b\,\omega_2 \sin(\phi-\beta) = 0$$

$$\therefore \qquad \omega_2 = \frac{-a\,\omega_1 \sin(\phi-\theta)}{b\sin(\phi-\beta)} \quad \ldots (xxviii)$$

From equations **(*xxv*)** and **(*xxviii*)**, we can find ω_3 and ω_2, if $a, b, c, \theta, \phi, \beta$ and ω_1 are known.

3. *Acceleration analysis*

Let α_1 = Angular acceleration of the link $AB = d\omega_1/dt$,

α_2 = Angular acceleration of the link $BC = d\omega_2/dt$, and

α_3 = Angular acceleration of the link $CD = d\omega_3/dt$.

Differentiating equation **(*xxi*)** with respect to time,

$$-a\left[\omega_1 \cos\theta\times\frac{d\theta}{dt} + \sin\theta\times\frac{d\omega_1}{dt}\right] - b\left[\omega_2 \cos\beta\times\frac{d\beta}{dt} + \sin\beta\times\frac{d\omega_2}{dt}\right]$$

$$+c\left[\omega_3 \cos\phi\times\frac{d\phi}{dt} + \sin\phi\times\frac{d\omega_3}{dt}\right] = 0$$

$$\ldots \left[\because \frac{d}{dx}(u\,v) = u\times\frac{dv}{dx} + v\times\frac{du}{dx}\right]$$

or
$$-a\,\omega_1^2 \cos\theta - a\sin\theta\,\alpha_1 - b\,\omega_2^2 \cos\beta - b\sin\beta\,\alpha_2$$

$$+c\,\omega_3^2 \cos\phi + c\sin\phi\,\alpha_3 = 0 \quad \ldots (xxix)$$

Again, differentiating equation **(*xxii*)** with respect to time,

$$a\left[\omega_1\times -\sin\theta\times\frac{d\theta}{dt} + \cos\theta\times\frac{d\omega_1}{dt}\right] + b\left[\omega_2\times -\sin\beta\times\frac{d\beta}{dt} + \cos\beta\times\frac{d\omega_2}{dt}\right]$$

$$-c\left[\omega_3\times -\sin\phi\times\frac{d\phi}{dt} + \cos\phi\times\frac{d\omega_3}{dt}\right] = 0$$

or
$$-a\omega_1^2 \sin\theta + a\cos\theta\alpha_1 - b\,\omega_2^2 \sin\beta + b\cos\beta\,\alpha_2$$

$$+c\,\omega_3^2 \sin\phi - c\cos\phi\,\alpha_3 = 0 \quad \ldots (xxx)$$

Multiplying equation **(*xxix*)** by $\cos\phi$, and equation **(*xxx*)** by $\sin\phi$,

$$-a\,\omega_1^2 \cos\theta\cos\phi - a\,\alpha_1 \sin\theta\cos\phi - b\,\omega_2^2 \cos\beta\cos\phi$$

$$-b\,\alpha_2 \sin\beta\cos\phi + c\,\omega_3^2 \cos^2\phi + c\,\alpha_3 \sin\phi\cos\phi = 0 \quad \ldots (xxxi)$$

and $$-a\,\omega_1^2 \sin\theta \sin\phi + a\,\alpha_1 \cos\theta \sin\phi - b\,\omega_2^2 \sin\beta \sin\phi$$
$$+ b\,\alpha_2 \cos\beta \sin\phi + c\,\omega_3^2 \sin^2\phi - c\,\alpha_3 \cos\phi \sin\phi = 0 \quad \text{... (xxxii)}$$

Adding equations (*xxxi*) and (*xxxii*),

$$-a\,\omega_1^2 (\cos\phi \cos\theta + \sin\phi \sin\theta) + a\,\alpha_1 (\sin\phi \cos\theta - \cos\phi \sin\theta)$$
$$- b\,\omega_2^2 (\cos\phi \cos\beta + \sin\phi \sin\beta) + b\,\alpha_2 (\sin\phi \cos\beta - \cos\phi \sin\beta)$$
$$+ c\,\omega_3^2 (\cos^2\phi + \sin^2\phi) = 0$$

$$-a\,\omega_1^2 \cos(\phi - \theta) + a\,\alpha_1 \sin(\phi - \theta) - b\,\omega_2^2 \cos(\phi - \beta) + b\,\alpha_2 \sin(\phi - \beta) + c\,\omega_3^2 = 0$$

$$\therefore \quad \alpha_2 = \frac{-a\,\alpha_1 \sin(\phi - \theta) + a\,\omega_1^2 \cos(\phi - \theta) + b\,\omega_2^2 \cos(\phi - \beta) - c\,\omega_3^2}{b \sin(\phi - \beta)} \quad \text{... (xxxiii)}$$

Again multiplying equation (*xxix*) by $\cos\beta$ and equation (*xxx*) by $\sin\beta$,

$$-a\,\omega_1^2 \cos\theta \cos\beta - a\,\alpha_1 \sin\theta \cos\beta - b\,\omega_2^2 \cos^2\beta - b\,\alpha_2 \sin\beta \cos\beta$$
$$+ c\,\omega_3^2 \cos\phi \cos\beta + c\,\alpha_3 \sin\phi \cos\beta = 0 \quad \text{... (xxxiv)}$$

and $$-a\,\omega_1^2 \sin\theta \sin\beta + a\,\alpha_1 \cos\theta \sin\beta - b\,\omega_2^2 \sin^2\beta + b\,\alpha_2 \cos\beta \sin\beta$$
$$+ c\,\omega_3^2 \sin\phi \sin\beta - c\,\alpha_3 \cos\phi \sin\beta = 0 \quad \text{... (xxxv)}$$

Adding equations (*xxxiv*) and (*xxxv*),

$$-a\,\omega_1^2 (\cos\beta \cos\theta + \sin\beta \sin\theta) + a\,\alpha_1 (\sin\beta \cos\theta - \cos\beta \sin\theta) - b\,\omega_2^2 (\cos^2\beta + \sin^2\beta)$$
$$+ c\,\omega_3^2 (\cos\phi \cos\beta + \sin\phi \sin\beta) + c\,\alpha_3 (\sin\phi \cos\beta - \cos\phi \sin\beta) = 0$$

$$-a\,\omega_1^2 \cos(\beta - \theta) + a\,\alpha_1 \sin(\beta - \theta) - b\,\omega_2^2 + c\,\omega_3^2 \cos(\phi - \beta) + c\,\alpha_3 \sin(\phi - \beta) = 0$$

$$\therefore \quad a_3 = \frac{-a\,\alpha_1 \sin(\beta - \theta) + a\,\omega_1^2 \cos(\beta - \theta) + b\,\omega_2^2 - c\,\omega_3^2 \cos(\phi - \beta)}{c \sin(\phi - \beta)} \quad \text{... (xxxvi)}$$

From equations (*xxxiii*) and (*xxxvi*), the angular acceleration of the links *BC* and *CD* (*i.e.* α_2 and α_3) may be determined.

25.3. Programme for Four Bar Mechanism

The following is a programme in Fortran for determining the velocity and acceleration of the links in a four bar mechanism for different position of the crank.

```
C       PROGRAM TO FIND THE VELOCITY AND ACCELERATION IN A FOUR-BAR
C       MECHANISM
        DIMENSION PH (2), PHI (2), PP (2), BET (2), BT (2), VELC (2), VELB (2), ACCC (2),
        ACCB (2), C1 (2), C2 (2), C3 (2), C4 (2), B1 (2), B2 (2), B3 (2), B4 (2)
        READ (*, *) A, B, C, D, VELA, ACCA, THETA
        PI = 4.0 * ATAN (1.0)
        THET = 0
        IHT = 180/THETA
        DTHET = PI/IHT
```

```
        DO 10 J = 1, 2 * IHT
        THET = (J – 1) * DTHET
        AK = (A * A – B * B + C * C + D * D) * 0.5)
        TH = THET * 180/PI
        AA = AK – A * (D – C) * COS (THET) – (C * D)
        BB = – 2.0 * A* C * SIN (THET)
        CC = AK – A * (D + C) * COS (THET) + (C * D)
        AB = BB * * 2 – 4 * AA * CC
        IF    (AB . LT . 0) GO TO 10
        PHH = SQRT (AB)
        PH (1) = – BB + PHH
        PH (2) = – BB – PHH
        DO 9 I =   1, 2
        PHI (I) = ATAN (PH (I) * 0.5/AA) * 2
        PP (I) = PHI (I) * 180/PI
        BET (I) = ASIN ((C * SIN (PHI (I)) – A * SIN (THET)) / B)
        BT (I) = BET (I) * 180/PI
        VELC (I) = A * VELA * SIN (BET (I) – THET) / (C * SIN (BET (I) – PHI (I)))
        VELB (I) = (A * VELA * SIN (PHI (I) – THET) ) / (B * SIN (BET (I) – PHI (I))))
        C1 (I) = A * ACCA * SIN (BET (I) – THET)
        C2 (I) = A * VELA * * 2 * COS (BET (I) – THET) + B * VELB (I) * * 2
        C3 (I) = C * VELC (I) * * 2 * COS (PHI (I) – BET (I) )
        C4 (I) = C * SIN (BET (I) – PHI (I))
        ACCC (I) = (C1 (I) – C2 (I) + C3 (I) ) / C4 (I)
        B1 (I) = A* ACCA* SIN (PHI (I) – THET )
        B2 (I) = A * VELA * * 2 * COS (PHI (I) – THET )
        B3 (I) = B * VELB (I) * * 2 * COS (PHI (I) – BET (I) ) – C * VELC (I) * * 2
        B4 (I) = B * (SIN (BET (I) – PHI (I))))
9       ACCB (I) = (B1 (I) – B2 (I) – B3 (I)) / B4 (I)
        IF (J . NE . 1) GO TO 8
        WRITE (*, 7)
7       FORMAT (4X,' THET', 4X,' PHI', 4X,' BETA', 4X,' VELC', 4X,' VELB', 4X,' ACCC', 4X,'
        ACCB')
8       WRITE (*, 6) TH, PP (1), BT (1), VELC (1), VELB (1), ACCC (1), ACCB (1)
6       FORMAT (8F8 . 2)
        WRITE (*, 5) PP (2), BT (2), VELC (2), VELB (2), ACCC (2), ACCB (2)
5       FORMAT (8X, 8F8 . 2)
10      CONTINUE
        STOP
        END
```

The various input variables are

A, B, C, D = Lengths of the links *AB, BC, CD,* and *DA* respectively in mm,

THETA = Interval of the input angle in degrees,

VELA = Angular Velocity of the input link *AB* in rad/s, and

ACCA = Angular acceleration of the input link in rad/s^2.

The output variables are :

THET = Angular displacement of the input link *AB* in degrees,

PHI = Angular displacement of the output link *DC* in degrees,

BETA = Angular displacement of the coupler link *BC* in degrees,

VELC = Angular velocity of the output link *DC* in rad/s,

VELB = Angular velocity of the coupler link *BC* in rad/s,

ACCC = Angular acceleration of the output link *DC* in rad/s^2,

ACCB = Angular acceleration of the coupler link *BC* in rad/s^2.

Example 25.1. *ABCD is a four bar mechanism, with link AD fixed. The lengths of the links are*

AB = 300 mm; BC = 360 mm; CD = 360 mm and AD = 600 mm.

The crank AB has an angular velocity of 10 rad/s and an angular retardation of 30 rad/s^2, both anticlockwise. Find the angular displacements, velocities and accelerations of the links BC and CD, for an interval of 30° of the crank AB.

Solution.

Given input :

A = 300, B = 360, C = 360, D = 600, VA = 10, ACCA = –30, THETA = 30

OUTPUT :

THET	PHI	BETA	VELC	VELB	ACCC	ACCB
.00	– 114.62	– 65.38	– 10.00	– 10.00	– 61.67	121.67
	114.62	65.38	– 10.00	– 10.00	121.67	– 61.67
30.00	– 144.88	– 82.70	– 8.69	– .84	101.52	181.43
	97.30	35.12	– .84	– 8.69	181.43	101.52
60.00	– 166.19	– 73.81	– 6.02	6.02	38.02	77.45
	106.19	13.81	6.02	– 6.02	77.45	38.02
90.00	174.73	– 47.86	– 8.26	12.26	– 180.18	216.18
	132.14	– 5.27	12.26	– 8.26	216.18	– 180.18
270.00	– 132.14	5.27	12.26	– 8.26	– 289.73	229.73
	– 174.73	47.86	– 8.26	12.26	229.73	– 289.73
300.00	– 106.19	– 13.81	6.02	– 6.02	– 113.57	– 1.90
	166.19	73.81	– 6.02	6.02	– 1.90	– 113.57
330.00	– 97.30	– 35.12	– .84	– 8.69	– 170.39	– 49.36
	144.88	82.70	– 8.69	– .84	– 49.36	– 176.39

25.4. Computer Aided Analysis For Slider Crank Mechanism

A slider crank mechanism is shown in Fig. 25.2 (*a*). The slider is attached to the connecting rod *BC* of length *b*. Let the crank *AB* of radius *a* rotates in anticlockwise direction with uniform

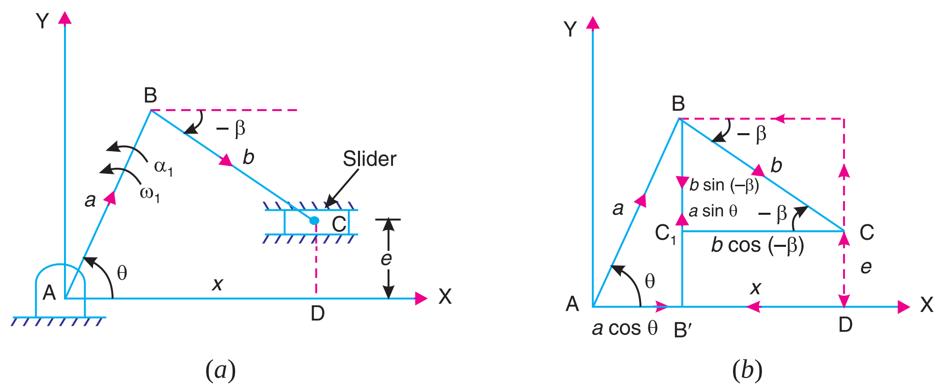


Fig. 25.2 Slider crank mechanism.

angular velocity ω_1 rad/s and an angular acceleration α_1 rad/s^2. Let the crank makes an angle θ with the *X*-axis and the slider reciprocates along a path parallel to the *X*-axis, *i.e.* at an eccentricity *CD* = *e*, as shown in Fig. 25.2 (*a*).

The expressions for displacement, velocity and acceleration analysis are derived as discussed below :

1. *Displacement analysis*

For equilibrium of the mechanism, the sum of the components along *X*-axis and along *Y*-axis must be equal to zero. First of all, taking the sum of the components along *X*-axis, as shown in Fig. 25.2 (*b*), we have

$$a\cos\theta + b\cos(-\beta) - x = 0 \quad \text{... (} \beta \text{ in clockwise direction from } X\text{-axis is taken} - ve)$$

or $$b\cos\beta = x - a\cos\theta \quad \text{... (i)}$$

Squaring both sides,

$$b^2\cos^2\beta = x^2 + a^2\cos^2\theta - 2xa\cos\theta \quad \text{... (ii)}$$

Now taking the sum of components along *Y*-axis, we have

$$b\sin(-\beta) + e + a\sin\theta = 0$$

or $$-b\sin\beta + e = a\sin\theta$$

∴ $$b\sin\beta = e - a\sin\theta \quad \text{... (iii)}$$

Squaring both sides,

$$b^2\sin^2\beta = e^2 + a^2\sin^2\theta - 2ea\sin\theta \quad \text{... (iv)}$$

Adding equations (*ii*) and (*iv*),

$$b^2(\cos^2\beta + \sin^2\beta) = x^2 + e^2 + a^2(\cos^2\theta + \sin^2\theta) - 2xa\cos\theta - 2ea\sin\theta$$

$$b^2 = x^2 + e^2 + a^2 - 2xa\cos\theta - 2ea\sin\theta$$

or $$x^2 + (-2a\cos\theta)x + a^2 - b^2 + e^2 - 2ea\sin\theta = 0$$

or $$x^2 + k_1 x + k_2 = 0 \quad \text{... (v)}$$

where $k_1 = -2a\cos\theta$, and $k_2 = a^2 - b^2 + e^2 - 2ea\sin\theta$ ... (*vi*)

The equation (*v*) is a quadratic equation in *x*. Its two roots are

$$x = \frac{-k_1 \pm \sqrt{k_1^2 - 4k_2}}{2} \quad \text{... (vii)}$$

From this expression, the output displacement *x* may be determined if the values of *a*, *b*, *e* and θ are known. The position of the connecting rod *BC* (*i.e.* angle β) is given by

$$\sin(-\beta) = \frac{a\sin\theta - e}{b}$$

or $$\sin\beta = \frac{e - a\sin\theta}{b}$$

∴ $$\beta = \sin^{-1}\left(\frac{e - a\sin\theta}{b}\right) \quad \text{... (viii)}$$

Note : When the slider lies on the *X*-axis, *i.e.* the line of stroke of the slider passes through the axis of rotation of the crank, then eccentricity, $e = 0$. In such a case, equations **(*vi*)** and **(*viii*)** may be written as

$$k_1 = -2a\cos\theta\,, \quad \text{and} \quad k_2 = a^2 - b^2$$

and
$$\beta = \sin^{-1}\left(\frac{-a\sin\theta}{b}\right)$$

2. *Velocity analysis*

Let ω_1 = Angular velocity of the crank $AB = d\theta/dt$,

ω_2 = Angular velocity of the connecting rod $BC = d\beta/dt$, and

v_S = Linear velocity of the slider = dx/dt.

Differentiating equation **(*i*)** with respect to time,

$$b\times -\sin\beta\times\frac{d\beta}{dt} = \frac{dx}{dt} - a\times -\sin\theta\times\frac{d\theta}{dt}$$

or
$$-a\,\omega_1\sin\theta - b\,\omega_2\sin\beta - \frac{dx}{dt} = 0 \qquad \text{... (ix)}$$

Again, differentiating equation **(*iii*)** with respect to time,

$$b\cos\beta\times\frac{d\beta}{dt} = -a\cos\theta\times\frac{d\theta}{dt}$$

or
$$a\,\omega_1\cos\theta + b\,\omega_2\cos\beta = 0 \qquad \text{... (x)}$$

Multiplying equation **(*ix*)** by $\cos\beta$ and equation **(*x*)** by $\sin\beta$,

$$-a\,\omega_1\sin\theta\cos\beta - b\,\omega_2\sin\beta\cos\beta - \frac{dx}{dt}\times\cos\beta = 0 \qquad \text{... (xi)}$$

and
$$a\,\omega_1\cos\theta\sin\beta + b\,\omega_2\cos\beta\sin\beta = 0 \qquad \text{... (xii)}$$

Adding equations **(*xi*)** and **(*xii*)**,

$$a\,\omega_1(\sin\beta\cos\theta - \cos\beta\sin\theta) - \frac{dx}{dt}\times\cos\beta = 0$$

$$a\,\omega_1\sin(\beta-\theta) = \frac{dx}{dt}\times\cos\beta$$

$$\therefore \quad \frac{dx}{dt} = \frac{a\,\omega_1\sin(\beta-\theta)}{\cos\beta} \qquad \text{... (xiii)}$$

From this equation, the linear velocity of the slider (v_S) may be determined.

The angular velocity of the connecting rod *BC* (*i.e.* ω_2) may be determined from equation **(*x*)** and it is given by

$$\omega_2 = \frac{-a\,\omega_1\cos\theta}{b\cos\beta} \qquad \text{... (xiv)}$$

3. *Acceleration analysis*

Let α_1 = Angular acceleration of the crank $AB = d\omega_1 / dt$,

α_2 = Angular acceleration of the connecting rod = $d\omega_2 / dt$, and

a_S = Linear acceleration of the slider = d^2x / dt^2

Differentiating equation **(*ix*)** with respect to time,

$$-a\left[\omega_1 \cos\theta \times \frac{d\theta}{dt} + \sin\theta \times \frac{d\omega_1}{dt}\right] - b\left[\omega_2 \cos\beta \times \frac{d\beta}{dt} + \sin\beta \times \frac{d\omega_2}{dt}\right] - \frac{d^2x}{dt^2} = 0$$

$$-a\left[\alpha_1 \sin\theta + \omega_1^2 \cos\theta\right] - b\left[\alpha_2 \sin\beta + \omega_2^2 \cos\beta\right] - \frac{d^2x}{dt^2} = 0 \quad \text{... (xv)}$$

The chain-belt at the bottom of a bulldozer provides powerful grip, spreads weight and force on the ground, and allows to exert high force on the objects to be moved.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Differentiating equation **(*x*)** with respect to time,

$$a\left[\omega_1 \times -\sin\theta \times \frac{d\theta}{dt} + \cos\theta \times \frac{d\omega_1}{dt}\right] + b\left[\omega_2 \times -\sin\beta \times \frac{d\beta}{dt} + \cos\beta \times \frac{d\omega_2}{dt}\right] = 0$$

$$a\left[\alpha_1 \cos\theta - \omega_1^2 \sin\theta\right] + b\left[\alpha_2 \cos\beta - \omega_2^2 \sin\beta\right] = 0 \quad \text{... (xvi)}$$

Multiplying equation **(*xv*)** by $\cos\beta$ and equation **(*xvi*)** by $\sin\beta$,

$$-a\left[\alpha_1 \sin\theta\cos\beta + \omega_1^2 \cos\theta\cos\beta\right] - b\left[\alpha_2 \sin\beta\cos\beta + \omega_2^2 \cos^2\beta\right]$$
$$- \frac{d^2x}{dt^2} \times \cos\beta = 0 \quad \text{... (xvii)}$$

and $$a\left[\alpha_1 \cos\theta\sin\beta - \omega_1^2 \sin\theta\sin\beta\right] + b\left[\alpha_2 \cos\beta\sin\beta - \omega_2^2 \sin^2\beta\right] = 0 \quad \text{... (xviii)}$$

Adding equations ***(xvii)*** and ***(xviii)***,

$$a\left[\alpha_1(\sin\beta\cos\theta-\cos\beta\sin\theta)-\omega_1^2(\cos\beta\cos\theta+\sin\beta\sin\theta)\right]$$

$$-b\,\omega_2^2(\cos^2\beta+\sin^2\beta)-\frac{d^2x}{dt^2}\times\cos\beta=0$$

$$a\,\alpha_1\sin(\beta-\theta)-a\,\omega_1^2\cos(\beta-\theta)-b\,\omega_2^2-\frac{d^2x}{dt^2}\times\cos\beta=0$$

$$\therefore\qquad \frac{d^2x}{dt^2}=\frac{a\,\alpha_1\sin(\beta-\theta)-a\,\omega_1^2\cos(\beta-\theta)-b\,\omega_2^2}{\cos\beta}\qquad \text{... }(xix)$$

From this equation, the linear acceleration of the slider (a_S) may be determined.

The angular acceleration of the connecting rod *BC* (*i.e.* α_2) may be determined from equation ***(xvi)*** and it is given by,

$$\alpha_2=\frac{a(\alpha_1\cos\theta-\omega_1^2\sin\theta)-b\,\omega_2^2\sin\beta}{b\cos\beta}\qquad \text{... }(xx)$$

25.5. Programme for a Slider Crank Mechanism

The following is a programme in Fortran to find the velocity and acceleration in a slider crank mechanism.

```
c      PROGRAM TO FIND THE VELOCITY AND ACCELERATION IN A SLIDER
c      CRANK MECHANISM
       READ (*, *) A, B, E, VA, ACC, THA
       PI = 4 * ATAN (1.)
       TH = 0
       IH = 180/THA
       DTH = PI / IH
       DO 10 I = 1, 2 * I H
       TH = (I – 1) * DTH
       BET = ASIN (E – A * SIN (TH) ) / B)
       VS = – A * VA * SIN (TH – BET) / (COS (BET) * 1000)
       VB = – A * VA * COS (TH) / B * COS (BET)
       AC1 = A * ACC * SIN (BET – TH) – B * VB * * 2
       AC2 = A * VA * * 2 * COS (BET – TH)
       ACS = (AC1 – AC2) / (COS (BET) * 1000)
       AC3 = A * ACC * COS (TH) – A * VA * * 2 * SIN (TH)
       AC4 = B * VB * * 2 * SIN (BET)
       ACB = – (AC3 – AC4) / (B * COS (BET) )
       I F (i . EQ . 1) WRITE (*, 9)
9      FORMAT (3X,' TH', 5X,' BET', 4X,' VS,' 4X,' VB,' 4X,' ACS', 4X,' ACB')
10     WRITE (*, 8) TH * 180 / P I , BET * 180 / P I, VS, VB, ACS, ACB
8      FORMAT (6 F 8 . 2)
       STOP
       END
```

The input variables are :

A, *B*, *E* = Length of crank *AB* (*a*), connecting rod *BC* (*b*) and offset (*e*) in mm,

VA = Angular velocity of crank *AB* (input link) in rad/s,

ACC = Angular acceleration of the crank *AB* (input link) in rad/s^2, and

THA = Interval of the input angle in degrees.

The output variables are :

THA = Angular displacement of the crank or input link *AB* in degrees,

BET = Angular displacement of the connecting rod *BC* in degrees,

VS = Linear velocity of the slider in m/s,

VB = Angular velocity of the crank or input link *AB* in rad/s,

ACS = Linear acceleration of the slider in m/s^2, and

ACB = Angular acceleration of the crank or input link *AB* in rad/s^2.

Example 25.2. *In a slider crank mechanism, the crank AB = 200 mm and the connecting rod BC = 750 mm. The line of stroke of the slider is offset by a perpendicular distance of 50 mm. If the crank rotates at an angular speed of 20 rad/s and angular acceleration of 10 rad/s², find at an interval of 30° of the crank,* ***1.*** *the linear velocity and acceleration of the slider, and* ***2.*** *the angular velocity and acceleration of the connecting rod.*

Solution.

Given input :

A = 200, B = 750, E = 50, VA = 20, ACC = 10, THA = 30

OUTPUT :

T H	B E T	V S	V B	ACS	AC B
.00	3.82	.27	– 5.32	– 101.15	– .78
30.00	– 3.82	– 2.23	– 4.61	– 83.69	49.72
60.00	– 9.46	– 3.80	– 2.63	– 35.62	91.14
90.00	– 11.54	– 4.00	.00	14.33	108.87
120.00	– 9.46	– 3.13	2.63	44.71	93.85
150.00	– 3.82	– 1.77	4.61	55.11	54.35
180.00	3.82	– .27	5.32	58.58	4.56
210.00	11.54	1.29	4.53	62.42	– 47.90
240.00	17.31	2.84	2.55	57.93	– 93.34
270.00	19.47	4.00	.00	30.28	– 113.14
300.00	17.31	4.09	– 2.55	– 21.45	– 96.14
330.00	11.54	2.71	– 4.53	– 75.44	– 52.61

25.6. Coupler Curves

It is often desired to have a mechanism to guide a point along a specified path. The path generated by a point on the coupler link is known as a ***coupler curve*** and the generating point is called a ***coupler point*** (also known as ***tracer point***). The straight line mechanisms as discussed in chapter 9 (Art. 9.3) are the examples of the use of coupler curves. In this article, we shall discuss the method of determining the co-ordinates of the coupler point in case of a four bar mechanism and a slider crank mechanism.

1. *Four bar mechanism*

Consider a four bar mechanism *ABCD* with an offset coupler point *E* on the coupler link *BC*, as shown in Fig. 25.3. Let the point *E* makes an angle α with *BC* in the anticlockwise direction and its co-ordinates are *E* (x_E, y_E).

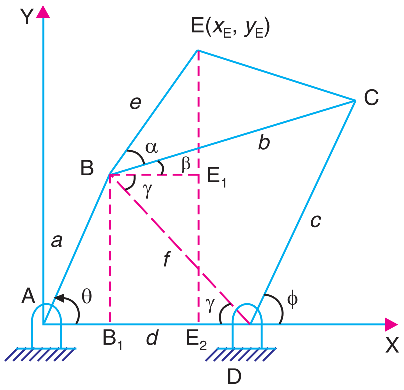


Fig. 25.3. Four bar mechainsm with a coupler point.

First of all, let us find the value of *BD*, γ and β. From right angled triangle BB_1D,

$$\tan\gamma = \frac{BB_1}{B_1D} = \frac{BB_1}{AD - AB_1} = \frac{a\sin\theta}{d - a\cos\theta}$$

or $$\gamma = \tan^{-1}\left(\frac{a\sin\theta}{d - a\cos\theta}\right)$$

and $$(BD)^2 = (BB_1)^2 + (B_1D)^2 = (BB_1)^2 + (AD - AB_1)^2$$

$$= (a\sin\theta)^2 + (d - a\cos\theta)^2$$

$$= a^2\sin^2\theta + d^2 + a^2\cos^2\theta - 2\,a\,d\cos\theta$$

$$= a^2(\sin^2\theta + \cos^2\theta) + d^2 - 2\,a\,d\cos\theta$$

$$= a^2 + d^2 - 2\,a\,d\cos\theta$$

Now in triangle *DBC*,

$$\cos(\gamma+\beta) = \frac{(BD)^2 + (BC)^2 - (CD)^2}{2\,BC \times BD} \quad \text{... (cosine law of triangle)}$$

$$= \frac{f^2 + b^2 - c^2}{2\,b\,f}$$

or $$\gamma + \beta = \cos^{-1}\left(\frac{f^2 + b^2 - c^2}{2\,b\,f}\right)$$

$$\therefore \quad \beta = \cos^{-1}\left(\frac{f^2 + b^2 - c^2}{2\,b\,f}\right) - \gamma \quad \text{... }(i)$$

Let us now find the co-ordinates x_E and y_E. From Fig. 25.3, we find that

$$x_E = AE_2 = AB_1 + B_1E_2 = AB_1 + BE_1 \quad \text{... } (\because B_1E_2 = BE_1)$$

$$= a\cos\theta + e\cos(\alpha+\beta) \quad \text{... }(ii)$$

and $$y_E = E_2E = E_2E_1 + E_1E = B_1B + E_1E \quad \text{... } (\because E_2E_1 = B_1B)$$

$$= a\sin\theta + e\sin(\alpha+\beta) \quad \text{... }(iii)$$

From the above equations, the co-ordinates of the point *E* may be determined if *a*, *e*, θ, α and β are known.

2. *Slider crank mechanism*

Consider a slider crank mechanism with an offset coupler point *E*, as shown in Fig. 25.4. Let the point *E* makes an angle α with *BC* in the anticlockwise direction and its co-ordinates are $E\ (x_E, y_E)$.

First of all, let us find the angle β. From right angled triangle BC_1C,

$$\sin\beta = \frac{BC_1}{BC} = \frac{BB_1 - B_1C_1}{BC} = \frac{a\sin\theta - e_1}{b}$$

$$\therefore \quad \beta = \sin^{-1}\left(\frac{a\sin\theta - e_1}{b}\right) \quad \text{... (iv)}$$

Now $$x_E = AE_1 = AB_1 + B_1E_1 = AB_1 + BB_2$$

$$= a\cos\theta + e\cos(\alpha - \beta) \quad \text{... (v)}$$

and $$y_E = E_1E = E_1B_2 + B_2E = B_1B + B_2E$$

$$= a\sin\theta + e\sin(\alpha - \beta) \quad \text{... (vi)}$$

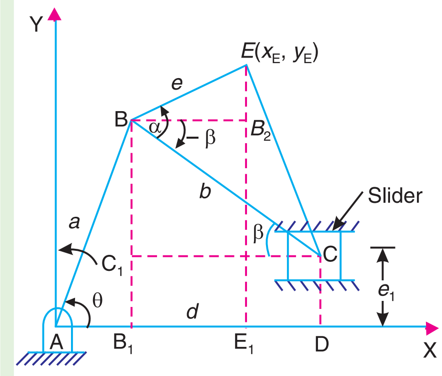


Fig. 25.4 Slider crank mechanism with coupler point.

From the above equations, the co-ordinates of the point *E* may be determined, if *a*, *b*, *e*, e_1, θ, α and β are known.

Note : When the slider lies on the *X*-axis, *i.e.* the line of stroke of the slider passes through the axis of rotation of the crank, then eccentricity $e_1 = 0$. In such a case equation ***(iv)*** may be written as

$$\beta = \sin^{-1}\left(\frac{a\sin\theta}{b}\right)$$

25.7. Synthesis of Mechanisms

In the previous articles, we have discussed the computer-aided analysis of mechanisms, *i.e.* the determination of displacement, velocity and acceleration for the given proportions of the mechanism. The synthesis is the opposite of analysis. The synthesis of mechanism is the design or creation of a mechanism to produce a desired output motion for a given input motion. In other words, the synthesis of mechanism deals with the determination of proportions of a mechanism for the given input and output motion. We have already discussed the application of synthesis in designing a cam (Chapter 20) to give follower a known motion from the displacement diagram and in the determination of number of teeth on the members in a gear train (Chapter 13) to produce a desired velocity ratio.


Roller conveyor.

Note : This picture is given as additional information and is not a direct example of the current chapter.

In the application of synthesis, to the design of a mechanism, the problem divides itself into the following three parts:

1. ***Type synthesis***, *i.e.* the type of mechanism to be used,

2. ***Number synthesis***, *i.e.* the number of links and the number of joints needed to produce the required motion, and

3. ***Dimensional synthesis***, *i.e.* the proportions or lengths of the links necessary to satisfy the required motion characteristics.

In designing a mechanism, one factor that must be kept in mind is that of the accuracy required of the mechanism. Sometimes, it is possible to design a mechanism that will theoretically generate a given motion. The difference between the desired motion and the actual motion produced is known as ***structural error***. In addition to this, there are errors due to manufacture. The error resulting from tolerances in the length of links and bearing clearances is known as ***mechanical error.***

25.8. Classifications of Synthesis Problem

The problems in synthesis can be placed in one of the following three categories :

1. Function generation ; **2.** Path generation ; and **3.** Body guidance.

These are discussed as follows :

1. ***Function generation.*** The major classification of the synthesis problems that arises in the design of links in a mechanism is a function generation. In designing a mechanism, the frequent requirement is that the output link should either rotate, oscillate or reciprocate according to a specified function of time or function of the motion of input link. This is known as function generation. A simple example is that of designing a four bar mechanism to generate the function $y = f(x)$. In this case, x represents the motion of the input link and the mechanism is to be designed so that the motion of the output link approximates the function y.

Note : The common mechanism used for function generation is that of a cam and a follower in which the angular displacement of the follower is specified as a function of the angle of rotation of the cam. The synthesis problem is to find the shape of the cam surface for the given follower displacements.

2. ***Path generation.*** In a path generation, the mechanism is required to guide a point (called a tracer point or coupler point) along a path having a prescribed shape. The common requirements are that a portion of the path be a circular arc, elliptical or a straight line.

3. ***Body guidance.*** In body guidance, both the position of a point within a moving body and the angular displacement of the body are specified. The problem may be a simple translation or a combination of translation and rotation.

25.9. Precision Points for Function Generation

In designing a mechanism to generate a particular function, it is usually impossible to accurately produce the function at more than a few points. The points at which the generated and desired functions agree are known as ***precision points*** or ***accuracy points*** and must be located so as to minimise the error generated between these points.

The best spacing of the precision points, for the first trial, is called ***Chebychev spacing***. According to Freudenstein and Sandor, the Chebychev spacing for n points in the range $x_S \le x \le x_F$ (*i.e.* when x varies between x_S and x_F) is given by

$$x_j = \frac{1}{2}(x_S + x_F) - \frac{1}{2}(x_F - x_S)\cos\left[\frac{\pi(2j-1)}{2n}\right] \quad \text{... (i)}$$

$$= \frac{1}{2}(x_S + x_F) - \frac{1}{2} \times \Delta x \times \cos\left[\frac{\pi(2j-1)}{2n}\right]$$

where $\qquad x_j$ = Precision points

Δx = Range in $x = x_F - x_S$, and

$j = 1, 2, \ldots n$

The subscripts $_S$ and $_F$ indicate start and finish positions respectively.

The precision or accuracy points may be easily obtained by using the graphical method as discussed below.

1. Draw a circle of diameter equal to the range $\Delta x = x_F - x_S$.

2. Inscribe a regular polygon having the number of sides equal to twice the number of precision points required, *i.e.* for three precision points, draw a regular hexagon inside the circle, as shown in Fig. 25.5.

3. Draw perpendiculars from each corner which intersect the diagonal of a circle at precision points x_1, x_2, x_3.

Now for the range $1 \le x \le 3,\ x_S = 1;\ x_F = 3$, and

$$\therefore \qquad \Delta x = x_F - x_S = 3 - 1 = 2$$

or radius of circle,
$$r = \Delta x / 2 = 2/2 = 1$$

$$\therefore \qquad x_2 = x_S + r = x_S + \frac{\Delta x}{2} = 1 + \frac{2}{2} = 2$$

$$x_1 = x_2 - r\cos 30^\circ = x_2 - \frac{\Delta x}{2}\cos 30^\circ$$

$$= 2 - \frac{2}{2}\cos 30^\circ = 1.134$$

and
$$x_3 = x_2 + r\cos 30^\circ = x_2 + \frac{\Delta x}{2}\cos 30^\circ$$

$$= 2 + \frac{2}{2}\cos 30^\circ = 2.866$$

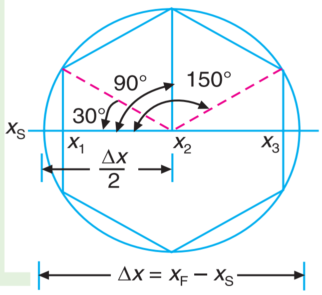
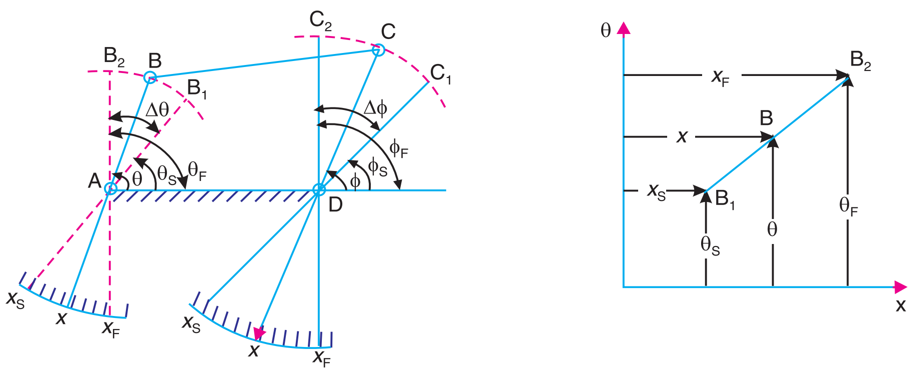


Fig. 25.5. Graphical method for determining three precision points.

25.10. Angle Relationships for Function Generation



(*a*) Four bar mechanism. (*b*) Linear relationship between *x* and θ.

Fig. 25.6

Consider a four bar mechanism, as shown in Fig. 25.6 (*a*) arranged to generate a function $y = f(x)$ over a limited range. Let the range in x is $(x_F - x_S)$ and the corresponding range in θ is $(\theta_F - \theta_S)$. Similarly, let the range in y is $((y_F - y_S)$ and the corresponding range in ϕ is $(\phi_F - \phi_S)$.

The linear relationship between x and θ is shown in Fig. 25.6 (*b*). From the figure, we find that

$$\theta = \theta_S + \frac{\theta_F - \theta_S}{x_F - x_S}(x - x_S) \quad \text{... (i)}$$

Similarly, the linear relationship between y and ϕ may be written as

$$\phi = \phi_S + \frac{\phi_F - \phi_S}{y_F - y_S}(y - y_S) \quad \text{... (ii)}$$

An automatic filling and sealing machine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

For n points in the range, the equation ***(i)*** and ***(ii)*** may be written as

$$\theta_j = \theta_S + \frac{\theta_F - \theta_S}{x_F - x_S}(x_j - x_S) = \theta_S + \frac{\Delta\theta}{\Delta x}(x_j - x_S)$$

and

$$\phi_j = \phi_S + \frac{\phi_F - \phi_S}{y_F - y_S}(y_j - y_S) = \phi_S + \frac{\Delta\phi}{\Delta y}(y_j - y_S)$$

where $j = 1, 2, \ldots n,$

$\Delta x = x_F - x_S$; $\Delta\theta = \theta_F - \theta_S$,

$\Delta y = y_F - y_S$; and $\Delta\phi = \phi_F - \phi_S$

Example 25.3. *A four bar mechanism is to be designed, by using three precision points, to generate the function*

$$y = x^{1.5}, \text{ for the range } 1 \le x \le 4.$$

Assuming 30° starting position and 120° finishing position for the input link and 90° starting position and 180° finishing position for the output link, find the values of x, y, θ and ϕ corresponding to the three precision points.

Solution : Given : $x_S = 1$; $x_F = 4$; $\theta_S = 30°$; $\theta_F = 120°$; $\phi_S = 90°$; $\phi_F = 180°$

Values of x

The three values of x corresponding to three precision points (*i.e.* for $n = 3$) according to Chebychev's spacing are given by

$$x_j = \frac{1}{2}(x_S + x_F) - \frac{1}{2}(x_F - x_S)\cos\left[\frac{\pi(2j-1)}{2n}\right], \quad \text{where } j = 1, 2 \text{ and } 3$$

$$\therefore \quad x_1 = \frac{1}{2}(1+4) - \frac{1}{2}(4-1)\cos\left[\frac{\pi(2\times1-1)}{2\times3}\right] = 1.2 \textbf{ Ans.} \quad ...(\because j = 1)$$

$$x_2 = \frac{1}{2}(1+4) - \frac{1}{2}(4-1)\cos\left[\frac{\pi(2\times2-1)}{2\times3}\right] = 2.5 \textbf{ Ans.} \quad ...(\because j = 2)$$

and

$$x_3 = \frac{1}{2}(1+4) - \frac{1}{2}(4-1)\cos\left[\frac{\pi(2\times3-1)}{2\times3}\right] = 3.8 \textbf{ Ans.} \quad ...(\because j = 3)$$

Note : The three precision points x_1, x_2 and x_3 may be determined graphically as discussed in Art. 25.9.

Values of y

Since $y = x^{1.5}$, therefore the corresponding values of y are

$$y_1 = (x_1)^{1.5} = (1.2)^{1.5} = 1.316 \textbf{ Ans.}$$

$$y_2 = (x_2)^{1.5} = (2.5)^{1.5} = 3.952 \textbf{ Ans.}$$

$$y_3 = (x_3)^{1.5} = (3.8)^{1.5} = 7.41 \textbf{ Ans.}$$

Note : $y_S = (x_S)^{1.5} = (1)^{1.5} = 1$ and $y_F = (x_F)^{1.5} = (4)^{1.5} = 8$

Values of θ

The three values of θ corresponding to three precision points are given by

$$\theta_j = \theta_S + \frac{\theta_F - \theta_S}{x_F - x_S}(x_j - x_S), \text{ where } j = 1, 2 \text{ and } 3$$

$$\therefore \quad \theta_1 = 30 + \frac{120-30}{4-1}(1.2-1) = 36° \textbf{ Ans.}$$

$$\theta_2 = 30 + \frac{120-30}{4-1}(2.5-1) = 75° \textbf{ Ans.}$$

and

$$\theta_3 = 30 + \frac{120-30}{4-1}(3.8-1) = 114° \textbf{ Ans.}$$

Values of ϕ

The three values of ϕ corresponding to three precision points are given by

$$\phi_j = \phi_S + \frac{\phi_F - \phi_S}{y_F - y_S}(y_j - y_S)$$

$\therefore$
$$\phi_1 = 90 + \frac{180-90}{8-1}(1.316-1) = 94.06° \textbf{ Ans.}$$

$$\phi_2 = 90 + \frac{180-90}{8-1}(3.952-1) = 127.95° \textbf{ Ans.}$$

and
$$\phi_3 = 90 + \frac{180-90}{8-1}(7.41-1) = 172.41° \textbf{ Ans.}$$

25.11. Graphical Synthesis of Four Bar Mechanism

The synthesis of four bar mechanism consists of determining the dimensions of the links in which the output link is to occupy three specified positions corresponding to the three given positions of the input link. Fig. 25.7 shows the layout of a four bar mechanism in which the starting angle of the input link AB_1 (link 2) of known length is θ. Let θ_{12}, θ_{23} and θ_{13} be the angles between the positions B_1B_2, B_2B_3 and B_1B_3 measured anticlockwise. Let the output link DC_1 (link 4) passes through the desired positions C_1, C_2 and C_3 and ϕ_{12}, ϕ_{23} and ϕ_{13} are the corresponding angles between the positions C_1C_2, C_2C_3 and C_1C_3. The length of the fixed link (link 1) is also known. Now we are required to determine the lengths of links B_1C_1 and DC_1 (*i.e.* links 3 and 4) and the starting position of link 4 (ϕ).

The easiest way to solve the problem is based on inverting the mechanism on link 4. The procedure is discused as follows :

1. Draw AD equal to the known length of fixed link, as shown in Fig. 25.8.
2. At A, draw the input link 1 in its three specified angular positions AB_1, AB_2 and AB_3.
3. Since we have to invert the mechanism on link 4, therefore draw a line B_2D and and rotate it clockwise (in a direction opposite to the direction in which link 1 rotates) through an angle ϕ_{12} (*i.e.* the angle of the output link 4 between the first and second position) in order to locate the point B_2'.

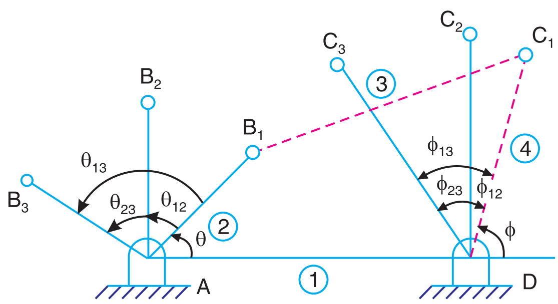


Fig. 25.7. Layout of four bar mechanism.

4. Similarly, draw another line B_3D and rotate it clockwise through an angle ϕ_{13} (*i.e.* angle of the output link between the first and third position) in order to locate point B_3'.

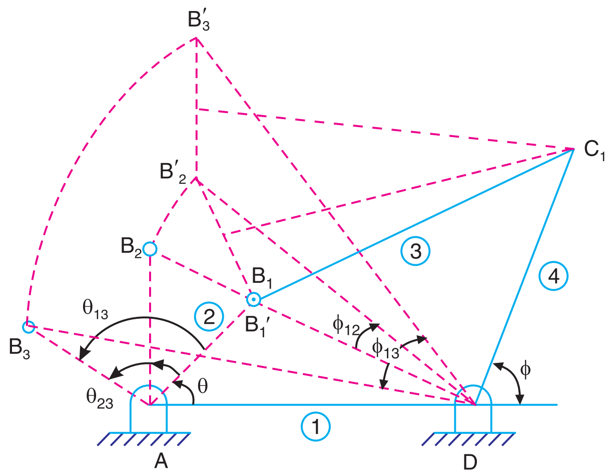


Fig. 25.8. Design of four bar mechanism (Three point synthesis).

5. Since the mechanism is to be inverted on the first design position, therefore B_1 and B_1' are coincident.

6. Draw the perpendicular bisectors of the lines $B_1'\ B_2'$ and $B_2'\ B_3'$. These bisectors intersect at point C_1.

7. Join $B_1'C_1$ and C_1D. The figure $AB_1'\ C_1D$ is the required four bar mechanism. Now the length of the link 3 and length of the link 4 and its starting position (ϕ) are determined.

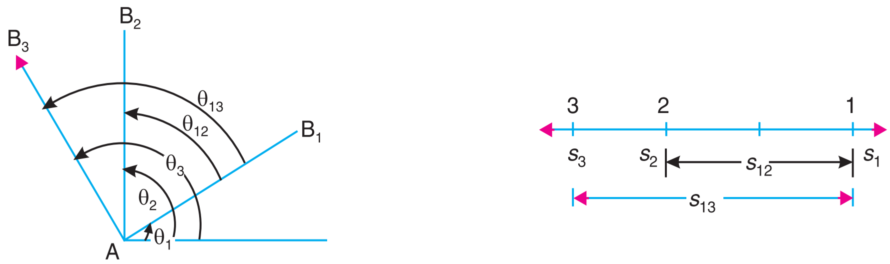
25.12. Graphical Synthesis of Slider Crank Mechanism

Consider a slider crank mechanism for which the three positions of the crank AB (*i.e.* θ_1, θ_2 and θ_3) and corresponding three positions of the slider C (*i.e.* s_1, s_2 and s_3) are known, as shown in Fig. 25.9.

In order to synthesis such a mechanism, the following procedure is adopted.

1. First of all, draw the crank AB_1 in its initial position. If the length of crank is not specified, it may be assumed.
2. Now find the *relative poles P_{12} and P_{13} as shown in Fig. 25.10. The relative poles are obtained by fixing the link A and observing the motion of the crank AB_1 in the reverse direction. Thus, to find P_{12}, draw angle YAP_{12} equal to half of the angle between the first and second position (θ_{12}) in the reverse direction and from AY draw IP_{12} equal to half of the slider displacement between the first and second position (*i.e.* s_{12}). Similarly P_{13} may be obtained.
3. From P_{12} and P_{13}, draw two lines $P_{12}\ Q_{12}$ and $P_{13}\ Q_{13}$ such that $\angle AP_{12}\ I = \angle B_1P_{12}\ Q_{12}$ and $\angle AP_{13}\ I = \angle B_1P_{13}\ Q_{13}$. The lines $P_{12}\ Q_{12}$ and $P_{13}\ Q_{13}$ intersect at C_1, which is the location of the slider at its first position. Now the length of the connecting rod B_1C_1 and the offset (e) may be determined.

* The relative pole is the centre of rotation of the connecting rod relative to the crank rotation and the corresponding slider displacement.



(*a*) Three positions of the crank. (*b*) Three positions of the slider.

Fig. 25.9

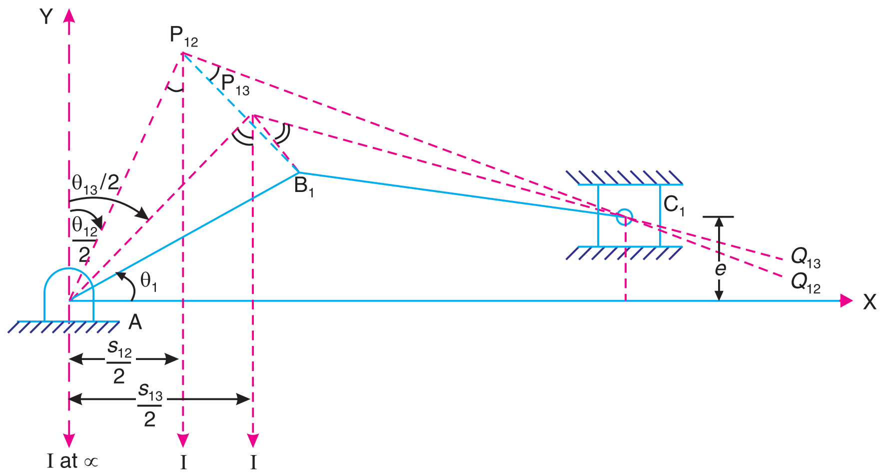
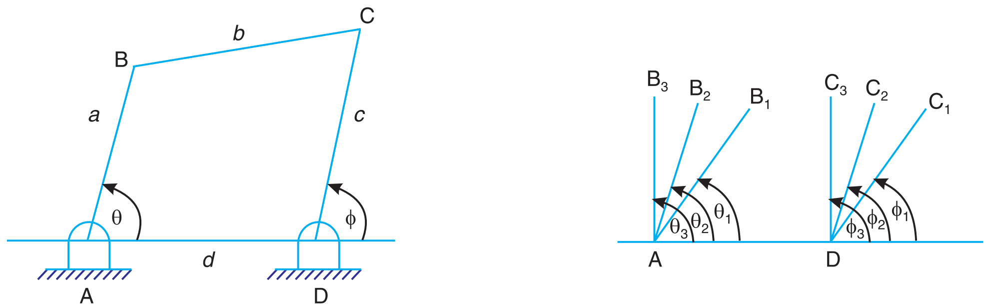


Fig. 25.10

25.13. Computer Aided (Analytical) Synthesis of Four Bar Mechanism



(*a*) Four bar mechanism. (*b*) Three positions of input and output link.

Fig. 25.11

Consider a four bar mechanism as shown in Fig. 25.11.

The synthesis of a four bar mechanism, when input and output angles are specified, is discussed below :

Let the three positions *i.e.* angular displacements (θ_1, θ_2 and θ_3) of the input link *AB* and the three positions (ϕ_1, ϕ_2 and ϕ_3) of the output link, as shown in Fig. 25.11 (*b*), are known and we have to determine the dimensions *a*, *b*, *c* and *d* of the four bar mechanism.

We have discussed in Art. 25.2 that the Freudenstein's equation is

$$k_1 \cos\phi - k_2 \cos\theta + k_3 = \cos(\theta - \phi) \quad \text{... (i)}$$

where $$k_1 = \frac{d}{a}\ ;\ k_2 = \frac{d}{c}\ ;\ \text{and}\ k_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac} \quad \text{... (ii)}$$

For the three different positions of the mechanism, the equation (*i*) may be written as

$$k_1 \cos\phi_1 - k_2 \cos\theta_1 + k_3 = \cos(\theta_1 - \phi_1) \quad \text{... (iii)}$$

$$k_1 \cos\phi_2 - k_2 \cos\theta_2 + k_3 = \cos(\theta_2 - \phi_2) \quad \text{... (iv)}$$

and $$k_1 \cos\phi_3 - k_2 \cos\theta_3 + k_3 = \cos(\theta_3 - \phi_3) \quad \text{... (v)}$$

An off-shore oil well.

Note : This picture is given as additional information and is not a direct example of the current chapter.

The equations (*iii*), (*iv*) and (*v*) are three simultaneous equations and may be solved for k_1, k_2 and k_3 either by elimination method (See Examples 25.4 and 25.5) or by using Cramer's rule of determinants as discussed below :

$$\Delta = \begin{vmatrix} \cos\phi_1 & \cos\theta_1 & 1 \\ \cos\phi_2 & \cos\theta_2 & 1 \\ \cos\phi_3 & \cos\theta_3 & 1 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} \cos(\theta_1 - \phi_1) & \cos\theta_1 & 1 \\ \cos(\theta_2 - \phi_2) & \cos\theta_2 & 1 \\ \cos(\theta_3 - \phi_3) & \cos\theta_3 & 1 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} \cos\phi_1 & \cos(\theta_1 - \phi_1) & 1 \\ \cos\phi_2 & \cos(\theta_2 - \phi_2) & 1 \\ \cos\phi_3 & \cos(\theta_3 - \phi_3) & 1 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} \cos\phi_1 & \cos\theta_1 & \cos(\theta_1 - \phi_1) \\ \cos\phi_2 & \cos\theta_2 & \cos(\theta_2 - \phi_2) \\ \cos\phi_3 & \cos\theta_3 & \cos(\theta_3 - \phi_3) \end{vmatrix}$$

Now the values of k_1, k_2 and k_3 are given by

$$k_1 = \frac{\Delta_1}{\Delta},\; k_2 = \frac{\Delta_2}{\Delta} \text{ and } k_3 = \frac{\Delta_3}{\Delta}$$

Once the values of k_1, k_2 and k_3 are known, then the link lengths *a*, *b*, *c* and *d* are determined by using equation ***(ii)***. In actual practice, either the value of *a* or *d* is assumed to be unity to get the proportionate values of other links.

Note : The designed mechanism may not satisfy the input and output angle co-ordination at positions other than these three positions. It is observed that a four bar mechanism can be designed precisely for five positions of the input and output links provided θ and ϕ are measured from some arbitrary reference rather than from the reference fixed link *AD*. In such cases, the synthesis equations become non-linear and some other means are required to solve such synthesis equations.

25.14. Programme to Co-ordinate the Angular Displacement of the Input and Output Links

The following is the programme in Fortran to co-ordinate the angular displacements of the input and output links.

```
C       PROGRAM TO COORDINATE ANGULAR DISPLACEMENTS OF
C       THE INPUT AND OUTPUT LINKS IN THREE POSITIONS
        READ (*, *) Q1, Q2, Q3, P1, P2, P3
        RAD = 4 * ATAN (1.0) / 180
        QA = COS (Q1 * RAD)
        QB = COS (Q2 * RAD)
        QC = COS (Q3 * RAD)
        PA = COS (P1 * RAD)
        PB = COS (P2 * RAD)
        PC = COS (P3 * RAD)
        AA = COS ( (Q1 – P1) * RAD )
        BB = COS ( (Q2 – P2) * RAD )
        CC = COS ( (Q3 – P3) * RAD )
        D = PA * (QB – QC) + QA * (PC – PB) + (PB * QC – PC * QB)
        D1 = AA * (QB – QC) + QA * (CC – BB) + (BB * QC – CC * QB)
        D2 = PA * (BB – CC) + AA * (PC – PB) + (PB * CC – PC * BB)
        D3 = PA * (QB * CC – QC * BB) + QA * (BB * PC – CC * PB) + AA * (PB * QC – PC * QB)
        A1 = D/D1
        A2 = SQRT (A1 * A1 + A3 * A3 + 1.0 – 2 * A1 * A3 * D3 / D)
        A3 = – D/D2
        WRITE (*, 1) A1, A2, A3, 1
1       FORMAT (6X, A1', 7X,' A2', 7X,' A3' 7X,' A4,' / 4F8 . 2)
        STOP
        END
```

The input variables are :

Q_1, Q_2, Q_3 = Angular displacement of the input link *AB* in degrees,

P_1, P_2, P_3 = Angular displacement of the output link *DC* in degrees.

The output variables are :

A, B, C, D = Ratio of length of the links *AB*, *BC*, *CD* and *AD* respectively

Grinding machine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Example 25.4. *Design a four bar mechanism to co-ordinate the input and output angles as follows :*

Input angles = 15°, 30° and 45° ; Output angles = 30°, 40° and 55°.

Solution. Given : $\theta_1 = 15°$; $\theta_2 = 30°$; $\theta_3 = 45°$; $\phi_1 = 30°$; $\phi_2 = 40°$; $\phi_3 = 55°$

The Freudenstein's equation for the first position of the input and output link (*i.e.* when $\theta_1 = 15°$ and $\phi_1 = 30°$) may be written as

$$k_1 \cos 30° - k_2 \cos 15° + k_3 = \cos (15° - 30°)$$

or $$0.866\, k_1 - 0.966\, k_2 + k_3 = 0.966 \quad \text{... (i)}$$

Similarly, for the second position (*i.e.* when $\theta_2 = 30°$ and $\phi_2 = 40°$),

$$k_1 \cos 40° - k_2 \cos 30° + k_3 = \cos (30° - 40°)$$

or $$0.766\, k_1 - 0.866\, k_2 + k_3 = 0.985 \quad \text{... (ii)}$$

and for the third position (*i.e.* when $\theta_3 = 45°$ and $\phi_3 = 55°$),

$$k_1 \cos 55° - k_2 \cos 45° + k_3 = \cos (45° - 55°)$$

or $$0.574\, k_1 - 0.707\, k_2 + k_3 = 0.985 \quad \text{... (iii)}$$

Solving the three simultaneous equations (*i*), (*ii*) and (*iii*), we get

$$k_1 = 0.905 \; ; \; k_2 = 1.01 \text{ and } k_3 = 1.158$$

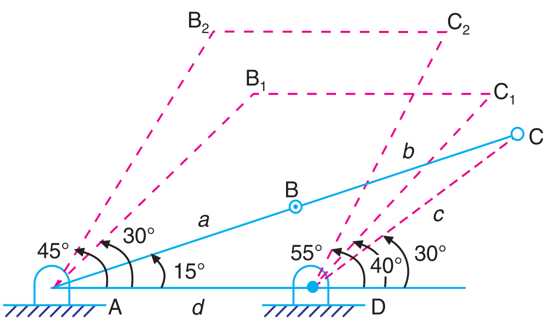


Fig. 25.12

Assuming the length of one of the links, say *a* as one unit, we get the length of the other links as follows :

We know that $k_1 = d / a$ or $d = k_1\, a = 0.905$ units **Ans.**

$k_2 = d / c$ or $c = d / k_2 = 0.905 / 1.01 = 0.896$ units **Ans.**

and $$k_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}$$

or $$-b^2 = k_3 \times 2ac - (a^2 + c^2 + d^2)$$
$$= 1.158 \times 2 \times 1 \times 0.896 - [1^2 + (0.896)^2 + (0.905)^2]$$
$$= 2.075 - 2.622 = -0.547 \quad \text{or} \quad b = 0.74 \text{ units}$$ **Ans.**

The designed mechanism with $AB = a = 1$ unit, $BC = b = 0.74$ units ; $CD = c = 0.896$ units and $AD = d = 0.905$ units, is shown in Fig. 25.12.

Example 25.5. *Determine the proportions of four bar mechanism, by using three precision points, to generate $y = x^{1.5}$, where x varies between 1 and 4. Assume $\theta_S = 30°$; $\Delta\theta = 90°$; $\phi_S = 90°$; and $\Delta\phi = 90°$. Take length of the fixed link AD as 25 mm.*

Solution. Given : $x_S = 1$; $x_F = 4$; $\theta_S = 30°$; $\Delta\theta = \theta_F - \theta_S = 90°$; $\phi_S = 90°$; $\Delta\phi = \phi_F - \phi_S = 90°$; $d = 25$ mm

We have already calculated the three values of x and y for the above given data in Example 25.3. These values are :

$$x_1 = 1.2 \; ; \quad x_2 = 2.5 \; ; \quad \text{and} \quad x_3 = 3.8$$
$$y_1 = 1.316 \; ; \quad y_2 = 3.952 \; ; \quad \text{and} \quad y_3 = 7.41$$

The corresponding values of θ and ϕ are

$$\theta_1 = 36° \; ; \; \theta_2 = 75° \; ; \text{ and } \theta_3 = 114°$$

$$\phi_1 = 94.06° \; ; \; \phi_2 = 127.95° \; ; \text{ and } \phi_3 = 172.41°$$

We know that the Freudenstein's equation is

$$k_1 \cos\phi - k_2 \cos\theta + k_3 = \cos(\theta - \phi) \qquad \text{... }(i)$$

where $$k_1 = \frac{d}{a} \; ; \; k_2 = \frac{d}{c} \; ; \quad \text{and} \quad k_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac} \qquad \text{... }(ii)$$

Now for the three different positions of the mechanism, the equation ***(i)*** may be written three times as follows :

$$k_1 \cos 94.06° - k_2 \cos 36° + k_3 = \cos(36° - 94.06°)$$

or $$-0.0708\, k_1 - 0.809\, k_2 + k_3 = 0.529 \qquad \text{... }(iii)$$

Similarly $$k_1 \cos 127.95° - k_2 \cos 75° + k_3 = \cos(75° - 127.95°)$$

or $$-0.615\, k_1 - 0.259\, k_2 + k_3 = 0.6025 \qquad \text{... }(iv)$$

and $$k_1 \cos 172.41° - k_2 \cos 114° + k_3 = \cos(114° - 172.41°)$$

$$-0.9912\, k_1 + 0.4067\, k_2 + k_3 = 0.5238 \qquad \text{... }(v)$$

Solving three simultaneous equations ***(iii)***, ***(iv)*** and ***(v)***, we get

$$k_1 = 0.6 \; ; \; k_2 = 0.453 \; ; \text{ and } k_3 = 0.12$$

Now from equation ***(ii)***,

$$a = \frac{d}{k_1} = \frac{25}{0.6} = 41.7 \text{ mm}$$ **Ans.**

$$c = \frac{d}{k_2} = \frac{25}{0.453} = 55.2 \text{ mm} \quad \textbf{Ans.}$$

and $$b = (a^2 + c^2 + d^2 - k_3 \times 2ac)^{1/2}$$

$$= \left[(41.7)^2 + (55.2)^2 + (25)^2 - 0.12 \times 2 \times 41.7 \times 55.2\right]^{1/2} = 69.7 \text{ mm} \quad \textbf{Ans.}$$

The designed four bar mechanism $AB_2\, C_2D$ in one position (*i.e.* for θ_2, x_2 and ϕ_2, y_2) is shown by thick lines in Fig. 25.13.

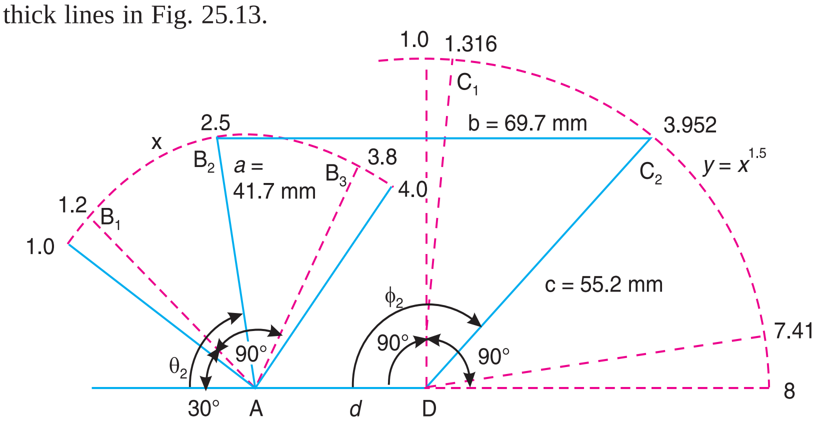


Fig. 25.13

The other two positions of the four bar mechanism may be drawn by joining B_1C_2 (*i.e.* θ_1, x_1 and ϕ_1, y_1) and B_3C_3 (*i.e.* θ_3, x_3 and ϕ_3, y_3).

Note : In the above example, the motion of input link and output link is taken clockwise.

Example 25.6. *Synthesize a four bar linkage, as shown in Fig. 25.14, using Freudenstein's equation to satisfy in one of its positions. The specification of position θ, velocity ω and acceleration α are as follows :*

$$\theta = 60°, \quad \omega_2 = 5 \text{ rad/s}; \; \alpha_2 = 2 \text{ rad/s}^2;$$
$$\phi = 90°; \quad \omega_4 = 2 \text{ rad/s}; \; \alpha_4 = 7 \text{ rad/s}^2.$$

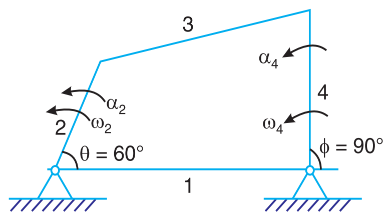


Fig. 25.14

Solution : Given : $\theta = 60°$; $\omega_2 = 5$ rad/s ; $\alpha_2 = 2$ rad/s^2; $\phi = 90°$, $\omega_4 = 2$ rad/s; $\alpha_4 = 7$ rad/s^2

The four bar linkages is shown in Fig. 25.15. Let

AB = Input link = *a*,

BC = Coupler = *b*,

CD = Output link = *c*, and

AD = Fixed link = *d*.

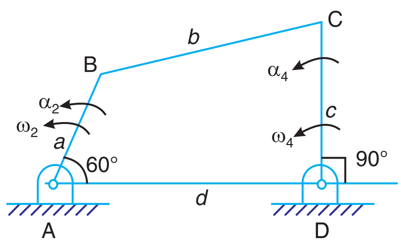


Fig. 25.15

The Freudenstein's equation is given by

$$k_1 \cos\phi - k_2 \cos\theta + k_3 = \cos(\theta - \phi) \quad \dots (i)$$

where $$k_1 = \frac{d}{a}; \quad k_2 = \frac{d}{c}; \quad \text{and} \quad k_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}$$

Substituting the value of θ and ϕ in equation ***(i)***,

$$k_1 \cos 90° - k_2 \cos 60° + k_3 = \cos(60° - 90°)$$

$$k_1 \times 0 - k_2 \times 0.5 + k_3 = 0.866$$

$$-0.5k_2 + k_3 = 0.866 \qquad \text{... (ii)}$$

Differentiating equation ***(i)*** with respect to time,

$$k_1 \times -\sin\phi \times \frac{d\phi}{dt} - k_2 \times -\sin\theta \times \frac{d\theta}{dt} = -\sin(\theta - \phi) \times \frac{d(\theta - \phi)}{dt}$$

$$-k_1 \sin\phi\,\omega_4 + k_2 \sin\theta\,\omega_2 = -\sin(\theta - \phi)(\omega_2 - \omega_4) \qquad \text{... (iii)}$$

$$\ldots\left(\because \frac{d\phi}{dt} = \omega_4;\ \text{and} \frac{d\theta}{dt} = \omega_2\right)$$

$$-k_1 \times \sin 90° \times 2 + k_2 \sin 60° \times 5 = -\sin(60° - 90°)\ (5 - 2)$$

$$-2k_1 + \frac{5\sqrt{3}}{2} k_2 = \frac{3}{2}$$

or $$k_1 = 2.165\, k_2 - 0.75 \qquad \text{.... (iv)}$$

Now differentiating equation ***(iii)*** with respect to time,

$$-k_1\left[\sin\phi \times \frac{d\omega_4}{dt} + \omega_4 \cos\phi \times \frac{d\phi}{dt}\right] + k_2\left[\sin\theta \times \frac{d\omega_2}{dt} + \omega_2 \cos\theta \times \frac{d\theta}{dt}\right]$$

$$= -\sin(\theta - \phi)\, \frac{d(\omega_2 - \omega_4)}{dt} + (\omega_2 - \omega_4)\cos(\theta - \phi) \times \frac{d(\theta - \phi)}{dt}$$

$$-k_1\left[\sin\phi \times \alpha_4 + \omega_4^2 \cos\phi\right] + k_2\left[\sin\theta \times \alpha_2 + \omega_2^2 \cos\theta\right]$$

$$= -\left[\sin(\theta - \phi)\,(\alpha_2 - \alpha_4) + (\omega_2 - \omega_4)^2 \cos(\theta - \phi)\right]$$

$$-k_1\left[\sin 90° \times 7 + 2^2 \cos 90°\right] + k_2\left[\sin 60° \times 2 + 5^2 \cos 60°\right]$$

$$= -\left[\sin(60° - 90°)\,(2 - 7) + (5 - 2)^2 \cos(60° - 90°)\right]$$

$$-k_1(7 + 0) + k_2(1.732 + 12.5) = -\ (2.5 + 7.794)$$

$$-7k_1 + 14.232\, k_2 = -10.294$$

or $$k_1 = 2.033\, k_2 + 1.47 \qquad \text{... (v)}$$

From equations ***(iv)*** and ***(v)***,

$$2.165\, k_2 - 0.75 = 2.033\, k_2 + 1.47 \quad \text{or} \quad k_2 = 16.8$$

From equation **(v)**

$$k_1 = 2.165 k_2 - 0.75 = 2.165 \times 16.8 - 0.75 = 35.6$$

and from equation **(ii),**

$$k_3 = 0.5\ k_2 + 0.866 = 0.5 \times 16.8 + 0.866 = 9.266$$

Assuming the length of one of links say *a* as one unit, we get the length of the links as follows :

We know that $k_1 = d / a$ or $d = k_1 \cdot a = 35.6$ units **Ans.**

$k_2 = d / c$ or $c = d / k_2 = 35.6 / 16.8 = 2.12$ units **Ans.**

and
$$k_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac} = \frac{1^2 - b^2 + (2.12)^2 + (35.6)^2}{2 \times 1 \times 2.12}$$

$$9.266 = \frac{1 - b^2 + 4.494 + 1267.36}{4.24} = \frac{1272.854 - b^2}{4.24}$$

$$b^2 = 1272.854 - 9.266 \times 4.24 = 1233.566$$

$\therefore$ $b = 35.12$ units **Ans.**

Example 25.7. *Synthesize a four-bar mechanism to generate a function y = sin x for $0 \le x \le 90°$. The range of the output crank may be chosen as 60° while that of inut crank be 120°. Assume three precision points which are to be abtained from Chebyshev spacing. Assume fixed link to be 52.5 mm long and $\theta_1 = 105°$ and $\phi_1 = 66°$.*

Solution. Given : $x_S = 0$; $x_F = 90°$; $\Delta\phi = 60°$; $\Delta\theta = 120°$; $d = 52.5$ mm; $\theta_1 = 1.05°$; $\phi_1 = 66°$

The three values of *x* corresponding to three precision points (*i.e.* for *n* = 3), according to Chebyshev spacing are given by

$$x_j = (x_S + x_F) - \frac{1}{2}(x_F - x_S)\cos\left[\frac{\pi(2j-1)}{2n}\right], \text{ where } j = 1, 2, 3$$

$\therefore$
$$x_1 = \frac{1}{2}(0 + 90) - \frac{1}{2}(90 - 0)\cos\left[\frac{\pi(2 \times 1 - 1)}{2 \times 3}\right]$$

$$= 45 - 45 \cos 30° = 6° \qquad \text{... } (\because\ j = 1)$$

$$x_2 = \frac{1}{2}(0 + 90) - \frac{1}{2}(90 - 0)\cos\left[\frac{\pi(2 \times 2 - 1)}{2 \times 3}\right]$$

$$= 45 - 45 \cos 90° = 45° \qquad \text{... } (\because\ j = 2)$$

and
$$x_3 = \frac{1}{2}(0 + 90) - \frac{1}{2}(90 - 0)\cos\left[\frac{\pi(2 \times 3 - 1)}{2 \times 3}\right]$$

$$= 45 - 45 \cos 150° = 84°$$

Since *y* = sin *x*, therefore corresponding values of *y* are

$$y_1 = \sin x_1 = \sin 6° = 0.1045$$

$$y_2 = \sin x_2 = \sin 45° = 0.707$$

and $$y_3 = \sin x_3 = \sin 84° = 0.9945$$

Also $$y_S = \sin x_S = \sin 0° = 0$$

and $$y_F = \sin x_F = \sin 90° = 1$$

The relation between the input angle (θ) and x is given by

$$\theta_j = \theta_S + \frac{\theta_F - \theta_S}{x_F - x_S}(x_j - x_S), \text{ where } j = 1, 2 \text{ and } 3.$$

The above expression may be written as

$$\theta_j = \theta_S + \frac{\Delta\theta}{\Delta x}(x_j - x_S)$$

The three values of θ corresponding to three precision points are given by

$$\theta_1 = \theta_S + \frac{\Delta\theta}{\Delta x} \times x_1 \quad \ldots (\because x_S = 0) \ldots (i)$$

$$\theta_2 = \theta_S + \frac{\Delta\theta}{\Delta x} \times x_2 \quad \ldots (ii)$$

and $$\theta_3 = \theta_S + \frac{\Delta\theta}{\Delta x} \times x_3 \quad \ldots. (iii)$$

From equations ***(i)***, ***(ii)*** and ***(iii)***,

$$\theta_2 - \theta_1 = \frac{\Delta\theta}{\Delta x}(x_2 - x_1) = \frac{120}{90}(45 - 6) = 52° \quad \ldots (iv)$$

$$\ldots (\because \Delta x = x_F - x_S = 90 - 0 = 90)$$

$$\theta_3 - \theta_2 = \frac{\Delta\theta}{\Delta x}(x_3 - x_2) = \frac{120}{90}(84 - 45) = 52° \quad \ldots (v)$$

and $$\theta_3 - \theta_1 = \frac{\Delta\theta}{\Delta x}(x_3 - x_1) = \frac{120}{90}(84 - 6) = 104° \quad \ldots (iv)$$

Since $\theta_1 = 105°$ (Given), therefore

$$\theta_2 = \theta_1 + 52° = 105° + 52° = 157°$$

$$\theta_3 = \theta_2 + 52° = 157° + 52 = 209°$$

The relation between the output angle (ϕ) and y is given by

$$\phi_j = \phi_S + \frac{\phi_F - \phi_S}{y_F - y_S}(y_j - y_S), \text{ when } j = 1, 2 \text{ and } 3$$

This expression may be written as

$$\phi_j = \phi_S + \frac{\Delta\phi}{\Delta y}(y_j - y_S)$$

The three values of ϕ corresponding to three precision points are given by

$$\phi_1 = \phi_S + \frac{\Delta\phi}{\Delta y} \times y_1 \qquad \text{.... } (\because\ y_S = 0) \text{ ... } (vii)$$

$$\phi_2 = \phi_S + \frac{\Delta\phi}{\Delta y} \times y_2 \qquad \text{... } (viii)$$

and

$$\phi_3 = \phi_S + \frac{\Delta\phi}{\Delta y} \times y_3 \qquad \text{... } (ix)$$

From equations **(*vii*)**, **(*viii*)** and **(*ix*),**

$$\phi_2 - \phi_1 = \frac{\Delta\phi}{\Delta y}(y_2 - y_1) = \frac{60}{1}(0.707 - 0.1045) = 36.15° \qquad \text{... } (x)$$

... $(\because\ \Delta y = y_F - y_S = 1 - 0 = 1)$

$$\phi_3 - \phi_2 = \frac{\Delta\phi}{\Delta y}(y_3 - y_2) = \frac{60}{1}(0.9945 - 0.707) = 17.25° \qquad \text{... } (xi)$$

$$\phi_3 - \phi_1 = \frac{\Delta\phi}{\Delta y}(y_3 - y_1) = \frac{60}{1}(0.9945 - 0.1045) = 53.4° \qquad \text{... } (xii)$$

Since $\phi_1 = 66°$ (Given), therefore

$$\phi_2 = \phi_1 + 36.15° = 66° + 36.15° = 102.15°$$

$$\phi_3 = \phi_2 + 17.25° = 102.15° + 17.25° = 119.40°$$

We have calculated above the three positions *i.e.* the angular displacements (θ_1, θ_2 and θ_3) of the input crank and the three positions (ϕ_1, ϕ_2 and ϕ_3) of the output crank. Now let us find the dimensions of the four bar mechanism.

Let a = Length of the input crank,

b = Length of the coupler,

c = Length of the output crank, and

d = Length of the fixed crank = 52.5 mm (Given)

We know that the Freudenstein displacement equation is

$$k_1 \cos\phi - k_2 \cos\theta + k_3 = \cos(\theta - \phi) \qquad \text{... } (xiii)$$

where $k_1 = \dfrac{d}{a}$; $k_2 = \dfrac{d}{c}$ and $k_3 = \dfrac{a^2 - b^2 + c^2 + d^2}{2ac}$

The equation **(*xiii*)** for the first position of input and output crank (*i.e.* when θ_1 = 45° and ϕ = 66°) may be written as

$$k_1 \cos\phi_1 - k_2 \cos\theta_1 + k_3 = \cos(\theta_1 - \phi_1)$$

$$k_1 \cos 66° - k_2 \cos 105° + k_3 = \cos(105° - 66°)$$

$$0.4067 k_1 + 0.2588 k_2 + k_3 = 0.7771 \qquad \text{... } (xiv)$$

Similarly, for the second position (*i.e.* when θ_2 = 157° and ϕ_2 = 102.15°),

$$k_1 \cos\phi_2 - k_2 \cos\theta_2 + k_3 = \cos(\theta_2 - \phi_2)$$

$$k_1 \cos 102.15° - k_2 \cos 157° + k_3 = \cos(157° - 102.15°)$$

$$-0.2105 k_1 + 0.9205 k_2 + k_3 = 0.5757 \quad \text{...}(xv)$$

and for the third position (*i.e.* when θ_3 = 209° and ϕ_3 = 119.4°),

$$k_1 \cos\phi_3 - k_2 \cos\theta_3 + k_3 = \cos(\theta_3 - \phi_3)$$

$$k_1 \cos 119.4° - k_2 \cos 209° + k_3 = \cos(209° - 119.4°)$$

$$-0.4909 k_1 + 0.8746 k_2 + k_3 = 0.007 \quad \text{...}(xvi)$$

Solving the three simultaneous equations **(*xiv*)**, **(*xv*)** and **(*xvi*)**, we get

k_1 = 1.8; k_2 = 1.375 and k_3 = – 0.311

Since the length of the fixed link (*i.e.* d = 52.5 mm) is known, therefore we get the length of other links as follows:

We know that

$k_1 = d / a$ or $a = d / k_1$ = 52.5 / 1.8 = 29.17 mm **Ans.**

$k_2 = d / c$ or $c = d / k_2$ = 52.5 / 1.375 = 38.18 mm **Ans.**

and $$k_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}$$

or $$b^2 = a^2 + c^2 + d^2 - k_3 \times 2ac$$

$$= (29.17)^2 + (38.18)^2 + (52.5)^2 - (-0.311) \times 2 \times 29.17 \times 38.18 = 5758$$

∴ b = 75.88 mm **Ans.**

25.15. Least Square Technique

Most of the mechanisms are not possible to design even for five positions of the input and output links. However, it is possible to design a mechanism to give least deviation from the specified positions. This is done by using least square technique as discussed below :

We have already discussed that the Freudenstein's equation is

$$k_1 \cos\phi - k_2 \cos\theta + k_3 - \cos(\theta - \phi) = 0$$

The angles θ and ϕ are specified for a position. If θ_i and ϕ_i are the angles for *i*th position, then Freudenstein's equation may be written as

$$k_1 \cos\phi_i - k_2 \cos\theta_i + k_3 - \cos(\theta_i - \phi_i) = 0$$

Let e be the error which is defined as

$$e = \sum_{i=1}^{n} [k_1 \cos\phi_i - k_2 \cos\theta_i + k_3 - \cos(\theta_i - \phi_i)]^2$$

For e to be minimum, the partial derivatives of e with respect to k_1, k_2, k_3 separately must be equal to zero, *i.e.*

$$\frac{\partial e}{\partial k_1} = 0 \;;\; \frac{\partial e}{\partial k_2} = 0, \text{ and } \frac{\partial e}{\partial k_3} = 0$$

$$\therefore \qquad \frac{\partial e}{\partial k_1} = 2\sum_{i=1}^{n}[k_1 \cos\phi_i - k_2 \cos\theta_i + k_3 - \cos(\theta_i - \phi_i)]\cos\phi_i = 0$$

or

$$k_1 \sum_{i=1}^{n} \cos^2 \phi_i - k_2 \sum_{i=1}^{n} \cos\theta_i \cos\phi_i + k_3 \sum_{i=1}^{n} \cos\phi_i = \sum_{i=1}^{n} \cos(\theta_i - \phi_i)\cos\phi_i \qquad \ldots (i)$$

Similarly,

$$\frac{\partial e}{\partial k_2} = -2\sum_{i=1}^{n}[k_1 \cos\phi_i - k_2 \cos\theta_i + k_3 - \cos(\theta_i - \phi_i)]\cos\theta_i = 0$$

or

$$k_1 \sum_{i=1}^{n} \cos\phi_i \cos\theta_i + k_2 \sum_{i=1}^{n} \cos^2 \theta_i + k_3 \sum_{i=1}^{n} \cos\theta_i = \sum_{i=1}^{n} \cos(\theta_i - \phi_i)\cos\theta_i \qquad \ldots (ii)$$

Now

$$\frac{\partial e}{\partial k_3} = 2\sum_{i=1}^{n}[k_1 \cos\phi_i - k_2 \cos\theta_i + k_3 - \cos(\theta_i - \phi_i)] = 0$$

or

$$k_1 \sum_{i=1}^{n} \cos\phi_i + k_2 \sum_{i=1}^{n} \cos\theta_i + k_3 \sum_{i=1}^{n} 1 = \sum_{i=1}^{n} \cos(\theta_i - \phi_i) \qquad \ldots (iii)$$

The equations *(i)*, *(ii)* and *(iii)* are simultaneous, linear, non-homogeneous equations in three unknowns k_1, k_2 and k_3. These equations can be solved by using Cramer's rule.

25.16. Programme Using Least Square Technique

The following is a programme in Fortrans to find the ratio of lengths for different links by using the least square technique.

The input variables are :

J = Number of specified positions

TH (*I*) = Angular displacements of the input link *AB* for *I* = 1 to *J* (degrees), and

PH (*I*) = Angular displacements of the output link *DC* for *I* = 1 to *J* (degrees).

The output variables are :

A, *B*, *C*, *D* = Ratio of the lengths of the links *AB*, *BC*, *CD* and *AD* respectively.

```
C    PROGRAM TO COORDINATE ANGULAR DISPLACEMENT OF THE
C    INPUT AND OUTPUT LINKS IN MORE THAN THREE POSITIONS TO
C    FIND RATIO OF DIFFERENT LINKS USING LEAST SQUARE TECHNIQUE

     DIMENSION
     READ (*, *) J
     READ (*, *) (TH (I), I = 1, J), PH (I), I = 1, J)
     RAD = 4 * ATAN (1.0) / 180
     DO 10 K = 1 . J
     A1 = A1 + (COS (PH (K) * RAD ) ) * * 2
     A2 = A2 + (COS (TH (K) RAD ) ) * (COS (PH (K) * RAD ) )
     A3 = A3 + (COS (PH (K) * RAD ) )
     B1 = A2
     B2 = B2 + (COS (TH (K) * RAD ) ) * * 2
```

```
      B3 = B3 + (COS (TH (K) * RAD ) )
      P1 = A3
      P2 = B3
      P3 = J
      TT = COS ( ( TH (K) – PH (K) * RAD )
      Q1 = Q1 + TT * COS (PH (K) * RAD )
      Q2 = Q2 + TT * COS (TH (K) * RAD
10    Q3 = Q3 + TT
      D = A1 * (B2 * P3 – B3 * P2) + B1 * (P2 * A3 – P3 * A2) + P1 * (A2 * B3 – A3 * B2)
      D1 = Q1 * (B2 * P3 – B3 * P2) + B1 * (P2 * Q3 – P3 * Q2) + P1 * (Q2 * B3 – Q3 * B2)
      D2 = A1 * (Q2 * P3 – Q3 * P2) + Q1 *  (P2 * A3 – P3 * A2) + P1 * (A2 * Q3 – A3 * Q2)
      D3 = A1 * (B2 * Q3 – B3 * Q2) + B1 * (Q2 * A3 – Q3 * A2) + Q1 * (A2 * B3 – A3 * B2)
      Q = D / D1
      R = – D / D2
      P = SQRT (Q * Q + r * r + 1. – 2. * r * r * 03 / D)
      WRITE (* , 9) Q, P, r, 1.
9     FORMAT (6X, ' Q', 7X,' P', 7X,' r', 7X,' D' / 4F8 . 2)
      STOP
      END
```

25.17. Computer Aided Synthesis of Four Bar Mechanism with Coupler Point

Consider a four bar mechanism *ABCD* with a couple point *E*, as shown in Fig. 25.16, which is specified by *r* and γ .

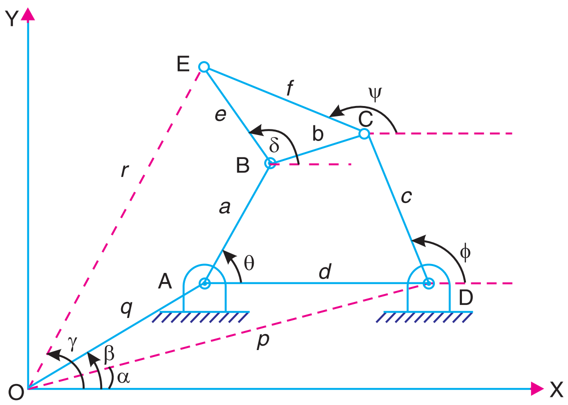


Fig. 25.16. Four bar mechanism with a couple point.

Let θ_1, θ_2 and θ_3 = Three positions of the input link *AB*,

r_1, r_2 and r_3 = Three positions of the coupler point *E* from point *O*, and

γ_1, γ_2 and γ_3 = Three angular positions of the coupler point *E* from *OX*.

The dimensions *a*, *c*, *e*, *f* and the location of points *A* and *D* specified by (q, β) and (p, α) respectively, may be determined as discussed below :

Considering the loop *OABE*, the horizontal and vertical components of vectors *q*, *a*, *e* and *r* are

$$q\cos\beta + a\cos\theta + e\cos\delta = r\cos\gamma \quad \text{... (i)}$$

and $$q\sin\beta + a\sin\theta + e\sin\delta = r\sin\gamma \quad \text{... (ii)}$$

Squaring equations ***(i)*** and ***(ii)*** and adding in order to eliminate angle δ, we have

$$q[2r\cos(\gamma-\beta)] + a[2r\cos(\theta-\gamma)] + e^2 - q^2 - a^2 = r^2 + q\,a[2\cos(\theta-\beta)] \quad \text{... (iii)}$$

Let $q = k_1$; $a = k_2$; $e^2 - q^2 - a^2 = k_3$; and $q\ a = k_4 = k_1\ k_2$ ... ***(iv)***

Now the equation ***(iii)*** may be written as

$$k_1[2r\cos(\gamma-\beta)] + k_2[2r\cos(\theta-\gamma)] + k_3 = r^2 + k_4[2\cos(\theta-\beta)] \quad \text{... (v)}$$

Since $k_4 = k_1 k_2$, therefore the equation ***(v)*** is difficult to solve for k_1, k_2, k_3 and k_4. Such type of non-linear equations can be solved easily by making them linear by some substitutions as given below :

Let $k_1 = l_1 + \lambda m_1$; $k_2 = l_2 + \lambda m_2$; and $k_3 = l_3 + \lambda m_3$ ... ***(vi)***

where $$\lambda = k_4 = k_1 k_2 = (l_1 + \lambda m_1)(l_2 + \lambda m_2)$$

$$= l_1 l_2 + l_1 \lambda m_2 + \lambda m_1 l_2 + \lambda^2 m_1 m_2$$

or $$m_1 m_2 \lambda^2 + (l_1 m_2 + l_2 m_1 - 1)\lambda + l_1 l_2 = 0$$

or $A\lambda^2 + B\lambda + C = 0$

$$\therefore \quad \lambda = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad \text{... (vii)}$$

where $A = m_1 m_2$; $B = (l_1 m_2 + l_2 m_1 - 1)$; and $C = l_1\ l_2$ ... ***(viii)***

Substituting the values of k_1, k_2, k_3 and k_4 in equation ***(v)***,

$$(l_1 + \lambda m_1)[2r\cos(\gamma-\beta)] + (l_2 + \lambda m_2)[2r\cos(\theta-\gamma)] + (l_3 + \lambda m_3)$$

$$= r^2 + \lambda([2\cos(\theta-\beta)]$$

Equating the terms with λ and without λ separately equal to zero, we get the components into two groups, one with λ and the other without λ. These components are

$$l_1[2r\cos(\gamma-\beta)] + l_2[2r\cos(\theta-\gamma)] + l_3 = r^2 \quad \text{... (ix)}$$

and $$m_1[2r\cos(\gamma-\beta)] + m_2[2r\cos(\theta-\gamma)] + m_3 = 2\cos(\theta-\beta) \quad \text{... (x)}$$

The equation ***(ix)*** for the three positions of θ, *r* and γ may be written three times as follows :

$$l_1[2r_1\cos(\gamma_1-\beta)] + l_2[2r_1\cos(\theta_1-\gamma_1)] + l_3 = r_1^2 \quad \text{... (xi)}$$

$$l_1[2r_2\cos(\gamma_2-\beta)] + l_2[2r_2\cos(\theta_2-\gamma_2)] + l_3 = r_2^2 \quad \text{... (xii)}$$

$$l_1[2r_3\cos(\gamma_3-\beta)] + l_2[2r_3\cos(\theta_3-\gamma_3)] + l_3 = r_3^2 \quad \text{... (xiii)}$$

Similarly, equation ***(x)*** for the three positions of θ, *r* and γ may be written three times as follows :

$$m_1[2r\cos(\gamma_1-\beta)]+m_2[2r\cos(\theta_1-\gamma_1)]+m_3=2\cos(\theta_1-\beta) \quad \text{... (xiv)}$$

$$m_1[2r\cos(\gamma_2-\beta)]+m_2[2r\cos(\theta_2-\gamma_2)]+m_3=2\cos(\theta_2-\beta) \quad \text{... (xv)}$$

$$m_1[2r\cos(\gamma_3-\beta)]+m_2[2r\cos(\theta_3-\gamma_3)]+m_3=2\cos(\theta_3-\beta) \quad \text{... (xvi)}$$

The equations ***(xi)***, ***(xii)*** and ***(xiii)*** are three linear equations in l_1, l_2, l_3. Similarly, equations ***(xiv)***, ***(xv)*** and ***(xvi)*** are three linear equations in m_1, m_2 and m_3. Assuming a suitable value of β, the values of l_1, l_2, l_3 and m_1, m_2, m_3 may be determined by using elimination method or Cramer's rule.

Knowing the values of l_1, l_2, l_3 and m_1, m_2, m_3, we can find the value of λ from equation ***(vii)***. Now the values of k_1, k_2 and k_3 are determined from equation ***(vi)*** and hence *q*, *a* and *e* are known from equation ***(iv)***. Using equation ***(i)*** or ***(ii)***, we can find the three valves of δ *i.e.* δ_1, δ_2 and δ_3. From equation ***(i)***, we have

$$e\cos\delta = r\cos\gamma - q\cos\beta - a\cos\theta$$

$$\therefore \quad \delta_1 = \cos^{-1}\left[\frac{r_1\cos\gamma_1 - q\cos\beta - a\cos\theta_1}{e}\right] \quad \text{... (xvii)}$$

Similarly,
$$\delta_2 = \cos^{-1}\left[\frac{r_2\cos\gamma_2 - q\cos\beta - a\cos\theta_2}{e}\right] \quad \text{... (xviii)}$$

and
$$\delta_3 = \cos^{-1}\left[\frac{r_3\cos\gamma_3 - q\cos\beta - a\cos\theta_3}{e}\right] \quad \text{... (xix)}$$

Thus by considering the loop *OABE*, we can find the values of *q*, *a*, *e*, β and δ.

Now considering the loop *ODCE* in order to find *p*, *c*, *f*, α and ψ. The horizontal and vertical components of vectors *p*, *c*, *f* and *r* are

$$p\cos\alpha + c\cos\phi + f\cos\psi = r\cos\gamma \quad \text{... (xx)}$$

and
$$p\sin\alpha + c\sin\phi + f\sin\psi = r\sin\gamma \quad \text{... (xxi)}$$

Since these equations are similar to equations ***(i)*** and ***(ii)***, therefore we shall proceed in the similar way as discussed for loop *OABE*.

Squaring equations ***(xx)*** and ***(xxi)*** and adding in order to eliminate angle ϕ, we have

$$p[2r\cos(\gamma-\alpha)]+f[2r\cos(\psi-\gamma)]+c^2-p^2-f^2=r^2+p\,f[2\cos(\psi-\alpha)] \quad \text{... (xxii)}$$

Let $p = k_5$; $f = k_6$; $c^2 - p^2 - f^2 = k_7$ and $p\,f = k_8 = k_5\,k_6$ ... ***(xxiii)***

Now equations ***(xxii)*** may be written as

$$k_5[2r\cos(\gamma-\alpha)]+k_6[2r\cos(\psi-\gamma)]+k_7=r^2+k_8[2\cos(\psi-\alpha)] \quad \text{... (xxiv)}$$

The equation ***(xxiv)*** is a non-linear equation and can be solved easily by making it linear by some substitutions as given below :

Let $k_5 = l_5 + \lambda_1 m_5$; $k_6 = l_6 + \lambda_1 m_6$; and $k_7 = l_7 + \lambda_1 m_7$ ... ***(xxv)***

where
$$\lambda_1 = k_8 = k_5\,k_6 = (l_5 + \lambda_1 m_5)(l_6 + \lambda_1 m_6)$$

$$= l_5 l_6 + l_5 \lambda_1 m_6 + \lambda_1 m_5 l_6 + \lambda_1^2 m_5 m_6$$

or
$$m_5 m_6 \lambda_1^2 + (l_5 m_6 + l_6 m_5 - 1)\lambda_1 + l_5 l_6 = 0$$

or $$D\lambda_1^2 + E\lambda_1 + F = 0$$

$$\therefore \quad \lambda_1 = \frac{-E \pm \sqrt{E^2 - 4DF}}{2D} \quad \text{... (xxvi)}$$

where $D = m_5\, m_6$; $E = (l_5\, m_6 + l_6\, m_5 - 1)$; and $F = l_5\, l_6$ ... **(xxvii)**

Substituting the values of k_5, k_6, k_7 and k_8 in equation **(xxiv)**,

$$(l_5 + \lambda_1\, m_5)[2r\cos(\gamma - \alpha)] + (l_6 + \lambda_1\, m_6)[2r\cos(\psi - \gamma)] + l_7 + \lambda_1\, m_7$$
$$= r^2 + \lambda_1[2\cos(\psi - \alpha)]$$

Equating the terms with λ and without λ separately equal to zero, we get the components into two groups, one with λ and the other without λ. These components are

$$l_5[2r\cos(\gamma - \alpha)] + l_6[2r\cos(\psi - \gamma)] + l_7 = r^2 \quad \text{... (xxviii)}$$

and $$m_5[2r\cos(\gamma - \alpha)] + m_6[2r\cos(\psi - \gamma)] + m_7 = 2\cos(\psi - \alpha) \quad \text{... (xxix)}$$

The equation **(xxviii)** for the three positions of r, γ and ψ may be written three times as follows :

$$l_5[2r_1\cos(\gamma_1 - \alpha)] + l_6[2r_1\cos(\psi_1 - \gamma_1)] + l_7 = r_1^2 \quad \text{... (xxx)}$$

$$l_5[2r_2\cos(\gamma_2 - \alpha)] + l_6[2r_2\cos(\psi_2 - \gamma_2)] + l_7 = r_2^2 \quad \text{... (xxxi)}$$

$$l_5[2r_3\cos(\gamma_3 - \alpha)] + l_6[2r_3\cos(\psi_3 - \gamma_3)] + l_7 = r_3^2 \quad \text{... (xxxii)}$$

Similarly, equation **(xxix)** for the three positions of r, γ and ψ may be written three times as follows :

$$m_5[2r_1\cos(\gamma_1 - \alpha)] + m_6[2r_1\cos(\psi_1 - \gamma_1)] + m_7 = 2\cos(\psi_1 - \alpha) \quad \text{... (xxxiii)}$$

$$m_5[2r_2\cos(\gamma_2 - \alpha)] + m_6[2r_2\cos(\psi_2 - \gamma_2)] + m_7 = 2\cos(\psi_2 - \alpha) \quad \text{.. (xxxiv)}$$

$$m_5[2r_3\cos(\gamma_3 - \alpha)] + m_6[2r_3\cos(\psi_3 - \gamma_3)] + m_7 = 2\cos(\psi_3 - \alpha) \quad \text{... (xxxv)}$$

The equations **(xxx)**, **(xxxi)** and **(xxxii)** are three linear equations in l_5, l_6 and l_7. Similarly, equations **(xxxiii)**, **(xxxiv)** and **(xxxv)** are linear equations in m_5, m_6 and m_7. Assuming a suitable value of α, the values of l_5, l_6, l_7 and m_5, m_6, m_7 may be determined by using elimination method or Cramer's rule.

Knowing the values of l_5, l_6, l_7 and m_5, m_6, m_7, we can find the value of λ_1 from equation **(xxvi)**. Now the values of k_5, k_6 and k_7 are determined from equation **(xxv)** and hence p, f and c are known from equation **(xxiii)**.

Assuming the value of ψ_1, the corresponding values of ψ_2 and ψ_3 may be calculated as follows :

Since the angular displacements of the coupler link *BCE* is same at the points *B* and *C*, therefore

$$\psi_2 - \psi_1 = \delta_2 - \delta_1$$

or $$\psi_2 = \psi_1 + (\delta_2 - \delta_1) \quad \text{... (xxxvi)}$$

Similarly, $$\psi_3 = \psi_1 + (\delta_3 - \delta_1) \quad \text{... (xxxvii)}$$

If the mechanism is to be designed for more than three positions of the input link *AB* and the same number of positions of the couple point E, then the least square technique is used. The error function from equations (*ix*) and (*x*) are defined as

$$e_1 = \sum [l_1\{2r\cos(\gamma-\beta)\} + l_2\{2r\cos(\theta-\gamma)\} + l_3 - r^2]^2 \quad \text{... (xxxviii)}$$

and
$$e_2 = \sum [m_1\{2r\cos(\gamma-\beta)\} + m_2\{2r\cos(\theta-\gamma)\} + m_3 - 2\cos(\theta-\beta)]^2 \quad \text{... (xxxix)}$$

An aircraft assembling plant.

Note : This picture is given as additional information and is not a direct example of the current chapter.

For e_1 and e_2 to be minimum, the partial derivatives of e_1 with respect to l_1, l_2, l_3 and partial derivatives of e_2 with respect to m_1, m_2, m_3 separately must be equal to zero, *i.e.*

and
$$\left.\begin{aligned} &\frac{\partial e_1}{\partial l_1} = 0\ ;\ \frac{\partial e_1}{\partial l_2} = 0\ ;\ \frac{\partial e_1}{\partial l_3} = 0 \\ &\frac{\partial e_2}{\partial m_1} = 0\ ;\ \frac{\partial e_2}{\partial m_2} = 0\ ;\ \frac{\partial e_2}{\partial m_3} = 0 \end{aligned}\right] \quad \text{... (xxxx)}$$

First consider when $\dfrac{\partial e_1}{\partial l_1} = 0$,

$$\sum_1^n 2\left[l_1 2r\cos(\gamma-\beta) + l_2\, 2r\cos(\theta-\gamma) + l_3 - r^2\right] 2r\cos(\gamma-\beta) = 0$$

or
$$l_1 \sum_1^n [2r\cos(\gamma-\beta)]^2 + l_2 \sum_1^n [2r\cos(\theta-\gamma)][2r\cos(\gamma-\beta)]$$
$$+ l_3 \sum_1^n [2r\cos(\gamma-\beta)] = \sum_1^n [2r\cos(\gamma-\beta)]\, r^2 \quad \text{... (xxxxi)}$$

Similarly, for $\dfrac{\partial e_1}{\partial l_2} = 0$,

$$l_1 \sum_1^n [2r\cos(\gamma-\beta)][2r\cos(\theta-\gamma)] + l_2 \sum_1^n [2r\cos(\theta-\gamma)]^2$$
$$+ l_3 \sum_1^n [2r\cos(\theta-\gamma)] = \sum_1^n [2r\cos(\theta-\gamma)]\, r^2 \quad \text{... (xxxxii)}$$

and for $\dfrac{\partial e_1}{\partial l_3} = 0$, $l_1 \sum_1^n [2r\cos(\gamma - \beta)] + l_2 \sum_1^n [2r\cos(\theta - \gamma)] + l_3 \sum_1^n 1 = \sum r^2$ ... ***(xxxxiii)***

The above three equations can be solved by using Cramer's rule to find l_1, l_2 and l_3.

In the similar way as discussed above, for $\dfrac{\partial e_2}{\partial m_1} = 0$

$$m_1 \sum_1^n [2r\cos(\gamma - \beta)]^2 + m_2 \sum_1^n [2r\cos(\theta - \gamma)][2r\cos(\gamma - \beta)]$$

$$+ m_3 \sum_1^n [2r\cos(\gamma - \beta)] = \sum_1^n [2r\cos(\theta - \beta)][2r\cos(\gamma - \beta)] \quad \text{... (xxxxiv)}$$

Similarly, for $\dfrac{\partial e_2}{\partial m_2} = 0,$

$$m_1 \sum_1^n [2r\cos(\gamma - \beta)] + [2r\cos(\theta - \gamma)] + m_2 \sum_1^n [2r\cos(\theta - \gamma)]^2$$

$$+ m_3 \sum_1^n [2r\cos(\theta - \gamma)] = \sum_1^n [2r\cos(\theta - \beta)][2r\cos(\theta - \gamma)] \quad \text{... (xxxxv)}$$

and for $\dfrac{\partial e_2}{\partial m_3} = 0$,

$$m_1 \sum_1^n [2r\cos(\gamma - \beta)] + m_2 \sum_1^n [2r\cos(\theta - \gamma)] + m_3 \sum_1^n 1 = \sum [2r\cos(\theta - \gamma)] \quad \text{... (xxxxvi)}$$

The above three equations can be solved by using Cramer's rule to find m_1, m_2 and m_3.

Knowing the values of l_1, l_2, l_3 and m_1, m_2, m_3, we can find the value of λ from equation ***(vii)*** and k_1, k_2, k_3 from equation ***(vi)***. Thus q, a and e are determined. Now $\delta_1, \delta_2, \delta_3$ may be determined by using equation ***(i)*** or ***(ii)***.

The values of p, c and f are obtained by solving equation ***(xxiv)*** in the similar way as discussed earlier.

25.18. Synthesis of Four Bar Mechanism For Body Guidance

Consider the three positions of a rigid planer body containing the points A and B as shown in Fig. 25.17 (*a*). The four bar mechanism for body guidance, considering the three positions of the body, may be designed graphically as discussed below.

1. Consider the three positions of the points A and B such as A_1, A_2, A_3 and B_1, B_2, B_3 as shown in Fig. 25.17 (*a*).

2. Find the centre of a circle which passes through three points A_1, A_2, A_3. This is obtained by drawing the perpendicular bisectors of the line segments $A_1 A_2$ and $A_2 A_3$. Let these bisectors intersect at O_A. It is evident that a rigid link $A\,O_A$ pinned to the body at point A and pinned to the ground at point O_A will guide point A through its three positions A_1, A_2 and A_3.

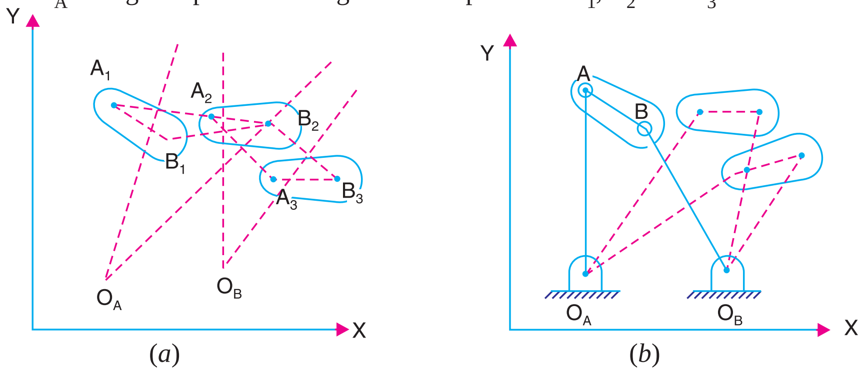


Fig. 25.17. Four bar mechanism for body guidance.

3. Similarly, find the centre O_B of a circle which passes through three points B_1, B_2, B_3. It is evident that a rigid link $B\,O_B$ pinned to the body at point B and pinned to the ground at point O_B will guide point B through its three positions B_1, B_2 and B_3.

4. The above construction forms the four bar mechanism $O_A\,ABO_B$ which guides the body through three specified positions. Fig. 25.17 (*b*) shows a four bar mechanism in these three positions.

The points O_A and O_B may be determined analytically as discussed below :

Consider the three positions of the point A such as A_1, A_2, A_3. Let the co-ordinates of these points are A_1 (x_1, y_1) ; A_2 (x_2 , y_2) and A_3 (x_3, y_3). Let the co-ordinates of the point O_A are (x, y). Now we know that

Distance between points A_1 and O_A,

$$A_1O_A = [(x_1 - x)^2 + (y_1 - y)^2]^{1/2} \quad \text{... (i)}$$

Similarly, distance between points A_2 and O_A,

$$A_2O_A = [(x_2 - x)^2 + (y_2 - y)^2]^{1/2} \quad \text{... (ii)}$$

and distance between points A_3 and O_A,

$$A_3O_A = [(x_3 - x)^2 + (y_3 - y)^2]^{1/2} \quad \text{... (iii)}$$

For the point O_A to be the centre of a circle passing through the points A_1, A_2 and A_3, the distances A_1O_A, A_2O_A and A_3O_A must be equal. In other words,

$$A_1O_A = A_2O_A = A_3O_A$$

Now considering $A_1O_A = A_2O_A$, we have

$$\left[(x_1 - x)^2 + (y_1 - y)^2\right]^{1/2} = \left[(x_2 - x)^2 + (y_2 - y)^2\right]^{1/2} \quad \text{... (iv)}$$

Similarly, considering $A_2O_A = A_3O_A$, we have

$$\left[(x_2 - x)^2 + (y_2 - y)^2\right]^{1/2} = \left[(x_3 - x)^2 + (y_3 - y)^2\right]^{1/2} \quad \text{... (v)}$$

Squaring both sides of the equations (*iv*) and (*v*) and simplifying, we get the following two equations in the unknowns x and y.

$$2x(x_2 - x_1) + 2y(y_2 - y_1) + (x_1^2 - x_2^2) + (y_1^2 - y_2^2) = 0 \quad \text{... (vi)}$$

and

$$2x(x_3 - x_2) + 2y(y_3 - y_2) + (x_2^2 - x_3^2) + (y_2^2 - y_3^2) = 0 \quad \text{... (vii)}$$

The equations **(*vi*)** and **(*vii*)** are simultaneous equations and may be solved to find the co-ordinates *x*, *y* of the point O_A. This point O_A becomes the location of the fixed pivot guiding the point *A*. The length of the guiding link O_AA may be determined by any of the equations **(*i*)**, **(*ii*)** or **(*iii*)**.

In the similar way, as discussed above, we can find the location of the fixed pivot point O_B and the length of the link O_BB.

Example 25.8. *Synthesize a four bar mechanism to guide a rod AB through three consecutive positions* A_1B_1, A_2B_2 *and* A_3B_3 *as shown in Fig. 25.18.*

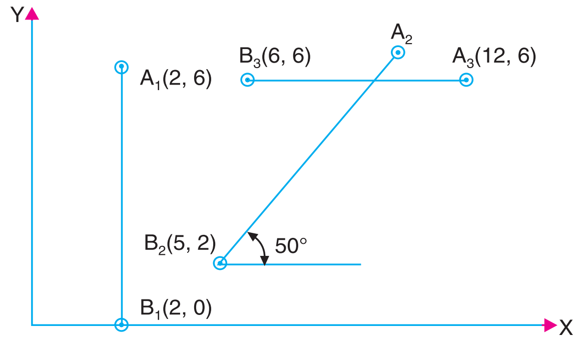


Fig. 25.18

Solution : In order to synthesize a four bar mechanism, we shall use the graphical method as discussed below :

1. Join points A_1, A_2 and A_2, A_3. Draw the perpendicular bisectors of line segments A_1A_2 and A_2A_3 to intersect at O_A, as shown in Fig. 25.19. It is evident that a rigid link O_AA_1 pinned to the body at point A_1 and pinned to the ground at point O_A will guide point A_1 through its three positions.

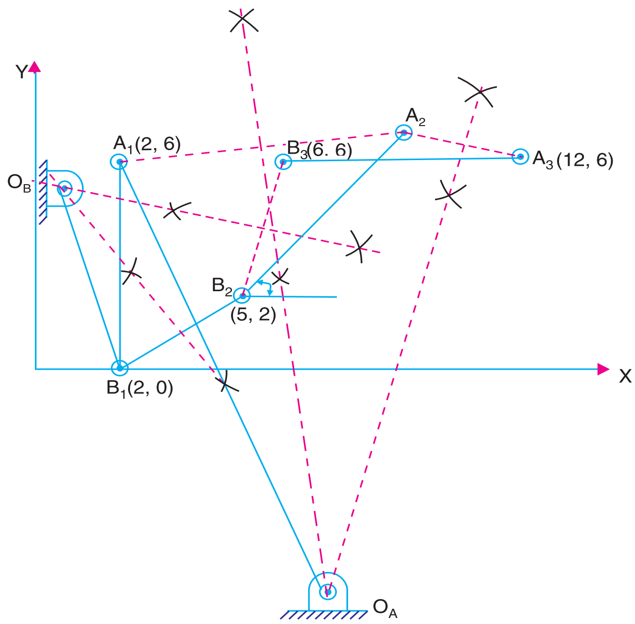


Fig. 25.19

2. Similarly, join points B_1, B_2 and B_2, B_3. Draw the perpendicular bisectors of line segments B_1B_2 and B_2B_3 to intersect at O_B as shown in Fig. 25.19. It is evident that a rigid link O_BB_1 pinned to the body at point B_1 and pinned to the ground at point O_B will guide point *B*, through its three positions.

3. From above we see that the points O_A and O_B are the required fixed points and $O_A\,A_1\,B_1\,O_B$ is one position of the four bar mechanism. The other two positions of the mechanism will be $O_A\,A_2\,B_2\,O_B$ and $O_A\,A_3\,B_3\,O_B$.

25.19. Analytical Synthesis for Slider Crank Mechanism

A slider crank mechanism is shown in Fig. 25.20. In the sysnthesis problem of the slider crank mechanism, the displacement (*s*) of the slider *C* has to co-ordinate with the crank angle (θ) in a specified manner. For example, consider that the displacement of the slider is proportional to crank angle over a given interval, *i.e.*

$$s - s_S = C(\theta - \theta_S)\text{, for } \theta_S \le \theta \le \theta_F \qquad \text{... }(i)$$

where C = Constant of proportionality, and

s = Displacement of the slider when crank angle is θ.

The subscripts $_S$ and $_F$ denote starting and finishing positions.

The synthesis of a slider crank mechanism for three precision points is obtained as discussed below.

The three positions of the crank (θ_1, θ_2 and θ_3) may be obtained in the similar way as discussed in Art. 25.10 and the corresponding three positions of the slider (s_1, s_2 and s_3) are obtained from the given condition as in equation **(*i*)**. Now the dimensions *a*, *b* and *c* may be determined as discussed below :

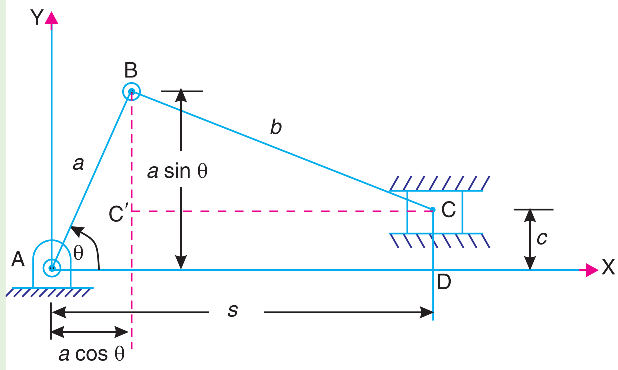


Fig. 25.20. Slider crank mechanism.

In a right angled triangle $BC'C$,

$$BC = b;\ BC' = a\sin\theta - c\ \text{, and } CC' = s - a\cos\theta$$

$$\therefore \quad b^2 = (a\sin\theta - c)^2 + (s - a\cos\theta)^2$$

$$= a^2\sin^2\theta + c^2 - 2ac\sin\theta + s^2 + a^2\cos^2\theta - 2sa\cos\theta$$

$$= a^2 + c^2 - 2ac\sin\theta - s^2 - 2as\cos\theta$$

or $$2as\cos\theta + 2ac\sin\theta + b^2 - a^2 - c^2 = s^2$$

$$k_1 s\cos\theta + k_2\sin\theta - k_3 = s^2 \qquad \text{... }(ii)$$

where $$k_1 = 2a\ ;\ k_2 = 2ac \text{ and } k_3 = a^2 - b^2 + c^2 \qquad \text{... }(iii)$$

For the three different positions of the mechanism *i.e.* for $(\theta_1, \theta_2\ \theta_3)$ and (s_1, s_2, s_3), the equation ***(ii)*** may be written as

$$k_1\, s_1 \cos\theta_1 + k_2 \sin\theta_1 - k_3 = s_1^2 \quad \text{... }(iv)$$

$$k_1\, s_2 \cos\theta_2 + k_2 \sin\theta_2 - k_3 = s_2^2 \quad \text{... }(v)$$

$$k_1\, s_3 \cos\theta_3 + k_2 \sin\theta_3 - k_3 = s_3^2 \quad \text{... }(vi)$$

The equations ***(iv)***, ***(v)*** and ***(vi)*** are three simultaneous equations and may be solved for three unknowns k_1, k_2 and k_3. Knowing the values of k_1, k_2 and k_3, the lengths *a*, *b* and *c* may be obtained from equations ***(iii)***.

Example 25.9. *Synthesize a slider crank mechanism so that the displacement of the slider is proportional to the square of the crank rotation in the interval* $45° \le \theta \le 135°$. *Use three precision points with Chebyshev's spacing.*

Solution : Given. $\theta_S = 45°$; $\theta_F = 135°$

First of all, let us find the three precision points (*i.e.* x_1, x_2 and x_3). We know that

$$x_j = \frac{1}{2}(x_S + x_F) - \frac{1}{2}(x_F - x_S)\cos\left[\frac{\pi(2j-1)}{2n}\right] \text{ ; where } j = 1, 2 \text{ and } 3$$

Assuming the starting displacement of the slider (s_S) = 100 mm and final displacement of the slider (s_F) = 30 mm. It may be noted that for the crank rotating in anticlockwise direction, the final displacement will be less than the starting displacement.

$$\therefore \quad x_1 = \frac{1}{2}(100+30) - \frac{1}{2}(30-100)\cos\left[\frac{\pi(2\times1-1)}{2\times3}\right] = 95.3 \text{ mm.}$$

$$\text{. . . } (\because \quad x_S = s_S \text{ ; } x_F = s_F \text{ and } n = 3)$$

$$x_2 = \frac{1}{2}(100+30) - \frac{1}{2}(30-100)\cos\left[\frac{\pi(2\times2-1)}{2\times3}\right] = 65 \text{ mm}$$

and

$$x_3 = \frac{1}{2}(100+30) - \frac{1}{2}(30-100)\cos\left[\frac{\pi(2\times3-1)}{2\times3}\right] = 34.7 \text{ mm}$$

The corresponding three values of θ are given by

$$\theta_j = \theta_S + \frac{\theta_F - \theta_S}{x_F - x_S}(x_j - x_S) \text{ ; } j = 1, 2, \text{ and } 3$$

$$\therefore \quad \theta_1 = 45 + \frac{135-45}{30-100}(95.3-100) = 51.04°$$

$$\theta_2 = 45 + \frac{135-45}{30-100}(65-100) = 90°$$

and

$$\theta_3 = 45 + \frac{135-45}{30-100}(34.7-100) = 128.96°$$

Since it is given that the displacement of the slider (*s*) is proportional to the square of the crank rotation (θ), therefore, for the displacement from initial position (s_S) to *s* when crank rotates from initial position (θ_S) to θ, we have

A belt-conveyor that can transport small components.
Note : This picture is given as additional information and is not a direct example of the current chapter.

$$s - s_S = C(\theta - \theta_S)^2 \qquad \text{... (} \theta \text{ is expressed in degrees)}$$

$$\therefore \qquad C = \frac{s - s_S}{(\theta - \theta_S)^2} = \frac{30 - 100}{(135 - 45)^2} = \frac{-7}{810} \qquad \text{... (Taking } s = s_F \text{ ; and } \theta = \theta_F \text{)}$$

Now the three positions for the slider displacement (s) corresponding to the three positions of the crank angle (θ) are given by

$$s_1 = s_S + C(\theta_1 - \theta_S)^2 = 100 - \frac{7}{810}(51.4 - 45)^2 = 99.7 \text{ mm}$$

$$s_2 = s_S + C(\theta_2 - \theta_S)^2 = 100 - \frac{7}{810}(90 - 45)^2 = 82.5 \text{ mm}$$

$$s_3 = s_S + C(\theta_3 - \theta_S)^2 = 100 - \frac{7}{810}(128.96 - 45)^2 = 39.08 \text{ mm}$$

Now the three equations relating the (θ_1, s_1), (θ_2, s_2) and (θ_3, s_3) are written as

$$k_1 \times 99.7 \cos 51.04° + k_2 \sin 51.04° - k_3 = (99.7)^2$$

or $$62.7\, k_1 + 0.7776\, k_2 - k_3 = 9940 \qquad \text{... } (i)$$

Similarly, $k_1 \times 82.5 \cos 90° + k_2 \sin 90° - k_3 = (82.5)^2$

or $$k_2 - k_3 = 6806 \qquad \text{... } (ii)$$

and $$k_1 \times 39.08 \cos 128.96° + k_2 \sin 128.96° - k_3 = (39.08)^2$$

or $$-24.57\, k_1 + 0.776\, k_2 - k_3 = 1527 \qquad \text{... } (iii)$$

The equations (*i*), (*ii*) and (*iii*) are three simultaneous equations in three unknowns k_1, k_2 and k_3. On solving, we get

$k_1 = 96.4$; $k_2 = 13\ 084$; and $k_3 = 6278$

We know that $k_1 = 2a$, or $a = k_1 / 2 = 96.4 / 2 = 48.2$ mm **Ans.**

$k_2 = 2a.c$ or $c = k_2 / 2a = 13\ 084 / 2 \times 48.2 = 135.7$ mm **Ans.**

and $$k_3 = a^2 - b^2 + c^2$$

$$b^2 = a^2 + c^2 - k_3 = (48.2)^2 + (135.7)^2 - 6278 = 14\ 460$$

or $b = 120.2$ mm **Ans.**

EXERCISES

1. In a four bar mechanism *PQRS*, the link *PS* is fixed. The length of the links are : *PQ* = 62.5 mm ; *QR* = 175 mm ; *RS* = 112.5 mm and *PS* = 200 mm. The crank *PQ* rotates at 10 rad/s clockwise. Find the angular velocity and angular acceleration of the links *QR* and *RS* for the values of angle *QPS* at an interval of 60°.

2. In a slider crank mechanism, the crank *AB* = 100 mm and the connecting rod *BC* = 300 mm. When the crank is at 120° from the inner dead centre, the crank shaft has a speed of 75 rad/s and an angular acceleration of 1200 rad/s^2 both clockwise. Find at an interval of 60° 1. the linear velocity and acceleration of the slider, and 2. the angular velocity and angular acceleration of the rod, when (*a*) the line of stroke of the slider is offset by 30 mm, and

(*b*) the line of stroke of the slider is along the axis of rotation of the crank.

3. A mechanism is to be designed to generate the function

$$y = x^{0.8}$$

for the range $1 \le x \le 3$, using three precision points. Find the three values of *x* and *y*.

[Ans. 1.134, 2, 2.866 ; 1.106, 1.741, 2.322]

4. Determine the three precision positions of input and output angles for a mechanism to generate a function

$$y = x^{1.8}$$

when *x* varies from 1 to 5, using Chebyshev's spacing. Assume that the initial values for the input and output crank are 30° and 90° respectively and the difference between the final and initial values for the input and output cranks are each equal to 90°.

[Ans. 36°, 75°, 94.48°; 91.22°, 144.57°, 181.22°]

5. Synthesize a four bar linkage using Freudenstein's equation to generate the function $y = x^{1.8}$ for the interval $1 \le x \le 5$. The input crank is to start from $\theta_S = 30°$ and is to have a range of 90°. The output follower is to start at $\phi_S = 0°$ and is to have a range of 90°. Take three accuracy points at *x* = 1, 3 and 5.

6. A four bar function generator is used to generate the function *y* = 1/*x* for $1 \le x \le 3$ between the input angle of a crank and the angle the follower makes with the frame. Find the three precision points from Chebyshev's spacing if the initial values of input angle (*i.e.* crank angle) and output angle (*i.e.* follower angle) are 30° and 200° respectively. The difference between the final and initial values of the crank and follower angles are each equal to 90°.

7. Synthesize a four bar linkage that will generate a function $y = x^{1.2}$ for the range $1 \le x \le 5$. Take three precision points : $\theta_S = 30°$; $\phi_S = 60°$ and $\Delta\theta = \Delta\phi = 90°$, where θ_S and ϕ_S represent respectively the initial angular positions of the input and output crank; $\Delta\theta$ and $\Delta\phi$ are respectively the ranges of the angular movements of the input and output crank.

8. Synthesize a four bar mechanism to generate the function *y* = log *x*, where *x* varies between 1 and 10. Use three accuracy points with Chebyshev's spacing. Assume θ_S = 45°; θ_F = 105°; ϕ_S = 135° and ϕ_F = 225°. Take the length of the smallest link equal to 50 mm.

9. Synthesize a four bar mechanism to move the rod *AB* as shown in Fig. 25.21, through the positions 1, 2 and 3. The end points *A* and *B* are used as moving pivot points.

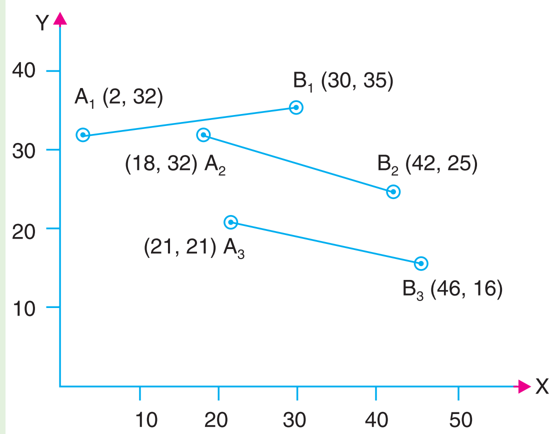


Fig. 25.21

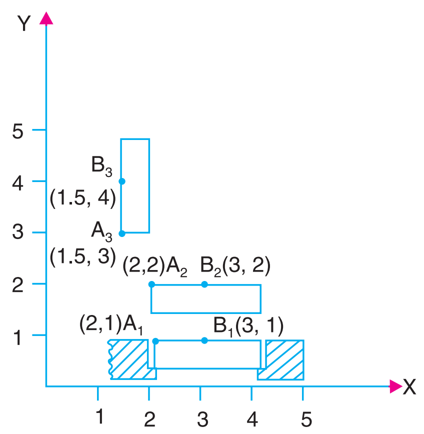


Fig. 25.22

10. Design a four bar mechanism to guide the door in and out with little rotation until it clears the surrounding structure, after which it swings fully open to one side. The three positions of such a door under going this type of motion is shown in Fig. 25.22. The points *A* and *B* are used as moving pivots that guides the body through the three positions.

DO YOU KNOW ?

1. Explain Freudenstein's method of three point synthesis of mechanisms.

2. Derive the expressions for displacement, velocity and acceleration of a four bar mechanism.

3. What do you understand by coupler curves ? Describe the method of obtaining the co-ordinates of a coupler point in a slider crank mechanism.

4. Explain synthesis of mechanism with examples. What do you understand by

(*a*) Type synthesis ; (*b*) Number synthesis ; and (*c*) dimensional synthesis.

5. Describe the classifications of synthesis problem.

6. Write an expression for determining the precision points.

7. Discuss the method of determining the angles for input and output link in a four bar mechanism for function generation.

8. Describe the method of designing a four bar mechanism as a function generation.

OBJECTIVE TYPE QUESTIONS

1. The analysis of mechanism deals with

(*a*) the determination of input and output angles of a mechanism

(*b*) the determination of dimensions of the links in a mechanism

(*c*) the determination of displacement, velocity and acceleration of the links in a mechanism

(*d*) none of the above

2. The synthesis of mechanism deals with

(*a*) the determination of input and output angles of a mechanism

(*b*) the determination of dimensions of the links in a mechanism

(*c*) the determination of displacement, velocity and acceleration of the links in a mechanism

(*d*) none of the above

3. The three precision points in the range $1 \le x \le 3$ are

(*a*) 1.1, 2, 2.6 (*b*) 1.6, 2.5, 2.95

(*c*) 1.134, 2, 2.866 (*d*) 1.341, 2 , 2.686

4. For a four bar mechanism, as shown in Fig. 25.23 the Freudenstein's equation is

(*a*) $k_1 \cos\theta + k_2 \cos\phi + k_3 = \cos(\theta - \phi)$

(*b*) $k_1 \cos\theta - k_2 \cos\phi + k_3 = \cos(\theta - \phi)$

(*c*) $k_1 \cos\phi + k_2 \cos\theta + k_3 = \cos(\theta - \phi)$

(*d*) $k_1 \cos\phi - k_2 \cos\theta + k_3 = \cos(\theta - \phi)$

where $k_1 = \dfrac{d}{a}$; $k_2 = \dfrac{d}{c}$; $k_3 = \dfrac{a^2 - b^2 + c^2 + d^2}{2ac}$

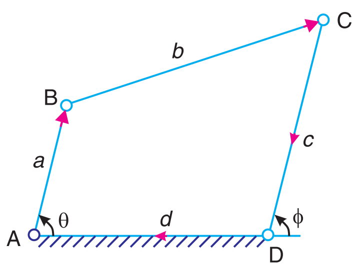


Fig. 25.23

ANSWERS

1. (*c*) **2.** (*b*) **3.** (*c*) **4.** (*d*)

GO To FIRST