Title : THE ELECTRON STRUCTURE & COULOMB INTERACTION

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ABSTRACT

Based on the Elastic Continuum Theory[1], a new model for the electron structure and its electrostatic wave field, has been introduced in this paper. Obtained as a spherically symmetric solution of elastic equilibrium equations, the electron and positron type particles are found to consist of an oscillating strain wave core surrounded by radial strain wave field. The concept of charge is related to the direction of propagation and intensity of radial strain wave field. The Coulomb interaction between two charge particles is computed through superposition of their strain wave fields and Coulomb law verified for separation distances greater than core diameter of the electron. The electromagnetic interaction is thus shown to be not mediated by any real or virtual particles.

Keywords. Electron Structure; Core; Field energy; Strain wave field; Coulomb interaction.

Action at a distance : **'That one body may act upon another at a distance through a vacuum, without the mediation of anything else, is to me so great an absurdity that I believe no man, who has in philosophical matters a competent faculty for thinking, can ever fall into.'**

Isaac Newton

1. INTRODUCTION

1.1 Out of thousands of elementary particles known so far, the electron was the first to be detected, most actively researched and studied, used in tremendous applications and most well known of all particles. As a part of our understanding about the electron, we have accurately determined its charge, mass, spin, angular momentum, magnetic moment and interaction characteristics. Apparently, everything that is worth knowing about the electron is already known. Yet, most scientists still regard the electron as a point charge, a point mass and a structure-less elementary particle. We are still not in a position to visualize the shape, size and inner structural details of the electron. We are still unable to pin point the structural peculiarity of the electron which endows it with the unique property of charge and the spin.

1.2 Ideally speaking, the mental visualization of a physical situation must precede the use of mathematical techniques for its logical analysis. The analytical techniques employed might be highly abstract, but the end result must always come within the purview of mental grasp. However, we can not develop a very clear and vivid mental picture of a physical situation as long as the fundamental concepts associated with that situation are themselves vague and hazy. For instance, in the present case we can't develop a very clear mental picture of Coulomb interaction between two electrons as long as the fundamental concepts of electron structure, its charge property, its electrostatic field etc. are themselves vague and unclear. Hence, based on the Elastic Continuum Theory[1] we will first examine some of the spherically symmetric solutions of equilibrium equations of elasticity in the Continuum to explore the structure of electron and its electrostatic wave field. With this framework, we shall then develop a model to compute the interaction energy from the superposition of effective strain wave fields of two electrons separated by a finite distance.

2. ELECTRON STRUCTURE

2.1 Electrostatic Wave Field. It is known that electrostatic field influence propagates or spreads out from the source particle at the velocity of light. It is also believed that the electromagnetic interaction is mediated through continuous exchange of 'virtual photons' between the charge particles. Even a 'virtual photon' field is often assumed to be surrounding all charge particles to account for the simultaneous Coulomb interaction among infinitely many such particles. Hence the electrostatic field seems to be inherently dynamic instead of being static in character. That is, instead of being a function f(R) of relative position vector **R** alone, it may have to depend on time as well, so that the field influence appears to be spreading at velocity of light. Therefore we may characterize the electrostatic wave field by a function of the type $f(R).\exp(\imath\,\kappa(R-ct))$, where $\imath=(-1)^{1/2}$, representing a sort of spherical phase wave of amplitude f(R) propagating radially at velocity of light c. Here κ represents the wave number. This hazy picture of the electrostatic field is further developed by using Elastic Continuum Theory (ECT).

2.2 As per ECT, our familiar space-time continuum, with characteristic properties of permittivity ε_0 and permeability μ_0, behaves as a perfect isotropic Elastic Continuum

with elastic constant $1/\varepsilon_0$ and inertial constant μ_0. The equilibrium equations of elasticity written in terms of displacement vector **U** in this Continuum turn out to be identical to the vector wave equation in electromagnetic theory. These equations in vector and tensor form are given below,

$$\partial^2\mathbf{U}/\partial x^2 + \partial^2\mathbf{U}/\partial y^2 + \partial^2\mathbf{U}/\partial z^2 = \nabla^2\mathbf{U} = (1/c^2)\,\partial^2\mathbf{U}/\partial t^2 \quad \text{.....(1)}$$

$$g^{11}u^i_{,11} + g^{22}u^i_{,22} + g^{33}u^i_{,33} = g^{jj}u^i_{,jj} = (1/c^2)\,\partial^2u^i/\partial t^2 \quad \text{..... (2)}$$

where the displacement vector components u^i are functions of space & time coordinates referred to a coordinate system (y^1, y^2, y^3). The following correlation exists between displacement vector field **U** or the corresponding temporal and spatial strain components and the electromagnetic field vectors **E** and **B**,

$$\mathbf{E} = -(1/\varepsilon_0).(1/c).\partial\mathbf{U}/\partial t \quad \& \quad \mathbf{B} = (1/c).(1/\varepsilon_0).(\nabla\times\mathbf{U}) \quad \text{..... (3)}$$

That means, the electromagnetic field in the so called 'vacuum' comes out to be a dynamic stress-strain field in the corresponding Elastic Continuum. As per the ECT, one of the spherically symmetric solutions of equilibrium equations (2), represent the electron and positron strain bubbles consisting of a small 'core' of standing strain wave oscillations, surrounded by propagating phase wave type 'strain wave field' or the 'electrostatic field'.

2.3 Spherically Symmetric Solution of Equilibrium Equation. The complete electron structure may be seen as a special solution of equilibrium equations (2) in spherical polar coordinate system (y^1=R, y^2=θ, y^3=ϕ). If we write equation (2) in terms of physical components u^R, u^θ and u^ϕ of displacement vector **U** in spherical coordinate system, we get a set of three simultaneous partial differential equations, the general solution of which is most intricate due to mutual coupling of these displacement components. One of the lowest order solutions of these equations is obtained when we restrict u^θ=0, u^ϕ to be independent of ϕ coordinate and u^R to be independent of both θ and ϕ coordinates. The resulting equations reduce to

$$\frac{\partial^2u^R}{\partial R^2} + \frac{2}{R}.\frac{\partial u^R}{\partial R} - \frac{2}{R^2}.u^R = \frac{1}{c^2}.\frac{\partial^2u^r}{\partial t^2} \quad \text{..... (4)}$$

$$\frac{\partial^2u^\phi}{\partial R^2} + \frac{2}{R}.\frac{\partial u^\phi}{\partial R} - \frac{1}{R^2\sin^2\theta}.u^\phi + \frac{1}{R^2}.\left(\frac{\partial^2u^\phi}{\partial\theta^2} + \cot\theta.\frac{\partial u^\phi}{\partial\theta}\right) = \frac{1}{c^2}.\frac{\partial^2u^\phi}{\partial t^2} \quad \text{..... (5)}$$

2.4 **The Core.** The oscillating wave type solution of the above equations (4) and (5) with a spherically symmetric boundary surface, is given for positron (+ve) and electron (-ve) cores by

$$u^R = \pm A_e.e\kappa.G_1(X).\,\mathrm{Cos}(\kappa ct); \quad \text{..... (6A)}$$

$$u^\phi = \pm A_e.e\kappa.G_1(X).\,\mathrm{Sin}(\theta).\,\mathrm{Sin}(\kappa ct); \quad \text{..... (6B)}$$

where $G_1(X) = (\cos X - \sin X / X)/X = -(\pi/2x)^{1/2}.J_{3/2}(X)$ and $X = \kappa r$.

Another similar solution which has a singularity at the origin and hence not admissible for the electron/positron core is given by

$$u^R = A_e.e\kappa.H_1(X).\ Sin(\kappa ct); \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ (7A)$$

$$u^\phi = -A_e.e\kappa.H_1(X).\ Sin\theta.\ Cos(\kappa ct); \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ (7B)$$

where $H_1(X) = (\sin X + \cos X / X)/X = -(\pi/2x)^{1/2}.\ J_{-3/2}(X)$

$X = \kappa R$; $b_1 \le X \le \infty$ with $J_{-1-1/2}(b_1) = 0$ & $b_1 = 2.7984$

2.5 **The Field.** However, if the above two solutions are combined together we get an oscillating core given by equations (6) for $0 \le X \le b_1$ and a propagating phase wave solution for the electron field for $X \ge b_1$ given by,

$$u^R = -A_e.e\kappa.\{G_1(X).\ Cos(\kappa ct) - H_1(X).\ Sin(\kappa ct)\} = -A_e.e\kappa.G_1(X,\psi_-)$$
$$\approx -(A_e.e\kappa/X).Cos(\psi_-) \qquad \ldots\ldots\ldots\ldots\ldots\ (8A)$$

$$u^\phi = -A_e.e\kappa.\{G_1(X).Sin(\kappa ct) + H_1(X).\ Cos(\kappa ct)\}.\ Sin(\theta) = -A_e.e\kappa.H_1(X,\psi_-).\ Sin(\theta)$$
$$\approx -(A_e.e\kappa/X).\ Sin(\theta).Sin(\psi_-) \qquad \ldots\ldots\ldots\ldots\ldots\ (8B)$$

$u^\theta = 0$; where $\psi_- = X + \kappa ct$ $G_1(X,\psi_-) = [Cos(\psi_-) - Sin(\psi_-)/X]/X$;
and $H_1(X,\psi_-) = [Sin(\psi_-) + Cos(\psi_-)/X]/X$.

This strain wave field consisting of phase waves propagating inwards from infinity to the core boundary, at the speed of light 'c', represents the electrostatic field of electron type charge particle. Another similar solution consisting of phase waves propagating outwards from the core boundary to infinity, at the speed of light 'c', representing the electrostatic field of positron type charge particle will be given by,

$$u^R = +A_e.e\kappa.\{G_1(X).\ Cos(\kappa ct) + H_1(X).\ Sin(\kappa ct)\} = A_e.e\kappa.G_1(X,\psi_+)$$
$$\approx (A_e.e\kappa/X).Cos(\psi_+) \qquad \ldots\ldots\ldots\ldots\ldots\ (9A)$$

$$u^\phi = +A_e.e\kappa.\{G_1(X).Sin(\kappa ct) - H_1(X).\ Cos(\kappa ct)\}.\ Sin(\theta) = -A_e.e\kappa.H_1(X,\psi_+).\ Sin(\theta)$$
$$\approx -(A_e.e\kappa/X).\ Sin(\theta).Sin(\psi_+) \qquad \ldots\ldots\ldots\ldots\ldots\ (9B)$$

$u^\theta = 0$; where $\psi_+ = X - \kappa ct$ $G_1(X,\psi_+) = [Cos(\psi_+) - Sin(\psi_+)/X]/X$;
and $H_1(X,\psi_+) = [Sin(\psi_+) + Cos(\psi_+)/X]/X$.

2.6 Core & Field Strain Energy. The strain energy of the 'core' and 'strain wave field' can now be computed from the corresponding strain components. Here the wave number κ is of the order of $10^{15}\ m^{-1}$, A_e is a dimensionless constant, and 'e' the magnitude of electron charge. The solutions u^R and u^ϕ are in phase quadrature in the core as well as in the field. The elements of strain tensor S^i_j can be computed by taking covariant space and time derivatives of displacement components u^i for spatial and temporal strain terms. The mixed tensor components S^i_j can be converted to the corresponding physical components by using the relation,

$$S^{y^i}_{y^j} = \sqrt{g_{ii}}.S^i_j.\sqrt{g^{jj}} \qquad \text{(No summation over i or j)} \quad \ldots\ldots\ldots\ (10)$$

where g^{ij} are the usual metric tensor components for the reference coordinate system. The physical strain components of **S** and the corresponding strain energy density W can therefore be directly computed from the following relations,

$$S_R^R = \frac{\partial u^R}{\partial R} \quad ; \quad S_\theta^R = \frac{1}{R}.\frac{\partial u^R}{\partial \theta} - \frac{u^\theta}{R} \quad ; \quad S_\phi^R = \frac{1}{R.\sin\theta}.\frac{\partial u^R}{\partial \phi} - \frac{u^\phi}{R} \qquad \text{.....(11A)}$$

$$S_R^\theta = \frac{\partial u^\theta}{\partial R} \quad ; \quad S_\theta^\theta = \frac{1}{R}.\frac{\partial u^\theta}{\partial \theta} + \frac{u^R}{R} \quad ; \quad S_\phi^\theta = \frac{1}{R.\sin\theta}.\frac{\partial u^\theta}{\partial \phi} - \frac{\cot\theta}{R}.u^\phi \qquad \text{....(11B)}$$

$$S_R^\phi = \frac{\partial u^\phi}{\partial R} \quad ; \quad S_\theta^\phi = \frac{1}{R}.\frac{\partial u^\phi}{\partial \theta} \quad ; \quad S_\phi^\phi = \frac{1}{R.\sin\theta}.\frac{\partial u^\phi}{\partial \phi} + \frac{\cot\theta}{R}.u^\theta + \frac{u^R}{R} \qquad \text{.....(11C)}$$

$$S_t^R = \frac{1}{c}.\frac{\partial u^R}{\partial t} \quad ; \quad S_t^\theta = \frac{1}{c}.\frac{\partial u^\theta}{\partial t} \quad ; \quad S_t^\phi = \frac{1}{c}.\frac{\partial u^\phi}{\partial t} \qquad \text{.....(11D)}$$

$$W = (1/2\varepsilon_0).\sum \left| S_{y^j}^{y^i} \right|^2 . \qquad \text{..... (12)}$$

2.7 Since in the strain wave field, surfaces of constant phase propagate outwards or inwards at the velocity of light 'c', without any associated transport of strain energy, we may also term this wave field as phase wave field. However, there is a special feature in this phase wave field. Whereas in the standing or oscillating wave solutions the temporal and spatial strain components are in quadrature; in propagating phase wave field, they are in phase opposition for $\text{Cos}(\psi_\pm)$ & $\text{Sin}(\psi_\pm)$ terms and get canceled out. Hence the energy density in the phase wave field will be governed only by the maximum amplitude of these phase waves or more precisely by their rms. values. The physical strain components for the electron core, computed from relations (6) and (11) are listed below in terms of functions G_1 and H_1 defined above.

$$S_R^R = -A_e.e\kappa^2.G_1'(X).\text{Cos}(\kappa ct) \; ; \qquad S_\theta^\theta = -A_e.e\kappa^2.\frac{G_1(X)}{X}.\text{Cos}(\kappa ct)$$

$$S_\phi^\phi = -A_e.e\kappa^2.\frac{G_1(X)}{X}.\text{Cos}(\kappa ct) \; ; \qquad S_\phi^R = A_e.e\kappa^2.\frac{G_1(X)}{X}.\text{Sin}(\theta).\text{Sin}(\kappa ct)$$

$$S_\phi^\theta = A_e.e\kappa^2.\frac{G_1(X)}{X}.\text{Cos}(\theta)\text{Sin}(\kappa ct); \qquad S_\theta^\phi = -A_e.e\kappa^2.\frac{G_1(X)}{X}.\text{Cos}(\theta).\text{Sin}(\kappa ct)$$

$$S_R^\phi = -A_e.e\kappa^2.G_1'(X).\text{Sin}(\theta).\text{Sin}(\kappa ct); \qquad S_t^R = A_e.e\kappa^2.G_1(X).\text{Sin}(\kappa ct)$$

$$S_t^\phi = -A_e.e\kappa^2.G_1(X).\text{Sin}(\theta).\text{Cos}(\kappa ct); \text{ where } G_1'(X) = -(\text{Sin}(X)+2.G_1(X))/X \quad \text{.... (13)}$$

The strain components for the positron core will be just the same as given above but with different sign. In the strain wave field of the electron represented by equations (8), the simplified strain components for large X, where $1/X^2$ terms could be neglected in comparison to 1/X terms, are given by

$$S_R^R = \frac{A_e.e\kappa^2}{X^2}.\text{Cos}(\psi_-); \qquad S_\theta^\theta = S_\phi^\phi = -\frac{A_e.e\kappa^2}{X^2}.\text{Cos}(\psi_-)$$

$$S_\phi^R = \frac{A_e.e\kappa^2.\text{Sin}(\theta)}{X^2}.\text{Sin}(\psi_-); \qquad S_R^\phi = \frac{A_e.e\kappa^2.\text{Sin}(\theta)}{X^2}.\text{Sin}(\psi_-)$$

$$S_{\phi}^{\theta} = \frac{A_e.e\kappa^2.Cos(\theta)}{X^2}.Sin(\psi_-); \qquad S_{\theta}^{\phi} = -\frac{A_e.e\kappa^2.Cos(\theta)}{X^2}.Sin(\psi_-) \qquad .....(14)$$

For a stationary particle the amplitude of these phase waves at any point will remain constant with time.

2.8 The strain energy density W, calculated by using equations (12) to (14), is given below as W_c for the core and W_f for the field,

$$W_c = \frac{A_e^2e^2\kappa^4}{2\varepsilon_0}.\left[\begin{array}{l}\left\{\left(G_1^{'}(X)\right)^2 + \frac{2G_1^2(X)}{X^2} + G_1^2(X).Sin^2(\theta)\right\}.Cos^2(\kappa ct) \\ + \left\{G_1^2(X) + \frac{G_1^2(X)}{X^2}.\left(Cos^2(\theta)+1\right) + \left(G_1^{'}(X)\right)^2.Sin^2(\theta)\right\}.Sin^2(\kappa ct)\end{array}\right] \quad ...(15)$$

$$W_f = \frac{A_e^2e^2\kappa^4}{2\varepsilon_0}\left[\frac{3.Cos^2(\psi_-)}{X^4} + \frac{2.Sin^2(\psi_-)}{X^4}\right] \qquad ..... (16)$$

In W_c above, the coefficients of $Cos^2(\kappa ct)$ and $Sin^2(\kappa ct)$ terms are not exactly equal, which indicates energy density fluctuations within the core. Possibly these minor fluctuations in the energy density as well as the total energy content in the core are accommodated through slight fluctuations in the core boundary during the period of each oscillation cycle. However, for the overall total energy computation, we may take the time averaged value of the energy density as,

$$W_c = \frac{A_e^2e^2\kappa^4}{4\varepsilon_0}.\left[\left(\left(G_1^{'}(X)\right)^2 + G_1^2(X)\right).\left(1+Sin^2(\theta)\right) + \frac{G_1^2(X)}{X^2}.\left(3+Cos^2(\theta)\right)\right] \qquad ..... (17A)$$

and $W_f = (1/4\varepsilon_0).A_e^2.e^2.\kappa^4[\,5/X^4\,]$ .....(17B)

The total energy of the electron is given as a sum of the core and field energies obtained from the volume integral of W_c over the core region ($0 \le X \le b_1$) and W_f over the field region ($X \ge b_1$), as

$$E_{total} = E_c + E_f = (1/\varepsilon_0).\pi.A_e^2.e^2.\kappa.[\,3.2533 + 1.7867\,]$$

$$= (5.04/\varepsilon_0).\pi.A_e^2.e^2.\kappa. \quad = \mathbf{m_e.c^2} \qquad .....(18)$$

This shows that almost 65 percent of the total mass energy of the electron is contained in its core and remaining 35 percent in its field. In the above relation (18) where $\mathbf{m_e}$ is the known mass of the electron, there are two unknown parameters A_e and κ. For unique determination of these parameters we need one more relation which will be obtained from the Coulomb interaction model.

2.9 Intrinsic Spin Effect. As already indicated above, the only difference in the wave fields of positron (ψ_+) and electron (ψ_-) is in the opposite directions of propagation of their phase waves. In both cases, as seen from equations (6) to (9), the displacement components u^R and u^{ϕ} are in quadrature to each other. If we denote Z-axis ($\theta = 0$) as the axis of the electron or positron strain bubble, then all planes perpendicular to this axis may

be referred as transverse planes. It can be easily seen from phase quadrature of displacement components that the resultant displacement vector in any transverse plane keeps continuously rotating with constant angular velocity $\omega=\kappa c$ whereas its magnitude remains constant or time invariant at any space point. Particularly in the principal transverse plane given by $\theta = \pi/2$, the magnitude of resultant displacement vector **U** in the wave field remains constant with $|\mathbf{U}| = \sqrt{2} \, . \, (A_e.e / R)$. Throughout this principal transverse plane, the constant magnitude vector **U** keeps rotating or 'spinning' with constant angular velocity $\omega=\kappa c$. Direction of this 'spin' of the displacement vector is obviously along the axis of the strain bubble and remains constant with time. This constant 'intrinsic spin' of the displacement vector found in the core as well as strain wave field of the electron/positron type strain bubbles may be identified with the conventional notion of 'Spin' in these particles. The phenomenon of this 'intrinsic spin' is a very unique feature in the ultra-microscopic realm of Nature. Thus we can see from equations (6) to (9) that in normal orientation a positron will have a +ve direction of intrinsic spin, along +ve Z axis, and radially outward propagating phase wave field identified with +ve charge. If the axis of the positron is reversed, direction of its spin will become -ve but the phase waves will still be propagating radially outwards. Similarly for an electron, in any orientation of intrinsic spin, its phase waves will keep propagating radially inwards corresponding to conventional -ve charge.

3. COULOMB INTERACTION

3.1 Effective 'rms.' Valued *Strain* Field Model. When the cores of two interacting strain bubbles get overlapped or superposed, their resulting strong interaction can be computed in a straight forward manner. Their actual strain components, referred to a common coordinate system, are directly superposed to compute the resulting interaction energy over the common overlapped region. However, computation of the interaction energy in the common overlapped region of strain wave fields of two charge particles, appears to be a complex problem due to the presence of phase waves. The intrinsic characteristics of these phase waves have not yet been studied. Even though these phase waves do not transport strain energy, yet they appear to display certain 'wave momentum' effects. In a way these strain waves could be compared with sinusoidal AC voltages. Because of the inherent phase opposition of strain components in the cores of positron and electron, leading to the opposite directions of propagation of their phase waves, the strain wave fields of electron and positron will show inherent opposition such that when superposed they will tend to cancel out each other. The field energy density in each case is governed by the 'rms.' values of the amplitudes of their respective phase waves. We might therefore, adopt the rms. value concept for the magnitude of the effective strain components in the field.

3.2 Further, going by the analogy of electrostatic field lines and considering the fact that Maxwell's electric displacement vector **D** and electric field **E** are proportional to $-\partial\mathbf{U}/\partial t$ (equation 3), we might assign +ve sign to the effective strain components of ψ_+ wave field ($-\partial\psi_+/\partial t$ is +ve) and -ve sign to the effective strain components of ψ_- wave field ($-\partial\psi_-/\partial t$ is -ve). Therefore, for the purpose of developing a simplified model of Coulomb interactions we consider only the rms. valued amplitudes of the respective strain

components and assign them +ve or -ve signs depending on whether the phase waves are propagating outwards from or inwards to the source particle. Corresponding to equations (14), we may specifically write out the effective strain components for electrostatic wave field of a positron as,

$$S^R_R = \frac{1}{\sqrt{2}}.\frac{A_e.e\kappa^2}{X^2}; \qquad S^\theta_\theta = S^\phi_\phi = \frac{1}{\sqrt{2}}.\frac{A_e.e\kappa^2}{X^2}$$

$$S^R_\phi = \frac{1}{\sqrt{2}}.\frac{A_e.e\kappa^2.Sin(\theta)}{X^2}; \qquad S^\phi_R = \frac{1}{\sqrt{2}}.\frac{A_e.e\kappa^2.Sin(\theta)}{X^2}$$

$$S^\theta_\phi = \frac{1}{\sqrt{2}}.\frac{A_e.e\kappa^2.Cos(\theta)}{X^2}; \qquad S^\phi_\theta = \frac{1}{\sqrt{2}}.\frac{A_e.e\kappa^2.Cos(\theta)}{X^2} \qquad .....(19)$$

The corresponding effective strain components for an electron field will have opposite signs to the ones given above.

3.3 To compute Coulomb interaction between two charge particles, [1,2], we have to superpose the effective strain tensor components S^i_j of their respective wave fields in a common coordinate system and then compute the total energy of their combined fields. This total field energy may be more or less than the sum of their isolated field energies. The difference is termed interaction energy. [$E_{int} = E_{sup} - (E_1 + E_2)$]. For two similar charges the respective strain components get added up and since the energy density is proportional to the sum of squares of strain components, total field energy will be more than the sum of their separate field energies. Conventionally the interaction energy is termed +ve in this case. Similarly for two dissimilar charges the combined field energy will reduce and the interaction energy will be termed -ve . The negative interaction energy implies that due to the superposition of fields, part of the initial total field energy of the system of interacting charges is released by the system and may get transformed to some other form. As a limiting case when the separation between the two charge particles is reduced to zero, their interaction energy will not become infinity but will be limited to the sum of their initial mass energies, both for +ve and -ve interaction.

3.4 Interaction Computations. Actual computation of interaction energy of two charges, separated by distance 'R' say, involves transformation of effective field strain tensor components from one coordinate system to another, for effecting the superposition in a common coordinate system. As an illustration let us compute the interaction energy of two positrons located at points O and A with their axes collinear and separated by distance 'R' along polar axis OZ (Fig. 1). Let any space point P be referred to two spherical polar coordinate systems, one (y^i) centered at point O and the other (x^i) centered at point A such that

$$y^1 \equiv r \; ; \qquad y^2 \equiv \theta \; ; \qquad y^3 \equiv \phi \qquad ..... (20)$$

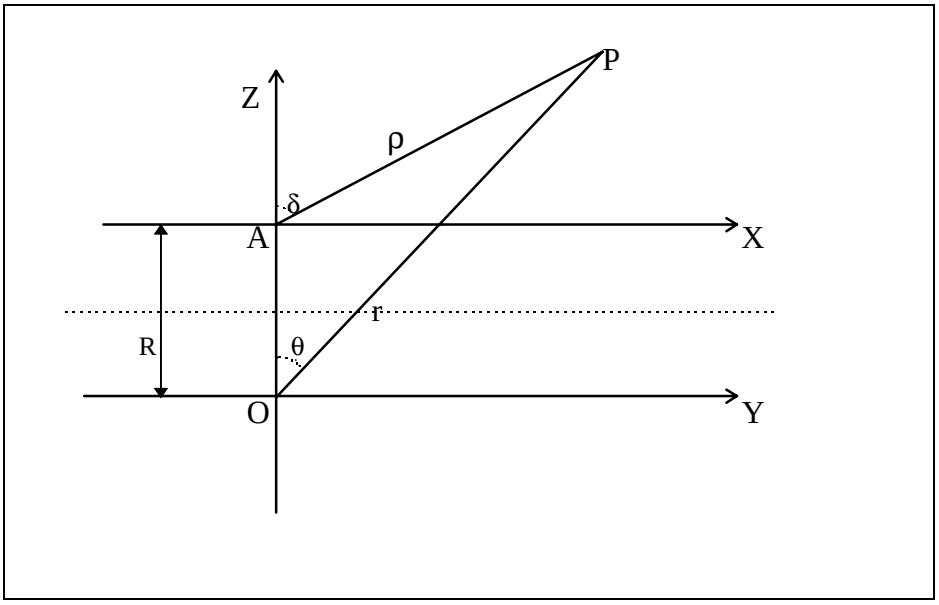
and

$$x^1 \equiv \rho \; ; \qquad x^2 \equiv \delta \; ; \qquad x^3 \equiv \phi \qquad ..... (21)$$

Let the components of any strain tensor referred to coordinate system (y^i) be represented by the symbols S^i_j and those referred to coordinate system (x^i) be represented by the symbols $\in^i_j$. Further, we may designate the field strain components due to the positron located at point O and referred to system (y^i) as $S^i_j(O)$ and those due to the positron located at point A and referred to system (y^i) as $S^i_j(A)$. The superposed or combined strain components may be designated as

$$S^i_j(C) = S^i_j(O) + S^i_j(A) \qquad \text{…….....…} (22)$$

Figure 1.



3.5 Transformation of Strain Components. However, before effecting the above superposition we have to first transform the mixed strain tensor components $\in^i_j(A)$ to $S^i_j(A)$ and convert them to corresponding physical components. The transformation of components $\in^i_j(A)$ in (x^i) coordinate system to $S^i_j(A)$ in (y^i) coordinate system is carried out through the relation

$$S^i_j(A) = \frac{\partial y^i}{\partial x^\alpha} \cdot \in^\alpha_\beta \cdot \frac{\partial x^\beta}{\partial y^j} \qquad \text{(summation over } \alpha \text{ and } \beta) \text{ ……..(23)}$$

For this transformation we need the coordinate transformation relations of the type $y^i = f^i(x^j)$ and $x^i = F^i(y^j)$ between two coordinate systems and the Jacobean matrices of their partial derivatives as under,

$$r^2 = R^2 + \rho^2 + 2\,R\,\rho\cos(\delta) \qquad \text{…… (24A)}$$
$$\rho^2 = r^2 + R^2 - 2\,R\,r\cos(\theta) \qquad \text{……(24B)}$$
$$r\sin(\theta) = \rho\sin(\delta) \qquad \text{……..(24C)}$$
$$r\cos(\theta) - R = \rho\cos(\delta) \qquad \text{……(24D)}$$
$$\tan(\delta) = r.\sin(\theta)/(r.\cos(\theta)-R) \qquad \text{……(24E)}$$

The Jacobean matrices of their partial derivatives obtained from above relations are,

$$\frac{\partial y^1}{\partial x^1}=\frac{\partial r}{\partial \rho}=\frac{r-R.\cos\theta}{\rho}\quad;\qquad \frac{\partial y^1}{\partial x^2}=\frac{\partial r}{\partial \delta}=-R.\sin\theta\quad;\qquad \frac{\partial y^1}{\partial x^3}=\frac{\partial r}{\partial \phi}=0$$

$$\frac{\partial y^2}{\partial x^1}=\frac{\partial \theta}{\partial \rho}=\frac{R.\sin\theta}{r.\rho}\quad;\qquad \frac{\partial y^2}{\partial x^2}=\frac{\partial \theta}{\partial \delta}=\frac{r-R.\cos\theta}{r}\quad;\qquad \frac{\partial y^2}{\partial x^3}=\frac{\partial \theta}{\partial \phi}=0$$

$$\frac{\partial y^3}{\partial x^1}=\frac{\partial \phi}{\partial \rho}=0\quad;\qquad \frac{\partial y^3}{\partial x^2}=\frac{\partial \phi}{\partial \delta}=0\quad;\qquad \frac{\partial y^3}{\partial x^3}=\frac{\partial \phi}{\partial \phi}=1 \qquad \text{.....(25)}$$

And

$$\frac{\partial x^1}{\partial y^1}=\frac{\partial \rho}{\partial r}=\frac{r-R.\cos\theta}{\rho}\quad;\quad \frac{\partial x^1}{\partial y^2}=\frac{\partial \rho}{\partial \theta}=\frac{r.R.\sin\theta}{\rho}\quad;\quad \frac{\partial x^1}{\partial y^3}=\frac{\partial \rho}{\partial \phi}=0$$

$$\frac{\partial x^2}{\partial y^1}=\frac{\partial \delta}{\partial r}=\frac{-R.\sin\theta}{\rho^2}\quad;\quad \frac{\partial x^2}{\partial y^2}=\frac{\partial \delta}{\partial \theta}=\frac{r.(r-R.\cos\theta)}{\rho^2}\quad;\quad \frac{\partial x^2}{\partial y^3}=\frac{\partial \delta}{\partial \phi}=0$$

$$\frac{\partial x^3}{\partial y^1}=\frac{\partial \phi}{\partial r}=0\quad;\quad \frac{\partial x^3}{\partial y^2}=\frac{\partial \phi}{\partial \theta}=0\quad;\quad \frac{\partial x^3}{\partial y^3}=\frac{\partial \phi}{\partial \phi}=1 \qquad \text{.....(26)}$$

3.6 From equations (19) the effective physical strain components due to the positron located at point O and referred to coordinate system (y^i) are given as,

$$S^r{}_r(O)=\frac{1}{\sqrt{2}}.\frac{A_e.e}{r^2}=S^\theta{}_\theta(O)=S^\phi{}_\phi(O)\;;\qquad S^r{}_\phi(O)=\frac{1}{\sqrt{2}}.\frac{A_e.e.Sin(\theta)}{r^2}=S^\phi{}_r(O)$$

$$S^\theta{}_\phi(O)=\frac{1}{\sqrt{2}}.\frac{A_e.e.Cos(\theta)}{r^2}=S^\phi{}_\theta(O)\;; \qquad \text{.....……(27)}$$

Similarly the effective physical strain components due to the positron located at point A and referred to coordinate system (x^i) are given as,

$$\in^\rho{}_\rho(A)=\frac{1}{\sqrt{2}}.\frac{A_e.e}{\rho^2}=\in^\delta{}_\delta(A)=\in^\phi{}_\phi(A)\;;\qquad \in^\rho{}_\phi(A)=\frac{1}{\sqrt{2}}.\frac{A_e.e.Sin(\delta)}{\rho^2}=\in^\phi{}_\rho(A)$$

$$\in^\delta{}_\phi(A)=\frac{1}{\sqrt{2}}.\frac{A_e.e.Cos(\delta)}{\rho^2}=\in^\phi{}_\delta(A)\;; \qquad \text{.....……(28)}$$

Before using equation (23) for transferring the strain components (28) from coordinate system (x^i) to the coordinate system (y^i), we have to first convert these physical components to the corresponding mixed tensor $\in^i{}_j(A)$ by using the relation (10). After thus obtaining $\in^i{}_j(A)$, we use equation (23) to transform $\in^i{}_j(A)$ to $S^i{}_j(A)$. The mixed tensor components $S^i{}_j(A)$ thus obtained are again converted to the corresponding physical components by using equation (10) in coordinate system (y^i) to finally obtain

$$S^{r}_{r}(A) = \frac{1}{\sqrt{2}}.\frac{A_e.e}{\rho^2} = S^{\theta}_{\theta}(A) = S^{\phi}_{\phi}(A)\ ; \qquad S^{r}_{\phi}(A) = \frac{1}{\sqrt{2}}.\frac{A_e.e.Sin(\delta)}{\rho^2} = S^{\phi}_{r}(A)$$

$$S^{\theta}_{\phi}(A) = \frac{1}{\sqrt{2}}.\frac{A_e.e.Cos(\delta)}{\rho^2} = S^{\phi}_{\theta}(A)\ ; \qquad \text{.....……(29)}$$

3.7 Interaction Energy Density. At any point P, the strain field energy density due to the positrons located at points O & A is obtained from equation (12) as,

$$W_O = \frac{1}{2.\varepsilon_0}.\sum\left|S^{y^i}_{y^j}(O)\right|^2 \ ; \qquad W_A = \frac{1}{2.\varepsilon_0}.\sum\left|S^{y^i}_{y^j}(A)\right|^2 \qquad \text{……(30)}$$

However, due to the superposition effect, the energy density of the combined field of both positrons located at points O and A is given by,

$$W_C = \frac{1}{2.\varepsilon_0}.\sum\left|S^{y^i}_{y^j}(O) + S^{y^i}_{y^j}(A)\right|^2 \qquad \text{……..(31)}$$

Therefore the interaction energy density W_{int} will be given by,

$$W_{int} = W_C - W_O - W_A = \frac{1}{\varepsilon_0}.\sum\left[S^{y^i}_{y^j}(O).S^{y^i}_{y^j}(A)\right] \qquad \text{……..(32)}$$

This interaction energy density can now be computed from summation of the product of corresponding physical strain components given by equations (27) and (29), which after a little simplification comes out to be,

$$W_{int} = \frac{(A_e.e)^2}{2\varepsilon_0}.\left[\frac{5}{r^2.\rho^2}\right] \qquad \text{……(33)}$$

The total interaction energy E_{int} can now be obtained from volume integral of W_{int} over the entire field of the two interacting charges.

3.8 On the other hand if we compute the interaction between a positron and an electron located at points O and A respectively, all the effective strain components given by equations (28) and (29) will be of opposite sign thereby attaching a negative sign to the W_{int} of equation (33). That is, the interaction energy E_{int} of two opposite charges will be negative. For $R > 0$, the magnitude of E_{int} will be less than twice the field strain energy E_f of one charge and residual field strain energy of the combined field E_C will be greater than zero. Ordinary bulk matter is considered electrically neutral. However, since all negative charges are not completely superposed over positive charges, residual field strain energy E_C of the bulk matter will be finite and greater than zero. This residual field energy might explain the origin of gravitational phenomenon in bulk matter.

3.9 Total Interaction Energy Corresponding to the field interaction energy density W_{int} given by equation (33), the total interaction energy E_{int} can be computed by taking volume integral of W_{int} over the entire field of the interacting positrons as,

$$E_{int} = \int_{\phi=0}^{2\pi}\left[\int_{\theta=0}^{\pi}\int_{r=0}^{\infty} W_{int}.r^2.\sin\theta.dr.d\theta\right]d\phi \qquad \text{…….. (34)}$$

Introducing a dimensionless parameter y = r/R in equations (33), (34) and (24B) we get,

$$E_{int} = 2\pi.R^3.\int_{\theta=0}^{\pi}\int_{y=0}^{\infty} W_{int}.y^2.\sin\theta.dy.d\theta = \frac{2\pi.(A_e.e)^2}{2\varepsilon_0.R}.\int_{\theta=0}^{\pi}\int_{y=0}^{\infty}\left[\frac{5.\sin\theta}{(y^2+1-2y.\cos\theta)}\right].dy.d\theta$$

$$= \frac{5\pi.(A_e.e)^2}{\varepsilon_0.R}.\int_{\theta=0}^{\pi}\left[\tan^{-1}\left(\frac{y-\cos\theta}{\sin\theta}\right)\right]_0^{\infty}d\theta = \frac{5\pi.(A_e.e)^2}{\varepsilon_0.R}.\int_{\theta=0}^{\pi}(\pi-\theta)d\theta$$

$$= \frac{5\pi^3.A_e^2.e^2}{2\varepsilon_0.R} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(35)$$

This verifies the Coulomb interaction law between two positrons as also between two electrons. Similarly the interaction energy between an electron and a positron can be shown to be given by equation (35) but with a negative sign. Mutual force between two charges (electrons or positrons) is given by the negative derivative of equation (35) with respect to relative separation R between them.

4. SALIENT PARAMETERS OF THE ELECTRON

4.1 Core Size and Oscillation Frequency. Comparing the interaction energy given by equation (35) with the Coulomb interaction energy or the so called Coulomb potential energy of $e^2/(4\pi\varepsilon_0 R)$ we can compute the dimensionless constant factor 'A_e', which works out to about (1/31.21) . Substituting this value of A_e in the total energy relation (18), with known mass of the electron $\mathbf{m_e} = 9.109\times10^{-31}$ kg, we obtain the value of κ to be equal to 1.73767×10^{15} m^{-1}. Finally, from the relation $x = \kappa.r_c = b_1$ for the core boundary, the electron and positron core radii are found to be $r_c = 1.61 \times 10^{-15}$ m. Beyond this core boundary of 1.61 f radius, the strain wave field of the electron extends to infinity and accommodates about 35 percent of its total mass energy. The characteristic frequency of oscillations of the electron/positron core as well as their wave fields, is given by $\nu_e = \kappa.c/2\pi = 8.291\times10^{22}$ Hz. This frequency plays a unique role in all charged particle interactions. The electron/positron type strain bubbles will be able to interact only with those stable/unstable strain bubbles, whose characteristic oscillation frequency matches with ν_e . That means the nucleons and all other charged particles must also be oscillating with this characteristic frequency ν_e .

4.2 Moment of Inertia of the Electron Core. As pointed out above, about 5.88×10^{-31} kg mass of electron is contained in the core region and about 3.229×10^{-31} kg of its mass is spread out in the radial wave field. As an obvious extension of the notion of mass-energy equivalence, we can associate the property of inertia to the total strain energy content as well as the strain energy density in any strain bubble. The inertial property of electron field energy density may have very important consequences. To begin with, we can easily calculate the moment of inertia I_{ec} of the electron core about the Z-axis, by using the mass density (W_c/c^2) from equation (17A), which works out to 5.98×10^{-61} kg m^2. Similarly, if we attempt to work out the moment of inertia for the electron mass spread out in its field, it tends to infinity.

4.3 Mechanical Spin & Magnetic moment of Electron. The electron core with its finite mass and moment of inertia, can be easily set in translational or rotational motion through various interactions. However, due to the inertial property, the strain field energy will tend to lag behind the moving core. This inertial lag of the 'field' is generally visualized for the translational motion as a consequence of finite velocity 'c' of spread of 'electromagnetic field' and gives rise to the well known magnetic field around charge particles in motion. But in the case of rotational motion of the electron core induced by certain interactions, the surrounding radial strain wave field will tend to lag behind due to inertia or equivalently the finite velocity 'c' of phase waves. The rotational motion of the electron core about its axis may be termed as 'mechanical spin' of the electron. Angular lag of the rotating strain wave field, associated with the mechanical spin of the electron, may give rise to the axial magnetic field and the already familiar magnetic moment of the electron. The effective total moment of inertia of the core and the deformed field may also be much higher.

5. SUMMARY & CONCLUSION

5.1 Based on the Elastic Continuum Theory, we have introduced a new model for the structure of the electron core and its wave field derived from spherically symmetric solutions of equilibrium equations of elasticity in the Continuum. The mass of electron, as of any other particle, is shown as the inertial equivalent of the total strain energy content 'locked up' in the oscillating strain bubble. The charge of positron / electron has been associated with the direction of propagation, outwards or inwards, of the strain phase waves linked with the oscillating core. The 'intrinsic spin' of electron is shown to be the result of rotation of the displacement vector **U** at a characteristic angular frequency κc. The Coulomb interaction has been derived from the superposition of strain wave fields of the interacting particles, resulting in overall change in combined field energy. Salient parameters of the electron have also been computed. The paper represents an important application of ECT.

References

[1] Elastic Continuum Theory of Electromagnetic Field & Strain Bubbles by the Author