Characteristic Polynomials of one Side Weighted Adjacency Matrices of Linear Chains

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Abstract

Characteristic polynomials of one side arithmetically weighted adjacency matrices of linear chains were calculated. The elements of the inverse of their matrix are derived from odd factorials.

Key words: adjacency matrices, Cluj matrices, eigenvalues, characteristic polynomials.

Diudea [1,2] introduced asymmetrically weighted distance matrices, Cluj matrices, by the Wiener weights $N_{i,(i,j)}$ and $N_{j,(i,j)}$ (the number of vertices on the end j of the path p_{ij} from the diagonal vertex ($i = j$) to the off-diagonal vertex j ($i \neq j$). I have studied [3] some properties of the direct (Hadamard) product of a Cluj matrix with the corresponding adjacency matrix **A**:

$$\mathbf{C}_e = \mathbf{C}_p \bullet \mathbf{A} \qquad (1)$$

which leaves only adjacent elements of the Cluj matrix $\mathbf{C}_e$ (or equivalently Cluj weighted adjacency matrix $\mathbf{A}_C$, for example for the linear chain L_4 (n-butane)

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

.

The eigenvalues of the linear chains L_n with odd n (from the inspection of the first chains) have values 0, $[2,\ 4,\ \dots (n-1)]$, the eigenvalues of the linear chains L_n with even n have values $[1,\ 3,\ \dots (n-1)]$.

In this paper, the characteristic polynomials of one side arithmetically weighted adjacency matrices of linear chains are studied

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{pmatrix}$$

The characteristic polynomials were calculated by counting weighted k-tuples. The results are tabulated:

Table 1: Coefficients of weighted linear chains adjacency matrices

n				
1	1			
2	1	-1		
3	1	-3		
4	1	-6	3	
5	1	-10	15	
6	1	-15	45	-15
7	1	-21	105	-105

The coefficients of the table are
$t_{i,1} = 1$, $t_{i,j} = (n - j + 1)t_{i-1,j-1} + t_{i-1,j}$
These coefficients can be tabulated in following matrix according to the powers of x terms

1	0	0	0	0	0	0
-1	1	0	0	0	0	0
0	-3	1	0	0	0	0
0	3	-6	1	0	0	0
0	0	15	-10	1	0	0
0	0	-15	45	-15	1	0
0	0	0	-105	105	-21	1

The inverse of this matrix is

1	2	3	4	5	6	7	$\sum$
1	0	0	0	0	0	0	1
1	1	0	0	0	0	0	2
3	3	1	0	0	0	0	5
15	15	6	1	0	0	0	37
105	105	45	10	1	0	0	266
945	945	420	105	15	1	0	2431
10395	10395	4725	1260	210	21	1	27007

The elements of the first column are the odd factorials $1x1x3x5x7\dots$ (the first 1 is 0!).

The recurrence of the matrix elements is $m_{1,1} = 1$, otherwise

$$[2(n-1) - j]m_{i-1,j} + m_{i-1,j-1} \tag{2}$$

The row sums S, except the first two, are obtained as

$$(2n-1)S_{n-1} + S_{n-2} \tag{3}$$

.

The characteristic polynomials of odd and even chains differ. It were better to include empty side diagonals and the epty graph. The reccurence is then:

1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
-1	0	1	0	0	0	0	0
0	-3	0	1	0	0	0	0
3	0	-6	0	1	0	0	0
0	15	0	-10	0	1	0	0
-15	0	45	0	-15	0	1	0
0	-105	0	105	0	-21	0	1

The inverse of this matrix has the same elements but they are all positive:

1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
1	0	1	0	0	0	0	0
0	3	0	1	0	0	0	0
3	0	6	0	1	0	0	0
0	15	0	10	0	1	0	0
15	0	45	0	15	0	1	0
0	105	0	105	0	21	0	1

The recurrence of the matrix elements is $m_{1,1} = 1$, otherwise

$$(n-1]m_{i-2,j} + m_{i-1,j-1} \tag{4}$$

The row sums S, except the first two, are obtained as

$$(n-1)S_{n-2} + S_{n-1} \tag{5}$$

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