Installation and Operation Manual
ProSeries Model SPS390
Dynamic Signal Analyzer
Part Eight

Legacy Manual

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This glossary contains definitions of common terms used in the collection and analysis of digital data relating to the dynamic properties of mechanical systems. Such data may include strain, vibration, sound pressure variations, system electrical characteristics, and other measurements well-suited to the SPS390. The glossary is based upon a list of terms and definitions compiled by the Institute of Environmental Sciences.

ACCELERANCE. *See* INERTANCE, MECHANICAL IMPEDANCE.

ACCELERATION SRS. *See* SHOCK RESPONSE SPECTRUM.

ACCURACY. The degree of conformity of a measure to a model, standard, or true value. Usually expressed as a percentage of a specified value. Should consider other dependent factors such as temperature, frequency, relative value (full scale vs. absolute), drift rates, range, etc.

ALIASING. When the sample rate f is less than two times the highest frequency in the data, the frequency is ambiguously represented. The frequencies above $f/2$ will be folded back into the lower frequencies to produce erroneous results.

AMPLITUDE DENSITY FUNCTION. *See* PROBABILITY DENSITY FUNCTION.

AMPLITUDE DISTRIBUTION FUNCTION. *See* PROBABILITY DISTRIBUTION FUNCTION.

ANALYSIS RANGE/BANDWIDTH. *See* FREQUENCY RANGE.

ANTI-ALIASING FILTER. *See* FILTER, ANTI-ALIASING.

APPARENT WEIGHT/APPARENT MASS. A term sometimes used for the preferred term EFFECTIVE MASS (*See* MECHANICAL IMPEDANCE).

APERTURE ERROR, *See* ERROR, APERTURE.

ARGAND DIAGRAM, *See* NYQUIST DIAGRAM.

ARRAY. An ordered set of numbers; in a sampled data system discussed in this glossary (*See* SAMPLING) they are equally spaced in time or frequency.

AUTOCORRELATION FUNCTION. A measure of the similarity that a function has with a time-displaced version of itself, as a function of the time displacement or lag. When the lag is zero, the value of the autocorrelation is equal to the mean square value of the function. The sample autocorrelation function $\hat{R}_x(j)$ of the sequence x_i is usually computed

$$\hat{R}_x(j) = \frac{1}{N-j}\sum_{i=1}^{N-j} x_i\, x_{i+j} \qquad j = 0, 1, \ldots, m$$

where

x_i = the function x(t) sampled at the time $i\Delta t$
Δt = the sample interval
$j\Delta t$ = the lag
N = the number of samples of x
$m\Delta t$ = the maximum lag

For a discrete "stationary" random quantity where N must be finite, the formula gives an estimate only with a statistical uncertainty which increases as N decreases. It is used to identify periodic components in the sample function x_i.

AUTOSPECTRAL DENSITY FUNCTION. *See* POWER SPECTRAL DENSITY FUNCTION.

AVERAGE, ARITHMETIC. An averaging technique which indicates the average value after N summations:

$$x = \frac{1}{N}\sum_{N-1}^{0} x_i$$

Note that Σx_i steadily grows until i = N, at which point it is divided by N.

AVERAGE, ENSEMBLE. A technique for averaging a set of data records. Each record (a member of the ensemble) is sampled at the same instant of time; the sampled values are then averaged. For an ergodic random process (*See* ERGODIC PROCESS) an ensemble average is equivalent to the time average along any record of the ensemble.

AVERAGE, EXPONENTIAL. An averaging technique which weights newer information more heavily than old. Computed as:

$$\hat{x}_{i+1} = \hat{x}_i + \frac{x_N - \hat{x}_i}{K}$$

where $\hat{x}_i$ = old average, x_N = latest value, K = averaging weighting factor.

AVERAGE, STABLE. An averaging technique which indicates the estimated value at each measurement, as opposed to an arithmetic average which gives the values only after division by the number of averages, Computed as

$$\hat{x}_{i+1} = \hat{x}_i + \frac{x_N - \hat{x}_i}{i+1}$$

where

$\hat{x}_i$ = old average,
xN = latest value (N = 1 to M)
i = current count index (O to M)
M = total number of averages

BANDPASS FILTER. *See* FILTER, BANDPASS.

BANDREJECT FILTER. *See* FILTER, BANDREJECT.

BANDWIDTH, 3-dB (B_{3dB}). The interval between the upper and lower frequencies at which the filter attenuates an applied signal by 3 dB (*See* DECIBEL) with respect to the gain at the center frequency of the filter.

BANDWIDTH, EQUIVALENT NOISE *See* EFFECTIVE (NOISE) FILTER BANDWIDTH.

BANDWIDTH, REAL TIME. The range of frequencies over which a real time analysis can be performed (*See* REAL TIME ANALYSIS).

BASEBAND ANALYSIS. *See* ZOOM ANALYSIS. Analysis over a frequency range from zero to a specified upper frequency.

BESSEL FILTER. *See* FILTER, BESSEL.

BIAS. A tendency of a statistical estimate to deviate in one direction away from a true value.

BLOCK. *See* FRAME, MEMORY LENGTH.

BLOCK SYNCHRONOUS. A data sampling and/or processing technique in which the beginning or ending of a data block is synchronized with an external event.

BLOCKED CONSTRAINTS. *See* MECHANICAL IMPEDANCE /MOBILITY.

BOXCAR FUNCTION. *See* WINDOW.

BUTTERWORTH FILTER. *See* FILTER, BUTTERWORTH.

CALIBRATION FACTOR (also called SCALE FACTOR). Data acquired by an ADC is in terms of binary counts representing some fraction of a specified full-scale voltage input. The calibration factor is the multiplier that must be applied to the digitized (*See* DIGITIZE) value to convert it to engineering units. For example, an acceleration signal of 5-g/v is to be digitized by a 12-bit (11 bits + sign) ADC having a full input of 10 V. The Calibration Factor is

$$CF = 5\frac{g}{volt} x \frac{10V}{2^{11} counts} = 24.414x10^{-3} g / count$$

An ADC count of 1562 would then be equivalent to

g = (1562) x (24. 414 x 10 3) = 38. 135g

CENTER FREQUENCY. A characteristic of a bandpass or band reject filter or a constant percentage filter of these types, the center frequency is the geometric mean of the upper (f_U) and the lower (f_L) cutoff frequencies:

$$f_C = \sqrt{f_L f_U}$$

For a constant bandwidth filter, the center frequency is the arithmetic mean (*See* AVERAGE, ARITHMETIC) of the upper and lower frequencies:

$$f_c = \frac{1}{2(f_L + f_U)}$$

CHEBYSHEV FILTER. *See* FILTER, CHEBYSHEV.

CHI SQUARE. *See* GOODNESS OF FIT.

CIRCULAR CONVOLUTION (PERIODIC CONVOLUTION). The periodic convolution z_i of two time sequences x_i and y_j (i = 0, 1, 2, . . . N-1) is defined as

$$z_i = \sum_{m=0}^{N-1} \overline{x}_m y_{i-m}$$

where $\overline{x}_m$ is a periodic extension of the sample set x_m, i. e., $\overline{x}_{i+kn} = x_i$ (n = 0, 1, 2,. . . N-1) (k = 0, ±1, ±2. . .). The inverse discrete Fourier transform (DFT^{-1}) of the product of two DFTs is a periodic convolution of the original time sequences (*See* DFT and WRAP-AROUND ERROR) .

CIRCULAR CORRELATION. A correlation function computed in terms of a circular convolution.

CLIPPING. The term applied to the generally undesirable (but sometimes intentional) circumstance when an output signal is limited in some sense by the full-scale range of an amplifier, ADC, or other device. Clipping may be hard, that is, when the signal is strictly limited at some level; or it may be soft, in which case the clipped signal continues to follow the input at some reduced gain (*See* GAIN).

CO (COINCIDENT, REAL). The real part of a complex function, or the component that is in phase with the input excitation. In frequency domain analysis, the coincident terms are the cosine terms of the Fourier transform (*See* FREQUENCY DOMAIN, FOURIER TRANSFORM) .

COHERENCE FUNCTION. A frequency domain function generally computed to show the degree of a linear, noise-free relationship between a system input and output. Values of coherence satisfy the relationship

$0 < \gamma 2(f) < 1$,

where a value of 0.0 indicates no causal relationship between an input and the output, and 1.0 indicates the existence of linear noise-free frequency response function between input and output (*See* FREQUENCY DOMAIN).

COHERENCE, MULTIPLE. For a system having multiple inputs $x_i(t)$ and one output y(t), the multiple coherence represents the fraction of power in the output accounted for by simultaneous linear filter relationships with all the inputs. This coherence function obeys the usual inequality, and will be unity under noise-free ideal conditions when there is a true linear relationship occurring in a multiple-input/single-output system.

COHERENCE, ORDINARY. For a system having a single input x(t) and output y(t), the ordinary coherence is defined as

$$\gamma^2_{xy}(f) = \frac{|G_{xy}(f)|^2}{G_x(f)G_y(f)}$$

where $G_x(f)$, $G_y(f)$ = power spectral density functions of x(t), y(t)

$G_{xy}(f)$ = cross spectral density function between x(t) and y(t)

For a linear system, $\gamma^2_{xy}(f)$ can be interpreted as the fractional portion of the power output y(t) which is contributed by the input x(t) at frequency f. The coherence function is a measure of the statistical validity of the transfer function estimate. A value of $\gamma^2_{xy} < 1.0$ indicates the existence of a nonlinear system, the presence of extraneous noise, or the existence of other uncorrelated inputs. Note that the coherence function is independently normalized at each frequency and is therefore independent of the shape of the frequency response function between measurement points (*See* FREQUENCY RESPONSE FUNCTION).

COHERENCE, PARTIAL. For a system having multiple inputs $x_i(t)$ and one output y(t), the partial coherence is the coherence computed between any individual input and the output when the effect of all other inputs is removed from the output by a linear least squares prediction. This coherence obeys the usual inequality, and will reveal the existence of a linear relationship between a particular residual input and the output even when the relationship is not apparent from the ordinary coherence function (*See* LEAST SQUARES PREDICTION) .

COMPLEX CONJUGATE. The result of multiplying the imaginary part of a complex quantity by −1 (*See* QUAD and COMPLEX FUNCTION).

COMPLEX DEMODULATION. (*See* ZOOM ANALYSIS).

COMPLEX FUNCTION. A complex function is any mathematically defined relationship of the form

y(x) = a (x) + ib(x) ,

where

x = real variable,
a(x) = the real part of y(x), a = Re{y(x)},
b(x) = the imaginary part of y(x), b = Im {y(x) }, and
i = √-1

Complex functions are often represented in terms of their amplitude

|y(x)| = [a2(x) + b2(x)]1/2

and phase

φy (x) = tan-1 [b(x)/a(x)]

CONFIDENCE BANDS. *See* CONFIDENCE INTERVAL.

CONFIDENCE INTERVAL. The range with a specified value of uncertainty within which the true value of a measured quantity will lie. For example, consider a random variable X with a true mean of x. Note that the true mean is never known, but can only be estimated. An estimate of the mean, x_{bar}' is then made with a 90% confidence interval of +a. The confidence statement means that if we made 100 such independent estimates of x, about 90 would be in the range of x±a, and about 10 would lie outside this range. Note that the confidence statement does not state how close a particular estimate x is to the true value x (a is a function of the numbered samples and the variance of the samples).

CONSTRAINTS, BLOCKED and FREE. (*See* MECHANICAL IMPEDANCE/MOBILITY.

CONTINUOUS SPECTRUM. The type of spectrum produced from nonperiodic data. The spectrum is continuous in the frequency domain (*See* LINE SPECTRUM, FREQUENCY DOMAIN).

CONVOLUTION. A mathematical concept. The convolution of continuous functions x(t) and y(t) is defined as

$$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

Let W(f),X(f), Y(f), and Z(f) be the Fourier transforms of w(t), x(t), y(t), and z(t), respectively. It can be shown that if z(t) = x(t)*y(t), then

Z (t) = X(f) · Y(f)

and if W(f) = X(f)*Y(f), then

x(t) · (t) = w(t).

If x(t) is the input to a linear system whose impulse response is h(t), then the output of the system is the convolution x(t)* h(t), (See CIRCULAR CONVOLUTION).

CORRELATION FUNCTION. *See* AUTOCORRELATION FUNCTION; CROSS CORRELATION FUNCTION.

COVARIANCE. A statistical concept. The covariance of two random variables, x and y, is defined as

$$Cov(x, y) = E[(x - E[x])(y - E[y])] = E[x, y] - E[x]E[y]$$

where

E[ ] is the expected value (mean) of the quantity in brackets.

The covariance is a measure of the correlation of the two variables.

If $Cov(x, y) = \sigma x \sigma y$

where $\sigma x \sigma y$ are the standard deviations of x and y, the signals are fully correlated. If

$Cov(x,y) = 0$, the signals are uncorrelated. A normalized quantity, ρxy,

where

$$\rho_{xy} = \frac{Cov(x, y)}{\sigma_x \sigma_y}$$

is called the correlation coefficient.

CRITICAL DAMPING COEFFICIENT. Critical damping is the smallest amount of damping at which a system will respond to a step function without overshoot (*See* DAMPING). The critical damping coefficient for a linear, viscously damped, single-degree-of-freedom mechanical system is defined as

$$C_c = 2\sqrt{km} = 4\pi m f_n$$

where

k = spring constant
m = mass
fn = undamped natural frequency.

(See DEGREES OF FREEDOM: VISCOUSLY DAMPED).

CROSS CORRELATION FUNCTION. A measure of the similarity of two functions with the time displacement (lag) between them used as an independent variable. The sample cross correlation function R (j) between two sequences x_i and Y_i is usually computed as:

$$\hat{R}_{xy}(j) = \frac{1}{N-j} \sum_{i=N-j-1}^{0} x_i y_{i+j} \qquad j = 0, 1, \ldots, m$$

where

xi, yi, $\hat{R}_{xy}(i)$	= the respective functions sampled at the time $i\Delta t$
Δt	= the sample interval
$j\Delta t$	= the lag
N	= the number of samples of x and y
$m\Delta t$	= the maximum lag.

For discrete "stationary" random quantities where N must be finite, the formula gives an estimate only with a statistical uncertainty which increases as N decreases.

CROSS SPECTRAL DENSITY FUNCTION. A measure in the frequency domain of the similarity of two functions. It is usually computed from the Fourier transforms of two discrete functions x_i and y_i to

$$\tilde{G}_{xy}(k) = \frac{2}{-}\tilde{G}_{xy} = \frac{2\Delta t}{N}[X_k^* Y_k]$$

where

Δt	= sample interval,
N	= the frame size of xi and Yi,
Xk*	= complex conjugate of Xk, and
Xk, Yk	= the discrete Fourier transform of xi, y

$\tilde{G}_{xy}$ is a "raw" cross spectral estimate. The cross spectral density is then estimated by averaging M frames of $\tilde{G}_{xy}$. It can also be computed from the "cross-correlation function" as

$$G_{xy}(k) = 2F[R_{xy}(j)]$$

where

F[ ] = Fourier transform of quantity within brackets
Rxy(j) = cross-correlation of xi and Yi

For discrete "stationary" random quantities, the formulas give an estimate only with a statistical uncertainty (error) which increases as M decreases. Typically, the data is also multiplied by a window and this will affect the effective resolution of the estimate. (See FOURIER TRANSFORM WINDOW).

CUMULATIVE PROBABILITY DISTRIBUTION FUNCTION. *See* PROBABILITY DISTRIBUTION F UNCTION.

CURVE FITTING. The process whereby coefficients of an arbitrary function (usually a polynomial) are computed such that the function approximates the values in a given data set. A mathematical function, such as the minimum mean squared error, is used to judge the goodness of fit (*See* MEAN SQUARED ERROR, TEST).

CUTOFF FREQUENCY. The frequency at which the rolloff skirt of the filter shape is down from the nominal unity gain passband level by a specified amount,

DAMPING. Dissipation of energy.

DAMPING FACTOR. *See* DAMPING RATIO,

DAMPING RATIO. For a linear, viscously damped, single-degree-of-freedom, mechanical system, the ratio of the actual damping coefficient, C, to the critical damping coefficient, C_c is called the damping ratio ζ.

where

$$\zeta = \frac{C}{C_c} = \frac{1}{2Q} \approx \frac{1}{2}\frac{B_{3dB}}{f_n}$$

$$C_c = 2\sqrt{km} = 4\pi\ mf_n$$

k	= spring constant,
m	= mass,
fn	= undamped natural frequency
Q	= Quality factor, and
B3dB	= half power frequency bandwidth,

(See VISCOUSLY DAMPED; DEGREES OF FREEDOM; DAMPING).

DECIBEL (dB). A measurement unit which denotes the ratio of the magnitude squared of a quantity with respect to an arbitrarily established reference value of the quantity, expressed as 10 times the logarithm to the base 10 of the ratio of the quantities, e. g.,

$$dB = 10\log_{10}\frac{V^2}{V_{ref}^2} = 20\log_{10}\frac{V}{V_{ref}}.$$

DECIMATION (of sampled data), A process used to decrease the amount of data for analysis. jth order decimation consists of keeping only every jth sample, discarding the remaining samples .

DEGREES OF FREEDOM. The minimum number of independent generalized coordinates required to define completely the positions of all parts of the system at any instant of time.

DELAY. In reference to filtering, refers to the time lag between the filter input and output. Delay shows up as a frequency-dependent phase shift between output and input, and depends on the type and complexity of the filter.

DELAY TIME. *See* TIME LAG.

DELTA F. *See* RESOLUTION.

DELTA FREQUENCY. *See* FREQUENCY INCREMENT.

DELTA FUNCTION. *See* IMPULSE FUNCTION.

DELTA TIME. *See* SAMPLING INTERVAL.

DETERMINISTIC FUNCTION. A function whose future values can be predicted on the basis of some known rules and of its past values.

DETERMINISTIC SYSTEM. Systems whose behavior can be described by an explicit mathematical relationship; that is, given the present state of a system, its exact state at any instant of time in the future can be calculated.

DFT, Discrete Fourier Transform (*See* FOURIER TRANSFORM).

DFT^{-1} , Discrete Inverse Fourier Transform (*See* FOURIER TRANSFORM).

DIFFERENCE SPECTRUM, (*See* ERROR SPECTRUM).

DISPLACEMENT SRS. *See* SHOCK RESPONSE SPECTRUM.

DISTRIBUTION FUNCTION. *See* PROBABILITY DISTRIBUTION FUNCTION.

DOMAIN OFF. *See* FUNCTION.

DOUBLE LENGTH TRANSFORM. A 2N point DFT of sample set x_i (i = 0, 1, ... N-1) extended with N zeros. This transform does not add any new information in the frequency domain, but is sometimes required to prevent circular convolution errors. Sometimes refers to a technique for obtaining the Fourier transform of a time series of length 2N by utilizing an N point fast Fourier transform algorithm (*See* FREQUENCY DOMAIN; CIRCULAR CONVOLUTION; FOURIER TRANSFORM).

DYNAMIC MASS. *See* MECHANICAL IMPEDANCE.

DYNAMIC RANGE. For spectrum measurements, the difference, in dB, between the overload level and the minimum detectable signal level (above the noise) within a measurement system. The minimum detectable signal of a system is ordinarily fixed by one or more of the following: noise level, low level distortion, interference, or resolution level. For transfer function measurements, the excitation, weighting, and analysis approaches taken can have a significant effect on resulting dynamic-range (*See* PROCESSING GAIN; TRANSFER FUNCTION).

EFFECTIVE MASS. A term sometimes used for the preferred term "dynamic mass" (*See* MECHANICAL IMPEDANCE; DYNAMIC MASS).

EFFECTIVE (NOISE) FILTER BANDWIDTH (Also called EQUIVALENT NOISE BANDWIDTH (ENBW)). The width of an ideal rectangular filter of unity gain having an area equal to the area under the square of the actual filter transmissibility characteristic.

EIGENVALUE/EIGENVECTOR. In structural dynamics the eigenvalues can be interpreted as the modal frequencies and eigenvectors as the "mode shapes." The basic concepts involved in the definition of eigenvalues and eigenvectors is as follows: Let A be an nxn matrix. If a non-zero vector X and a non-zero scalar λ exist, such that

$$AX = \lambda X$$

then λ is said to be an eigenvalue of A, and X an eigenvector. To obtain the eigenvalues, λi , find the roots of the equation

$$\det(A-\lambda\ I) = O,$$

and to find the corresponding eigenvectors Xi solve

$$(A-\lambda i)Xi = O.$$

ENERGY SPECTRAL DENSITY. The square of the magnitude of the Fourier spectrum of a transient waveform. That is, if F(ω) is the Fourier transform of f(t), the energy spectral density, A(ω), is given by

$$A(\omega) = |F(\omega)|2.$$

Note that the energy spectral density is not necessarily related to physical energy (See FOURIER TRANSFORM).

ENSEMBLE AVERAGING. *See* AVERAGE, ENSEMBLE.

ERGODIC PROCESS. A random process is ergodic if all its statistics can be determined from a single time history of that process. It follows that the statistics of every time history in an ensemble of that process must be the same, and that time averages will equal ensemble averages.

ERGODICITY. Property of an ergodic process.

ERROR, APERTURE. An amplitude error arising from the uncertainty about the exact time when a signal input was at the value represented by the output sample; occurs in an analog-to-digital converter since samples are taken over a finite period of time rather than instantaneously. In general, the aperture is equal to the ADC time and may be reduced by the use of sample and hold circuits. (*See* ERROR QUANTIZATION; JITTER.)

ERROR-SPECTRUM. A spectrum computed that is the difference between the reference spectrum and the control spectrum.

ERROR, MEAN SQUARE. The mean square error, e, of an estimate is the expected value of the difference squared of the true parameter P, and its estimate $\hat{P}$; or

$$e^2 = E[(P - \hat{P})^2].$$

The estimate, $\hat{P}$, is usually a function of the record length, T, (the 'sample interval' times the number of samples) and the mean square error will approach zero as T approaches infinity.

ERROR, NORMALIZED SPECTRUM. The error spectrum divided by the reference spectrum (*See* definition).

ERROR, NORMALIZED STANDARD. An error, e_N, defined as the positive square root of the ratio of the mean square error, e, and the square of the parameter being estimated, P; or

$$e_N = \sqrt{\frac{e^2}{P^2}}$$

ERROR, QUANTIZATION. When a continuous time history is sampled, each sample is represented by a word conveying n bits of information. The difference between the continuous function evaluated at the sample time and the sampled representation (n bits of information) is called the quantization error. The maximum quantization error is q/2 where q is the value of the least significant bit. This assumes that the analog-to-digital converter (ADC) comes as close as possible to the correct sample value. If it does not, it is a linearity error, not a quantization error.

ERROR, ROUNDOFF. Error resulting from deleting the least significant digits of a quantity and applying some rule of correction to the part retained.

ERROR, STATISTICAL. The uncertainty in the estimation (computation) of a parameter, such as a power spectral density function, because of the amount of data gathered, the underlying probabilistic nature of the data, and the method used in determining the desired parameter (*See* POWER SPECTRAL DENSITY FUNCTION).

ERROR, TRUNCATION. Error resulting from reducing the precision of the representation of a number. This occurs in a fixed word-length digital computer when a number is converted to its binary form.

EXPONENTIAL AVERAGING. *See* AVERAGE, EXPONENTIAL.

EXTERNAL SAMPLING (EXTERNAL CLOCK). In data acquisition hardware, refers to the use of an externally generated sequence of trigger pulses to control the data acquisition. These pulses are usually related, through a constant, to some physical phenomena such as shaft rotation.

FAST FOURIER TRANSFORM (FFT). One of a collection of algorithms for computing discrete Fourier transforms (DFTs) in an optimum fashion. The fast Fourier transform (FFT) algorithm reduces the number of complex multiplications required to compute the transform. This is accomplished by factoring the weighted trigonometric summations required in the computation of the discrete Fourier transform into a sequence of shorter weighted summations by taking advantage of the periodicities and symmetries of the periodic weighting factors and of the transform itself.

FFT. *See* FAST FOURIER TRANSFORM.

FILTER, ANTI-ALIASING. A filter used to remove the signals whose frequencies exceed one half the sample rate $f_s/2$. Ideal filters are not realizable so that the cutoff frequency, transition band, and out-of-band rejection need to be considered.

FILTER, BAND-PASS. A filter which passes all signal information between two cutoff frequencies and sharply attenuates all signal information outside that range.

FILTER, BAND-REJECT. A filter which attenuates all information between two cutoff frequencies and passes all signal information outside that range.

FILTER, BESSEL. An efficient stable filter characterized by excellent phase linearity at the expense of sharpness of cutoff and passband flatness. Commonly used for shock and other transient waveform phenomenon and often referred to as a linear phase filter.

FILTER, BUTTERWORTH. A filter having flat passband and moderately sharp cutoffs, but also moderately nonlinear phase response. Commonly used for acquisition of data from random processes.

FILTER, CHEBYSHEV. Similar to the Butterworth filter, it achieves faster rolloff near cutoff, but at the expense of significant passband ripple and a nonlinear phase response.

FILTER, DIGITAL. A weighted sum that can be defined as

$$g_m = \sum_{n=-N}^{N} b_n f_{m-n} - \sum_{n=1}^{N} a_n g_{m-n}$$

where

gm are the filter output samples,
fm are the filter input samples, and
bn and an are the filter weights or coefficients.

If all the weights a are zero, the filter is called a nonrecursive filter. If one or more of the weights a_n are nonzero, the filter is called a *recursive* filter. If any of the weights b_n are nonzero for n < 0, the filter is said to be *nonrealizable* since future values of the input samples, f_m , will be required. If all the filter weights b_n are zero for n < 0, the filter is said to be *realizable*. If we define a *digital unit impulse* as

d = 0 n≠ 0,

dn = 1/Δt .

where Δt is the sampling interval, then the response of a digital filter to this impulse is called the *impulse response* of the filter. The impulse response for any nonrecursive filter will be finite for any finite N. Hence, nonrecursive filters are sometimes called *finite impulse response* filters. Recursive filters are called *infinite impulse response* filters because the impulse responses are not necessarily finite.

FILTER, GAUSSIAN. A low-pass filter with maximally linear phase response at the expense of sharpness of cutoff and passband flatness. Commonly used for shock and other transient waveform phenomenon (*See* FILTER LOW-PASS).

FILTER, HIGH-PASS. A filter which passes all signal information above the cutoff frequency, and sharply attenuates everything below that frequency.

FILTER, LOW-PASS. A filter which passes all signal information below the cutoff frequency, and sharply attenuates everything above that frequency.

FILTER, NONRECURSIVE. *See* FILTER, DIGITAL.

FILTER RINGING. The tendency of nonlinear phase filters to exhibit lightly damped oscillatory outputs in response to an impulsive input.

FILTER RIPPLE. The peak-to-peak variation, in dB, of the gain of the filter in the passband.

FILTER ROLLOFF RATE. The best straight line fit to the slope of the filter transmissibility characteristic in the transition band, usually expressed in dB per octave.

FILTER SHAPE. *See* FILTER TRANSMISSIBILITY CHARACTERISTIC.

FILTER SHAPE FACTOR. *See* FILTER TRANSMISSIBILITY CHARACTERISTIC.

FILTER TRANSITION BAND. The difference in frequency between the filter cutoff frequency and the point at which the gain reaches the first peak of the out-of-band ripple.

FILTER TRANSMISSIBILITY CHARACTERISTIC (FILTER SHAPE). The magnitude of the frequency response function of a filter relating the output to the input of the filter.

FINITE IMPULSE RESPONSE FILTER. *See* FILTER, DIGITAL.

FIXED POINT. A number represented in such a way that the binary or decimal point is fixed with respect to one end of the numerals.

FLOATING POINT, BLOCK. A set of numbers represented in memory by their mantissa and 2 common exponents for the block of numbers. Often used in FFTs to make better use of the memory word size when the incoming data consist of small integers. (*See* FFT).

FLOATING POINT NOTATION. A technique for representing data values as a digital word. Each datum is divided into four fields: mantissa, mantissa sign, base ten, or base two exponent; and exponent sign. Frequently, a floating point value will use two or more computer words. The precision to which values can be represented is determined by the number of bits allotted to the mantissa field; the dynamic range, by the number of bits in the exponent field.

FOLDING FREQUENCY. Equal to half the sampling frequency, above which higher signal frequencies are folded or aliased back into the analysis band.

FOURIER TRANSFORM. A bilateral transformation typically used to convert quantities from time domain to frequency domain and vice versa, usually derived from the Fourier integral of a periodic function when the period grows without limit, often expressed as a Fourier transform pair. In the classic sense, a Fourier transform takes the form of

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

where

f(t) = continuous time waveform
(ω) = Fourier transform
ω = analysis frequency expressed in radians/sec

In the discrete or sampled sense (See DFT), this can be expressed as

$$F_k = \sum_{n=0}^{N-1} f_n e^{\frac{-j2\pi}{N} kn}$$

where

fn = samples of time waveform
n = running sample index
N = total number of samples or "frame size"
k = finite analysis frequency, corresponding to "FFT bin centers"
Fk = discrete Fourier transform, and can be associated with the frequencies
$\omega k = 2\pi k/(N\Delta t)$.

(*See* INVERSE FOURIER TRANSFORM, TIME DOMAIN, FREQUENCY DOMAIN, DFT, DFT $^{-1}$, FAST FOURIER TRANSFORM.)

FRAME. Discrete set of elements (numbers) representing a time or frequency domain function. The numerical size of the element set is called the frame, block, or record size and is generally a power of 2, such as 64, 128, 256, etc., The term frame length, or block length, is used to describe the length of the element set in seconds or milliseconds and is equal to NΔt where N is the frame size and Δt is the sampling interval (*See* TIME DOMAIN, FREQUENCY DOMAIN, MEMORY LENGTH).

FREE CONSTRAINTS. (*See* MECHANICAL IMPEDANCE/MOBILITY.)

FREE RUNNING. Term used to describe the operation of an analyzer or processor which operates continuously at a fixed rate, not in synchronism with some external reference event. Analyzers, processors and computing systems are often thought to be operating in a triggered, block synchronous, or free running mode of operation.

FREQUENCY CELL. *See* FREQUENCY INCREMENT.

FREQUENCY DOMAIN. Any parameter which is expressed as a function of frequency.

FREQUENCY FUNCTION. *See* PROBABILITY DENSITY FUNCTION.

FREQUENCY INCREMENT (Δf). The spacing between frequency components obtained by performing a discrete Fourier transform operation on a sequence of time samples, which in turn have a sampling interval Δt

$$\Delta f = \frac{1}{T} = \frac{1}{N\Delta t} = \frac{2f_h}{N}$$

where

T	= total sampling interval
N	= total number of time samples
fh	= $1/(2\Delta t)$, highest frequency or Nyquist frequency

FREQUENCY RANGE. The frequency range (bandwidth) over which the performance of the device remains within acceptable limits. Typical analyzers have selectable ranges. As applied to analyzers it usually refers to upper frequency limit of analysis, considering zero as the lower analysis limit (*See* ZOOM ANALYSIS).

FREQUENCY RESPONSE FUNCTION. A property of a physically realizable stable linear system expressed as a function of the frequency ω; defined as the Fourier transform of the impulse response function. Several of its important properties are:

$H(\omega) = Y(\omega) / X(\omega)$,

where $Y(\omega)$ and $X(\omega)$ are the Fourier transform of the output and input to the system.

$H(\omega) = Gxy/Gxx$,

where Gxy Gxx are the cross spectral density between the output and input and the auto spectral density of the input. For a noise-free linear system

$|H(\omega)|2 = Gyy / Gxx$,

where Gyy and Gxx are the auto spectral density functions of the output and input to the system,

$H(\omega) = A(\omega) ej\varphi(\omega)$

where $A(\omega)$ and $\varphi(\omega)$ are the gain and phase of the response of the system to a sinusoidal input at the frequency ω.

The frequency response function is a special case of the transfer function H(s) where the transfer function is evaluated along the line $s = j\omega$.

$H(\omega) = H(s)|s{=}j\omega$.

The frequency response function can replace the transfer function with no loss of useful information.

FUNCTION. A mathematical concept. A variable y is said to be a single-valued function of x, $y = f(x)$, for a certain range of values of x, if to each value of x, one and only one value of y is assigned. In this case, x is termed the independent variable and y, the dependent variable. The set of values over which the function f is defined is known as the domain of f. The set of values taken on by y is known as the range of f.

FUNDAMENTAL. The lowest frequency periodic component present in a complex waveform. At least one complete period of a signal must be present for it to qualify as the fundamental.

GAIN. Increase in signal power.

GAUSSIAN DISTRIBUTION. The samples of a random variable x are said to belong to a Gaussian distribution, provided its probability density function is of the form

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where

σ	= the standard deviation
μ	= the mean

As a result of the central limit theorem, the distribution of a random variable which is the composite of a large number of independent random processes will approach a Gaussian distribution independent of the distribution of the individual random processes.

GAUSSIAN FILTER. *See* FILTER, GAUSSIAN.

GIBBS PHENOMENON. In representing a discontinuous signal, such as a square wave, as a finite Fourier series, the discontinuities exhibit a characteristic 9% overshoot. Phenomenon often appears in trying to synthesize a "perfect" frequency domain filter using FFT techniques, and is minimized by applying various windows to the data (*See* FOURIER TRANSFORM, FREQUENCY DOMAIN, FFT, WINDOW).

GOODNESS OF FIT TEST (CHI-SQUARED). A hypothesis test often used to test the equivalence of a "histogram" of experimental data and the "histogram" of some theoretical "probability density function". The number of observations in each subinterval of the "histogram" for the experimental data (r_i) is compared with the expected value from the theoretical "histogram" (P_i). The result is a probability statement concerning the likelihood that the true distribution of the experimental processes is the same as the theoretical distribution. A measure of the total discrepancy between the experimental and theoretical observations is formed.

$$X^2 = \sum_i (r_i - P_i)^2 / P_i$$

The theoretical distribution which minimizes X2 is assumed to be the most likely distribution of the experimental processes.

HARMONIC. A frequency component which is an integer multiple of the fundamental component of a complex spectrum. Harmonic components often represent unwanted distortion and are classified by their level and harmonic number (multiple of the fundamental), i. e. first, second, third harmonic.

HISTOGRAM. A plot showing the number of occurrences in each interval, used as a discrete approximation of a probability density function. An interval in the range of a variable is divided into subintervals; all the data are examined and the number of occurrences in each interval is then tabulated (*See* PROBABILITY DENSITY FUNCTION).

IMAGINARY. *See* QUAD.

IMPULSE FUNCTION. A mathematical definition of a time pulse with infinitesimal width δ, infinite magnitude $1/\delta$, and unity area. The major attribute of this function is its broad spectrum width which is, theoretically, infinite, making it attractive as a system excitation function to measure frequency response functions. In practice, this function can be approximated, but never fully achieved, due to the finite bandwidth and gain of instrumentation which is used to create and process the impulse.

IMPULSE RESPONSE. *See* FILTER, DIGITAL.

IMPULSE RESPONSE FUNCTION. A characteristic of a linear system representing how the system would respond in the time domain to an input which is a perfect impulse function. Typically calculated as the inverse Fourier transform of the frequency response function. May also be computed through cross-correlation using a white noise excitation source (*See* TIME DOMAIN, FREQUENCY RESPONSE FUNCTION, WHITE NOISE).

INERTANCE. A term sometimes used for the preferred term accelerance (*See* MECHANICAL IMPEDANCE) .

INFINITE IMPULSE RESPONSE FILTER. *See* FILTER, DIGITAL, INVERSE FOURIER TRANSFORM.

INTERPOLATION. Using given data at two or more points to estimate a data value at a point within the range of the given data.

INVERSE FOURIER TRANSFORM. Typically used as the transformation of a frequency domain parameter back to the time domain. Can be defined in the sense of

$$f(t) = \int_{-\infty}^{\infty} F(\omega)e^{+j\omega t}dt$$

where

f(t)	= continuous time waveform
F(ω)	= Fourier transform
ω	= analysis frequency expressed in radians/sec.

In the discrete or sampled sense, this can be expressed as

$$f_n = \frac{1}{N}\sum_{n=0}^{N-1} F_k e^{\frac{+j2\pi}{N}kn}$$

where

fn	= samples of time waveform
n	= running sample index
N	= total number of samples or "frame size"
k	= finite analysis frequency, corresponding to "FFT bin centers"
Fk	= discrete Fourier transform

See FOURIER TRANSFORM.

JITTER. Abrupt and spurious shifts in time, amplitude, frequency, or phase with waveforms of either a pulse or continuous nature. Can also be introduced by design as in the case of sample pulse dither.

KURTOSIS. The fourth moment of the probability density function (or mass function for the discrete case). A measure of the curvature (convexity, concavity) of the function at the mean. A kurtosis of 3.0 is characteristic of a perfect Gaussian distribution.

LEAKAGE. Unwanted spectrum distortion caused by artificial truncation of a sampled data signal. Typically occurs in the computation of discrete Fourier transforms within a finite memory system. Problem is usually reduced by windowing (weighting) the data frame. Worst case occurs when period of a periodic waveform fails to match length of time sampling block.

LEAST SQUARES PREDICTION. A least squares prediction is y(c, t) where c has been chosen to minimize the total square error E where

$$E^2(c) = \int_{-\infty}^{\infty} [x(t) - y(c,t)]^2 dt,$$

where x(t) is some desired function to be approximated by y(c, t), and where c is an adjusted parameter.

LINEAR PHASE FILTER. A filter where the phase shift is a linear function of frequency.

LINE SPECTRUM. The discrete frequency spectrum produced by the analysis of a periodic time function. Typically presented with fixed bandwidth resolution and normally contains neither broadband noise nor transient characteristics. Not necessarily given as a line or bar display.

LINEAR SYSTEM. A system is linear if for every element in the system the response is proportional to the excitation. This definition implies that the dynamic properties of each element in the system can be represented by a set of linear differential equations with constant coefficients and that, for the system as a whole, superposition holds.

MAXIMUM (MAXIMAX) SRS. *See* SHOCK RESPONSE SPECTRUM.

MEAN SQUARE ERROR. *See* ERROR, MEAN SQUARE.

MEAN VALUE. The average of all values.

MECHANICAL IMPEDANCE/MOBILITY. The dynamic characteristics of structures can be determined from frequency response functions in any general form of the complex ratio of a motion measurement (acceleration, velocity, or displacement) and a force measurement. For practical reasons most mechanical impedance/mobility measurements are limited to driving point and response measurements of rectilinear motion. The terminology recommended, as well as the resulting measurements, depend heavily on the boundary conditions and constraints of the system being considered. It is generally assumed that if the energy sources are present, they should be removed and replaced with the source impedance. The boundary conditions imposed on the exterior of the system as a result of its natural placement and usage should be clearly stated. The system constraints are those conditions that are a result of the choice of a matrix modeling and/or external subsystem connection points. The constraint conditions of all points of the system other than the driving point are implied by the definition of the type of measurement being made. If the constraint conditions of all other points in the system cannot be described in either of the following two ways they should be explicitly stated. A point in a system is defined as one of the set of measurement points in a distributed system, or at the nodes of a lumped system.

Blocked Constraints — The driving point is driven with a motion source. The response point is constrained with an infinite impedance and the restraining force is measured. This implies that the response point motion is zero except for the case where the response point and the driving point are the same. All other points on the system not constrained by explicitly-stated boundary conditions are constrained with an infinite impedance, i. e., their motion is zero. This set of constraints is usually very difficult to implement in a test laboratory, but is commonly used in modeling. This measurement is the measurement of blocked impedance.

Free Constraints — The driving point is driven with a force source, and the resulting motion is measured. This implies that the response point motion is measured when the external dynamic force at the response point is zero except for the case where the response point and the driving point are the same. All other points on the system not constrained by explicitly-stated boundary conditions are constrained with the zero impedance, i. e., the applied dynamic force is zero. This is easy to do; it is the measurement of a free mobility.

The recommended terminology for each of these measurements is given below, but because the listed terminology has not been used consistently in the past, a reader should exercise caution in interpreting reported data. The blocked impedance matrix [Z] (a matrix giving the relations between the complete set of measurement/drive points) is needed for modeling but is difficult, if not impossible to obtain by experiment. In contrast the mobility matrix [Y] is easier to measure, but is not as useful in the modeling of a system. However, it can be shown that

$[Y] = [Z]^{-1}$

where $[\ ]^{-1}$ indicates a matrix inversion. Note that the individual elements are not reciprocal except in very special cases.

Quantity*	Symbol	Recommended Terminology Free Constraints	Blocked Constraints
		Mobility Measurements	Impedance Measurements
a/F		Accelerance	Blocked Accelerance
F/a		Dynamic Mass	Blocked Effective Mass
v/F	***Y***	Mobility	Blocked Mobility
F/v	***Z***	Free Impedance	Blocked Impedance
x/F		Dynamic Compliance	Blocked Dynamic Compliance
F/x		Free Dynamic Stiffness	Blocked Dynamic Stiffness

a* - acceleration
F - Force
v - velocity
x - displacement

MEMORY LENGTH (PERIOD). The size of storage, typically expressed in units of time for a specified sampling rate. Usually refers to the input memory section of an FFT processing system. Also, sometimes referred to as block or frame length (*See* FRAME). Defined as the sampling interval (Δt) times the number of samples (N) in the data block. (*See* FFT, SAMPLING INTERVAL.)

MODAL ANALYSIS. Measurement and interpretation of the dynamic properties of a mechanical system, including, but not limited to, the following: frequency response functions; mode shapes; modal stiffness; mass and damping; modal coupling coefficients; and orthogonality (*See* MODE SHAPE, FREQUENCY RESPONSE FUNCTION, MODAL STIFFNESS, DAMPING, MODEL COUPLING).

MODAL DAMPING. The generalized damping coefficient in a set of uncoupled differential equations of motion of a mechanical system (*See* MODAL MASS).

MODAL MASS. The generalized mass coefficient in a set of uncoupled differential equations of motion of a mechanical system. When a linear lumped parameter system has been transformed into a set of uncoupled differential equations of the form

$m_i x_i + c_i x_i + k_i x_i = F e^{j\omega t}$,

mi = the modal mass,

ci = the modal damping, and

ki = the modal stiffness

where

xi	= generalized displacement coordinate of the ith mode
F	= generalized force
ω	= driving frequency
t	= time
j	= $\sqrt{-1}$

MODAL STIFFNESS. The generalized stiffness coefficient in a set of uncoupled differential equations of motion of a mechanical system (*See* MODAL MASS).

MODE SHAPE. The characteristic shape assumed by an elastic structure when vibrating at one of its natural frequencies. The mode shapes are proportional to the eigenvectors which satisfy the system equations of motion when evaluated at one of the roots of the characteristic equations (*See* EIGENVALUES).

MULTIPLE COHERENCE. *See* COHERENCE, MULTIPLE.

NICHOLS DIAGRAM. A plot showing magnitude and phase contours of a system transfer function where the ordinate is the logarithmic loop gain and the abscissa is the loop phase angle. Used in control systems and feedback system design.

NOISE. Any unwanted signal. Can also be used to denote a random signal source (*See* WHITE NOISE).

NONLINEAR SYSTEM. A system which is not linear (*See* LINEAR SYSTEM).

NONRECURSIVE FILTER. *See* FILTER, DIGITAL.

NONREALIZABLE FILTER. *See* FILTER, DIGITAL.

NONSTATIONARY. *See* STATIONARY.

NORMAL DISTRIBUTION. *See* GAUSSIAN DISTRIBUTION.

NORMALIZED ERROR SPECTRUM. *See* ERROR, NORMALIZED SPECTRUM.

NORMALIZED STANDARD ERROR. *See* ERROR, NORMALIZED STANDARD.

NYQUIST DIAGRAM. A polar display in which the real part of a measured frequency response function is plotted versus the imaginary part of the same frequency response function with frequency as the independent parameter. Often used in modal analysis and servo system design. Also referred to as an Argand Diagram (*See* FREQUENCY RESPONSE FUNCTION, MODAL ANALYSIS).

NYQUIST FREQUENCY. One-half the sampling frequency, that is 1/2 of $1/\Delta t$ (*See* SAMPLING INTERVAL). The Nyquist frequency establishes maximum frequency which can be present in the data and still avoid aliasing (*See* SAMPLING FREQUENCY, ALIASING).

OCTAVE. An increase in frequency by a factor of two. The number of octaves, n, between two frequencies, f_1 and f_2 is given by $f_2 = 2^n f_1$ where $f_2 > f_1$.

ON-LINE PROCESSING. *See* REAL TIME ANALYSIS.

ORDINARY COHERENCE. *See* COHERENCE, ORDINARY.

OVERLAP PROCESSING. The concept of performing a new analysis on a segment of data in which only a portion of the signals has been updated (some old data, some new data). (*See* FFT.)

PADDING WITH ZEROS. To append zero values on the end of an array. Used when taking an FFT of an array whose length is not a whole power of two when the algorithm requires a power-of-two array size. Also, used to avoid circular convolution errors (*See* FFT).

PARTIAL COHERENCE. *See* COHERENCE, PARTIAL.

PASSBAND ANALYSIS. Analysis of signals (information) that occur in a known, usually restricted bandwidth. Normally applies to frequency domain analysis which does not include zero. (*See* BASEBAND ANALYSIS, FREQUENCY DOMAIN.)

PEAK CHANNEL HOLD (PEAK HOLD). A frequency domain "averaging" method which saves the highest response measured in each quantized increment during a specified time internal or number of spectral averages. The resultant spectrum is a composite of the highest spectral values measured during the averaging process (*See* FREQUENCY DOMAIN).

PEAK LEVEL HOLD (OVERALL LEVEL). A frequency domain measurement where a series of spectra are compared as to overall level (total power in band of interest, usually rms) and the spectrum with the highest overall level is retained (*See* FREQUENCY DOMAIN, RMS) .

PEAK SPECTRA. A frequency domain measurement where, in a series of spectral measurements, the one spectrum with the highest magnitude at a specified frequency is retained.

PERIODIC CONVOLUTION. *See* CIRCULAR CONVOLUTION.

PERIODIC IN THE WINDOW. Term applied to a situation where the data being measured in a sampled data system is exactly periodic (repeats an integer number of times) within the frame length. It results in a leakage-free measurement in digital analysis instrumentation if a rectangular window is used. Real signals are typically not periodic in the window unless sampling is synchronized to the data periodically.

PERIODIC RANDOM NOISE. A type of noise generated by digital measurement systems that satisfies the conditions for a periodic signal, yet changes with time so that devices under test respond as though excited in a random manner. When transfer function estimates are measured with this type of noise for the excitation, each individual measurement is leakage-free and, by ensemble averaging, the effects of system nonlinearities are reduced, thus providing benefits of both pseudo-random and true-random excitation.

PERIODIC WAVEFORM. A waveform which repeats itself over some fixed period of time.

PERIODICITY. The r repetitive characteristic of a signal. If the period is T (sec), then this results in a discrete frequency or line spectrum with energy only at frequencies spaced at 1/T (Hz) intervals.

PHASE ANGLE. The time displacement between two sinusoidal quantities measured relating to the time of one complete cycle of the sinusoidal. The phase can be expressed in degrees (or radians) where 360° (or 2π) represents one complete cycle. A phase angle can also be defined as the angle, A, given by

$$A = \tan^{-1} (y / x)$$

where x and y are the real and imaginary parts of a complex number.

POLES. The roots (possibly complex) of the denominator of a ratio of polynomials (possibly complex). They are used in continuous or discrete system transfer function analysis and describe the impulse response characteristics of systems. *See* ZEROS.

POWER. A function which varies as the square of a measured quantity. Power in this sense is not necessarily physical power.

POWER SPECTRAL DENSITY (PSD) FUNCTION. Also called the AUTO SPECTRAL DENSITY. A real-valued continuous function of frequency, presented with frequency on the horizontal axis and density on the vertical axis, which is a special case of a "cross-spectral density function" where the signal is compared with itself. The following is a method for measuring the PSD, which can lead to a definition of PSD: a signal is passed through a contiguous set of bandpass filters with an effective noise bandwidth B. The mean square output of each filter divided by the bandwidth B is measured with an effective averaging time T. A plot of this normalized mean square output as a function of the filter center frequencies is an estimate of the PSD. The PSD can be defined as the limit of this process where the time T is increased without limit and the bandwidth B is reduced to zero.

An equivalent definition is: the Fourier transform of the autocorrelation function. It is also the expected value of the squared magnitude of the Fourier transform of a sample time history divided by the effective (noise) filter bandwidth. Note that for a finite time history the value of the power spectral density can only be estimated. The statistical error and the effective filter shape are essential parts of that estimate.

PRIMARY SRS. *See* SHOCK RESPONSE SPECTRUM.

PROBABILITY DENSITY FUNCTION. An accumulation of data samples plotted as a function of an independent variable of the samples, such as amplitude, and normalized for a specified integral value is termed a histogram. For a time-varying function x(t), the probability density, p(x), can be defined as the limiting case of a histogram as

$$p(x) = \lim_{\substack{\Delta x_i \to 0 \\ N \to \infty}} \frac{N_i}{N \Delta x_i}$$

where

Δxi	= the quantization intervals of x
N	= the total number of samples
Ni	= the number of samples within Δxi .

A term, p(x), the probability density function, is also called the frequency function of a sample distribution. To find the probability that a value falls within a specific range x1 to x2, we integrate through the range. Symbolically,

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} p(x)dx.$$

Since it is certain that every measurement must yield some real value, we must have

$$P(-\infty < x < \infty) = \int_{-\infty}^{\infty} p(x)dx = 1.$$

Note that the probability density function is always positive, being the derivative of the probability distribution function.

PROBABILITY DISTRIBUTION FUNCTION. The probability that the value of a variable x(t) is less than some specific x_v as given by

$$P(x_v) = \int_{-\infty}^{x_v} p(x)dx.$$

From the nonnegative character of the density function p(x), we See that P(x) cannot decrease with increasing x; and also, that $P(-\infty) = 0$ and $P(\infty) = 1$.

PROCESSING GAIN. In a digital Fourier analysis system, the improvement in signal-to-noise ratio between periodic components and broadband noise obtained by transformation to the frequency domain and observation in that domain. The effect is caused by the noise power being spread out over all frequencies while the discrete signal power remains constant at fixed frequencies. Doubling the number of frequency resolution lines provides 3 dB of processing gain; i. e., the noise floor will appear to be reduced by 3 dB in each cell (*See* FOURIER TRANSFORM).

PSD. *See* POWER SPECTRAL DENSITY FUNCTION.

PSEUDO-RANDOM NOISE. A periodic signal generated by repeating a data record consisting of a series of random values. This noise has a discrete spectrum with energy at frequencies spaced at 1/record length (sec).

QUAD (QUADRATURE OR IMAGINARY). The imaginary component of a complex function, the component that is shifted in phase by 90° from the input excitation. In frequency domain analysis, it refers to the magnitude of the sine terms of the "Fourier transform." (*See* FREQUENCY DOMAIN, FOURIER TRANSFORM.)

QUADRATURE SPECTRAL DENSITY FUNCTION. The imaginary (sine) part of the complex valued cross spectral density function (*See* SPECTRAL DENSITY FUNCTION).

QUALITY FACTOR. A measure of damping in a linear, viscously damped, single-degree-of freedom mechanical system. The Q factor is defined as

$$Q = 1/(2\zeta)$$

where ζ= damping ratio.

Q can be approximated by

$$Q \approx \frac{f_n}{B_{3dB}}$$

where

fn = undamped natural frequency
B3dB = half power frequency bandwidth

(*See* DAMPING, VISCOUSLY DAMPED, DEGREES OF FREEDOM.)

QUANTIZATION. A process in which the continuous range of values of an input signal is divided into nonoverlapping subranges, and to each subrange a discrete value of the output is uniquely assigned.

QUANTIZATION ERROR. *See* ERROR, QUANTIZATION.

RANDOM. Describing a variable whose value at a particular future instant cannot be predicted exactly.

RANDOM INPUT, UNCORRELATED. A random input is uncorrelated for a lag t if for every lag greater than t the autocorrelation function is zero. White random noise (equal energy at all frequencies, a constant PSD) will be uncorrelated for all lags greater than zero. (*See* AUTOCORRELATION, WHITE NOISE, PSD.)

RANDOM VIBRATION. Vibration whose instantaneous value cannot be predicted with complete certainty for any given instant of time. Rather, the instantaneous values are specified only by probability distribution functions which give the probable fraction of the total time that the instantaneous values lie within a specified range.

Notes: "random" means not deterministic, "white" means uncorrelated (flat PSD), "Gaussian" describes the shape of the probability density function, and "noise" usually means not the signal, but can mean the random process. These are all different, though related.

RANGE OF F. *See* FUNCTION.

REAL. In a complex signal, the component that is in phase with the excitation. In frequency domain analysis, it is the magnitude of the COSINE terms of the Fourier series, "Coincident, Co", as in CO-QUAD analyzer (*See* FREQUENCY DOMAIN).

REAL FUNCTION. *See* FUNCTION. A function whose range is the subset of real numbers.

REALIZABLE FILTER. *See* FILTER, DIGITAL.

REAL-TIME ANALYSIS. Analysis for which, on the average, the computing associated with each sampled record can be completed in a time less than, or equal to, the record length. In digital analyzers, the functions accomplished during the computing time should be specified; e.g., Fourier transform, calibration, normalizing by the effective filter bandwidth, averaging, display, etc. (*See* FOURIER TRANSFORM).

RECURSIVE FILTER. *See* FILTER, DIGITAL.

REFERENCE SPECTRUM. The desired PSD or SRS of a control signal.

RESIDUE. A coefficient in the numerator of the partial fraction expansion of a transfer function. The residue can be associated with the mode shape.

RESIDUAL SRS. *See* SHOCK RESPONSE SPECTRUM.

RESOLUTION. The discernible difference between one value and adjacent values in a measurement. In a digital signal analyzer it usually refers to the smallest time or frequency increment that can be discerned. In frequency domain measurements the frequency resolution is also called delta f (Δf) and is equal to the analysis bandwidth divided by the number of spectral lines measured. Since only periodic signals can be resolved to within
Δf, the "effective noise bandwidth" of a digital analyzer is probably a more meaningful measure of resolving ability in the frequency domain (*See* FREQUENCY DOMAIN).

RESONANCE. The enhancement of the response of a physical system to an excitation. The resonant frequencies are usually defined as those frequencies where a small change in the frequency of excitation in either direction will cause the system response to decrease. The term resonant frequency is also used (sometimes erroneously) to denote the imaginary coordinate of the poles of a transfer function, the undamped natural frequencies of a system, and the damped natural frequencies.

RINGING, FILTER. *See* FILTER, RINGING

RMS. (ROOT-MEAN-SQUARE). The square root of the average of the square of a function taken through one interval T

$$x_{rms} = \sqrt{\frac{1}{T}\int_{t_1}^{t_1+T} x(t)^2\,dt}$$

T = the averaging time

tl = any arbitrary time

xrms = the root-mean-square value of x(t) .

ROLL-OFF RATE. Usually refers to a filter characteristic. The best straight-line fit to the slope of the "filter transmissibility characteristic" in the "transition band," usually expressed in dB per octave.

ROUND OFF ERROR. *See* ERROR ROUNDOFF.

SAMPLING. The process of obtaining a sequence of instantaneous values of a function at regular or intermittent intervals.

SAMPLING FREQUENCY. The frequency, in samples-per-second, at which analog signals are sampled and then digitized. The inverse of the sampling interval (*See* NYQUIST FREQUENCY).

SAMPLING INTERVAL (Δt). The time between samples in a sampled data system;

$$\Delta t = \frac{T}{N} = \frac{1}{N\Delta f} = \frac{1}{2f_N} = \frac{1}{F_s}$$

where

T	= frame length in sec.
N	= total number of time samples
fN	= Nyquist frequency
Δf	= frequency increment
Fs	= sampling frequency = sampling rate (samples/second).

SAMPLING THEOREM. Shannon's sampling theorem states that for a bandlimited signal, slightly more than two samples per period of the bandlimiting frequency are required to completely reconstruct the continuous signal from its samples. In sampled data systems, the sampling rate must be greater than two times the highest frequency present in the data, to be analyzed without aliasing.

F > 2 fh,

fh = highest frequency of interest.

Fs = sampling frequency

SCALE FACTOR. *See* CALIBRATION FACTOR.

SDOF. SINGLE DEGREE OF FREEDOM (*See* DEGREE OF FREEDOM).

SHOCK RESPONSE SPECTRUM (SRS). The maximum responses of a series of uniformly damped single-degree-of-freedom (SDOF) systems to an applied transient waveform, plotted as a function of the natural frequencies of the SDOF systems.

Primary SRS -- Maximum responses are determined *during* the application of the non-zero input waveform.

Residual SRS -- Maximum responses are determined during the period *after* the input waveform has been removed,

Maximum (Maximax) SRS -- Maximum responses are determined from the maximum values of both the primary and the residual SRS.

Acceleration SRS -- Maximum responses are measured in acceleration units.

Velocity SRS -- Maximum responses are measured in velocity units.

Displacement SRS -- Maximum responses are measured in displacement units.

SIDE LOBE. A secondary response separated by a notch from the main or desired response. Usually refers to a filter spectral shape, particularly in digital filters that have complex structure (many notches and peaks) in the filter transition band (*See* FILTER, DIGITAL).

SIGNAL ANALYSIS. Process of extracting information about a signal's behavior in the time domain and/or frequency domain, Describes the entire process of filtering, sampling, digitizing, computation, and display of results in a meaningful format (*See* TIME DOMAIN, FREQUENCY DOMAIN).

SIGNAL-TO-NOISE RATIO. A measure of signal quality. Typically, the ratio of voltage or power of a desired signal to the undesired noise component measured in corresponding units.

SIGNATURE ANALYSIS. The method whereby a physical process or device is identified in terms of the frequency characteristics of the signal it generates.

SIMULTANEOUS SAMPLE & HOLD. In data acquisition systems, the technique of using separate sample and hold amplifiers for each channel. This allows simultaneous sampling on all channels, thereby eliminating any skew due to use of a multiplexer (*See* SKEW).

SINGLE DEGREE OF FREEDOM. *See* DEGREES OF FREEDOM.

SKEW. Difference between sample times of sampled data in different channels of a data acquisition device. This results from multiplexing a single sample and hold amplifier among the channels (e.g., if 10 channels are scanned in 5 ms, there is a 0.5 ms skew between channels). Can also be due to head misalignment in a tape recorder.

SKEWNESS. The third moment of the probability density function of a random variable. It is a measure of the asymmetry of the function about its mean. A Gaussian distribution has zero skewness.

SMOOTHING. The general process of removing noise from data. Typically, it is performed by filtering a signal (provided that noise components are different in frequency than the desired signal).

SPECTRAL DENSITY FUNCTION. *See* POWER SPECTRAL DENSITY FUNCTION.

STABLE AVERAGING. *See* AVERAGE, STABLE.

STATIONARY (and NON-). A condition of a random process identified by the invariance of its statistical moments with time; if only the first two moments (mean and mean square) do not change, then the random process is weakly stationary; if all moments vary, the process is nonstationary. The nth moment of a random process X is defined as the expected value of X .

STATISTICAL ERROR. *See* ERROR, STATISTICAL.

STOCHASTIC PROCESS. A random process where the current value of the process cannot be deterministically related to the past values.

STRUCTURAL ANALYSIS. The analysis of the loads and motions of the physical structure of a mechanical system. The analysis includes both dynamic and static analysis, although in the context of this document the dynamic analysis is emphasized.

STRUCTURAL IDEALIZATION. *See* FINITE ELEMENT MODELING.

SYNC PULSE. A trigger pulse which is used to synchronize two or more processes.

SYSTEM IDENTIFICATION. The process of modeling a dynamic system and experimentally determining values of parameters in the mathematical model which best describe the behavior of the system.

TAPER. To reduce the data values smoothly as the edge of a data frame is approached to reduce leakage. Another word is usually used with taper to describe the manner in which the data is diminished at the frame edge; for example, cosine taper (*See* WINDOW).

THIRD-OCTAVE ANALYSIS. A spectral analysis where the data are plotted as the rms level in a band of frequencies 1/3-octave wide as a function of the center frequency of the band. The center frequencies, bandwidth, and filter shapes are almost always a set of standard values described in American National Standards Institute S1.6-1967(R.1971) and S1.ll(R.1971). In digital data analysis, a third-octave analysis is usually accomplished in one of two ways: by combining lines of a narrowband analysis, or by filtering the data with a digital filter which simulates the standard third-octave filter shape.

Note that a third-octave analysis of white noise will have an upwards slope of 3dB/octave because of the increasing bandwidths of the filters as filter center frequency is increased (*See* RMS, WHITE NOISE).

TIME DOMAIN. A description of a function as a function of time (*See* TIME HISTORY).

TIME HISTORY (SERIES). A function which describes how a variable changes with time; any function which assigns a value to the dependent variable for each independent value of time. In a sense, this function gives a history in time of the dependent variable. If a function is continuous and band-limited, the function can be sampled to give a sample set which describes the function. This sample set is also called a time history (series).

TIME LAG. The time difference between two signals in correlation analysis when one calculates an integral of the product of one signal and a temporally displaced signal.

TIME RECORD LENGTH. The total length of time over which a time history is observed. This total time may be broken up into several shorter data blocks.

TIME WINDOW. *See* WINDOW.

TRANSDUCER. A device that is actuated by power from one system and supplies power to a second system. In vibration, a transducer usually converts vibrational energy to electrical energy. The voltage output of a vibration transducer is usually proportional to acceleration, velocity, or displacement. The actual power extracted from the vibrating system is usually kept very small.

TRANSFER FUNCTION. A property of a constant parameter linear system expressed in the complex s domain. It is defined as the Laplace transform of the impulse response function. An important property of the transfer function is that it can be expressed as the ratio of the Laplace transforms of the output and input. If the transfer function of a physically realizable stable system is evaluated along the line $s = j\omega$ the resulting function of frequency, ω, is called the "frequency response function."

TRANSIENT. A finite duration change from one steady state condition to a second steady state condition. Usually the initial and final steady state conditions are zero.

TRANSIENT ANALYSIS. The analysis of the data when the excitation of a system is of finite duration; can be used to study the change from one steady-state to a second steady-state condition.

TREND. The deterministic signal when a long-period deterministic signal is added to a time history. Trends are usually unwanted signals which are removed from the data approximating them with polynomials.

TRIGGER. A stimulus that initiates a process.

TRUNCATION ERROR. *See* ERROR, TRUNCATION.

VARIANCE. The square of the standard deviation. The variance of a time history, x(t), is given by

$$Var(x) = \sigma_x^2 = \lim_{T \to \infty} \int_{-\infty}^{T} [x(t) - \mu_x]^2 dt$$

where

μx = the mean value of x(t)
σx = the standard deviation of x(t).

VARIANCE REDUCTION FACTOR. The ratio of the variance of a statistical estimate computed with overlap processing to the variance computed without overlap processing.

VECTOR. A quantity that has magnitude, direction, and sense. A vector is commonly represented by a directed line segment whose length represents the magnitude and whose orientation in space represents the direction and sense.

VISCOUSLY DAMPED. Damping that is proportional to velocity.

VELOCITY SRS. *See* SHOCK RESPONSE SPECTRUM.

WEIGHTING FUNCTION. *See* WINDOW.

WHITE NOISE. Uncorrelated, flat PSD noise (*See* RANDOM VIBRATION).

WINDOW. When only a portion of a record is analyzed the record is truncated by a window. A window can be expressed in either the time domain or in the frequency domain, although the former is more common. To reduce the edge effects, which cause leakage, a window is often given a shape or weighting function. For example, a window can be defined as

$w(t) = g(t) \quad -T/2 < t < T/2 = 0$ elsewhere

where

$g(t)$ = the window weighting function,
T = the window duration
$w(t)$ = the time window.

The data analyzed, $x(t)$, are then given by

$x(t) = w(t)f(t+t')$

where

$f(t)$ = the time history
t' = the start of the data window.

A window in the time domain is represented by a multiplication and, hence, is a convolution in the frequency domain. A convolution can be thought of as a smoothing function. This smoothing can be represented by an effective filter shape of the window; i.e., energy at a frequency in the original data will appear at other frequencies as given by the filter shape. Since time domain windows can be represented as a filter in the frequency domain, the time domain windowing can be accomplished directly in the frequency domain (*See* TIME DOMAIN, FREQUENCY DOMAIN).

Listed below are some of the common windows used in digital vibration signal analysis:

Rectangular (boxcar)	1
Triangle (Bartlett)	$1 - 2\|t\|/T$
Cosine	$(1/2)[1 - \{\cos(2\pi t/T)\}^n]$
Tukey (n=1)	
Hanning (n=2)	
Hamming	$0.54 + 0.46 \cos(2\pi t/T)$
Parzen	$1 - 2(2t/T)^2 + 6(2\|t\|/T)^3 \quad \|t\| \le T/4$
	$2(1 - 2\|t\|/T)^3 \quad T/4 \le \|t\| \le T/2$
10% cosine taper	$0.5(1 + \cos(\pi + 20\pi t/T) \quad 0.45T \le \|t\| < T/2$
	$1.0 \quad \|t\| \le 0.45T$
Kaiser-Bessel	$I_0[\alpha \pi \sqrt{1-(2t/T)^2}]/I_0(\alpha \pi)$
	I_0 = Modified Bessel function of order zero,
	α = constant, typically 3.0

WRAP-AROUND ERROR (CIRCULAR CONVOLUTION ERROR). When the convolution in the time domain (multiplication in the frequency domain) is performed on two transient signals, the result has a duration equal to the sum of the durations of the individual waveforms. When a discrete Fourier transform (DFT) is used to approximate the continuous transform of a transient, and the convolution is performed by multiplying DFTs, the time frame must be at least as long as the sum of the durations of the individual transients or an error known as "circular convolution" occurs. A DFT can be considered as one period of a periodic waveform. The response in excess of one period is "wrapped-around" into the beginning of the period (*See* TIME DOMAIN, DFT).

ZEROS. The complex frequencies for which the numerator of a system's transfer function becomes zero (*See* POLES, TRANSFER FUNCTION).

ZOOM ANALYSIS (COMPLEX DEMODULATION). A technique for examining the frequency content of a signal with a fine resolution over a relatively narrow band of frequencies. The technique basically takes a band of frequencies and translates them to a lower band of frequencies (demodulation) where the signals can be decimated to reduce the sample size. A standard DFT can then be used to analyze the data.

Note that the increased resolution of this technique requires a corresponding increase in the time record length. The sample rate is decreased by decimation, to reduce the number of samples in the time window, only after the demodulation.

NOTES