

**Title :**                    **A FRESH LOOK AT**  
**FUNDAMENTAL CONCEPTS OF**  
**QUANTUM MECHANICS**

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**ABSTRACT**

With reference to the Elastic Continuum Theory, fundamental concepts of kinetic energy and potential energy of a particle have been reviewed in relation to the development of Schrodinger's wave equation. Distinction between the particle and the accompanying wave packet, has been emphasized. Consequently an error in the Schrodinger's wave equation has been brought out and highlighted, wherein the potential energy term  $V(r)$  has not been made dependent on particle coordinates. Some of the resulting inconsistencies noticed in a typical solution of the Schrodinger's wave equation, are critically examined. For example, one of the results indicate an overall 23.81 % probability that the electron in 1s Hydrogen orbital will possess negative kinetic energy, which is absurd. Rectification of this error may have far reaching consequences on further development and refinement of Quantum Mechanics.

**Keywords.** Potential energy; Schrodinger's wave equation; Interaction energy.

<p><b>Uncertainty Principle : 'A basic indeterminacy in the behavior of nature, like the one that is being advocated, is due to an incompleteness of the theory.'</b> Albert Einstein</p>
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## 1. INTRODUCTION

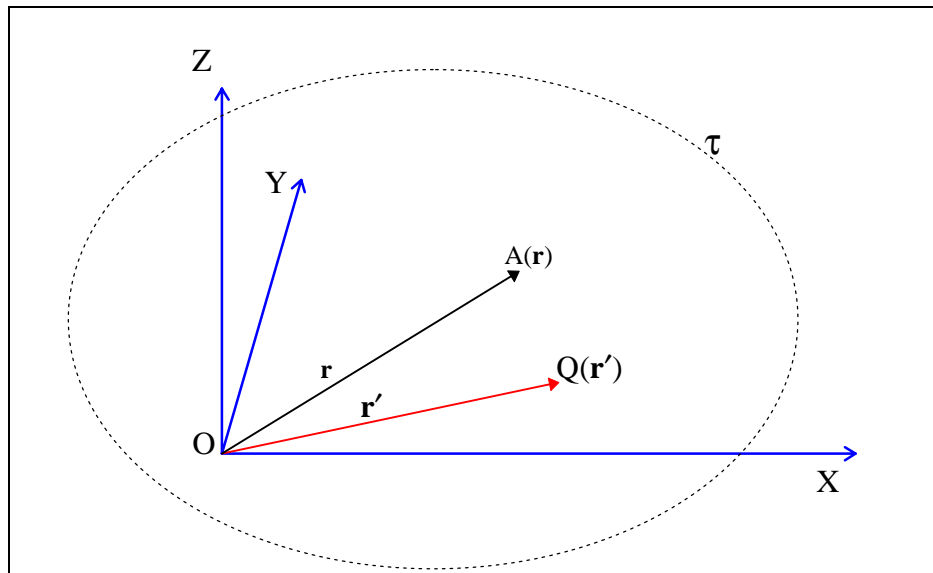
1.1 The Schrodinger's wave equation of Quantum Mechanics may be considered as founded in the suggestion of L. de Broglie that some sort of waves accompanied electrons and other micro particles in motion. These waves were assumed to represent the crucial dynamic characteristics of motion of the particle, namely the momentum  $\mathbf{p}$  and total energy  $E$ , through following two relations adapted from the photon wave packet.

$$p = h/\lambda \quad \dots\dots\dots (1)$$

$$E = h \nu \quad \dots\dots\dots (2)$$

Here  $h$  is the Planck's constant,  $\lambda$  is the wavelength and  $\nu$  the frequency of the above mentioned motion induced waves accompanying the particle. Of course, unlike the photon wave packet which as a whole behaves like a particle, the motion induced wave packet accompanying a material particle is a separate entity, an appendage to the particle. The material particle (i.e. a particle with non-zero rest mass) itself does not 'become' or transform itself into a wave packet. Let us consider the motion of a free particle, say an electron, located at point  $Q$  in a Cartesian coordinate system with center at point  $O$  as shown in figure 1. At any instant  $t$ , let  $\mathbf{r}'$  be the position vector of point  $Q$  and let  $r'$  be the magnitude of this position vector. Since the particle at  $Q(\mathbf{r}')$  is assumed to be in motion, it will be accompanied by a motion induced wave packet spread over or extending into a region of space of say, volume  $\tau$ , around the location of the particle.

Figure 1.



1.2 Let this motion induced wave packet be characterized by the parameter  $\psi$  known as wave function. The  $\psi$  is non-zero and finite at all points within the region ' $\tau$ ' and vanishes at the boundary of ' $\tau$ '. The whole region of space  $\tau$  where

the wave function  $\psi$  is defined or 'exists', may be termed as the  $\psi$  wave field . At any space point  $A(\mathbf{r})$ , with position vector  $\mathbf{r}$ , the value of wave function  $\psi$  will keep varying with time  $t$  and hence may be written as  $\psi(\mathbf{r},t)$ . The  $\psi(\mathbf{r},t)$  in general will be a complex function of space and time coordinates. The intensity of the wave function will be given by

$$|\psi|^2 = |\psi \cdot \psi^*| = P(\mathbf{r},t) \quad \dots\dots\dots (3)$$

The wave intensity  $P(\mathbf{r},t)$  is also known as the probability density. The wave function  $\psi$  is normalized by equating the integral of  $P(\mathbf{r},t)$ , over the entire volume  $\tau$  of the  $\psi$  wave field, to unity. The intensity or probability density  $P(\mathbf{r},t)$  is expected to be maximum in a region of space where the moving particle is actually expected to be located at that instant of time. If we consider a small element of volume  $\delta\tau$  around point  $A(\mathbf{r})$  where the probability density is  $P(\mathbf{r},t)$ , then the actual probability of locating the point  $Q(\mathbf{r}')$ , representing the moving particle, within this volume  $\delta\tau$  is given by  $[P(\mathbf{r},t) \cdot \delta\tau]$  . In other words the wave intensity  $P(\mathbf{r},t)$  is expected to be maximum in a region of space around point  $Q(\mathbf{r}')$ . Only in a limiting or extreme case when the volume  $\tau$  of the wave field tends to diminish to zero, that is when the  $\psi$  field collapses, will the field point  $A(\mathbf{r})$  tend to merge with the particle location point  $Q(\mathbf{r}')$ .

## 2. SCHRODINGER'S EQUATION FOR FREE PARTICLE

2.1 At any particular instant  $t$  , the point  $A(\mathbf{r})$  may be considered as a general field point within the wave field  $\tau$  and the point  $Q(\mathbf{r}')$  may be considered as a fixed point representing the instantaneous location of the particle within this  $\psi$  wave field. During the motion of the particle, the position vector  $\mathbf{r}'$  of point  $Q$  will keep changing with time. That is,  $\mathbf{r}'$  can be expressed as a function of time  $t$ .

$$\mathbf{r}' = \mathbf{f}(t) \quad \dots\dots\dots (4)$$

Equation (4) will therefore, represent the trajectory of the moving particle. At the given instant, let us examine the wave function  $\psi(\mathbf{r},t)$  within the entire wave field  $\tau$  when the moving particle is located at point  $Q(\mathbf{r}')$ . This wave function  $\psi(\mathbf{r},t)$  is obtained as a solution of Schrodinger's wave equation. The Schrodinger's equation in turn is derived from the energy conservation principle as applied to the moving particle, by making use of the following two operators which form the core of Quantum Mechanics.

$$\mathbf{p} \rightarrow -i\hbar \nabla \quad \dots\dots\dots (5)$$

$$E \rightarrow i\hbar \partial/\partial t \quad \dots\dots\dots (6)$$

where  $i = \sqrt{-1}$  and  $\hbar = h/2\pi$ . Using the total energy relation for a moving free particle of mass  $m$ , Total Energy = Kinetic Energy + Potential Energy

$$\text{Or} \quad E = T + V \quad \dots\dots\dots (7)$$

$$\text{and with } V=0; \quad E = p^2/2m + 0 \quad \dots\dots\dots (8)$$

Multiplying equation (8) with  $\psi$  and then applying the operators (5) and (6), we get

$$i\hbar \partial\psi/\partial t = -(\hbar^2/2m) \cdot \nabla^2\psi + 0 \quad \dots\dots\dots (9)$$

which is the well known Schrodinger's wave equation for a free particle. Before considering a similar approach for arriving at the Schrodinger's wave equation for a moving particle with force constraints or with non-zero potential energy, let us first examine our basic concepts of kinetic energy (T) and potential energy (V), with special reference to the Elastic Continuum Theory<sup>[1]</sup>. Henceforth we shall consider an electron to be the moving particle whose motion induced wave function  $\psi(\mathbf{r},t)$  is under discussion.

### 3. KINETIC ENERGY

3.1 If the kinetic energy (T) of a free particle is reduced to zero by bringing it to rest we can see from equations (5) and (9) that  $\nabla\psi$ ,  $\nabla^2\psi$ ,  $\partial\psi/\partial t$  and  $\psi$  will all reduce to zero. That is, in the absence of kinetic energy, the  $\psi$  wave field of the particle will collapse to zero. It implies that the  $\psi$  field of a moving particle is highly dependent on the kinetic energy of the particle. We know from the 'Elastic Continuum Theory' that for a micro particle or 'strain bubble' of rest mass  $m$ , the entire mass energy  $mc^2$  is stored or contained in the form of dynamic strain energy within the specific boundary of that particle. Further, just as all other forms of energy exist in the Elastic Continuum as strain energy of various strain bubbles or strain wave fields, the kinetic energy associated with the motion of any strain bubble or micro particle also must be 'existing' or 'contained' in some sort of 'strain wave field' associated with or induced by its motion. Therefore, combining the above mentioned concepts that the  $\psi$  field of a moving particle is highly dependent on kinetic energy of the particle and that the kinetic energy must be existing as strain energy of the associated strain wave field; we may consider it most likely that the kinetic energy of a moving particle could be actually 'existing' or 'contained' within the  $\psi$  field of the particle. In fact the intensity of the normalized wave function  $P(\mathbf{r},t)$  or  $|\psi|^2$  given by equation (3), may actually represent the normalized kinetic energy density of the particle. This also corresponds to the concept of energy density in electromagnetic field being proportional to the squares of electric and magnetic field strengths or to the sum of squares of strain components.

3.2 We can generally say that any change in motion of a particle will induce a corresponding change in the kinetic energy of that particle and vice-versa. Extending this notion to the  $\psi$  wave field of a moving particle, we can say that any change in the motion of the particle will induce a corresponding change in the overall  $\psi$  wave field of that particle and vice-versa. That is, any change in the overall  $\psi$  wave field of a particle will induce a corresponding change in the motion of that particle. Thus we may appreciate that high energy interactions of micro particles could be governed by the superposition interactions of their  $\psi$  wave fields. This precisely is the reason of phenomenal success of Quantum Mechanics in the study of micro particle interactions, especially the high energy interactions. However, the normalization of wave function  $\psi$  has effectively rendered Quantum Mechanics particularly suitable for statistical and qualitative applications.

### 4. POTENTIAL ENERGY

4.1 Let us consider an isolated electron. As per the Elastic Continuum Theory<sup>[1]</sup>, it consists of a small spherically symmetric 'core' of standing strain wave oscillations<sup>[2]</sup>, surrounded by propagating phase wave type 'strain wave field'. The electrostatic field of

electron with radially decaying electric field strength can be identified with its strain wave field with radially decaying amplitude. About 35% of the ‘mass’ energy of the electron is actually stored or ‘contained’ in this electrostatic or strain wave field. The energy density in this wave field of the electron is proportional to the sum of squares of the amplitudes of its strain components. The wave field energy component of the electron mass is an integral part of the electron and is not dependent on the existence of any other charge or field in its vicinity. Now let us consider a proton and electron pair separated by distance ‘d’ from each other. Their respective positive and negative electrostatic fields or strain wave fields will get ‘overlapped’ or superposed almost throughout their spatial extension. As a result of mutual cancellation of superposed +ve and -ve fields from two charges, the resultant or combined field strength of the two charges will get slightly reduced. Consequently the combined field energy of the proton-electron system, being proportional to the square of the resultant field strength, will be slightly less than the total sum of the individual field energies of the isolated charges. This reduction in the combined field energy of the proton-electron system, is precisely the negative interaction energy due to the Coulomb interaction<sup>[2]</sup> of the system and is known as the negative potential energy of the proton electron pair. Therefore,

$$\text{Potential energy of proton-electron pair} = V = -e^2/4\pi\epsilon_0d \quad \dots\dots\dots (10)$$

$$\text{or Interaction energy released by the system} = |V| = e^2/4\pi\epsilon_0d \quad \dots\dots\dots (11)$$

4.2 Obviously the interaction energy released by the proton-electron pair is zero when they are separated by infinite distance. As their separation is reduced to say ‘d’, the interaction energy released by the system, as given by equation (11), is continuously converted into the kinetic energy of the proton-electron pair. Of course, during this conversion or transfer of released interaction energy to the kinetic energy of the system, the overall conservation of energy, momentum and angular momentum is automatically ensured. As already noted, the kinetic energy of a particle is ‘stored’ or ‘contained’ in the  $\psi$  wave field of the particle. Therefore, the potential energy or the interaction energy of the proton-electron pair may be considered as a parameter signifying the transfer of energy between the  $\psi$  wave field and the combined field energy of the system. Thus a +ve potential energy of a particle signifies the transfer of  $\psi$  wave field energy (i.e. K.E.) to the field strain energy (i.e. mass energy) of the particle. Similarly a -ve potential energy signifies the transfer of a part of the field strain energy to the  $\psi$  wave field energy of the interacting particles. The Schrodinger’s wave equation is intended to describe the variations in  $\psi$  wave field of a moving particle as a result of such energy transfers.

4.3 It is therefore obvious that the potential energy of an electron with respect to a proton at distance d, represented as  $V(d)$ , can not be regarded as a field parameter in the sense that it does not represent any entity distributed in space. For example, the electrostatic field strength or strain wave field energy density can be regarded as field parameters because they represent the entities which are ‘existing’ or defined at all space points of the associated field at any instant of time. The potential energy, on the other hand, is the interaction energy depending entirely on relative location of the electron with respect to the proton at any particular instant and is not ‘defined’ or ‘existing’ at any other space point at that instant. However, the term potential energy, indicating the transfer of

energy between the electrostatic field and the  $\psi$  wave field, is not applicable for a single isolated particle where such transfer can not occur due to absence of any interaction. It has a meaning only for two or more interacting particles, wherein such transfer of energy could take place between the combined electrostatic or strain wave field energy (i.e. total rest mass energy) and the total  $\psi$  wave field energies of the interacting particles, without violating the principle of conservation of momentum and energy. If at any instant  $t$ , the proton (considered almost stationary) is located at point  $O$ , the origin of coordinate system (Fig. 1) and the moving electron is located at point  $Q(\mathbf{r}')$  with position vector  $\mathbf{r}'$ , then the potential energy of the electron will depend on the magnitude of the position vector  $|\mathbf{r}'|$  or simply  $r'$  and may be represented by  $V(r')$ . It will not be a function of the coordinates of field point  $A(\mathbf{r})$ . That is, when the wave function is represented as  $\psi(\mathbf{r},t)$ , the potential energy term can not be represented as  $V(\mathbf{r})$ .

## 5. TOTAL ENERGY

5.1 The total energy  $E$  of a system of two interacting particles (the proton-electron pair in the present case) is defined through equation (7), that is,

$$E = T + V(\mathbf{r}') = T + [-e^2/4\pi\epsilon_0 r'] \quad \dots\dots\dots (12)$$

Truly speaking the term  $E$  in equation (7) or (12) represents the total external energy supplied to the system.  $E$  is +ve when this amount of energy is externally added or supplied to the system of interacting particles and is -ve when it is extracted, or taken out of the system. The externally supplied energy may either get directly added to the kinetic energy of the system or may get added to their electrostatic field energies through the potential energy term and vice-versa. Generally a -ve  $E$  will represent a bound state of the system of interacting particles. Even though the name implies total energy, it actually does not include the rest mass energies of the interacting particles. Therefore, a constant total energy  $E$  or a stationary energy state of an isolated system of interacting particles, implies the constancy of sum of K.E. and potential energy of the system.

## 6. Schrodinger's Equation with Wrong Potential Energy Term

6.1 The total external energy, whether supplied to or extracted from the system will naturally influence the kinetic energy  $T$  and hence the  $\psi$  wave field of the moving particle. The complex relationship between the variations of total & potential energy and the corresponding space-time variations of the  $\psi$  wave field representing the kinetic energy, is reflected through the Schrodinger's wave equation involving the potential energy term. Multiplying equation (12) with  $\psi(\mathbf{r},t)$  throughout and then applying the operators (5) and (6), we get,

$$E \cdot \psi(\mathbf{r},t) = (p^2/2m) \cdot \psi(\mathbf{r},t) + V(\mathbf{r}') \cdot \psi(\mathbf{r},t) \quad \dots\dots\dots (13)$$

and 
$$i \hbar \partial\psi/\partial t = -(\hbar^2/2m) \cdot \nabla^2\psi + V(\mathbf{r}') \cdot \psi \quad \dots\dots\dots (14)$$

But the Schrodinger's wave equation is normally written in the form

$$i \hbar \partial\psi/\partial t = -(\hbar^2/2m) \cdot \nabla^2\psi + V(\mathbf{r}) \cdot \psi \quad \dots\dots\dots (15)$$

The difference in equations (14) and (15) is in the potential energy terms  $V(\mathbf{r}')$  and  $V(\mathbf{r})$ .

6.2 In the Schrodinger's original wave equation (15), the potential energy is expressed as a function of the coordinates of general field point  $A(\mathbf{r})$ , instead of the coordinates of instantaneous location  $Q(\mathbf{r}')$  of the particle (Figure 1). That means the Schrodinger's wave equation (15) is founded on the total energy relation

$$E = T + V(r) = T + [-e^2/4\pi\epsilon_0r] \dots\dots\dots (16)$$

instead of equation (12). As already discussed above, the potential energy of an electron-proton pair is strictly a function of their instantaneous relative distance  $r'$  and is not defined at any other space point  $A(\mathbf{r})$ . This discrepancy is not a simple or inadvertent mistake in the Schrodinger's original wave equation (15) but rather a serious conceptual mistake with far reaching consequences. This mistake is continued with throughout Quantum Mechanics, where the potential energy term  $V(r)$  is often replaced by  $e.\phi(r)$ ; with scalar potential  $\phi(r)$  treated as a function of coordinates of general field point  $A(\mathbf{r})$  rather than a function of coordinates of instantaneous location  $Q(\mathbf{r}')$  of the particle. Probably the greatest temptation for permitting this mistake, must have been the consequent ease of solving the Schrodinger's equation (15) by treating the potential energy term  $V(r)$  as spherically symmetric and independent of time. Even though most weaknesses of Quantum Mechanics could be attributed to this conceptual mistake, yet for want of timely rectification, the mistake had to be 'swept under the uncertainty carpet'.

## 7. Consequential Errors in Established Solutions

7.1 Let us now examine a few consequential errors in the established solutions of Schrodinger's wave equation (15), arising out of the above mentioned mistake in the potential energy term  $V(r)$ . For this let us consider the ground state 1s orbital of Hydrogen atom, the normalized wave function  $\psi_{nlm} = \psi_{100}(\mathbf{r},t)$  of which is,

$$\psi_{100}(\mathbf{r},t) = (1/\pi a_0^3)^{1/2} . \exp(-r/a_0) . \exp(-i E_1 t/\hbar) \dots\dots\dots (17)$$

where  $E_1 = -13.6 \text{ eV}$  is the total energy of the 1s orbital  
and  $a_0 = 0.53 \text{ \AA}$  is the corresponding Bohr radius.

7.2 Oscillating  $\psi$  Wave Field. In accordance with the original suggestion of L. de Broglie, the solution for ground state 1s orbital of Hydrogen was expected to yield some sort of waves or wave packet, i.e.  $\psi$  wave packet, accompanying the orbiting electron. But equation (17) represents a spherically symmetric standing wave oscillations of the  $\psi$  field, which does not correspond to the physical situation. Equation (17) can not represent any traveling wave or a wave group which could describe the orbiting motion of the electron. Hence, fundamentally this solution is unsuitable to represent the physical situation and should have been rejected. This error could be attributed to the wrong potential energy term  $V(r)$  used in the Schrodinger's equation (15), as discussed above.

7.3 Spherically Symmetric  $\psi$  Wave Field. As discussed earlier, at any instant the intensity of  $\psi$  wave function ( i.e.  $P(\mathbf{r},t)$  of equation (13) ) is expected to be maximum in the vicinity of location of the electron at that instant. But from equation (17) we get,

$$P(\mathbf{r},t) = |\psi_{100}|^2 = (1/\pi a_0^3) . \exp(-2r/a_0) \dots\dots\dots (18)$$

Equation (18) shows that the intensity of the  $\psi$  wave function is spherically symmetric with its maximum value at the center where the proton is located. The obvious conclusion from this result could be that the electron and proton are both located at the center, which of course is physically impossible. The physical situation demanded that the  $\psi$  wave packet should not only have accompanied the orbiting electron but also should have been centered at and spread around the instantaneous location of the electron. Therefore, it could be concluded that solution (17) does not represent the physical situation and should have been rejected as invalid. This error too could be attributed to the wrong potential energy term  $V(r)$  used in the Schrodinger's equation (15). It is, of course, a different matter that use of correct potential energy function  $V(r')$  might have rendered the Schrodinger's equation (14) analytically unsolvable. Ideally speaking, the Schrodinger's equation (14) should be solved in conjunction with pre-set initial and boundary conditions to yield a complete solution consisting of,

- (a) The electron trajectory  $\mathbf{r}' = \mathbf{f}(t)$
- (b) The wave function  $\psi(\mathbf{r}, \mathbf{r}', t)$  or  $\psi(\mathbf{r}, t)$ .

In fact the electron trajectory can be computed even without using the Schrodinger's wave equation. Therefore we must reject equation (17) as an invalid solution.

**7.4 Zero Orbital Angular Momentum of the Electron.** The ground state 1s Hydrogen orbital solution  $\psi_{100}(\mathbf{r}, t)$  given by equation (17), corresponds to zero orbital angular momentum. The zero angular momentum, by virtue of its basic definition and its fundamental physical concept, will represent either a stationary electron or an electron passing through the nucleus. Both of these alternatives do not correspond to the physical situation of an orbiting electron and hence invalid. This error also seems to have occurred due to the wrong potential energy term  $V(r)$  used in the Schrodinger's equation (15).

**7.5 Negative Kinetic Energy of the Electron.** As per the usual terminology in Quantum Mechanics, equation (17) represents the  $\psi$  wave function for the lowest stationary state of electron in Hydrogen atom. In this stationary state, the total energy eigenvalue  $E_1$  is equal to  $-13.6 \text{ eV}$ . For this stationary state, the probability density  $P(r)$  is given by equation (18). The integral of this probability density over the entire  $\psi$  field (i.e. for  $r$  varying from zero to infinity) works out to unity, as expected, since  $\psi$  is normalized. This result is interpreted as the overall probability of finding the electron within the entire  $\psi$  field is 100 % (again as expected, since, after all the electron has to be somewhere !). Let us now work out the overall probability of finding the electron within a spherical shell of inner radius  $r_1 = 0.5 a_0$  and outer radius  $r_2 = 1.5 a_0$  where  $a_0$  is the Bohr radius.

$$\begin{aligned}
 P_{12} &= \int_0^{2\pi} \int_0^{\pi} \int_{r_1}^{r_2} P(r) \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi \\
 &= 4\pi \int_{r_1}^{r_2} P(r) \cdot r^2 \, dr = 0.4965 \quad \text{by using equation (18).}
 \end{aligned}$$



That means the overall probability of finding the electron within a spherical shell of radii  $0.5 a_0$  and  $1.5 a_0$  is 49.65 %. We may interpret this result as follows. If somehow we could conduct a very large number of electron location determining experiments, then in about 49.65 percent cases we are likely to locate the electron within a spherical shell of radii  $0.5 a_0$  and  $1.5 a_0$ . Apparently this is quite a reasonable result and in the backdrop of uncertainty principle it is generally accepted quite faithfully.

7.6 Now let us carry out one more computation. This time let us work out the overall probability of finding the electron outside a sphere of radius  $r_3 = 2 a_0$ . Proceeding on the same lines as above, we get this probability as

$$\begin{aligned} P_3 &= 4\pi \int_{r_3}^{\infty} P(r).r^2 dr \\ &= \frac{4}{a_0^3} \int_{2a_0}^{\infty} \exp(-2r / a_0).r^2 dr = 0.2381 \end{aligned}$$

That means the overall probability of finding the electron outside a sphere of radius  $2a_0$  is about 23.81 percent. We might be tempted to accept this result too as quite reasonable essentially due to our immense faith in Quantum Mechanics. But a little closer look will show that this result is most unreasonable, absurd and totally wrong. For this let us once again consider the parent equations from which this result is derived, namely the original Schrodinger's equation (15) and the total energy equation (16). Obviously, since the result (17) is valid or true in the region of space outside a sphere of radius  $2a_0$ , the parent equations (15) and (16) must also be valid in that region. However for a stationary state represented by equation (17), the total energy  $E_1 = -13.6 \text{ eV} = 2.176 \times 10^{-18} \text{ J}$ , is known to be constant. Therefore, from equation (16) we get,

$$T = E_1 - [-e^2/4\pi\epsilon_0 r] = e^2/4\pi\epsilon_0 r - 2.176 \times 10^{-18} \quad \text{Joules} \quad \dots\dots\dots (19)$$

This shows that kinetic energy  $T$  of the electron, in the stationary state (17), keeps reducing with increasing  $r$ . Equation (19) shows that  $T$  reduces to zero at  $r = 2a_0$ . That means when the electron is located outside a sphere of radius  $2a_0$ , its kinetic energy will become negative. But we have seen above that probability of finding the electron outside a sphere of radius  $2a_0$  is 23.81 percent. Hence we draw the conclusion that as per Quantum Mechanics, based on original Schrodinger's equation (15), there is 23.81 % probability that the electron, in ground state of Hydrogen atom, will exist in a negative kinetic energy state with imaginary velocity components. Since this is patently an absurd conclusion, we must review the situation to find what went wrong and where? Therefore we come back to our previous observation that the potential energy term  $V$  in the original Schrodinger's equation (15) has been wrongly taken as a function of coordinates of general field point  $A(\mathbf{r})$ , instead of taking it as a function of coordinates of point  $Q(\mathbf{r}')$ , the instantaneous location of the electron.

7.7 From equation (19) we can easily see that for the ground state of Hydrogen, the electron will always remain bound well within a bounding sphere of radius  $2a_0$ . That is whatever be the shape of trajectory or electron orbits ( $\mathbf{r}' = \mathbf{f}(t)$ ) in the ground state of Hydrogen, these orbits will always be located well within this bounding sphere of radius  $2a_0$  i.e.  $r' < 2a_0$ . Now, let us assume for the time being that we succeed in obtaining analytical solutions of correct Schrodinger's equation (14). Then the  $\psi$  wave field will have to be restricted within this bounding sphere, so that the overall probability of finding the electron outside this sphere is zero. If however, the  $\psi$  wave field could not be reduced to zero outside this bounding sphere, the wave function intensity  $|\psi|^2$  will have to be re-interpreted as the normalized kinetic energy density instead of electron probability density as already discussed at para 4 above.

7.8 In spite of the error in potential energy term  $V$ , as brought out above, the Quantum Mechanics has been extremely useful in the study of high energy micro particle interactions. Even for those applications where either the effect of potential energy term is negligible or the potential energy function  $V$  is made more dependent on particle locations, the Quantum Mechanics has been of immense value. It is hoped that through rectification of the error in potential energy term  $V$  in the Schrodinger's wave equation, we may be in a better position to further enhance the efficacy and utility of Quantum Mechanics. We must keep striving to further develop, improve and refine this extremely important branch of Physics. Possibly, the already complex wave function  $\psi$  might ultimately turn out to be much more complex; possibly a longitudinal strain wave packet. Therefore, the necessity of strengthening the logical foundations of Quantum Mechanics, so as to bring it within the grasp of imagination, can not be overemphasized. After all, from purely philosophical standpoint, the end result of any intellectual pursuit must come within the mental grasp, within the perceptible limits of human mind. That is, any deeper understanding of the physical reality of the micro world, that we may develop through an intellectual process by using simple or mathematical logic, must be fully perceptible to the human mind and we should be in a position to mentally comprehend and visualize the same.

## 8. SUMMARY AND CONCLUSION

8.1 Considering the Schrodinger's wave equation as founded on L. de Broglie's suggestion that some sort of waves or wave packets accompany the electrons and other micro particles in motion, we must emphasize the distinction between the moving particle and the waves that accompany it. At any instant  $t$ , if  $\psi(\mathbf{r},t)$  is the wave function that describes the accompanying wave or wave packet, it must be defined at infinitely many space points (i.e. variable  $\mathbf{r}$ ) that constitute the  $\psi$  field, while at that instant the moving particle can be located at only one space point say  $Q(\mathbf{r}')$ . The potential energy  $V$  or the interaction energy of the proton-electron pair is a precise function of their relative separation distance  $r'$  and hence should be represented by the function  $V(r')$  and not by  $V(r)$ . This precisely is the error in the Schrodinger's wave equation, where the potential energy function used is  $V(r)$  instead of  $V(r')$ . Obviously this has led to wrong solutions for the wave function  $\psi(\mathbf{r},t)$ . In this paper we have highlighted some of the glaring errors

in a typical solution for  $\psi$ . For example, one of the results indicate an overall 23.81 % probability that the electron in 1s Hydrogen orbital will possess negative kinetic energy, which is obviously absurd. In addition to pointing out a gross conceptual mistake in the formulation of Schrodinger's wave equation and certain inconsistencies in the established solutions, we have also attempted to present a comprehensive conceptual picture of the kinetic energy, potential energy and the total energy of a pair of interacting particles. Most significant of these is the concept of  $\psi$  wave field carrying the kinetic energy of the particle in motion, adapted from the 'Elastic Continuum Theory'. The dynamic interactions of all micro particles are therefore controlled or governed by their  $\psi$  wave field interactions. The Quantum Mechanics provides an excellent and indispensable tool for studying such  $\psi$  wave field interactions of high energy particles. It is hoped that the rectification of the mistake in potential energy term of Schrodinger's wave equation will further enhance the utility and efficacy of Quantum Mechanics.

### **Reference**

- [1] Elastic Continuum Theory of Electromagnetic Field  
and Strain Bubbles by the Author
- [2] The Electron Structure and Coulomb Interaction „